Engineering students' use of scaffolding elements within digital tasks concerning planar integration domains

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Abstract. In this paper, we focus on a sequence of digital tasks involving the representation of subsets of the plane as normal domains. We investigate the role of scaffolding elements provided to supports Engineering students' activities with the tasks. The study focuses on 63 volunteer Engineering freshman. Using qualitative methods, we carried on the combined analysis of video recordings of the students' interaction with the tasks in small groups and their individual answers to a survey after the activity. The main result of the data analysis is the identification of three categories of students' behaviours concerning their use of the provided scaffolding elements. These categories reflect different ways of grasping the scaffolding, purposes and levels of awareness by the students.

Keywords: design of digital tasks, scaffolding, feedback, double integrals, engineering students.

MSC 2020: 97I50, 97U50, 97C70.

1. Introduction

This study sets in the stream of research concerning the use of domain-specific software in teaching Mathematics to Engineering students (see [1], [2], [3], [13], [14] and [18]). In particular, in [2] and [3], Alessio et al. presented two sequences of digital tasks concerning the conversion between different representations of subsets of the plane. These sequences of tasks have been designed taking into account the typical difficulties in Multivariable Calculus discussed in the literature, and aiming at the overall educational goal of improving students' understanding of problems involving double integrals. Indeed, to fruitfully face this kind of

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problems, students should be able to a) analytically and graphically represent the integration domain; b) recognize and describe the limits of integration in order to write a double integral as iterated integrals; c) choose the most effective description of the integration domain towards the calculation of a given integral (see [3]).

In this paper, we focus on the sequence of digital tasks about representation of subsets of the plane as normal domains described in [3], thus addressing points a) and b). Precisely, we investigate the role of scaffolding elements provided to support the students' activity with the tasks. In particular, we analyze the students' use of the scaffolding elements provided within the tasks and their perception of the impact of these elements on their learning.

This paper should be a contribution in two directions, both theoretical and practical. On the theoretical side, we aim to deepen how to design effective scaffolding in digital tasks and to grasp the elements allowing or preventing a fruitful use of it by the students. On the practical side, we would like to provide students with a non-human tutor supporting their learning of advanced mathematical concepts in the delicate phase of transition to university.

2. Theoretical framework

Recent research highlighted the essential role of scaffolding in supporting students' learning, especially within digital environments (see [4], [5], [11], [16]).

According to Holton and Clarke [11], scaffolding is a teaching act supporting the immediate construction of knowledge and, at the same time, fostering the students' future independent learning. In tune with [16], each scaffolding element should address two components: a *procedural one*, allowing students to overcome a contingent difficulty to perform a task correctly, by reducing the cognitive load; and a *metacognitive one*, having long-term effects and allowing students to face similar difficulties and tasks in the future.

In some contexts, characterized by large and heterogeneous classes, is very useful to provide scaffolding in digital environments. Indeed, although one-to-one scaffolding is very effective, it cannot be done in university classes with hundreds of students with different learning needs ([5]). So, using digital environments could allow to largely share the scaffolding and adapt it to the individual needs. Computer-based scaffolding (CBS) is defined as *"computer-based support that helps students engage in and gain skill at tasks that are beyond their unassisted abilities"* ([5], p. 26). Differently from face-to-face scaffolding, which can be progressively negotiated, CBS requires a careful design, that considers in advance students' possible difficulties and learning needs ([16]).

CBS is particularly relevant for our study since the careful use of technology is the key aspect for distributing scaffolding, adapting it to the specific difficulties of the learners. In this sense, and according to [9] and [17], scaffolding is intended throughout the study as a form of formative feedback, that is as a piece of information clarifying the students where they are in the learning process, where they are going, and what following steps should be done to achieve the goal [19].

Hattie and Timperley in [10] discussed the following levels of formative feedback: feedback on the task; feedback on the processing of the task; feedback on self-regulation; feedback on the self as a person.

From the literature (see [16], [17], [21], [22]), some design principles (DP) for scaffolding can be extracted, favouring its effectiveness. In particular:

(DP_a) its content (information suggested or provided), function (cognitive, meta-cognitive, ...) and form of presentation (timing, semiotic register, ...) should take into account the context and learner characteristics;

(DP_b) it should be separated from the assessment;

(DP_c) it should be written or communicated via computer, and provided in a not too formal way so that a dialogue between the sender and the receiver of the scaffolding could be triggered;

(DP_d) it should contain a limited quantity of information;

(DP_e) it should be customized to the individual learning needs;

(DP_f) it should be presented in nested sequences of tasks;

(DP_g) it should be progressively faded, i.e. gradually reduced.

All the previous design principles have been used for designing the scaffolding elements provided in the sequence of our digital tasks [3] and will be explicitly recalled in the next section.

