# A learning experience in and from practice using digital technologies: the mathematical discussion

**Antonella Montone**
*University of Bari Aldo Moro*
*Italy*
*antonella.montone@uniba.it*

**Michele Giuliano Fiorentino***
*University of Bari Aldo Moro*
*Italy*
*michele.fiorentino@uniba.it*

**Abstract.** In this contribution we report on an experimental research developed with Pre-Service primary Teachers (PSTs) in the Project Digimath, with the aim to give PSTs the opportunity to learn in and from the practice. For doing this we foster the Mathematical Discussion (MD) theoretical framework's learning (Bartolini Bussi, 1989). We explored the effectiveness of a new theoretical approach that combines Shulman's (1986) research about the Pedagogical Content Knowledge (PCK), Ball's Mathematical Knowledge for Teaching (MKT) (2008) and further research on how practice can be used for teachers' learning and what is relevant to learn in and from practice (Ball & Even, 2009), in a digital environment.

We noticed that to become aware of professional education and development of mathematics teachers and of specific characteristics of the teachers' actions, playing different roles (student - PST) is fundamental.

Our work was developed with 160 PSTs attending the Mathematics Education Course at the University of Bari. The MD has been at the same time the subject of the lecture and the methodology used during the lecture, both in a digital environment and personally. This choice has been done with two different aims: the construction of the MD's characteristics and the way to manage MD in the future professional practice. The experimental activities have been held using hybrid teaching and were videotaped. Conversations have been transcribed and have been used to analyze the teaching experiments.

As we expected, from the transcriptions' analysis, it emerged that the PTSs became aware of the MD's value. In and from their own practice they learned how to manage MD in class, in a future perspective.

**Keywords:** mathematical discussion, digital technologies, pre-service teachers.
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## 1. Introduction

This contribution aims to provide new insight on how to train PSTs, focusing on pedagogical models and their implementation in a class. Indeed. we asked

*. Corresponding author

ourselves how to train the prospective teacher on a specific pedagogical model, and in this contribution, we propose a new training mode, designed in order to introduce PSTs to the pedagogical model of Mathematical Discussion, both from a theoretical and practical point of view. In this study, we propose to let the PSTs experience the pedagogical model at stake in the role of student, then to involve them in a reflective situation guided by the trainer, consisting in a collective discussion focusing on the teacher's role previously played by the trainer. We call such a reflective discussion, Meta Discussion on a Pedagogical model (M-DPm). Our research has involved 160 PSTs attending the undergraduate 'mathematics teaching' course in primary education. Data analysis showed that the training activity carried out by experiencing in first person the pedagogical theories fostered the theory's conceptualisation itself and the future teaching profession's awareness. In the following, we outline the conceptual background, in particular the pedagogical model in focus, then we present the experimental design, the research methodology, the analysis of some of the data collected and the analysis' results.

## 2. Conceptual background

In last years, several research and theories emerged in the teacher's professional training area. Shulman (1986) claimed that to teach a discipline is necessary to tackle both the specific knowledge and pedagogical knowledge. The specific knowledge is not simply intended as a disconnected set of disciplinary knowledge. Indeed, in order to foster effective teaching-learning processes (e.g., a Pedagogical Content Knowledge, (PCK)), teachers need an integrated view of disciplinary content and issues related to its teaching. Moreover, for how concerns Mathematics, different conceptualisations and models of specialised mathematical knowledge for teaching have been proposed. Among them the Mathematical Knowledge for Teaching (MKT) model (Ball et al., 2008) or the more recent Mathematics Teacher's Specialised Knowledge (MKTS) model (Carrillo-Yanez et al., 2018). For the implementation of educational intervention Ball and Even (2009) provide us an interesting prospective that takes into account the complex articulation of different contents and their relationships.

By doing this they suggest focusing teacher training on practice – and on the problem of doing it effectively. From this perspective, on the one hand, it should be obvious that teaching is a practice and that, therefore, teacher education must provide systematic and lasting opportunities for teachers to learn and develop their effectiveness with that practice – with the complex tasks of their work. It follows the idea of training future teachers through practice and providing them an approach to a given task that can serve as a starting point for forming themselves as teachers through understanding and interpreting the actions of their future students. On the other hand, they highlight the lack of a shared articulation of a "curriculum" that would underlie teachers' opportunities to learn in and from their practice. Therefore increasingly necessary is

the development of ways to support teachers in learning in and from practice, making it a focus of discussion (Ball and Even, 2009).

