

Hypervaluations on ternary semihyperrings

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Abstract. This article aims to present the concept of hypervaluation on a commutative ternary semihyperring mapped onto an ordered abelian group. It examines various properties of hyperideals within the ternary semihyperring that correspond to the valuation map. Additionally, the article explores results which are similar to those found in classical valuation rings, but within the framework of hypervalued ternary semihyperrings.

Keywords: ternary hypervaluation, ternary semihyperring, ternary semihyperring homomorphism.

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1. Introduction

Generalizing the notion of binary operation in groups to hyperoperation, Marty [22] introduced hypergroups in 1934. A hypergroupoid is an ordered pair $(M, \circ)$, where $\circ : M \times M \rightarrow P^*(M)$ is a hyperoperation on a nonempty set M . Let L_1 and L_2 be two nonempty subsets of M and $m \in M$, then

$$L_1 \circ L_2 = \bigcup l_1 \circ l_2, \quad L_1 \circ m = L_1 \circ \{m\} \text{ and } m \circ L_2 = \{m\} \circ L_2,$$

where $l_1 \in L_1, l_2 \in L_2$.

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A hypergroupoid $(M, \circ)$ earns the title of a semihypergroup if, for every $m_1, m_2, m_3 \in M$, the equation $(m_1 \circ m_2) \circ m_3 = m_1 \circ (m_2 \circ m_3)$ holds, indicating that

$$\bigcup_{u \in m_1 \circ m_2} u \circ m_3 = \bigcup_{v \in m_2 \circ m_3} m_1 \circ v.$$

By a hypergroup, we mean a semihypergroup $(M, \circ)$ for which $m_1 \circ M = M \circ m_1 = M$, for all $m_1 \in M$.

The definitions of relevant algebraic structures follow the order: (binary) hyperring (Definition 1.1), ternary hyperring (Definition 1.2) and ternary semihyperring (Definition 1.3).

Krasner [20] initiated the concept of hyperrings and hyperfields, taking addition to be a hypercomposition and retaining multiplication as a binary composition.

Definition 1.1 ([20]). *A hyperring according to Krasner is described as a mathematical structure $(\mathcal{R}, +, \cdot)$ that adheres to the following set of axioms:*

- (i) $(\mathcal{R}, +)$ constitutes a canonical hypergroup, i.e.,
 - (1) for every $r_1, r_2, r_3 \in \mathcal{R}$, $r_1 + (r_2 + r_3) = (r_1 + r_2) + r_3$,
 - (2) for every $r_1, r_2 \in \mathcal{R}$, $r_1 + r_2 = r_2 + r_1$,
 - (3) there exists $0 \in \mathcal{R}$ such that $0 + r_1 = \{r_1\} = r_1 + 0$ for each $r_1 \in \mathcal{R}$,
 - (4) for each $r_1 \in \mathcal{R}$, there is precisely one element r'_1 in $\mathcal{R}$, such that $0 \in r_1 + r'_1$, (we will denote $-r_1$ as r'_1 and refer to it as the inverse of r_1),
 - (5) $r_3 \in r_1 + r_2$ implies $r_2 \in -r_1 + r_3$ and $r_1 \in r_3 - r_2$.
- (ii) $(\mathcal{R}, \cdot)$ forms a semigroup with zero acting as a bilaterally absorbing element, i.e., $r_1 \cdot 0 = 0 \cdot r_1 = 0$.
- (iii) The multiplication distributes with regard to the hyperoperation $+$.

Example 1.1 ([20]). Consider $\mathcal{R}$ as a ring with identity that satisfies commutativity. We define $\overline{\mathcal{R}} = \{\bar{r} = \{r, -r\}, r \in \mathcal{R}\}$. Then, $\overline{\mathcal{R}}$ becomes a hyperring with respect to the hyperoperation $\bar{r}_1 \oplus \bar{r}_2 = \{\overline{r_1 + r_2}, \overline{r_1 - r_2}\}$ and multiplication $\bar{r}_1 \circ \bar{r}_2 = \overline{r_1 \cdot r_2}$.

Let us suppose that $(\mathcal{R}, +, \cdot)$ is a hyperring and A is a nonempty subset of $\mathcal{R}$. We define A as a subhyperring of $\mathcal{R}$ if $(A, +, \cdot)$ forms a hyperring in its own right. A subhyperring A of a hyperring $\mathcal{R}$ is a left (right) hyperideal of $\mathcal{R}$ if $r_1 \cdot r_2 \in A$ ($r_2 \cdot r_1 \in A$), $\forall r_1 \in \mathcal{R}, r_2 \in A$. A is termed a hyperideal if it satisfies the conditions of being both a left and a right hyperideal. A hyperideal P of a hyperring $\mathcal{R}$ is considered prime if, for any pair of hyperideals A and B of $\mathcal{R}$, the inclusion $AB \subseteq P$ implies either $A \subseteq P$ or $B \subseteq P$. In the case of

a commutative hyperring $\mathcal{R}$, a hyperideal P is prime if $P \neq \mathcal{R}$ and for every $r_1, r_2 \in \mathcal{R}$, if $r_1 \cdot r_2 \in P$, then either $r_1 \in P$ or $r_2 \in P$.

Significant literature has evolved in the theory of hyperstructures till date, viz., hypergroupoids [16], hypergroups [17, 34, 18], semihypergroups [13], hyper-rings [20, 14, 27, 28, 29, 30, 9, 6, 33], semihyperrings [1, 2, 4], ternary hyperrings [8, 7, 32, 31, 15] and ternary semihyperings [11, 26, 25, 5].

Alajbegović and Močkoř [3] studied m -rings with a multivalued addition satisfying certain conditions and commutative associative binary multiplication. In 2010, Davvaz and Mirvakili [12] introduced a new class of multialgebra called (m, n) -hyperring $\mathcal{R}$ in which m -ary addition makes $\mathcal{R}$ into a canonical hypergroup whereas $\mathcal{R}$ with n -ary multiplication is a semigroup.

Krasner ternary hyperrings were investigated by Castillo and Paradero-Vilela [8] in 2014. For a Krasner ternary hyperring $(\mathcal{R}, +, \cdot)$, the symbol $+$ refers to a binary hyperoperation, while $\cdot$ denotes a ternary multiplication.