With regard to (DP_e), CBS presents a critical feature in relation to one-on-one scaffolding provided by an expert tutor: CBS could not be completely individualized to the student's learning needs nor completely adaptively provided ([16]). In this perspective, the challenge of CBS is to be responsive, that is such to induce effective consequences ([12]), to typical difficulties and approaches (that can be revealed by the student's behaviours within the digital environment), rather than to the specific individual's difficulties. As recalled by point (DP_g), a specific feature distinguishing scaffolding from other forms of support is fading. i.e. *"the gradual reduction and eventual elimination of scaffolding"* ([16], p. 29). Thus, each scaffolding element should be such that: *"(a) support learners in the achievement of tasks beyond their unassisted capacity; (b) when the support structure is removed, learners continue to function competently on their own; and (c) removing the support structure does not reduce learning or functioning – instead, learners continue to function at the elevated plane reached via scaffolding"* ([16], p. 29).

Also as concerns fading, in digital environments it could be less responsive with respect to the fading of scaffolding enacted by an expert in a one-to-one relationship. So, the challenge is to design a responsive fading of scaffolding ([16]) on the basis of typical students' behaviours. From this perspective, scaffolding as formative feedback turns out to be also a crucial tool for improving the individualization of teaching and learning (see [7], [8]).

According to Withney [20], scaffolding, intended as a form of formative feedback ([17]), is afflicted by a paradox: although generally the students consider it helpful, they do not use it. Winstone et al. in [21] investigated the reasons preventing students from using feedback and identified four main barriers: difficulties in decoding feedback content and purpose; poor knowledge of suitable strategies and theoretical elements addressed by the feedback; sense of disempowerment and difficulty in translating feedback in actions; lack of receptiveness and engagement.

In this paper, according to the previuos considerations, we investigate whether and how Engineering freshmen use scaffolding elements designed by us to support their interaction with a sequence of digital tasks. Our specific research questions are:

(1) do students utilize the available scaffolding elements?

(2) if students use these elements, in what way do they mostly use them?

(3) does scaffolding in a digital environment influence the subsequent work of students who utilized it?

3. The tasks and the scaffolding elements

We focus on a sequence of digital tasks concerning the representations of subsets of the plane as normal domains. It has been designed within wider research about the teaching-learning of double integrals to Engineering students (see [2], [3] and [8]). The tasks and their scaffolding elements are designed according to the principles listed in Section 2. In particular, the tasks (T1-T6) are nested in a sequence of increasing difficulty (according to (DP_f)): the first three tasks concern normal domains in the x or y-direction; the last three ones are about normal domains in polar coordinates (see [3] for more details about the concept of normal domain). According to (DP_b)), the sequence of tasks is intended for a free use by the students, who can interact with it in their own pace, and it is not envisaged any assessment.

All the tasks have some common elements: they provide a planar region with some points allowing to obtain its analytical description (Figure 1, right side of the screen), and the reminder of the definition of the normal domains (Figure 1, left side of the screen).

Students are expected to establish whether the planar region is a normal domain, in which direction, and identify functions that describe it as a normal domain (as open-ended questions). In the last three tasks, further competence is addressed, i.e. decision-making. Indeed, students are faced with planar regions

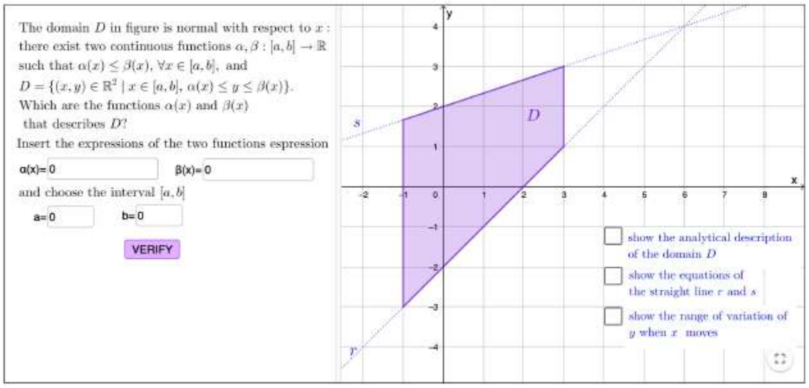


Figure 1: The starting screenshot of the task T1

that can be described in different ways as normal domains; they are required to compare these different descriptions and establish the most convenient description of the domain to set up the calculation of the double integral of a given function (see [3] for more details).

The scaffolding elements provided within the digital tasks are of two kinds: some of them are teacher-driven and others are student-driven. As discussed by Alessio et al. in [3], all the tasks are designed so that automatic, immediate and facilitative feedback in different semiotic registers appears when the student submits an answer, according to (DP_a). In particular, in case of a wrong answer, a verbal message signals the mistake (feedback at the task level), and the region corresponding to the analytical description submitted by the student, if it is not empty, becomes shaded (feedback at the process level and the self-regulation level). This is a response-specific feedback, designed so that individual learning needs are addressed (according to (DP_e)). It is obviously teacher-driven, communicated via computer and refers to a little piece of information (according to (DP_c)).