Our main hypothesis is aligned with these general suggestions, and it concerns the effectiveness of a specific training mode to introduce PSTs to a definite Pedagogical Model (PM). This training mode is characterised by combining the living experience of such a PM and a reflective activity on it. In particular, we introduce a new construct which is the focus of our study that we named Meta Discussion on a Pedagogical model (M-DPm) which is described in the following section.

## 3. A new theoretical construct: the M-DPm

The M-DPm consists in a collective discussion which involves the PSTs and their instructor on the previous didactical experience of the specific pedagogical model of the mathematical discussion that they lived with the role of students (Fiorentino et al., 2023). In other words, such a reflective experience involves the same PSTs in a discussion, still conducted by their instructor, but with a different motive. That is of reflecting on their previous lived experience and identifying the key aspects characterising the pedagogical model that is at stake. A specific pedagogical model, has been selected in order to explore the proposed training mode, which is the Mathematical Discussion (MD).

MD was firstly elaborated by Bartolini Bussi (1998) and was originally defined as a 'polyphony of voices articulated about a mathematical object (concept, problem, procedure, etc.), which constitutes a motive of the teaching-learning activity'. In this model, the teacher has the responsibility of "orchestrating the polyphony", the voices (represented by the signs produced by the students) have to be coordinated with the voice of the mathematical culture (witnessed by the teacher herself)" (Bartolini Bussi and Mariotti, 2008, p. 763). The teacher plays two fundamental roles during a MD, namely those of mediator and moderator. To play this role, the teacher acts the main following actions: the "back to the task" action ( with the aim of reconstructing the context and fostering the (re)emergence of meanings and processes related to the task); "focalizing" action (with the aim of focus on aspects consistent with the didactic objective); "request of synthesis" action (with the aim of supporting students in the process of de-contextualisation and generalisation with respect to specific tasks); "offer of synthesis" action (with the aim of providing a formulation introducing the desired terms; ratifying the acceptability and mathematical status of a specific meaning). A mathematical discussion activity is differentiated according to its objective (motive).

Different types of discussion can be distinguished: the Balance Discussion (MDb) and to a Conceptualisation Discussion (MDc). MDb is defined as the collective process of informing, analysing and evaluating the individual solutions proposed to a given problem. MDc is understood as the collective process of constructing mathematical concepts, building up suitable connections between

already lived experiences and particular mathematical terms (Bartolini Bussi et al., 1995, pp. 11-12). Sometimes it may happen that a Balance Discussion naturally develops into a Conceptualization Discussion, and this is actually what happened in our experiment. Thus, in the following, for clarity reasons, we will simply use the acronym MD to refer to the combination of these two types of discussions.

Our hypothesis is that an effective training mode can be based on the combination of a didactical experience lived by PSTs as the students in participating to a MD (orchestrated by their instructor with the motive of conceptualising a specific mathematical content) and the reflective experience involving the same PSTs in a new discussion, still orchestrated by their instructor. This new experience has a new motive: reflecting on their previous lived experience, identifying the key theoretical aspects characterising the pedagogical model constituted by the MD.

In this new perspective the previous actions characterising the teacher's role, are redefined with new aims in the new construct:

- the "back to the experience" action ( with the aim of reconstructing the context and fostering the (re)emergence of characteristics and interactions, among students and with the instructor, related to the PM);

- the "meta-focusing" action (with the aim to make explicit the characterising elements that made it possible to recognise a MD in the experienced activity);

- the "request of synthesis" action (with the aim of supporting students in the process of de-contextualisation and generalisation with respect to the PM, in order to acquire awareness about its practice through distancing process);

- the "offer of synthesis" action (with the aim of providing a formulation introducing the PM's characteristics; ratifying the acceptability and mathematics teachers' role in a specific PM).

Within such a complex conceptual framework the specific hypothesis of this study can be synthesised as follows: Through the combined experience of MD and M-DPm, PSTs can be introduced to the pedagogical model of MD, and develop, in and from practice, both theoretical knowledge and practical experience of this model.

In this combined experience the PSTs shift their experience from living the experience themselves to observe it and recognize the characteristics of the PM, through the actions characterizing the teacher's role.

