

Understanding bipartite soft semigraph structures

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Abstract. Soft set theory functions as a flexible mathematical instrument designed to handle uncertain data by aiding in the categorization of universe elements according to predefined parameters. Unlike hypergraphs, semigraphs present a wider interpretation of conventional graphs, allowing for a finer representation of relationships. Through the integration of soft set principles, the notion of soft semigraphs arises, enhancing the adaptability and versatility of semigraphs in addressing uncertainty. This paper sets out to reveal different forms of bipartite soft semigraphs, meticulously examining their varied structures and delving into their inherent characteristics.

Keywords: soft graph, soft semigraph, bipartite soft semigraph.

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1. Introduction

Most conventional approaches in formal modelling, reasoning, and computation are characterized by determinism, clarity, and precision. However, complex challenges in fields like engineering, medicine, economics, and social sciences often involve uncertain data. Various uncertainties in these areas make the application of traditional methods challenging. This led to the emergence of soft set theory in 1999 by Molodtsov [18]. Soft set theory proves more practical than other established theories like probability or fuzzy set theory due to its

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versatility. Authors such as Maji, Biswas, and Roy [16], [17] have expanded on soft set theory, applying it to resolve decision-making problems.

The concept of soft graphs was introduced by Thumbakara and George [24]. George, Thumbakara and Jose [25], [26], [27] investigated some properties of soft graphs. In 2015, Akram and Nawas [1], [2] modified the definition of a soft graph. Further enhancing the field, Akram and Nawas [3], [4] introduced fuzzy soft graphs, strong fuzzy soft graphs, complete fuzzy soft graphs, and regular fuzzy soft graphs, delving into their properties and potential applications. Akram and Zafar [5], [6] pioneered the notions of soft trees and fuzzy soft trees.

Contributions to the study of soft graphs have been made by Thenge, Jain, and Reddy [21], [22], [23]. Owing to their utility in handling parameterization, soft graphs represent a burgeoning domain within graph theory. George, Thumbakara, and Jose introduced soft hypergraphs [7], soft directed graphs [14], [15], and soft directed hypergraphs [13] and studied their properties. The concept of semigraphs, which are an expanded version of graphs, was initially proposed by E. Sampathkumar [19], [20]. Unlike hypergraphs, semigraphs maintain a defined sequence of vertices within their edges. When depicted on a two-dimensional plane, semigraphs bear a visual similarity to traditional graphs. In their research, George, Jose, and Thumbakara [11] introduced soft semigraphs by applying soft set principles to semigraphs and defined some soft semigraph operations. Moreover, they introduced connectedness [10] and various degrees, graphs, and matrices linked to soft semigraphs [12]. This paper introduces different types of bipartite soft semigraphs and investigates their properties.

2. Preliminaries

2.1 Semigraph

The notion of semigraph was introduced by E. Sampathkumar [19], [20] as follows. “A *semigraph* G is a pair (V, X) where V is a nonempty set whose elements are called vertices of G , and X is a set of n -tuples, called edges of G , of distinct vertices, for various $n \geq 2$, satisfying the following conditions.

1. Any two edges have at most one vertex in common;
2. Two edges $(u_1, u_2, \dots, u_n)$ and $(v_1, v_2, \dots, v_m)$ are considered to be equal if and only if:
 - (a) $m = n$;
 - (b) either $u_i = v_i$ for $1 \leq i \leq n$, or $u_i = v_{n-i+1}$ for $1 \leq i \leq n$.