The students' activity with the tasks is supported by some other scaffolding elements, also called *hints* throughout the paper, which the students can require if they are in difficulty. These hints scaffold the students' work within the digital environment and represent a form of feedback that makes the students responsible for their learning. Indeed, differently from the automatic and immediate feedback previously described, this form of feedback is student-driven and appears only if the students select it. This choice is in tune with Winston at al. [21] and due to our aim to make the students progressively aware of their learning needs and the kind of support useful for them. In the following, we describe in detail these hints, in relation to the DP listed in Section 2.

In **task T1**, the planar region to be described is a quadrilateral (region D in Figure 2). The student could obtain the expressions of the two functions defining D as a normal domain in the x-direction from the graph, using the

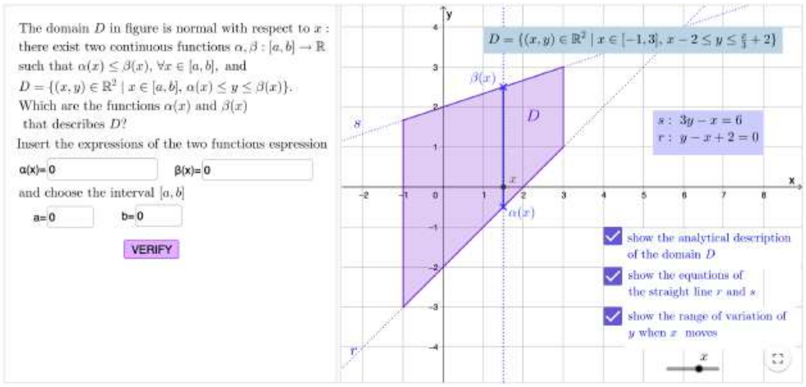


Figure 2: A screenshot of the task T1

coordinates of points in the graphical representation to identify the equations of the straight lines r and s and the interval where x varies.

The hints available for the students are shown on the right side of the screen (Figure 2). Two of them are analytical ("show the analytical description of D", which we will call *domain hint*; "show the equations of the straight lines r and s", *curves equation hint*) and one is geometrical ("show the range of variation of y when x moves", *dynamic variation hint*). The hints are listed in the Table 1 and their different features are in tune with (DP_a).

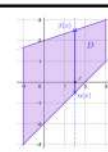
Name of the hint	Description of the hint in the digital task T1	What the hint provides in the digital task T1
domain hint	show the analytical description of the domain D	$D = \{(x, y) \in \mathbb{R}^2 \mid x \in [-1, 3], x - 2 \leq y \leq \frac{x}{3} + 2\}$
curves equation hint	show the equations of the straight line r and s	$s: 3y - x = 6$ $r: y - x + 2 = 0$
dynamic variation hint	show the range of variation of y when x moves	

Table 1: The available hints in task T1

They have been designed to scaffold the students' work with different degrees of guidance and to take into account different needs, according to (DP_d) and (DP_f). Indeed, the *domain hint*, provides explicit support to the student's work, giving the range of variation of the x-coordinate and the functions between which the y-coordinate varies. So the student should only recognize in the writing of D an instantiation of the general writing of an x-normal domain shown on the left side of the screen and suitably substitute the extreme points of the range of x and the expressions of the functions in the insert fields.

The *curves equation hint*, is less guiding since it gives the (implicit) equations of the straight lines. In this case, the student, to correctly perform the task, should first autonomously find the range of variation of the x-coordinate, hence make explicit the equations of the straight lines and recognize the obtained expressions as the lower and upper functions delimiting the y-coordinate of the domain D.

The *dynamic variation hint*, is even less guiding since it does not provide quantitative information, but addresses the geometrical meaning of the concept of normal domain as explained by the teacher during the lessons (i.e. as a domain for which the interception with each vertical straight line is a segment). In this case, the student is induced to focus on the fact that for each fixed value of x in a suitable range, the y-coordinate varies between the straight lines r and s, that is, between the graphs of the functions $y = \alpha(x)$ and $y = \beta(x)$ which identify D as an x-normal domain. Thus, he/she should autonomously find the range of variation of both the coordinates describing D as a normal domain.

All the presented scaffolding elements, either those teacher-driven, i.e. the response-specific feedback, and those student-driven, i.e. the hints, act at both the procedural and metacognitive levels, making the student reflect on the steps to switch from the graphical to the analytical representation of a normal domain. Among the hints, the *dynamic variation hint* is the most metacognitive one, since it does not provide explicit support to perform the task, but creates conditions for significant learning and linking between the theoretical knowledge and the solution of the problem. We expected some students to choose hints with higher level of guidance in the first tasks of the sequence and successively hints with less level of guidance or no hints, having taken advantage of the first ones. This would be in tune with the idea of fading the scaffolding (DP_g).