Definition 1.2 ([8]). *A hyperring $(\mathcal{R}, +, \cdot)$ is called a Krasner ternary hyperring if it meets the following conditions:*

- (i) $(\mathcal{R}, +)$ constitutes a canonical hypergroup;
- (ii) $(r_1 \cdot r_2 \cdot r_3) \cdot r_4 \cdot r_5 = r_1 \cdot (r_2 \cdot r_3 \cdot r_4) \cdot r_5 = r_1 \cdot r_2 \cdot (r_3 \cdot r_4 \cdot r_5)$;
- (iii) $(r_1 + r_2) \cdot r_3 \cdot r_4 = r_1 \cdot r_3 \cdot r_4 + r_2 \cdot r_3 \cdot r_4$, $r_1 \cdot (r_2 + r_3) \cdot r_4 = r_1 \cdot r_2 \cdot r_4 + r_1 \cdot r_3 \cdot r_4$,
 $r_1 \cdot r_2 \cdot (r_3 + r_4) = r_1 \cdot r_2 \cdot r_3 + r_1 \cdot r_2 \cdot r_4$,
- (iv) $0 \cdot r_1 \cdot r_2 = r_1 \cdot 0 \cdot r_2 = r_1 \cdot r_2 \cdot 0 = 0$,

$\forall r_1, r_2, r_3, r_4, r_5 \in \mathcal{R}$.

The concept of ternary semihyperrings was introduced by Davvaz [11] in 2009, extending the idea of semirings.

Definition 1.3 ([11]). *A set $\mathcal{R}$ with a binary hyperoperation $+$ and a ternary multiplication $\cdot$ is termed a ternary semihyperring if $(\mathcal{R}, +)$ constitutes a commutative semihypergroup and fulfills the subsequent conditions:*

1. $(r_1 \cdot r_2 \cdot r_3) \cdot r_4 \cdot r_5 = r_1 \cdot (r_2 \cdot r_3 \cdot r_4) \cdot r_5 = r_1 \cdot r_2 \cdot (r_3 \cdot r_4 \cdot r_5)$;
2. $(r_1 + r_2) \cdot r_3 \cdot r_4 = r_1 \cdot r_3 \cdot r_4 + r_2 \cdot r_3 \cdot r_4$;
3. $r_1 \cdot (r_2 + r_3) \cdot r_4 = r_1 \cdot r_2 \cdot r_4 + r_1 \cdot r_3 \cdot r_4$;
4. $r_1 \cdot r_2 \cdot (r_3 + r_4) = r_1 \cdot r_2 \cdot r_3 + r_1 \cdot r_2 \cdot r_4$.

$\forall r_1, r_2, r_3, r_4, r_5 \in \mathcal{R}$. We simply write $r_1 \cdot r_2 \cdot r_3 = r_1 r_2 r_3$.

Example 1.2 ([11]). Consider $\mathbb{Z}$ as the set of all integers. We establish a binary hyperoperation and ternary multiplication on $\mathbb{Z}$ as follows: $r_1 \oplus r_2 = \{r_1, r_2\}$, and $r_1 \cdot r_2 \cdot r_3$ denotes the standard ternary multiplication of integers. Thus, $(\mathbb{Z}, \oplus, \cdot)$ forms a ternary semihyperring.

Example 1.3 ([11]). Consider $(\mathcal{R}, +, \cdot)$ as a semiring. We define a binary hyperoperation by $r_1 \oplus r_2 = \langle r_1, r_2 \rangle$ (the ideal generated by r_1, r_2), and a ternary multiplication by $r_1 \circ r_2 \circ r_3 = r_1 \cdot r_2 \cdot r_3$. Consequently, $(\mathcal{R}, \oplus, \circ)$ constitutes a ternary semihyperring.

A nonempty subset A of a ternary semihyperring $\mathcal{R}$ is a ternary subsemihyperring of $\mathcal{R}$ if $(A, +)$ is an additive subsemihypergroup of $(\mathcal{R}, +)$, i.e., $r_1 + r_2 \subseteq A$, for any $r_1, r_2 \in A$ and $AAA \subseteq A$, i.e., $r_1 r_2 r_3 \in A$, for any $r_1, r_2, r_3 \in A$. $\mathcal{R}$ is said to have a zero element if there exists an element $0 \in \mathcal{R}$ such that for all $r_1, r_2 \in \mathcal{R}$, $0r_1r_2 = r_10r_2 = r_1r_20 = 0$. An element e of $\mathcal{R}$ is called a unital element if for all $r_1 \in \mathcal{R}$, $eer_1 = er_1e = r_1ee = r_1$. An element $r_2 \in \mathcal{R}$ is called an inverse of $r_1 \in \mathcal{R}$ if $r_1r_2x = xr_1r_2 = r_2r_1x = xr_2r_1 = x$ for all $x \in \mathcal{R}$. A nonempty additive subsemihypergroup I of $\mathcal{R}$ is called a left hyperideal of $\mathcal{R}$ if $\mathcal{R}RI \subseteq I$, lateral hyperideal of $\mathcal{R}$ if $\mathcal{R}IR \subseteq I$ and right hyperideal of $\mathcal{R}$ if $IR\mathcal{R} \subseteq I$. If I is both a left and right hyperideal as well as a lateral hyperideal of $\mathcal{R}$ then I is called a hyperideal of $\mathcal{R}$. A proper hyperideal P of $\mathcal{R}$ is called prime hyperideal of $\mathcal{R}$ if $I_1I_2I_3 \subseteq P$ implies $I_1 \subseteq P$ or $I_2 \subseteq P$ or $I_3 \subseteq P$, for any three hyperideals I_1, I_2, I_3 of $\mathcal{R}$. A ternary semihyperring $\mathcal{R}$ is called commutative if $r_1r_2r_3 = r_2r_3r_1 = r_3r_1r_2 = r_2r_1r_3 = r_3r_2r_1 = r_1r_3r_2$, $\forall r_1, r_2, r_3 \in \mathcal{R}$. Let $\mathcal{R}$ be a commutative ternary semihyperring and P be a hyperideal of $\mathcal{R}$. Then, P is a prime hyperideal if and only if $r_1r_2r_3 \in P$ implies $r_1 \in P$ or $r_2 \in P$ or $r_3 \in P$, for all $r_1, r_2, r_3 \in \mathcal{R}$.