In summary, our research questions are: can the direct experience of implementing a mathematical discussion make PSTs aware of the functioning of this

teaching methodology/theory? How the combined experience, lived in first person by students, of MD and M-DPm could introduce the PSTs to the PM and to enact it? How technology could support the development of PSTs' training?

According with this hypothesis and with the research questions a teaching experiment has been designed, with the aim of exploring and deepening such hypothesis.

## 4. The role of technology

The role of technology is fundamental, as the activities are designed to be held online. In this project technology plays a major role, as it allows to support the processes involved in the teaching experiment in its three main functions (Albano et al., 2020): sending and viewing; processing and analysis of the data collected during the lessons; provide an interactive environment, where students can interact to work individually or in groups on a task or to explore mathematical/scientific content (Perry et al., 2021). The use of the digital resources in organizing the activities of the teaching experiment allows to solve the tasks in small online groups; to make a MDb all together to share the solutions and identify a common solution; to make a M-DPm on the MD; to customize both the types of action and the interaction among the PSTs (Fiorentino et al., 2022).

In detail, the environment technology is useful to stimulate collaborative processes and encourage the active participation of PSTs. In particular, it allows each participant to solve the problem individually and to send the solutions in a form shared first in a small group and subsequently, a shared solution in the large group; it encourages listening and reflection; it allows the participation of all students both in small groups (subdivision into rooms) and in large groups with screen sharing where necessary; it freed the students from the embarrassment of intervening; it allows video and audio recording of all discussions; it makes the materials immediately available. (Albano et al., 2020)

In the following section the research methodology will be described.

## 5. The research methodology and experimental setting

This paper focuses on a specific experimental path of a general project. We intend to foster the PSTS' professional development, in and from practice, of both practical experience and theoretical knowledge on a specific pedagogical model. For doing this, we focus on the design and the implementation of a training mode in which we involve the PSTs into two kinds of activities that develop at two different levels: the practical level where PSTs play the role of students and the instructor applies the pedagogical model at stake; the reflective level where, within the frame of the M-DPm, the PSTs reflect on their previous experience under the guidance of the instructor. From the huge amount of data collected during the experimentation, we consider the M-DPm transcript and the personal reflective diaries. The excerpts we present in the following sections, have been

chosen because they reveal evidence about effectiveness of the training mode and particularly of the construct of the M-DPm. According to the criteria of credibility, dependability, transferability and confirmability (Guba, 1981), to ensure trustworthiness a qualitative analysis of the transcriptions was carried out. The research involved 180 pre-service PSTs in mathematics, fourth-year students in Primary Education, and attending Mathematics Education Course. The training mode and in particular both MD and M-DPm take place synchronously on Microsoft Teams platform. Both the discussions take place with the whole class group. These discussions have been recorded and then transcribed for the analysis. In the following section, we describe the general setting of the specific experimental path in focus.

### 5.1 The structure of experimented training mode

The training mode is designed by a succession of steps identified by activities informed by our hypothesis on the educational relationship between experiencing a MD and participating in a M-DPm on such an experience.

Step 1: introduction of the MD theoretical model.

The instructor introduces the pedagogical model through a lecture, in which she gives the theoretical elements characterising the MD, in particular the teachers' actions to be performed to orchestrate the discussion.

Step 2: The open-ended problem and the MD (workshop online).

In order to activate a MD, it is necessary to start by solving an open-ended mathematics problem sufficiently elementary to be understood by anyone and included in school curricula. As a matter of fact, the use of an open-ended problem (Pehkonen, 1997) is crucial for triggering a rich and effective mathematical discussion: it is not solved mechanically, but requires different solving strategies, it is sufficiently challenging to make students engaged both cognitively and emotionally, as solvers of the problem. The authors selected the topic of equi-extension which is a complex one. It is generally based on the procedural activity of calculating the measure of equi-extended areas, and not on the idea of extension as a quantity related to equi-decomposability.

The following problem was proposed to be solved in groups, online, through Teams platform: "Two brothers receive a rectangular piece of land as inheritance. In order to divide it into two parts of the same size, one of them suggests planting a stake anywhere on the land and joining it to the four stakes driven into the four vertices of the rectangular land. One of the brothers will take two non-adjacent triangles, the other the remaining part. Are the two parts really equal? Justify your answer."