In **task T2** a y-normal domain is shown (see Figure 3). Also in this case the student can find the equations of the parabola and the two straight lines delimiting D by the graph, and she can ask the same hints previously described. However, according to the idea to progressively fade the scaffolding (see Section 2, (DP_g)), in this case, the *domain hint* does not provide the range of variation of the y-coordinate, leaving the student to find them. Analogously, the *curves equation hint* provides only the equation of the parabola and not the equations of the straight lines. The *dynamic variation hint* is similar to that of task T1.

Differently from the previous task, in T2 when the student describes D correctly, the question *"Is D a normal domain in the x-direction?"* appears. If the student selects the wrong answer *"Yes"*, a verbal feedback is provided, inviting her to dynamically explore the possibility of describing D as an x-normal domain by moving a slider. This is similar to the *dynamic variation hint* but, in this case, the segment linked to the slider becomes red when it splits into

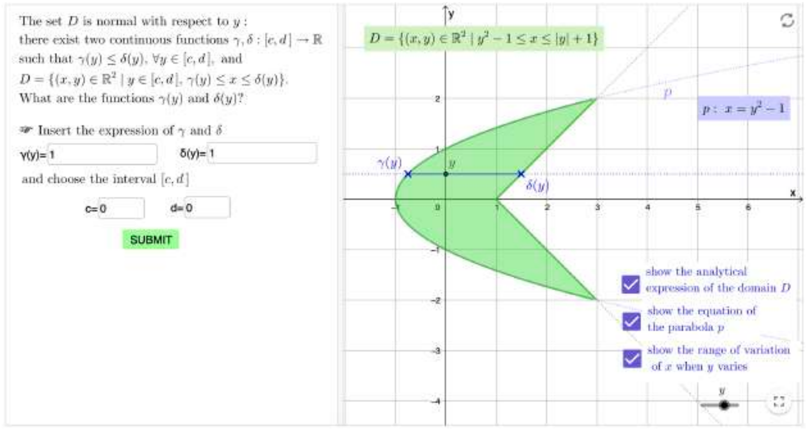


Figure 3: A screenshot of the task T2

two parts, as depicted in Figure 4. This has been designed as another feedback given in the graphical semiotic register and acting at the task level and at the processing of the task level.

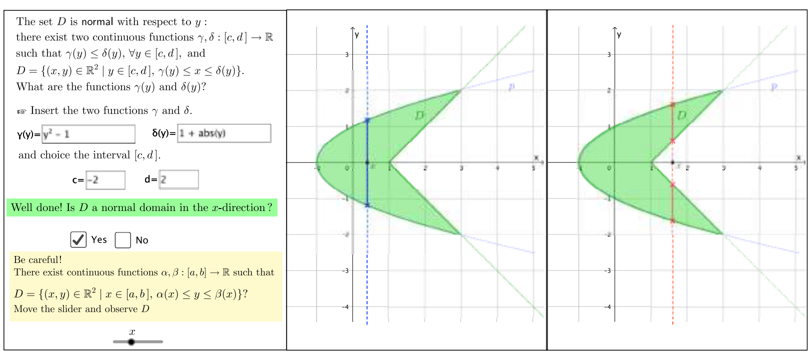


Figure 4: Another screenshot of task T2

In **task T3** a planar region that is x-normal but not y-normal is given. The student is required to choose whether the given planar region is a normal domain in the x or the y-direction (see Figure 5). At this step, only the *domain hint* is available before the student's choice, not revealing the correct one. Indeed, in the analytical description of the domain only the (dis)equations of the circumference and the ellipse delimiting the domain D are provided (see Figure 5).

Since D is x-normal but not y-normal, if the student makes the wrong choice, a piece of feedback information appears, not revealing but inducing the student to understand that the domain is not y-normal, but is the union of two disjoint y-normal domains. Indeed, the student is invited to dynamically explore the possibility of describing D as a normal domain in the y-direction by moving a slider (the segment linked to the slider turns red when it splits into two parts,