The notion of m -valuation on m -rings was put forward by Alajbegović and Močkoř [3] in 1985. The idea of hypervaluation was explored in 2006 by Davvaz and Salsi [10]. Harijani and Anvariye [19] introduced hypervaluation of a hyperfield onto a totally ordered canonical hypergroup. In 2020, Linzi and Stojalowska [21] studied that any hypervaluation from a hyperfield onto an ordered canonical hypergroup is the composition of a hypervaluation onto an ordered abelian group and an order preserving homomorphism of hypergroups. Nikmehr, Nikandish and Yassine [23] studied the notion of hypervaluation hyperideals and then found relations between hypervaluations, integral closure of hyperideals and primary hyperideals.

A group G can be partially ordered [10] by $\leq$ if $(G, \leq)$ is a poset in which $\leq$ is compatible with the binary composition, i.e., if $a_1 < b_1$ then $ga_1 < gb_1$ and $a_1g < b_1g$ for all $g \in G$. If $a_1 < a'_1$ and $b_1 < b'_1$ then we obtain $a_1b_1 < a'_1b'_1$. So $P = \{g \in G : 1 \leq g\}$, called the positive cone of G , is a submonoid of G . Setting $P^{-1} = \{a^{-1} : a \in P\}$ yields

1. $P \cap P^{-1} = \{1\}$, and
2. if $\leq$ is a total order, then $P \cup P^{-1} = G$.

The ordered group G is adjoined with ∞ fulfilling the conditions: $\infty > a$ and $\infty \cdot \infty = \infty = a \cdot \infty = \infty \cdot a$ for all $a \in G$. We write G_∞ to mean $G \cup \{\infty\}$

which is a hyperring with hyperoperation $\oplus$ defined as follows: if $a < b$ then $a \oplus b = \{a\}$ for all $a, b \in G_\infty$ and $a \oplus a = \{g \in G_\infty : a \leq g\}$. The multiplication is given by $a \circ b = a \vee b$ for all $a, b \in G_\infty$.

Definition 1.4 ([10]). *If $\mathcal{R}$ is a hyperring, then a hypervaluation on $\mathcal{R}$ is a mapping $v : \mathcal{R} \rightarrow G_\infty = G \cup \{\infty\}$, where G is a totally ordered abelian group, satisfying the following conditions:*

1. $v(0) = \infty$;
2. $v(xy) = v(x) \cdot v(y)$, for all $x, y \in \mathcal{R}$;
3. $v(-x) = v(x)$, for all $x \in \mathcal{R}$;
4. $z \in x + y \Rightarrow v(x) \geq \min\{v(x), v(y)\}$, for all $x, y, z \in \mathcal{R}$.

For a hypervaluation v , we say that $(\mathcal{R}, v, G)$ is a hypervalued hyperring. We define two sets $\mathcal{A}_v = \{x \in \mathcal{R} : v(x) \geq 1\}$ and $P_v = \{x \in \mathcal{R} : v(x) > 1\}$. Then, $\mathcal{A}_v$ becomes a subhyperring of $\mathcal{R}$ and P_v becomes a prime hyperideal of $\mathcal{A}_v$. Additionally, the set $v^{-1}(\infty)$ forms a prime hyperideal of $\mathcal{R}$ which is contained in P_v .

Valuation on a ternary semiring has been studied by Pal et. al [24]. In this paper, we introduce and explore the notion of hypervaluation on a ternary semihyperring, finally proving that any hypervaluation from a ternary semihyperring onto an ordered abelian multiplicative group is the composition of a hypervaluation onto an ordered abelian multiplicative group and an order preserving isomorphism of ternary semirings under certain conditions.

2. Hypervaluation on ternary semihyperring

We consider a totally ordered (multiplicative) abelian group G . Throughout this section, we consider G_∞ as $G \cup \{\infty\}$.

Definition 2.1. *Let $\mathcal{R}$ be a commutative ternary semihyperring with a unital element e and zero element 0 . By a hypervaluation, we mean a mapping $\mathfrak{v} : \mathcal{R} \rightarrow G_\infty$ which satisfies the conditions as follows: for $x, y, z \in \mathcal{R}$,*

- (i) $\mathfrak{v}(xyz) = \mathfrak{v}(x) \cdot \mathfrak{v}(y) \cdot \mathfrak{v}(z)$;
- (ii) if $z \in x + y$ then $\mathfrak{v}(z) \geq \min\{\mathfrak{v}(x), \mathfrak{v}(y)\}$;
- (iii) $\mathfrak{v}(e) = 1$;
- (iv) $\mathfrak{v}(0) = \infty$.

Example 2.1. Consider the ternary semihyperring $\mathcal{R} = (\mathbb{Z}, \oplus, \cdot)$ as shown in Example 1.2. We take $G = (\mathbb{Q}^+, \cdot)$. Define a mapping $\mathfrak{v} : \mathcal{R} \rightarrow G_\infty$ by

1. $\mathfrak{v}(x) = |x|, x \neq 0$,

2. $\mathfrak{v}(0) = \infty$.

Then, $\mathfrak{v}$ becomes a hypervaluation on $\mathcal{R}$.

Henceforth, by $\mathcal{R}$, we will mean a ternary semihyperring with a unital element e and a zero element 0 .

We consider the set of all hypervaluations of $\mathcal{R}$ which is denoted by $\text{val}(\mathcal{R})$. For any hypervaluation $\mathfrak{v} \in \text{val}(\mathcal{R})$, we set

$$\begin{aligned}\mathcal{A}_{\mathfrak{v}} &= \{x \in \mathcal{R} : \mathfrak{v}(x) \geq 1\}, \\ \mathcal{P}_{\mathfrak{v}} &= \{y \in \mathcal{R} : \mathfrak{v}(y) > 1\}, \\ \mathcal{I}_{\mathfrak{v}} &= \{z \in \mathcal{R} : \mathfrak{v}(z) = \infty\}.\end{aligned}$$

We see that $e \in \mathcal{A}_{\mathfrak{v}}$ and 0 is in $\mathcal{I}_{\mathfrak{v}}$ as well as in $\mathcal{P}_{\mathfrak{v}}$.

Theorem 2.1. $\mathcal{A}_{\mathfrak{v}}$ is a ternary subsemihyperring in $\mathcal{R}$.