In the following activity, starting from the solutions of the above problem a MD was developed. The aim of the MD was bringing out all the problems' solutions strategies, converging towards a shared solution, and making explicit the equi-extension's concept. The instructor who orchestrated the discussion performed the actions expected from the Pedagogical model referring to the

patterns described by the MD pedagogical model. In so doing the authors intended to make the prospective teacher experience the effect of these actions as students. In this way, PSTs directly 'experience' the putting into practice of a DM as students and, at the same time, have the opportunity to observe the teacher's actions, recognizing the characteristics of the teacher's role in the practice of this PM.

By doing so they could also observe these actions as performed by the 'teacher'.

The technological tool is fundamental in this activity because the student, standing in front of the screen, can reflect on the development of the activity, have a relaxed time to elaborate answers and listen to the interventions of colleagues. Furthermore, the MD conducted online by the teacher is supported by the technological tool in the management of the intervention by all participants in the discussion and in the possibility of keeping all participants at the same distance.

Step 3: M-DPm on the previous MD (workshop online).

In this step PSTs are asked to reflect on the activity carried out in step 2, by participating to the M-DPm: identify the actions performed by the teacher and characterising the pedagogical model, that is the case of the MD. In this M-DPm Mathematics remains in the background, although it is not ignored by the participants. MD, as a pedagogical model and in particular the teacher' actions, become the subject of discussion. A key aspect for the PSTs' professional development lies in the transition from the role of student to that of teacher, projecting himself/herself in the actions performed by the instructor. Indeed, through the observation and the analyisis of the teacher's action, the PST live his/her future role by distancing himself/herself from the student's role.

Even in this phase, technology plays a fundamental role, because through the sharing of observations on the teacher's actions during the DM, the students implement a distancing from the experience lived in practice, recognizing and generalizing the characteristics of the theoretical construct.

## 6. Analysis of data with respect to the M-DPm

In this section we present the analysis concerning the M-DPm carried out in step 3. The following analysis, according to the main hypothesis inspiring the research study, attempts to highlight the effect of performing such a meta-discussion: how the M-DPm makes the characteristics of the pedagogical model MD emerge and are made explicit by the PSTs.

The transcript of the M-DPm was analysed with the twofold aim: highlighting the PSTs' recognition of MD's characteristics through the recognition of the teacher's actions; and at same time highlighting the importance of experience in practice of PSTs during MD. In the following we report the analysis of some excerpts consistent with this twofold aim.

(0:00) Instructor: What did we do in the last lesson [refers to the lesson in which the MD on solving the mathematical open-ended problem on equi-extension was carried out]. Why did I do this lesson with you? What was the teacher's aim?

(1:42) Stefania: we retrace everything that we did together, [...] so both an analysis from the point of view of Mathematical Discussion, as a pedagogical model and on the solutions of the problem [...] so the discussion's steps, the teacher's role who has to act as ... especially as a mediator within the discussion.

(2:20) Francesca: in my opinion, when the problem has been proposed to us, when we have been divided into groups and when we had to discuss firstly with our peers, and then with the teacher... for all this I suppose we did a Mathematical Discussion, [...] as a pedagogical model and on the solutions of the problem [...] in addition your role was especially that of mediator of the discussion

The instructor starts the M-DPm, with the aim to bring out the MD's characteristics. Indeed, she proposes to discuss the previous lesson in which the PSTs participated. This action carried out by the instructor is the new action: "Back to the experience". It is aimed at going back over the discussion experience in order to recognise the MD as a pedagogical model. The question is formulated with no longer reference to the problem to be solved by the instructor: "Why did I do this lesson with you?". The question is aimed to problematize the actions of the teacher in the attempt to make the characteristics of MD emerge. Stefania's answer shows the effectiveness of the instructor's intervention: she recognises and shares with the others her recognition of the MD pedagogical model in action. Following the question that induces and promotes shifting the discussion's focus from the mathematical content to the pedagogical model Francesca's answer shows a recognition of the MDPm's aim to make PSTs discuss about MDb-c. Through the Francesca's and Stefania's words to answer to the questions of the instructor, we highlight their recognition of the MD as a PM.

Their answers probably follows a question that induces shifting the discussion's focus from the mathematical content to the pedagogical model. Furthermore, they detect the specific role of the teacher, as mediator. The MDPm proceeds and the teacher invites Eleonora to speak.

(3:22) Instructor: OK, so let's see, Eleonora wanted to intervene.

(3:30) Eleonora: Yes, the first thing that came to my mind was that...ehmmm that mathematics can be discussed, precisely that through reasoning guided by you and... the various ideas emerged, the various hypotheses that we articulated to solve the problem.