Let $G = (V, X)$ be a semigraph and $E = (v_1, v_2, \dots, v_n)$ be an edge of G . Then, v_1 and v_n are the *end vertices* of E and $v_i, 2 \leq i \leq n-1$ are the *middle vertices* (or *m-vertices*) of E . If a vertex v of a semigraph G appears only as an end vertex then it is called an *end vertex*. If a vertex v is only a middle

vertex then it is a *middle vertex* or *m-vertex* while a vertex v is called *middle-cum-end vertex* or *(m,e)-vertex* if it is a middle vertex of some edge and an end vertex of some other edge. A *subedge* of an edge $E = (v_1, v_2, \dots, v_n)$ is a k -tuple $E' = (v_{i_1}, v_{i_2}, \dots, v_{i_k})$, where $1 \leq i_1 < i_2 < \dots < i_k \leq n$ or $1 \leq i_k < i_{k-1} < \dots < i_1 \leq n$. We say that the subedge E' is *induced* by the set of vertices $\{v_{i_1}, v_{i_2}, \dots, v_{i_k}\}$. A *partial edge* of $E = (v_1, v_2, \dots, v_n)$ is a $(j-i+1)$ -tuple $E(v_i, v_j) = (v_i, v_{i+1}, \dots, v_j)$, where $1 \leq i < j \leq n$. $G' = (V', X')$ is a *partial semigraph* of a semigraph G if the edges of G' are partial edges of G . Two vertices u and v in a semigraph G are said to be *adjacent* if they belong to the same edge. If u and v are adjacent and consecutive in order then they are said to be *consecutively adjacent*. u and v are said to be *e-adjacent* if they are the end vertices of an edge and *1e-adjacent* if both the vertices u and v belong to the same edge and at least one of them, is an end vertex of that edge”.

2.2 Soft set

In 1999, D. Molodtsov [18] initiated the concept of soft sets. Let U be an initial universe set and let A be a set of parameters. A pair (F, A) is called a soft set (over U) if and only if F is a mapping of A into the set of all subsets of the set U . That is, $F : A \rightarrow \mathcal{P}(U)$.

2.3 Soft semigraph

B. George, R. K. Thumbakara and J. Jose [11], [12] introduced soft semigraph by applying the concept of soft set in semigraph as follows: “Let $G^* = (V, X)$ be a semigraph having vertex set V and edge set X . Consider a subset V_1 of V . Then, a partial edge formed by some or all vertices of V_1 is said to be a *maximum partial edge* or *mp edge* if it is not a partial edge of any other partial edge formed by some or all vertices of V_1 .

Let X_p be the collection of all partial edges of the semigraph G and A be a nonempty set. Let a subset R of $A \times V$ be an arbitrary relation from A to V . We define a mapping Q from A to $\mathcal{P}(V)$ by $Q(x) = \{y \in V | xRy\}$, $\forall x \in A$, where $\mathcal{P}(V)$ denotes the power set of V . Then, the pair (Q, A) is a soft set over V . Also define a mapping W from A to $\mathcal{P}(X_p)$ by $W(x) = \{\text{mp edges} < Q(x) >\}$, where $\{\text{mp edges} < Q(x) >\}$ denotes the set of all mp edges that can be formed by some or all vertices of $Q(x)$ and $\mathcal{P}(X_p)$ denotes the power set of X_p . The pair (W, A) is a soft set over X_p . Then, we can define a soft semigraph as follows:

The 4-tuple $G = (G^*, Q, W, A)$ is called a *soft semigraph* of G^* if the following conditions are satisfied:

1. $G^* = (V, X)$ is a semigraph having vertex set V and edge set X ;
2. A is the nonempty set of parameters;
3. (Q, A) is a soft set over V ;
4. (W, A) is a soft set over X_p ;

5. $H(a) = (Q(a), W(a))$ is a partial semigraph of G^* , $\forall a \in A$.

Let $G^* = (V, X)$ be a semigraph and $G = (G^*, Q, W, A)$ be a soft semigraph of G^* which is also given by $\{H(x) : x \in A\}$. Then, the partial semigraph $H(x)$ corresponding to any parameter x in A is called a *p-part* of the soft semigraph G . An edge present in a soft semigraph G of G^* is called an *f-edge*. It may be a partial edge of some edge in G^* or an edge in G^* . A partial edge of any *f-edge* of a soft semigraph G is called a *p-edge* of G . An *f-edge* is a *p-edge* of itself. An *f-edge* or a *p-edge* of a soft semigraph G is called an *fp-edge* of G ."

3. Different types of bipartite soft semigraphs

In this section we extend the concepts of bipartite semigraphs given in [20] to soft semigraphs.