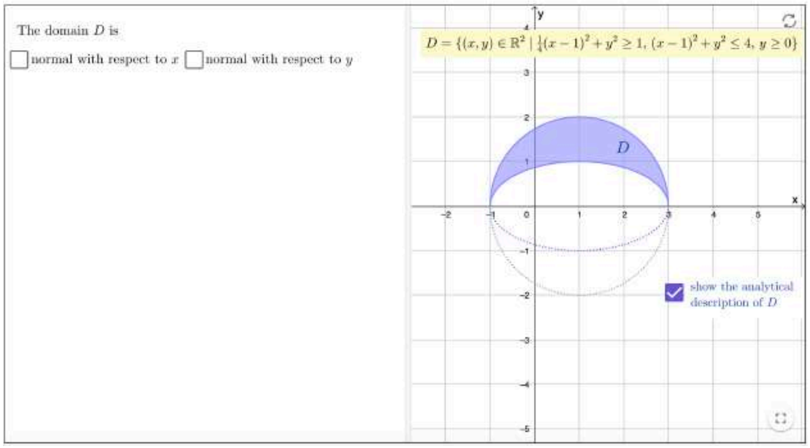


Figure 5: A screenshot of task T3

as depicted in Figure 6). This feedback at the task and process of the task

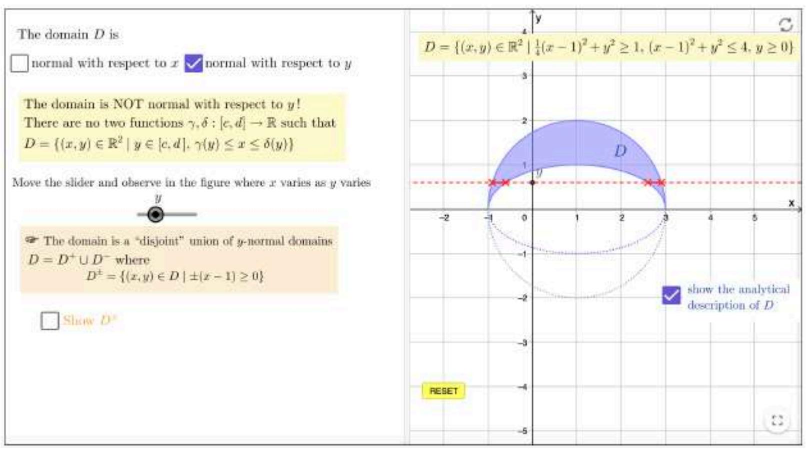


Figure 6: Another screenshot of task T3

levels should make clear that there are more possible ways to describe the given domain, for example, either as an x-normal domain or as a union of y-normal domains.

If the student selects x-normal, the program requires the description of D as an x-normal domain and the other two hints become available (Figure 7). The *curves equation hint* makes the equations of the circumference and the ellipse visible. According to the idea of progressively fading the scaffolding (DP_g), in this case, the *domain hint* and the *curves equation hint* give the equations of the whole circumference and ellipse, from which the student should make explicit the y variable to autonomously find the expression of the functions whose graphs (a semicircumference and a semiellipse) delimit D.

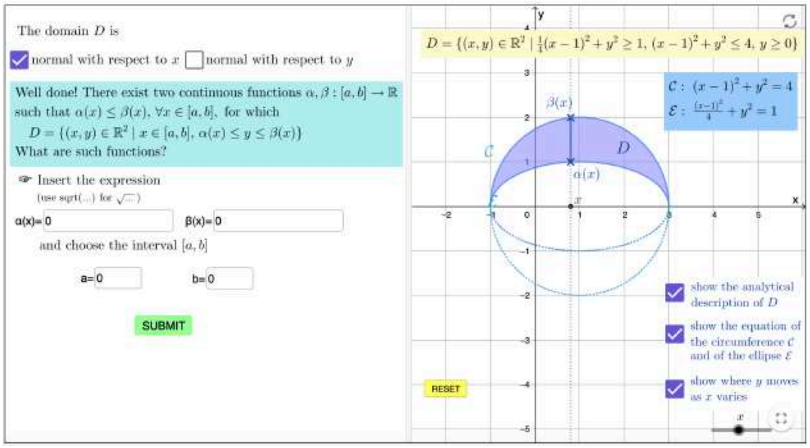


Figure 7: Another screenshot of task T3

In the next three **tasks T4-T6**, the hints proposed are of the same type as the previous ones, and they are provided according to the idea of progressively fading the scaffolding, increasing the activation of the student (see Table 2).

For the sake of completeness and in order to make visible all the potential of

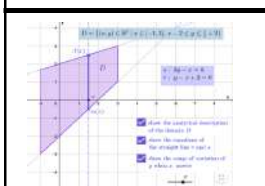
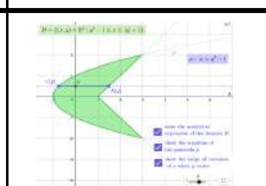
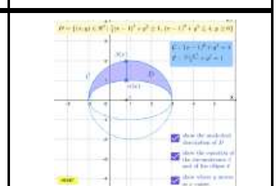
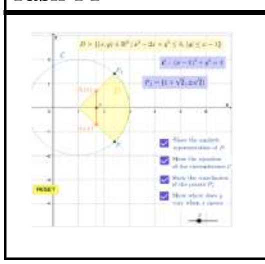
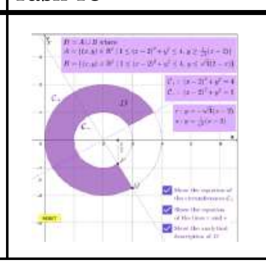
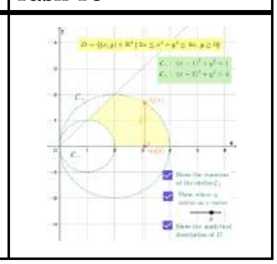
Task T1	Task T2	Task T3
		
Task T4	**Task T5**	**Task T6**
		

Table 2: The available hints in tasks T1-T6

the *dynamic variation hint*, we briefly illustrate it in **task T6**. Here the domain is normal with respect to both the x and y coordinates, but also with respect to polar coordinates. Considering the domain as x-normal or y-normal, the student has to enter the expression of the functions describing D as piecewise-defined functions. In this case, the *dynamic variation hint* is useful both for identifying the different function expressions and the point (in this case, $x = 2$) where the function changes expression (see Figure 8).