Proof of Theorem 2.1. First we show that $\mathcal{A}_{\mathfrak{v}}$ becomes an additive subsemihypergroup. Let $x, y \in \mathcal{A}_{\mathfrak{v}}$, then $\mathfrak{v}(x) \geq 1, \mathfrak{v}(y) \geq 1$. Let $z \in x + y$, then $\mathfrak{v}(z) \geq \min\{\mathfrak{v}(x), \mathfrak{v}(y)\} \geq 1$ which gives us $z \in \mathcal{A}_{\mathfrak{v}} \Rightarrow x + y \subseteq \mathcal{A}_{\mathfrak{v}}$. Therefore $\mathcal{A}_{\mathfrak{v}}$ is an additive subsemihypergroup. Again, for $x, y, z \in \mathcal{A}_{\mathfrak{v}}$, $\mathfrak{v}(xyz) = \mathfrak{v}(x) \cdot \mathfrak{v}(y) \cdot \mathfrak{v}(z) \geq 1 \Rightarrow xyz \in \mathcal{A}_{\mathfrak{v}}$. Thus, $\mathcal{A}_{\mathfrak{v}}$ is a ternary subsemihyperring of $\mathcal{R}$.

Theorem 2.2. $\mathcal{P}_{\mathfrak{v}}$ is a prime hyperideal of $\mathcal{A}_{\mathfrak{v}}$.

Proof of Theorem 2.2. It is clear that $\mathcal{P}_{\mathfrak{v}}$ is an additive subsemihypergroup of $\mathcal{A}_{\mathfrak{v}}$. Let $a, b \in \mathcal{A}_{\mathfrak{v}}$ and $x \in \mathcal{P}_{\mathfrak{v}}$ then $\mathfrak{v}(a) \geq 1, \mathfrak{v}(b) \geq 1$ and $\mathfrak{v}(x) > 1$. Then, $\mathfrak{v}(abx) = \mathfrak{v}(a) \cdot \mathfrak{v}(b) \cdot \mathfrak{v}(x) > 1 \Rightarrow abx \in \mathcal{P}_{\mathfrak{v}}$. We can show that $axb \in \mathcal{P}_{\mathfrak{v}}$ and $xab \in \mathcal{P}_{\mathfrak{v}}$ in a similar way. Hence $\mathcal{P}_{\mathfrak{v}}$ is a hyperideal of $\mathcal{A}_{\mathfrak{v}}$. Again let $a, b, c \in \mathcal{A}_{\mathfrak{v}}$ and $abc \in \mathcal{P}_{\mathfrak{v}}$, then $\mathfrak{v}(abc) > 1 \Rightarrow \mathfrak{v}(a) \cdot \mathfrak{v}(b) \cdot \mathfrak{v}(c) > 1$ which further implies either $\mathfrak{v}(a) > 1$ or $\mathfrak{v}(b) > 1$ or $\mathfrak{v}(c) > 1$. Hence $\mathcal{P}_{\mathfrak{v}}$ is a prime hyperideal of $\mathcal{A}_{\mathfrak{v}}$.

Theorem 2.3. $\mathcal{I}_{\mathfrak{v}}$ is a prime hyperideal of $\mathcal{R}$.

Proof of Theorem 2.3. Let $x, y \in \mathcal{I}_{\mathfrak{v}}$ then $\mathfrak{v}(x) = \infty, \mathfrak{v}(y) = \infty$. Let $z \in x + y$, then $\mathfrak{v}(z) \geq \min\{\mathfrak{v}(x), \mathfrak{v}(y)\} \Rightarrow \mathfrak{v}(z) = \infty \Rightarrow z \in \mathcal{I}_{\mathfrak{v}}$. Therefore $x + y \subseteq \mathcal{I}_{\mathfrak{v}}$. So $\mathcal{I}_{\mathfrak{v}}$ is an additive subsemihypergroup. Let $a, b \in \mathcal{R}$ and $x \in \mathcal{I}_{\mathfrak{v}}$ then $\mathfrak{v}(abx) = \mathfrak{v}(a) \cdot \mathfrak{v}(b) \cdot \mathfrak{v}(x) = \infty \Rightarrow abx \in \mathcal{I}_{\mathfrak{v}}$. Similarly we can show that $axb \in \mathcal{I}_{\mathfrak{v}}$ and $xab \in \mathcal{I}_{\mathfrak{v}}$. Therefore $\mathcal{I}_{\mathfrak{v}}$ is a hyperideal of $\mathcal{R}$. Now, let $abc \in \mathcal{I}_{\mathfrak{v}}$ for any $a, b, c \in \mathcal{R}$, then $\mathfrak{v}(abc) = \infty$ and so $\mathfrak{v}(a) \cdot \mathfrak{v}(b) \cdot \mathfrak{v}(c) = \infty$. Then, at least one of $\mathfrak{v}(a), \mathfrak{v}(b)$ or $\mathfrak{v}(c)$ is ∞ . That is, either $a \in \mathcal{I}_{\mathfrak{v}}$ or $b \in \mathcal{I}_{\mathfrak{v}}$ or $c \in \mathcal{I}_{\mathfrak{v}}$. Hence $\mathcal{I}_{\mathfrak{v}}$ becomes a prime hyperideal of $\mathcal{R}$.

In the results that follow, we present some properties of a hypervalued ternary semihyperring $\mathcal{R}$ where $\mathcal{A}_{\mathfrak{v}}$, $\mathcal{P}_{\mathfrak{v}}$ and $\mathfrak{v}^{-1}(\infty) = \mathcal{I}_{\mathfrak{v}}$ play a significant role.