3.1 Bipartite soft semigraph

Let $G^* = (V, X)$ be a semigraph and $G = (G^*, Q, W, A)$ be a soft semigraph of G^* represented by $\{H(x) : x \in A\}$. Then, G is called a bipartite soft semigraph if each of its *p-parts* $H(x)$ is a bipartite partial semigraph of G^* . That is, $Q(x)$ can be partitioned into sets $\{Q_1(x), Q_2(x)\}$ such that both $Q_1(x)$ and $Q_2(x)$ are independent for all x in A . That is, no *f-edge* in $W(x)$ is an *mp edge* $\langle Q_1(x) \rangle$ or an *mp edge* $\langle Q_2(x) \rangle$ for all x in A . The term *mp edge* $\langle Q_i(x) \rangle$ denotes a maximum partial edge that can be formed by some or all vertices of $Q_i(x)$.

Example 3.1. Let $G^* = (V, X)$ be a semigraph given in Figure 1, where $V = \{v_0, v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9\}$ and $X = \{(v_0, v_1, v_2), (v_2, v_3, v_4), (v_1, v_6, v_5), (v_3, v_9, v_8), (v_7, v_8)\}$.

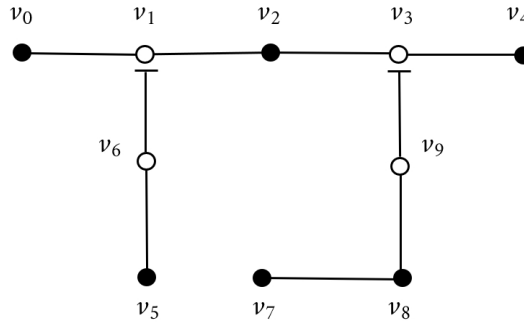


Figure 1: Semigraph $G^* = (V, X)$

Let $A = \{v_1, v_8\} \subseteq V$ be a parameter set. Define Q from A to $\mathcal{P}(V)$ by $Q(x) = \{y \in V | xRy \Leftrightarrow x = y \text{ or } x \text{ and } y \text{ are consecutively adjacent}\}, \forall x \in A$ and W from A to $\mathcal{P}(X_p)$ by $W(x) = \{mp \text{ edges} \langle Q(x) \rangle\}, \forall x \in A$. That is, $Q(v_1) =$

$\{v_0, v_1, v_2, v_6\}$ and $Q(v_8) = \{v_7, v_8, v_9\}$. Also, $W(v_1) = \{(v_1, v_6), (v_0, v_1, v_2)\}$ and $W(v_8) = \{(v_7, v_8), (v_8, v_9)\}$. Then, $H(v_1) = (Q(v_1), W(v_1))$ and $H(v_8) = (Q(v_8), W(v_8))$ are partial semigraphs of G^* as shown below in Figure 2. Hence, $G = \{H(v_1), H(v_8)\}$ is a soft semigraph of G^* . Here, $Q(v_1) = \{v_0, v_1, v_2, v_6\}$