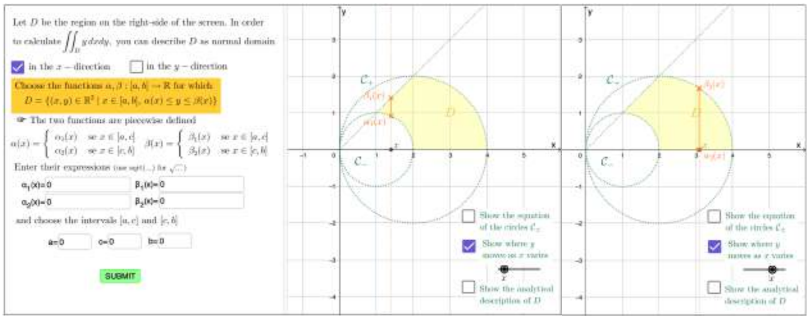


Figure 8: A screenshot of task T6

Finally, to describe the domain using polar coordinates, the same *dynamic variation hint* shows how the variable ρ varies as the angular variable t moves (Figure 9).

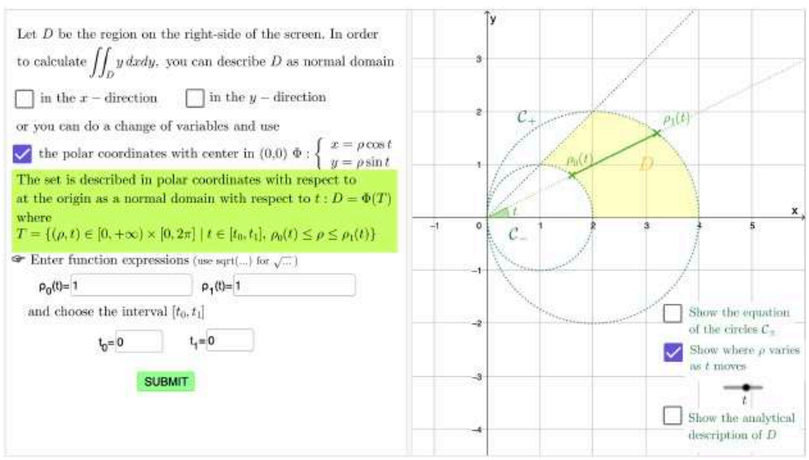


Figure 9: Another screenshot of task T6

4. Research context and methodology

We submitted the sequence of tasks T1-T6 to 63 volunteer Engineering freshmen from the Polytechnic University of Marche (Ancona, Italy), dividing them into small groups of three or four students for the interaction with tasks. They attended the Calculus II course during the 2019-20 academic year and had passed the exam of Calculus I. The first author of the paper was the teacher of the course. Due to the pandemic, the entire course, including the interaction with the sequence of digital tasks, was conducted online. In particular, the students interacted with the sequence of digital tasks for about 2 hours. The authors of the paper were present in the online room as observers to support the groups of students for possible technical troubles. We want to remark that, at the time of the teaching experiment, the students were already familiar with the concepts of normal domains and reduction formulas for the calculation of double integrals. Specifically, they had already learned how to set up the calculation of double integrals in theoretical and practical lessons and had faced traditional pen-and-paper tasks on this topic.

After the interaction with the sequence of digital tasks, students were asked to individually complete a survey regarding: 1) any difficulties they encountered with the tasks; 2) their use of hints (if they used them, why they did so, which hints they used, and for what purpose); 3) their perception of the usefulness of the hints in solving the tasks and in tackling subsequent ones; and 4) their perception of the impact of using hints on their learning.

To give answers to the research questions, we collected two kinds of data: video recordings of the student groups' interaction with the sequence of tasks and individual students' answers to the survey. A cross-analysis of these data has been conducted, comparing the students' behaviour during the interaction with the tasks, specifically about the use of hints, and their answers to the survey. The researchers individually reviewed and analyzed the video recordings, highlighting the key moments where students required and used the hints. In particular, after a first explorative view of video recordings, the focus has been put on some aspects: the choice of using the hints or not in case of difficulty, the consistency between the kind of possible difficulty and the chosen hint and the effects of the use of the hints on the subsequent work. Moreover, the researchers independently read and coded all the answers to the survey according to the theoretical framework, focusing on the students' motivations for using or not using the hints, their specific goals when requiring a hint and their perception of the usefulness of the hints at the procedural level (to correctly perform the task) and at the metacognitive one (on the subsequent learning). Finally, the researchers discussed their analyses, until an agreement was reached (see [15]).