Theorem 2.4. *Consider a ternary subsemihyperring A containing e within a commutative ternary semihyperring $\mathcal{R}$. Assume that every nonzero element in $\mathcal{R}$ has a multiplicative inverse and P represents a proper prime hyperideal of A . In this scenario, the following assertions hold true equivalently:*

- (I) *for each $x \in \mathcal{R} \setminus A$, there exists $y \in P$ such that $xye \in A \setminus P$,*
- (II) *there exists a nontrivial onto ternary hypervaluation $\mathbf{v} : \mathcal{R} \rightarrow G_\infty$ such that $\mathcal{A}_{\mathbf{v}} = A$ and $\mathcal{P}_{\mathbf{v}} = P$.*

Proof of Theorem 2.4. (II) $\Rightarrow$ (I). Suppose there exists an onto ternary hypervaluation $\mathbf{v} : \mathcal{R} \rightarrow G_\infty = G \cup \{\infty\}$ such that $\mathcal{A}_{\mathbf{v}} = A$ and $\mathcal{P}_{\mathbf{v}} = P$. If $x \in \mathcal{R} \setminus \mathcal{A}_{\mathbf{v}}$, then $\mathbf{v}(x) < 1$ and therefore $\mathbf{v}(x) \neq \infty$, i.e., $\mathbf{v}(x) \in G$. Thus, we get $\mathbf{v}(x)^{-1} = \mathbf{v}(y) > 1$ for some $y \in \mathcal{R}$, which implies $y \in \mathcal{P}_{\mathbf{v}}$. Again we get $\mathbf{v}(x) \cdot \mathbf{v}(y) = 1$. Now, $\mathbf{v}(xye) = \mathbf{v}(x) \cdot \mathbf{v}(y) \cdot \mathbf{v}(e) = 1 \Rightarrow xye \in \mathcal{A}_{\mathbf{v}} \setminus \mathcal{P}_{\mathbf{v}}$.

(I) $\Rightarrow$ (II). We see that (I) implies the property: for all $x, y \in \mathcal{R}$, $xye \in P \Rightarrow x \in P$ or $y \in P$.

Next, for every element x belonging to the set $\mathcal{R}$, we set

$$(P : x)_{\mathcal{R}} = \{z \in \mathcal{R} : xze' \in P, \text{ for some unital } e' \text{ in } \mathcal{R}\}.$$

We define a relation ρ on $\mathcal{R}$ by $x\rho y$ if and only if $(P : x)_{\mathcal{R}} = (P : y)_{\mathcal{R}}$. Then, ρ is an equivalence relation on $\mathcal{R}$. Let us denote the equivalence class of an element $x \in \mathcal{R}$ by $x\rho$. As $\mathcal{R}$ is commutative, all the unital elements belong to the same equivalence class, say, $e\rho$. We define multiplication on $\mathcal{R}/\rho$ by $(x\rho)(y\rho) = (xye)\rho$. Consider the set $G = \{x\rho : x \in \mathcal{R}\} \setminus \{0\rho\}$. Then, G is a group with respect to multiplication. In fact, $e\rho$ is the identity element in G and for an inverse y of x in $\mathcal{R}$, $y\rho$ is the inverse of $x\rho$ in G .

Also, G is a totally ordered group where the ordering is given by $x\rho \leq y\rho$ iff $(P : x)_{\mathcal{R}} \subseteq (P : y)_{\mathcal{R}}$. Now, we define $\mathbf{v} : \mathcal{R} \rightarrow G \cup \{\infty\}$ by

$$\mathbf{v}(x) = x\rho, \text{ for } x \in \mathcal{R}, \mathbf{v}(0) = \infty \text{ and } \mathbf{v}(e) = e\rho.$$

It is clear that $\mathbf{v}(xyz) = \mathbf{v}(x) \cdot \mathbf{v}(y) \cdot \mathbf{v}(z)$. Next we prove that for any $z \in x + y$, $\mathbf{v}(z) \geq \min\{\mathbf{v}(x), \mathbf{v}(y)\}$.

Let $\mathbf{v}(x) < \mathbf{v}(y)$ then $(P : x)_{\mathcal{R}} \subsetneq (P : y)_{\mathcal{R}}$ and first we suppose that $\mathbf{v}(z) < \mathbf{v}(x)$, then we get $zue \notin P$ and $uxe = xue \in P$ for some $u \in \mathcal{R}$. We now show that $yue \notin P$. Suppose that $yue \in P$ then $zue \in (x + y)ue = xue + yue \subseteq P + P \subseteq P \Rightarrow zue \in P$, this leads to a contradiction, implying that $yue \notin P$. Given the assumption $\mathbf{v}(x) < \mathbf{v}(y)$, we deduce that $yte \in P$ and $xte \notin P$ for some $t \in \mathcal{R}$. Consequently, we derive $(xte)(yue)e = (xte)y(uee) = (xue)(yte)e \in PPe \subseteq P$. Hence, we encounter a contradiction, as either $xte \in P$ or $yte \in P$. Similarly, we reach a contradiction for the case $\mathbf{v}(x) < \mathbf{v}(z)$. Therefore, we have successfully demonstrated that $\mathbf{v}(z) = \mathbf{v}(x), \forall z \in x + y$ if $\mathbf{v}(x) \neq \mathbf{v}(y)$. If $\mathbf{v}(x) = \mathbf{v}(y)$, we show that $\mathbf{v}(z) \geq \mathbf{v}(x)$ for any $z \in x + y$, that

is, $\{u \in \mathcal{R} : uxe \in P\} \subseteq \{u \in \mathcal{R} : uze \in P\}$. Let $u \in \mathcal{R}$ and $uxe \in P$. Since $\mathfrak{v}(x) = \mathfrak{v}(y)$, we have $uye \in P$ and $uze \in uxe + uye \subseteq P + P \subseteq P$. Hence $(P : x)_{\mathcal{R}} \subseteq (P : z)_{\mathcal{R}}$, that is, $\mathfrak{v}(z) \geq \mathfrak{v}(x)$. So $\mathfrak{v}$ is a ternary hypervaluation.

Theorem 2.5. *Consider $\mathcal{R}$ as a commutative ternary semihyperring, and let $\mathfrak{v} : \mathcal{R} \rightarrow G_{\infty}$ represent a non-trivial hypervaluation on $\mathcal{R}$. In this context, we observe:*

$$(I) \quad \mathfrak{v}^{-1}(\infty) = \{x \in \mathcal{R} : xye \in \mathcal{A}_{\mathfrak{v}}, \forall y \in \mathcal{R} \setminus \mathcal{A}_{\mathfrak{v}}\}.$$

(II) *If P is a proper prime hyperideal of $\mathcal{A}_{\mathfrak{v}}$ such that $P \subseteq \mathcal{P}_{\mathfrak{v}}$ and $P \not\subseteq \mathfrak{v}^{-1}(\infty)$, then $\mathfrak{v}^{-1}(\infty) \subseteq P$.*

Proof of Theorem 2.5. (I) Let $x \in \mathcal{R}$ be an element such that $\mathfrak{v}(x) = \infty$ and $y \in \mathcal{R} \setminus \mathcal{A}_{\mathfrak{v}}$. Then, we have $\mathfrak{v}(xye) = \mathfrak{v}(x) \cdot \mathfrak{v}(y) \cdot \mathfrak{v}(e) = \infty$. Hence $xye \in \mathcal{A}_{\mathfrak{v}}$.