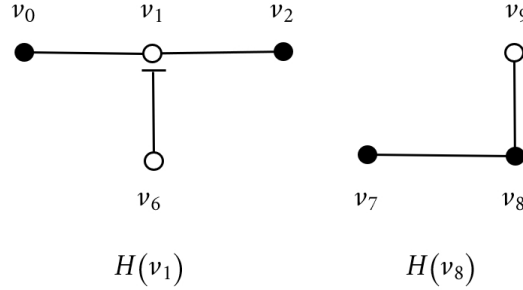


Figure 2: Soft Semigraph $G = \{H(v_1), H(v_8)\}$

can be partitioned into sets $\{Q_1(v_1), Q_2(v_1)\}$, where $Q_1(v_1) = \{v_0, v_2, v_6\}$ and $Q_2(v_1) = \{v_1\}$. Then, $Q_1(v_1)$ and $Q_2(v_1)$ are independent since, no edge in $W(v_1)$ is an mp edge $\langle Q_1(v_1) \rangle$ or an mp edge $\langle Q_2(v_1) \rangle$. Also, $Q(v_8) = \{v_7, v_8, v_9\}$ can be partitioned into sets $\{Q_3(v_8), Q_4(v_8)\}$, where $Q_3(v_8) = \{v_7, v_9\}$ and $Q_4(v_8) = \{v_8\}$. Then, $Q_3(v_8)$ and $Q_4(v_8)$ are independent since, no edge in $W(v_8)$ is an mp edge $\langle Q_3(v_8) \rangle$ or an mp edge $\langle Q_4(v_8) \rangle$. Therefore, $H(v_1)$ and $H(v_8)$ are bipartite partial semigraphs of G^* and hence, $G = \{H(v_1), H(v_8)\}$ is a bipartite soft semigraph.

3.2 e -bipartite soft semigraph

Definition 3.1. Let $G^* = (V, X)$ be a semigraph and $G = (G^*, Q, W, A)$ be a soft semigraph of G^* represented by $\{H(x) : x \in A\}$. Then, G is called an e -bipartite soft semigraph if each of its p -parts $H(x)$ is an e -bipartite partial semigraph of G^* . That is, $Q(x)$ can be partitioned into sets $\{Q_1(x), Q_2(x)\}$ such that both $Q_1(x)$ and $Q_2(x)$ are e -independent for all x in A . That is, no two end vertices or partial end vertices of an f -edge in $W(x)$ belong to $Q_1(x)$ or $Q_2(x)$ for all x in A .

Example 3.2. Consider the soft semigraph G given in Figure 2. Here, $Q(v_1) = \{v_0, v_1, v_2, v_6\}$ can be partitioned into sets $\{Q_1(v_1), Q_2(v_1)\}$, where $Q_1(v_1) = \{v_0, v_1\}$ and $Q_2(v_1) = \{v_2, v_6\}$. Then, $Q_1(v_1)$ and $Q_2(v_1)$ are e -independent since no two end vertices or partial end vertices of an f -edge in $W(v_1)$ belong to $Q_1(v_1)$ or $Q_2(v_1)$. Also, $Q(v_8) = \{v_7, v_8, v_9\}$ can be partitioned into sets $\{Q_3(v_8), Q_4(v_8)\}$, where $Q_3(v_8) = \{v_7, v_9\}$ and $Q_4(v_8) = \{v_8\}$. Then, $Q_3(v_8)$ and $Q_4(v_8)$ are e -independent since no two end vertices or partial end vertices of an f -edge in $W(v_8)$ belong to $Q_3(v_8)$ or $Q_4(v_8)$. Therefore, $H(v_1)$ and $H(v_8)$

are e -bipartite partial semigraphs of G^* and hence, $G = \{H(v_1), H(v_8)\}$ is an e -bipartite soft semigraph.

3.3 Strongly bipartite soft semigraph

Definition 3.2. Let $G^* = (V, X)$ be a semigraph and $G = (G^*, Q, W, A)$ be a soft semigraph of G^* represented by $\{H(x) : x \in A\}$. Then, G is called a strongly bipartite soft semigraph if each of its p -parts $H(x)$ is a strongly bipartite partial semigraph of G^* . That is, $Q(x)$ can be partitioned into sets $\{Q_1(x), Q_2(x)\}$ such that both $Q_1(x)$ and $Q_2(x)$ are strongly independent for all x in A . That is, no two adjacent vertices in $H(x)$ belong to $Q_1(x)$ or $Q_2(x)$ for all x in A .

Example 3.3. Let be a semigraph given in Figure 3, where $V = \{v_1, v_2, v_3, v_4, v_5, v_6\}$ and $X = \{(v_1, v_2), (v_1, v_3, v_6, v_4), (v_4, v_5)\}$.