Eleonora acknowledges that the activity in the previous lesson was a discussion and in particular a MD because the motive was that of making the "various ideas" emerge and be shared, and this is a characteristic of MD.

In our opinion, already emerges that having experienced in first person the previous MD on the mathematics problem, allows Eleonora, Stefania and Francesca to simpler recognize the PM's characteristics.

A clearer articulation of MD' characteristics is shown in the following excerpt.

(4:16) Instructor: Eleonora said we discussed Mathematics, but can you tell me what were the very elements that allowed you to recognise this activity as a discussion?

(4:45) Ornella: Another thing I'd like to add is that... you had another role, specifically that of moderator. In particular when you gave the opportunity to most of us to intervene during the discussion... you never stopped our speech or you never came to a conclusion and we also came to an uncertainty... Let's say you never said "this is the right solution", "this is another right solution" ... you replicated our words...

(6:22) Daniele: I agree with Ornella, in the last lesson we did a MD and, furthermore, I had the impression that ... that mathematics can be discussed, precisely that through reasoning guided by you teachers...the various ideas emerged...

Here, the instructor performs the "meta-focusing" action, which can be considered a key action for M-DPm, aiming to make explicit the characterising elements that made it possible to recognise a MD in the experienced activity. Ornella refers to the experience related to the concept of moderator as characteristic of the teacher's roles in a MD. From Ornella's words it emerges that, at the moment when the student talks about his/her solution, the teacher intervenes to let most of them intervene during the MD and she never says if the solution is correct/incorrect. Ornella also when she says "you replicated our words", highlight an her personal sign with which she recognize the action's teacher of replicating students' words which we can classify as the action of "mirroring" as a characteristic teacher's action of a MD.

Daniele acknowledges that the activity in the previous lesson was a MD; he also states that a discussion can evolve a mathematical meaning, which is a particular aspect that characterises MD.

(10:21) Instructor: So... synthesising... Daniele said we discussed Mathematics, Francesca said, the role of the teacher is seen to be one of mediator, Ornella added that it is also that of moderator... but can you tell me what were the very elements that allowed you to recognise this activity as a discussion?

(11:57) Rossana: Well, while Daniele was speaking and outlining what was his problem's solution [she refers to the MD], you made an explicit request as to why... you tried to rework that concept several times, so that it wasn't just clear to Daniele but to the whole class. You wanted to make sure that you led everyone on that path.

Here, the instructor makes an "offer of synthesis" action by explaining what emerged from the previous interventions. In particular, referring to Francesca's words, she recalls the teacher's role of mediator, which is another characteristic

of the MD model. In the first part of the intervention, Rossana makes explicit reference to her experience during MD, highlighting specific teacher's actions as mediator in always asking why. In the second part, Rossana recognises another mediator's action related to the teacher's concern about making all the students involved. Referring to the summary intervention and Rossana's answer, we can observe the evolution from direct reference to the lived experience - what was specifically done and said - to reference to the pedagogical model. It seems to us that Rossana shifts from the student's perspective when she refers to the problem's solutions, to the teacher's perspective, when she focuses on the teacher's questions aimed at involving all students in the shared solution.

As in the following intervention (31:55) by Stefania, the PSTs become increasingly aware of the teacher's role through distancing themselves from the lived situation.

(31:55) Stefania: So, in the MD the cognitive conflict emerged... there was a constant constructing and deconstructing of what we were saying, going to investigate why, through your questions "But I didn't understand... but are you sure? But can you make me understand? No, but I don't see it...". At one point when I was describing my solution, I distanced myself from what I was saying; that is, I realised that I was able to treat the subject, the solution... with more detachment; I realised that I was treating it with distance and therefore could be more aware and impartial in the discussion. Distancing oneself from oneself and cognitive conflict are the two fundamental aspects of mathematical discussion realised thanks to the teacher's mediation and moderation. Even now in this discussion, I realise that I distanced myself from the fact that I was involved in the discussion in the previous lesson, thanks to your questions and the interventions of my peers.

Stefania states that she experienced a situation in which it was possible for her, thanks to the teacher's interventions, to become aware of the MD pedagogical model, through a distancing process. Moreover, as she describes, she becomes aware of the teacher's role, that is how the teacher's guidance enabled a process of de-contextualisation of the solution with respect to the problem.