Now, we show that for any $x \in \mathcal{R}$ such that for every $y \in \mathcal{R} \setminus \mathcal{A}_{\mathfrak{v}}$, whenever $xye \in \mathcal{A}_{\mathfrak{v}}$, then $\mathfrak{v}(x) = \infty$. Suppose $\mathfrak{v}(x) \neq \infty$. If $\mathfrak{v}(x) < 1$, we can take $y = x \in \mathcal{R} \setminus \mathcal{A}_{\mathfrak{v}}$, then $\mathfrak{v}(xye) = \mathfrak{v}(x) \cdot \mathfrak{v}(y) \cdot \mathfrak{v}(e) < 1 \Rightarrow xye \notin \mathcal{A}_{\mathfrak{v}}$, which is a contradiction. Next we consider the case $1 \leq \mathfrak{v}(x) < \infty$. If $\mathfrak{v}(x) > 1$, then for any $y \in \mathcal{R}$ such that $\mathfrak{v}(y) = \mathfrak{v}(x)^{-1} < 1$ implies that $y \in \mathcal{R} \setminus \mathcal{A}_{\mathfrak{v}}$. Now, $\mathfrak{v}(yye) = \mathfrak{v}(y) \cdot \mathfrak{v}(y) \cdot \mathfrak{v}(e) < 1$. Therefore $\mathfrak{v}(x(yye)e) = \mathfrak{v}(x) \cdot \mathfrak{v}(yye) \cdot \mathfrak{v}(e) < 1 \Rightarrow x(yye)e \notin \mathcal{A}_{\mathfrak{v}}$ where $yye \in \mathcal{R} \setminus \mathcal{A}_{\mathfrak{v}}$, which is a contradiction. If $\mathfrak{v}(x) = 1$, let any $y \in \mathcal{R} \setminus \mathcal{A}_{\mathfrak{v}}$, $\mathfrak{v}(xye) = \mathfrak{v}(x) \cdot \mathfrak{v}(y) \cdot \mathfrak{v}(e) = \mathfrak{v}(y) < 1 \Rightarrow xye \notin \mathcal{A}_{\mathfrak{v}}$, which is a contradiction. Hence we can conclude that $\mathfrak{v}(x) = \infty$. Therefore $\mathfrak{v}^{-1}(\infty) = \{x \in \mathcal{R} : xye \in \mathcal{A}_{\mathfrak{v}}, \forall y \in \mathcal{R} \setminus \mathcal{A}_{\mathfrak{v}}\}$.

(II) Let P be a prime hyperideal of $\mathcal{A}_{\mathfrak{v}}$, with $P \subseteq \mathcal{P}_{\mathfrak{v}}$ such that $P \not\subseteq \mathfrak{v}^{-1}(\infty)$. Let $p \in P$ with $1 < \mathfrak{v}(p) < \infty$ and let $z \in \mathfrak{v}^{-1}(\infty) = \{x \in \mathcal{R} : xye \in \mathcal{A}_{\mathfrak{v}}, \forall y \in \mathcal{R} \setminus \mathcal{A}_{\mathfrak{v}}\}$. Then, $\mathfrak{v}(x) = \mathfrak{v}(p)^{-1} < 1$, for some $x \in \mathcal{R} \setminus \mathcal{A}_{\mathfrak{v}}$. Now, $\mathfrak{v}(zxe) = \mathfrak{v}(z) \cdot \mathfrak{v}(x) \cdot \mathfrak{v}(e) = \infty \Rightarrow zxe \in \mathcal{P}_{\mathfrak{v}}$. Further we have $xpe \in \mathcal{A}_{\mathfrak{v}} \setminus \mathcal{P}_{\mathfrak{v}} \subseteq \mathcal{A}_{\mathfrak{v}} \setminus P \Rightarrow xpe \notin P$. Now, $(xpe)ze = x(pez)e = x(zep)e = xz(epe) = (xze)pe = (zxe)pe \in \mathcal{P}_{\mathfrak{v}}Pe \subseteq P \Rightarrow z \in P$, since $xpe \notin P$. Therefore $\mathfrak{v}^{-1}(\infty) \subseteq P$.

Suppose $(G, \cdot, \leq)$ is a totally ordered abelian group, it can be made into a ternary semiring in an obvious way by defining

1. $a + b = \max\{a, b\}$,
2. $a \cdot b \cdot c = a \wedge b \wedge c = \min\{a, b, c\}$,

for all $a, b, c \in G$. Then, $(G, +, \cdot)$ becomes a ternary semiring.

Theorem 2.6. *Let $\mathcal{R}$ be a commutative ternary semihyperring, G be a totally ordered abelian group and $\mathfrak{v}_1 : \mathcal{R} \rightarrow G_{\infty}$ be a nontrivial onto ternary hypervaluation on $\mathcal{R}$. Then, for any totally ordered abelian group H and any nontrivial onto ternary hypervaluation $\mathfrak{v}_2 : \mathcal{R} \rightarrow H_{\infty}$ on $\mathcal{R}$, $\mathcal{A}_{\mathfrak{v}_1} = \mathcal{A}_{\mathfrak{v}_2}$ and $\mathcal{P}_{\mathfrak{v}_1} = \mathcal{P}_{\mathfrak{v}_2}$ if and only if there is an order preserving isomorphism $\mathfrak{f} : G_{\infty} \rightarrow H_{\infty}$ of ternary semirings satisfying $\mathfrak{v}_2 = \mathfrak{f} \circ \mathfrak{v}_1$ and $\mathfrak{f}$ carries identity of G to identity of H .*