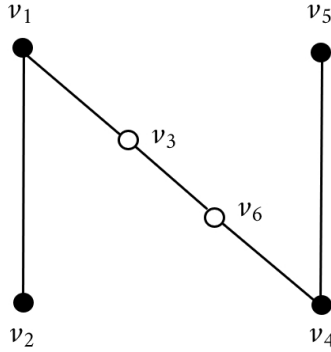
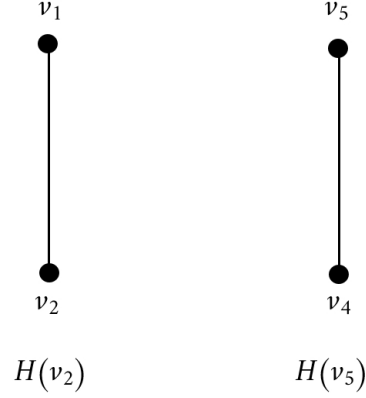


Figure 3: Semigraph $G^* = (V, X)$

Let $A = \{v_2, v_5\} \subseteq V$ be a parameter set. Define Q from A to $\mathcal{P}(V)$ by $Q(x) = \{y \in V \mid xRy \Leftrightarrow x = y \text{ or } x \text{ and } y \text{ are adjacent}\}, \forall x \in A$ and W from A to $\mathcal{P}(X_p)$ by $W(x) = \{mp \text{ edges}\langle Q(x) \rangle\}, \forall x \in A$. That is, $Q(v_2) = \{v_1, v_2\}$ and $Q(v_5) = \{v_4, v_5\}$. Also, $W(v_2) = \{(v_1, v_2)\}$ and $W(v_5) = \{(v_4, v_5)\}$. Then, $H(v_2) = (Q(v_2), W(v_2))$ and $H(v_5) = (Q(v_5), W(v_5))$ are partial semigraphs of G^* as shown below in Figure 4. Hence, $G = \{H(v_2), H(v_5)\}$ is a soft semigraph of G^* . Here, $Q(v_2) = \{v_1, v_2\}$ can be partitioned into sets $\{Q_1(v_2), Q_2(v_2)\}$, where $Q_1(v_2) = \{v_1\}$ and $Q_2(v_2) = \{v_2\}$. Then, $Q_1(v_2)$ and $Q_2(v_2)$ are strongly independent since, no two adjacent vertices in $H(v_2)$ belong to $Q_1(v_2)$ or $Q_2(v_2)$. Also, $Q(v_5) = \{v_4, v_5\}$ can be partitioned into sets $\{Q_3(v_5), Q_4(v_5)\}$, where $Q_3(v_5) = \{v_4\}$ and $Q_4(v_5) = \{v_5\}$. Then, $Q_3(v_5)$ and $Q_4(v_5)$ are strongly independent since, no two adjacent vertices in $H(v_5)$ belong to $Q_3(v_5)$ or $Q_4(v_5)$. Therefore, $H(v_2)$ and $H(v_5)$ are strongly bipartite partial semigraphs of G^* and hence, $G = \{H(v_2), H(v_5)\}$ is a strongly bipartite soft semigraph.

Figure 4: Soft Semigraph $G = \{H(v_2), H(v_5)\}$

Theorem 3.1. *If a soft semigraph G is strongly bipartite then its p -parts are bipartite graphs if we treat both end vertices and partial end vertices as vertices of the bipartite graph.*

Proof. Let $G = (G^*, Q, W, A)$ be a strongly bipartite soft semigraph of G^* represented by $\{H(x) : x \in A\}$. Then, each of its p -parts $H(x)$ is a strongly bipartite partial semigraph of G^* . That is, $Q(x)$ can be partitioned into sets $\{Q_1(x), Q_2(x)\}$ such that both $Q_1(x)$ and $Q_2(x)$ are strongly independent for all x in A . That is, no two adjacent vertices in $H(x)$ belong to $Q_1(x)$ or $Q_2(x)$ for all x in A . So, each $H(x)$ can contain f -edges with 2 vertices only. Therefore, this partition $\{Q_1(x), Q_2(x)\}$ will satisfy all the criteria for a bipartite graph in graph theory, if we treat both end vertices and partial end vertices as vertices and f -edges as edges. $\square$

Theorem 3.2. *If a soft semigraph G is e -bipartite, then it is also a bipartite soft semigraph.*