We notice that after the activities were completed by the participants involved in this research, the tasks were made available to all the students (about 80) enrolled in the course of Calculus II on the Moodle platform of the university, as a self-assessment tool.

5. Results

The analysis of data allowed us to identify three categories of students' behaviours with respect to their use of the scaffolding elements. In the following, we will refer to some excerpts which are paradigmatic for the relative category. The first category (about 17% of the total) includes behaviours of students who did not ask for hints, even in case of need, or did not take advantage of the hints. Students whose behaviour is in this category did not realize the opportunities offered by the scaffolding elements in the sequence of digital tasks to overcome procedural difficulties nor to deepen their understanding of the topic at stake.

Typically, the focus of students whose behaviors is in this category is on giving the correct answers, putting in the background the theoretical knowledge and understanding. Some of these students preferred to perform the sequence without any scaffolding. One of these students declared *"I did not ask for hints, since the challenge was to solve the tasks only with my own knowledge and skills"*. Other students asked for some hints, but were not able to take advantage of them.

According to the students' declarations, the non-use of the hints happened for different reasons, which are in tune with those identified by Winston et al. in [21] and recalled in Section 2. Some students had difficulties in decoding the scaffolding elements' content (*"the hints give too vague suggestions"*, *"the hint was not easily understandable for me"*, *"[the hints] are not enough to understand where I was wrong"*); other students lacked strategies to translate the scaffolding elements into actions to be performed (*"I did not see how to use the hint"*). Other students recognized that their lack of theoretical knowledge made the hints not sufficient for them to perform the task correctly (*"I asked all the hints, but I did not study all the theory, so I was not able to use the hints"*). In our opinion, these behaviours suggest a lack of metacognitive control by these students, who sometimes remained blocked in performing the task rather than asking for hints or did not were able to decode the scaffolding into strategies to be enacted.

A second category (about 40%) includes behaviours of students who used the hints and took advantage of the scaffolding at the procedural level. The analysis of the answers to the survey highlights that some students used the hints to quickly solve the problem, without making calculations. Some of them declared *"the hints make quick the solving process"*, *"the hints saved me to make calculations"*, *"I asked for the equation hint, so that I had yet the equations"*.

Typically, students whose behaviour is in this category required repeatedly the same kind of hint in different tasks (mainly, the domain or curves equation hints) and used it only to overcome their temporary impasse. Indeed, their interaction within the digital environment and their answers to the survey showed that they did not take advantage of the required hint in the subsequent tasks and did not go beyond the task level of feedback. Moreover, their main focus seemed to be to find the correct solution to the tasks, rather than deepen their knowledge. One of them declared *"the hints allowed me to make the calculations quickly [...] the messages on the screen told me if I was wrong"*.

According to the students' answers, some different reasons induced them to ask for the hints: some students had the goal to check their answers before submitting them (*"to confirm my conclusions"*, *"they helped me to check that my equations were correct"*, *"they avoid calculation mistakes"*) or after the submission, to modify the given answer (*"the hints suggested me where I was wrong"*, *"the hints made me understand what I was wrong and then correct my answer"*). We can highlight a fairly superficial level of awareness by these students, who mainly recognize the sequence of tasks as assignments to be correctly performed, but not as an occasion to review the mathematical contents and reflect on them.

Finally, we identified a third category of students' behaviours (about 43%), who displayed of having grasped the scaffolding provided by the hints not only at the procedural level, but at the metacognitive one. During the interaction, typically these students ask different hints according to their specific needs and take advantage of the asked scaffolding element for the subsequent tasks. One student said *"I was in difficulty [...] I needed the hint showing me the equations of the curves to describe D as a normal domain [...]. In the next, thanks to the previous hint, I could recover by myself the equations of the curves. Subsequently, I asked for the geometrical hint [dynamic variation hint], since I could not visualize what the boundaries of the normal domain were"*.

From the answers of students whose behaviour is in this category, it arose that their focus during the interaction in the digital environment was not only on giving the correct answer to the tasks, but also on deepening the knowledge of the topic at stake. Indeed, they used words such as *"understand"*, *"clarify"*, *"reasoning"*, revealing their reflection of the provided scaffolding beyond the task to be performed (*"all the hints have been useful, because they helped me to clarify some concepts"*, *"they [the hints] helped me fully understand the exercise"*). Some of these students clearly linked the scaffolding provided within the digital environment with their general difficulties in learning this topic (*"the geometrical hint allowed me to understand something about my typical difficulties with problems of this kind"*).