Proof of Theorem 2.6. Let us assume that $\mathcal{A}_{\mathbf{v}_1} = \mathcal{A}_{\mathbf{v}_2}$ and $\mathcal{P}_{\mathbf{v}_1} = \mathcal{P}_{\mathbf{v}_2}$. We define $\mathbf{f} : G_\infty \rightarrow H_\infty$ by $\mathbf{f}(\mathbf{v}_1(x)) = \mathbf{v}_2(x)$, for all $\mathbf{v}_1(x) \neq \infty_G$ and $\mathbf{f}(\infty_G) = \infty_H$. This definition is well-defined, in fact, we show that

$$\mathbf{v}_1(x) = \mathbf{v}_1(y) \neq \infty_G \Rightarrow \mathbf{v}_2(x) = \mathbf{v}_2(y) \neq \infty_H.$$

Now, $\mathbf{v}_1(y)^{-1} \in G$, so $\mathbf{v}_1(y)^{-1} = \mathbf{v}_1(z)$ for some $z \in \mathcal{R}$. Thus, we have $1 = \mathbf{v}_1(y) \cdot \mathbf{v}_1(y)^{-1} = \mathbf{v}_1(x) \cdot \mathbf{v}_1(z) \cdot \mathbf{v}_1(e) = \mathbf{v}_1(xze)$ which implies that $xze \in \mathcal{A}_{\mathbf{v}_1} \setminus \mathcal{P}_{\mathbf{v}_1} = \mathcal{A}_{\mathbf{v}_2} \setminus \mathcal{P}_{\mathbf{v}_2}$. This gives us $\mathbf{v}_2(xze) = 1 \Rightarrow \mathbf{v}_2(x) \cdot \mathbf{v}_2(z) \cdot \mathbf{v}_2(e) = 1 \Rightarrow \mathbf{v}_2(x) \cdot \mathbf{v}_2(z) = 1 \Rightarrow \mathbf{v}_2(x) = \mathbf{v}_2(z)^{-1}$. Also, $\mathbf{v}_1(yze) = 1$. Proceeding in a similar way, it follows that $\mathbf{v}_2(yze) = 1 \Rightarrow \mathbf{v}_2(y) = \mathbf{v}_2(z)^{-1}$. Thus, $\mathbf{v}_2(x) = \mathbf{v}_2(y) \neq \infty_H$. Also, from Theorem 2.5, we have $\mathbf{v}_1^{-1}(\infty_G) = \mathbf{v}_2^{-1}(\infty_H)$.

In order to show that $\mathbf{f}$ is an order preserving isomorphism, first we show that if $\mathbf{v}_1(x) < \mathbf{v}_1(y)$ then $\mathbf{v}_2(x) < \mathbf{v}_2(y)$. Otherwise $\mathbf{v}_1(x) < \mathbf{v}_1(y)$ and $\mathbf{v}_2(y) \leq \mathbf{v}_2(x)$ imply that $\mathbf{v}_1(x) \neq \infty_G$, so $\mathbf{v}_2(x) \neq \infty_H$ and $\mathbf{v}_2(y) \neq \infty_H$. Now, there exists $z \in \mathcal{R}$ such that $\mathbf{v}_2(y)^{-1} = \mathbf{v}_2(z) \Rightarrow 1 \leq \mathbf{v}_2(x) \cdot \mathbf{v}_2(y)^{-1} = \mathbf{v}_2(x) \cdot \mathbf{v}_2(y)^{-1} \cdot \mathbf{v}_2(e) = \mathbf{v}_2(x) \cdot \mathbf{v}_2(z) \cdot \mathbf{v}_2(e) = \mathbf{v}_2(xze) \Rightarrow xze \in \mathcal{A}_{\mathbf{v}_2} = \mathcal{A}_{\mathbf{v}_1} \Rightarrow \mathbf{v}_1(xze) \geq 1$. But, $uze \in \mathcal{A}_{\mathbf{v}_2} \setminus \mathcal{P}_{\mathbf{v}_2} = \mathcal{A}_{\mathbf{v}_1} \setminus \mathcal{P}_{\mathbf{v}_1}$. Hence $1 = \mathbf{v}_1(y) \cdot \mathbf{v}_1(z) \cdot \mathbf{v}_1(e) \Rightarrow \mathbf{v}_1(y) = \mathbf{v}_1(z)^{-1} \leq \mathbf{v}_1(x)$, since $1 \leq \mathbf{v}_1(xze)$. Thus, we get $\mathbf{v}_1(y) \leq \mathbf{v}_1(x)$ which is a contradiction.

We can show, in a similar manner, that if $\mathbf{v}_2(x) < \mathbf{v}_2(y)$ then $\mathbf{v}_1(x) < \mathbf{v}_1(y)$ and so $\mathbf{f}$ is order preserving.

Now, G and H can be made into ternary semirings. To prove that $\mathbf{f} : G_\infty \rightarrow H_\infty$ is a homomorphism, we need to show that $\mathbf{f}(a + b) = \mathbf{f}(a) + \mathbf{f}(b)$ and $\mathbf{f}(abc) = \mathbf{f}(a)\mathbf{f}(b)\mathbf{f}(c)$, where $a = \mathbf{v}_1(x)$, $b = \mathbf{v}_1(y)$ and $c = \mathbf{v}_1(z)$ for some $x, y, z \in \mathcal{R}$. Since $a + \infty_G = \infty_G$ for any a in G_∞ and $a_1 + \infty_H = \infty_H$ for any a_1 in H_∞ , it is sufficient to consider the case when $a, b \in G$. Let $a, b \in G$ and $x, y \in \mathcal{R}$ such that $a = \mathbf{v}_1(x)$, $b = \mathbf{v}_1(y)$. Then, $\mathbf{f}(a) + \mathbf{f}(b) = \mathbf{f}(\mathbf{v}_1(x)) + \mathbf{f}(\mathbf{v}_1(y))$. Then, if $\mathbf{v}_1(x) < \mathbf{v}_1(y)$ we have $\mathbf{v}_2(x) < \mathbf{v}_2(y)$ and $\mathbf{v}_1(x) + \mathbf{v}_1(y) = \mathbf{v}_1(y)$. Hence $\mathbf{f}(a + b) = \mathbf{f}(\mathbf{v}_1(x) + \mathbf{v}_1(y)) = \mathbf{f}(\mathbf{v}_1(y)) = \mathbf{v}_2(y) = \mathbf{f}(\mathbf{v}_1(x)) + \mathbf{f}(\mathbf{v}_1(y)) = \mathbf{f}(a) + \mathbf{f}(b)$. If $\mathbf{v}_1(x) = \mathbf{v}_1(y)$ then $\mathbf{v}_2(x) = \mathbf{f}(\mathbf{v}_1(x)) = \mathbf{f}(\mathbf{v}_1(y)) = \mathbf{v}_2(y)$. Therefore $\mathbf{f}(a + b) = \mathbf{f}(a) + \mathbf{f}(b)$.