Proof. Let $G = (G^*, Q, W, A)$ be an e -bipartite soft semigraph represented by $\{H(x) : x \in A\}$. Then, each p -part $H(x)$ of G will be an e -bipartite partial semigraphs of G^* . That is, $Q(x)$ can be partitioned into sets $\{Q_1(x), Q_2(x)\}$ such that both $Q_1(x)$ and $Q_2(x)$ are e -independent for all x in A . That is, no two end vertices or partial end vertices of an f -edge in $W(x)$ belong to $Q_1(x)$ or $Q_2(x)$, for all x in A . Definitely, no edge in $W(x)$ is an mp edge $< Q_1(x) >$ or an mp edge $< Q_2(x) >$, for all x in A . Therefore, G is also a bipartite soft semigraph. $\square$

Remark 3.1. The converse of this theorem need not be true. This is clear from the following example.

Example 3.4. Let be a semigraph given in Figure 5, where $V = \{v_0, v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9\}$ and $X = \{(v_0, v_2), (v_0, v_1, v_3), (v_2, v_3), (v_3, v_4, v_5), (v_6, v_5, v_9), (v_6, v_7), (v_5, v_7), (v_7, v_8, v_9)\}$.

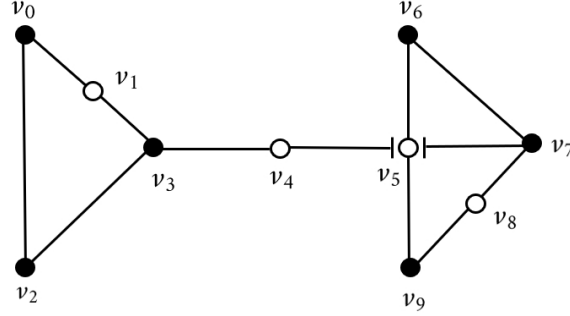


Figure 5: Semigraph $G^* = (V, X)$

Let $A = \{v_0, v_7\} \subseteq V$ be a parameter set.

Define mappings Q from A to $\mathcal{P}(V)$ by $Q(x) = \{y \in V | xRy \Leftrightarrow x = y \text{ or } x \text{ and } y \text{ are adjacent}\}, \forall x \in A$ and W from A to $\mathcal{P}(X_p)$ by $W(x) = \{mp \text{ edges}\langle Q(x) \rangle\}, \forall x \in A$. That is, $Q(v_0) = \{v_0, v_1, v_2, v_3\}$ and $Q(v_7) = \{v_5, v_6, v_7, v_8, v_9\}$. Also, $W(v_0) = \{(v_0, v_2), (v_2, v_3), (v_0, v_1, v_3)\}$ and $W(v_7) = \{(v_6, v_7), (v_6, v_5, v_9), (v_5, v_7), (v_7, v_8, v_9)\}$. Then, $H(v_0) = (Q(v_0), W(v_0))$ and $H(v_7) = (Q(v_7), W(v_7))$ are partial semigraphs of G^* as shown below in Figure 6. Hence, $G = \{H(v_0), H(v_7)\}$ is a soft semigraph of G^* .

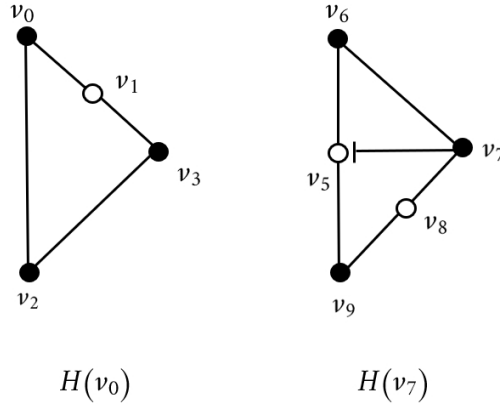


Figure 6: Soft Semigraph $G = \{H(v_0), H(v_7)\}$

Here, $Q(v_0) = \{v_0, v_1, v_2, v_3\}$ can be partitioned into sets $\{Q_1(v_0), Q_2(v_0)\}$, where $Q_1(v_0) = \{v_0, v_3\}$ and $Q_2(v_0) = \{v_2, v_1\}$. Then, $Q_1(v_0)$ and $Q_2(v_0)$ are independent, since no edge in $W(v_0)$ is an mp edge $\langle Q_1(v_0) \rangle$ or an