Furthermore, by the analysis of the personal diaries written by the students, has been possible to observe the significant role of technology in this activity. Moreover it is also possible to highlight the importance to have lived in first person the MD to make the PSTs aware of their future professional prospective. In the following sentences we report some expressions:

Daniele: What struck me about today's workshop is that each of us reasoned differently and being at a distance initially it was not possible to influence each other. I also had time to look for a solution, which I couldn't find initially. We identified proposals that referred to completely different ways of reasoning. Thanks to the initial discussion between us, without the teacher, we felt free to express our solution, without the fear of being evaluated.

Francesca: The language used in the discussion within the rooms and during the lesson in which we were all together on Teams was different. In the rooms

the discussion was between peers and therefore the language was less formal, in the discussion of May 6th however the language was more formal, especially when the professor claimed that a certain explanation had not convinced her and therefore, each of us, tried to explain more clearly, acting on language, what he wanted to express.... Without the fear of making mistakes!

Rossana: One thing that made me think a lot, even when the lesson was over, was the perception of serenity that was experienced during the discussion. I never expected that everything would be so orderly from a distance and despite the fact that most of us had never taken part in a discussion collective it seemed we were organized in a collaborative dimension perceiving the importance of exchange.

Eleonora: The online discussions made me understand how complicated it is to listen...listening is a serious practice that requires respect. Listening does not mean nodding in silence but understanding, leaving free space to offer it to what the other is communicating and keeping free space for oneself to connect with the other. This was possible because we were at a distance, each in our own space... Perhaps in presence we would not have had the time to reflect and listen that we had while at a distance.

From the reflections that the PSTs wrote freely in the diaries, the value of using technology to carry out the activity seems to emerge. Indeed, Daniele underlines the impossibility of influencing each others by distance, the possibility of having time to develop a solution and to express it freely without the fear of feeling judged. Furthermore, Francesca underlines the importance of discussion between peers, carried out online, without the presence of the teacher, using informal language typical of an environment of discussion among students. Francesca underlines that this language becomes formal with the presence of the teacher, moving from the perspective of a student to that of a future teacher. Rossana also reflects on the value of remote working, declaring that she perceived a serene and well-organised working climate, in which the exchange of opinions was important for a shared choice. Even Rossana, distancing herself from the lesson experienced in first person, observes an unexpected working climate by placing herself in the perspective of a teacher. Finally, Eleonora highlights a potential offered by the technological environment to be able to listen to the other and understand his opinion, "leaving free space" for reflection in order to share the one other's idea. All this, as Eleonora states, is possible because everyone worked remotely in "their own space".

## 7. Preliminary results and concluding remarks

From the analysis, it seems that our main hypothesis, implemented in the experimented training mode, has been corroborated. The new theoretical construct M-DPm, applied to the MD pedagogical model, made it possible for PSTs to reflect on previous lived experience, identifying the key theoretical aspects characterising the MD's pedagogical model. The instructor's interventions trigger the

recognition of theoretical aspects from the experienced situation, as it arose from the analysis of the transcripts; particularly the identification of the teacher's actions recognised through a detachment process that allows the PSTs to leave the student's role and assume the role of future teacher. Indeed, Eleonora in her intervention (03:30) recognises the MD as an appropriate pedagogical model that is constructed. In particular, the student identifies and makes it own the patterns of the model, recognises the effectiveness of the model with respect to its function and finally, distinguishes two specific functions: 1. solving the problem and constructing the mathematical concept; 2. recognising the nature of mathematics as something that can be discussed.

Furthermore, data analysis seems to show that involving PSTs in training activities could foster the theoretical conceptualisation and the practical implementation of a pedagogical model. This happens because PSTs can experience and reflect in first person and in so doing, they contribute to their effective professional development.

Moreover, also technology has been a fundamental role, since it allows students to collaborate in a stimulative environment, sometimes also without the judger presence of the teacher. Also the tools of the online platform revealed very useful to better understand each others during both the MD and the M-DPm.

In this research study, the implementation of the direct experience provides a specification of what kind of experience has to be lived (activities designed according to the pedagogical model) and how reflection can be promoted (M-DPm). The specific training mode presented here, proposes a specific interpretation of the suggestive expression "in and from practice": through the M-DPm, PSTs, after having personally experienced the teacher's actions as students, recognise them according to the specific pedagogical model.

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