The metacognitive level of the scaffolding emerged in many answers of these students, who seemed to project the received scaffolding on their future learning. Some of them declared *"the hints supported me in understanding what I should have found"*, *"the hints supported me in finding by myself the correct reasoning"*, *"the hint induced me to reflect on the given answer"*, *"I will put the same questions to myself in the future"*, *"they directed me towards the correct execution of the exercise and helped me to think"*, *"especially in the last exercise, they [the hints] allowed me to understand what effects the choice I made would have had on the calculation of an integral"*.

6. Conclusive discussion

In this paper, we investigated the use of computer-based scaffolding elements provided in a sequence of digital tasks by Engineering students. We designed teacher-driven scaffolding elements, that is immediate and response-specific facilitative feedback, and student-driven scaffolding elements, that is hints of different kinds. According to [10], we interpreted the scaffolding as a form of formative feedback, that students could grasp at the procedural or metacognitive levels (see [16]). In tune with Winstone et al. in [21], we wanted to explore whether and how students used the scaffolding elements provided within a sequence of digital tasks concerning planar integral domains. Moreover, we wanted to deepen the reasons preventing a fruitful use of these scaffolding elements, if this is the case. For students who took advantages of the provided scaffolding, we investigated the specific purpose inducing them to ask for hints and at what level (procedural or metacognitive, [16]) they interiorized the scaffolding.

The combined analysis of video recordings of the students' group interaction with the digital tasks and their individual answers to a survey concerning their use of the provided scaffolding elements allowed us to identify three categories of behaviours, corresponding to different levels of awareness and ways of grasping the scaffolding. This categorization gives answers to the research questions set in Section 2.

The obtained categories can be put in relation to the functions of a particular form of scaffolding described by Cusi and Telloni ([8]). The authors focused on the use of sliders in some GeoGebra applets, supporting the students' activity within the digital environment. Investigating the instrument-mediated action schemes emerging by students' use of these tools, the authors identified three different functions of the sliders: the *replacement function*, arising when students *"rely completely on the information provided by the digital tools, using them to find out the answers"*; the *diagnostic function*, arising *"when students use the tool to control the correctness of their answers [...], or to detect where the mistake is, and, possibly, how it to be corrected"*; finally, the *elaboration function*, arising *"when the students refer to the tool to deepen their understanding of the theoretical knowledge subtended to the task and their interpretation of the representations involved in it"* (see [8], p. 5).

The second category highlighted by our analysis, characterized by the students' grasping of scaffolding mainly at the procedural level, includes some behaviours linked to the *replacement function* of the scaffolding. They are the behaviours of students who rely on the hints to save time or to not make calculations. Other behaviours in the second category, characterized by the use of hints to check the correctness of the answers, are related to the *diagnostic function* of the scaffolding. Finally, the third category highlighted by our analysis is related to the *elaboration function* of the scaffolding, allowing a fruitful integration between the use of the hints and the deepening of the theoretical knowledge.

The results obtained from this study are expected to contribute theoretically to the design of effective scaffolding elements for digital tasks, as well as to enhance our understanding of the factors that enable or hinder their successful use by university students. Additionally, from an educational standpoint, the sequence of tasks could serve as a tool, acting as a non-human tutor that supports students in learning advanced mathematical concepts.

The outcomes of the study suggest trajectories for ongoing and future research. On the basis of the students' difficulties in fruitfully using the scaffolding elements and the motivations for this, we would like to exploit the potential of digital environments to eliminate those barriers preventing students from taking advantages of the scaffolding. In particular, to support students who do not know what kind of scaffolding could be useful with respect to their specific difficulties, we are planning to design digital tasks where the students in difficulties are guided by suitable stimulus questions to choose the correct hint or, alternatively, where the hints are presented one by one, from the less guiding to the most guiding. Moreover, to support students who are not able to translate the scaffolding into actions, some guiding messages could be designed, allowing them to take advantage step-by-step of the scaffolding element. In this way, another crucial educational goal, i.e. making the students sensitive to the importance of scaffolding and feedback, could be pursued. The awareness of both the levels at which each scaffolding element acts, i.e. the procedural one and the metacognitive one, is an essential element improving the students' learning, according to [21]. In particular, this should be a key strategy to progressively move the behaviour of the largest possibile number of students from the first and second categories before described to the third one.

We conclude by observing that the study presents some clear limitations. Firstly, the interaction with the digital tasks is on volunteer basis and not all students chose to participate. This could be linked to the context of the research, i.e. Engineering degree course, where Mathematics is a service subject. Often in this context Mathematics is viewed as a goal-oriented activity (see [6]) and students may be reluctant to engage in non-mandatory activities. In this perspective, we should foster the participation of more students in the future. Moreover, students' behaviour might have been influenced by factors not investigated in this study, such as potential technical difficulties in interacting with the digital tasks. Another critical issue could concern the pandemic context during which the study has been conducted, which could have afflicted the obtained results. Further theoretical research and experimentation are needed to understand the possible dependence of the obtained results by the context of the study and possibly to validate the obtained classification of the students' behaviours.

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