mp edge $< Q_2(v_0) >$. Also, $Q(v_7) = \{v_5, v_6, v_7, v_8, v_9\}$ can be partitioned into sets $\{Q_3(v_7), Q_4(v_7)\}$, where $Q_3(v_7) = \{v_5, v_6, v_8\}$ and $Q_4(v_7) = \{v_7, v_9\}$. Then, $Q_3(v_7)$ and $Q_4(v_7)$ are independent, since no edge in $W(v_7)$ is an mp edge $< Q_3(v_7) >$ or an mp edge $< Q_4(v_7) >$. Therefore, $H(v_0)$ and $H(v_7)$ are bipartite partial semigraphs of G^* and hence, $G = \{H(v_0), H(v_7)\}$ is a bipartite soft semigraph. But, $H(v_0)$ and $H(v_7)$ are not e -bipartite partial semigraphs of G^* because, $Q(v_0)$ and $Q(v_7)$ are not e -independent. That is, there exists no partition for $Q(v_0)$ and $Q(v_7)$ such that the two end vertices or partial end vertices of an f -edge in $W(v_0)$ and $W(v_7)$ are in two different sets of the partition.

Theorem 3.3. *A soft semigraph G is e -bipartite if and only if all of its p -part end vertex graphs $H(x)_e$ are bipartite where $H(x)_e$ is a graph having vertex set $Q(x)$ and two vertices u and v in $H(x)_e$ are adjacent if they are the end vertices or a partial end vertices of an f -edge containing these vertices in the p -part $H(x)$.*

Proof. Assume that a soft semigraph $G = (G^*, Q, W, A)$ represented by $\{H(x) : x \in A\}$ is an e -bipartite soft semigraph. Then, all of its p -parts $H(x)$ are e -bipartite partial semigraphs of G^* . That is, $Q(x)$ can be partitioned into sets $\{Q_1(x), Q_2(x)\}$ such that both $Q_1(x)$ and $Q_2(x)$ are e -independent for all x in A . That is, no two end vertices or partial end vertices of an f -edge in $W(x)$ belong to $Q_1(x)$ or $Q_2(x)$ for all x in A . We know that the p -part end vertex graph $H(x)_e$ is $Q(x)$ and two vertices u and v in $H(x)_e$ are adjacent if they are the end vertices or a partial end vertices of an f -edge $W(x)$ for all x in A . So, if we give the same partition $\{Q_1(x), Q_2(x)\}$ to $Q(x)$ in $H(x)_e$ then each edge has one end in $Q_1(x)$ and the other end in $Q_2(x)$. Therefore, $H(x)_e$ is a bipartite graph for all x in A .

Conversely, assume that all p -part end vertex graph $H(x)_e$ of G is bipartite. That is, if $Q(x)$ is the vertex set of $H(x)_e$, then it can be partitioned into two nonempty sets $Q_1(x)$ and $Q_2(x)$ such that each edge in $H(x)_e$ has one end in $Q_1(x)$ and the other end in $Q_2(x)$ for all x in A . In the corresponding p -part $H(x)$, end vertices or partial end vertices of the f -edge are the same as the end vertices of the corresponding edge in $H(x)_e$. Therefore, if we use the same partition of $Q(x)$ in $H(x)$, $Q_1(x)$ and $Q_2(x)$ will be e -independent for all x in A . Therefore, $H(x)$ is an e -bipartite partial semigraph for all x in A . That is, G is an e -bipartite soft semigraph. $\square$

4. Conclusion

This paper has explored the intersection of soft set theory and semigraphs, demonstrating the efficacy of soft set principles in enhancing the adaptability of semigraphs to address uncertainty. The paper focused on different kinds of bipartite soft semigraphs, studying their structures and features closely. This research helps us better understand how to use these tools to tackle real-world

problems with uncertain data, which can be important for making better decisions in various fields.

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