Also, $\mathbf{f}(abc) = \mathbf{f}(a)\mathbf{f}(b)\mathbf{f}(c)$ and $\mathbf{f}(1) = 1$ follows from the order preserving property of $\mathbf{f}$. Obviously $\mathbf{f}$ is onto.

We now show that $\mathbf{f}$ is injective. Let $\mathbf{f}(\mathbf{v}_1(x)) = \mathbf{f}(\mathbf{v}_1(y)) \neq \infty_H$, that is, $\mathbf{v}_2(x) = \mathbf{v}_2(y) \neq \infty_H$. If possible, let $\mathbf{v}_1(x) \neq \mathbf{v}_1(y)$. Then, either $\mathbf{v}_1(x) < \mathbf{v}_1(y)$ or $\mathbf{v}_1(y) < \mathbf{v}_1(x)$. Consider the case when $\mathbf{v}_1(x) > \mathbf{v}_1(y)$. Since $\mathbf{v}_2(x) \neq \infty_H$, we must have $\mathbf{v}_1(x) \neq \infty_G$ and so $\mathbf{v}_1(y) \neq \infty_G$. Now, there exists $z \in \mathcal{R}$ such that $\mathbf{v}_1(y)^{-1} = \mathbf{v}_1(z) \neq \infty_G$. Then, $\mathbf{v}_1(x) \cdot \mathbf{v}_1(y)^{-1} > 1 \Rightarrow \mathbf{v}_1(x) \cdot \mathbf{v}_1(z) \cdot \mathbf{v}_1(e) > 1 \Rightarrow xze \in \mathcal{P}_{\mathbf{v}_1} = \mathcal{P}_{\mathbf{v}_2} \Rightarrow \mathbf{v}_2(xze) > 1$. Again $uze \in \mathcal{A}_{\mathbf{v}_1} \setminus \mathcal{P}_{\mathbf{v}_1} = \mathcal{A}_{\mathbf{v}_2} \setminus \mathcal{P}_{\mathbf{v}_2}$. Hence $\mathbf{v}_2(yze) = 1$. But, $\mathbf{v}_2(z) \neq \infty_H$. Now, $\mathbf{v}_2(y) \cdot \mathbf{v}_2(z) \cdot \mathbf{v}_2(e) = 1 \Rightarrow \mathbf{v}_2(y) = \mathbf{v}_2(z)^{-1} < \mathbf{v}_2(x)$, which is a contradiction. Similarly contradiction is obtained for the case $\mathbf{v}_1(x) < \mathbf{v}_1(y)$. Hence $\mathbf{f}$ is injective, making $\mathbf{f}$ an order preserving isomorphism.

For the converse part, let $x \in \mathcal{A}_{\mathbf{v}_1} \Rightarrow \mathbf{v}_1(x) \geq 1 \Rightarrow \mathbf{f}(\mathbf{v}_1(x)) \geq \mathbf{f}(1) \Rightarrow \mathbf{v}_2(x) \geq 1 \Rightarrow x \in \mathcal{A}_{\mathbf{v}_2}$. Therefore $\mathcal{A}_{\mathbf{v}_1} \subseteq \mathcal{A}_{\mathbf{v}_2}$. Similarly, we can show that $\mathcal{A}_{\mathbf{v}_2} \subseteq \mathcal{A}_{\mathbf{v}_1}$, hence $\mathcal{A}_{\mathbf{v}_1} = \mathcal{A}_{\mathbf{v}_2}$. In a similar way we can show that $\mathcal{P}_{\mathbf{v}_1} = \mathcal{P}_{\mathbf{v}_2}$.

In the above theorem, the hypothesis of surjectivity of the hypervaluations cannot be dispensed with, as supported by Example 2.2 below.

Example 2.2. Consider the ternary semihyperring $\mathcal{R} = (\mathbb{Z}, \oplus, \cdot)$ (as in Example 1.2) and the totally ordered group $G = (\{1\}, \cdot)$. Then, $\mathbf{v}_1 : \mathcal{R} \rightarrow G_\infty$ defined by

1. $\mathbf{v}_1(x) = 1$, if $x \neq 0$,
2. $\mathbf{v}_1(0) = \infty$,

is an onto hypervaluation on $\mathcal{R}$.

We take $H = (\mathbb{Q}^+, \cdot)$ and the hypervaluation $\mathbf{v}_2$ on $\mathcal{R}$ to be the hypervaluation as defined in Example 2.1.

Here, $\mathcal{A}_{\mathbf{v}_1} = \mathcal{A}_{\mathbf{v}_2} = \mathbb{Z}$ and $\mathcal{P}_{\mathbf{v}_1} = \{0\}$, $\mathcal{P}_{\mathbf{v}_2} = \mathbb{Z} \setminus \{1\}$. We define $\mathbf{f} : G_\infty \rightarrow H_\infty$ by

1. $\mathbf{f}(1) = 1$,
2. $\mathbf{f}(\infty) = \infty$,

which is a homomorphism of ternary semirings satisfying $\mathbf{v}_2 = \mathbf{f} \circ \mathbf{v}_1$. But, $\mathbf{f}$ is not an isomorphism. We also note that $\mathbf{v}_2$ is not onto.

Conclusion

The impetus for studying hypervaluation on ternary semihyperrings arises from its potential to generalize and expand the theory of hypervaluation on hyperrings. A necessary and sufficient condition for the existence of a ternary hypervaluation is provided in Theorem 2.4. The set $\mathbf{v}^{-1}(\infty) = \mathcal{I}_{\mathbf{v}}$ is characterised thereafter. We conclude by proving (in Theorem 2.6) that any hypervaluation is the composition of a hypervaluation and an order preserving isomorphism (of ternary semirings).

This study may be continued with the study of hyperideal theory in ternary semihyperrings. The concept of regularity in these structures along with study in relation to hypervaluation is another area which may be explored. Further, one may investigate the notion of product hypervaluation and attempt to characterise the set of all hypervaluations on ternary semihyperrings into suitable algebraic system, as open issues.

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