Prospective mathematics teachers' subjectification processes: joint labor in a Moodle environment through Quick Chat plugin

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Abstract. In this paper, we describe a study involving master's degree students in mathematics taking the "Mathematics Education" course. The students, potential prospective teachers, worked in presence on an activity designed to make them reflect on teaching practices and methodologies. They first responded individually to an online questionnaire on misconceptions in mathematics. Then, in small groups, they discussed in the Moodle environment through multiple communication channels using the Quick Chat plugin. The plugin allowed the teacher to monitor all conversations in real time. We analyzed a specific context of practice characterized by the introduction of the Quick Chat plugin within the framework of Objectification Theory, resorting to the notions of joint labor and subjectification.

Keywords: computer-based environment, prospective teachers education, theory of objectification, joint labor and subjectification, Moodle.

MSC 2020: 97B50, 97C70, 97D40, 97M20, 97U50.

1. Introduction

In the last few years, many mathematics learning activities have been designed and implemented in online platforms on the basis of computer-based collaborative scripts. Such activities, called Vygotskian computer-based learning activities-VCBLAs ([1]; [2]), promote collaboration and discussion among students and foster online exchanges through multiple communication channels (e.g., [3]; [4]; [5]). For collaborating and discussing to lead to effective learning, it is necessary for the technological environment to be accessible to reduce the cognitive load for students to cognitively focus on the task ([6]). Furthermore, it

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is important to increase the degree of user acceptance of the environment itself and hence the usability ([7]). A VCBLA is typically made up of a narrative flow that guides the learning activity, that is, the task, and a set of collaborative tools (e.g., communication channels). Within a VCBLA, students work in small groups and communicate at various levels (peer-to-peer, whole class, and/or with an expert/teacher) through chats. The most common way to implement a VCBLA is with Moodle, which has all of the tools needed to implement both the narrative flow and all of the communication channels. However, when many collaborative tools are needed at the same time, Moodle has some limitations. Users are forced to open multiple browser windows or tabs: one for the flow of the narrative, and one for each chat or additional tool. The problem is not only for the students, but also for the teacher, who may have a hard time keeping track of all the conversations in real time.

The Quick Chat plugin was developed as part of the LIME (Learning Interface for Mathematics Education) project at the University of Campania "L. Vanvitelli" to improve the user experience and make the Moodle environment more accessible. The goal is to overcome the accessibility and usability limitations of the Moodle platform by being able to group both the main course page and all the chats the user is a part of according to their role within the script into a single browser tab. Once installed, the plugin is available from the navigation block on the course home page. It allows students and teachers to manage all the tools and activities provided for a VCBLA, i.e. the task and all the communication channels in which each user (student or teacher) is involved, from a single browser page. Dello Iacono and Fiorentino ([8]; [9]) showed the effectiveness of the plugin in promoting student interactions in the university setting. In particular, they showed that students in all groups interacted using all communication channels, even in parallel, and that the teacher was able to monitor all conversations to intervene effectively in real time.

The aim of this study is to envision teacher training beyond professional development marked by subject matter and pedagogical content knowledge ([24]) whose importance we do not underestimate. We would like to enlarge the approach to teacher training taking into account the production of new identities as individuals establish a reflexive relationship with the sociocultural context of mathematics teaching and learning. This means that in their training trajectory prospective teachers in addition to being affected by their cultural-historical context they react agentically to such a context thereby producing a new identity as a mathematics teacher. Their professional development is related to the production and transformation of subjectivities and to the continuous co-production of singular and unique individuals against the backdrop of their cultural-historical domain. It allows prospective teachers to critically position themselves in the cultural-historical domain of mathematics teaching-learning practices, realizing new ways of thought and action. In this study we continue in the line of Ferretti and Santi ([11]) looking at the role of Quick chat plugin in molding the structure of reflexive activity that accomplishes the relationship between individuals and

the sociocultural context. In light of the TO, the sociocultural context is seen as symbolic superstructures called *semiotic systems of cultural signification*that include cultural conceptions surrounding mathematical objects (their nature, their way of existing, their relation to the concrete world, etc.) and social patterns of meaning production (beliefs about conceptual objects, conceptions about truths, methods of inquiry). In regard to teacher training we have singled out the following: epistemology and ontology of mathematics; systems of truths; forms of rationality; accepted mathematical teaching practices problems and situations; conception of teachers and students; socio-mathematical norms; conceptions about education; language and languages; assessment of learning.

In this paper we describe a study involving 20 undergraduate mathematics students taking a course in "Mathematics Education". The aim of our research is to scrutinize how mathematics education courses for prospective teachers are affected in a mixed learning environment (in person and online) that makes use of the Quick Chat plugin. According to a Vygotskian perspective ([12]), we analyzed the potential of the Quick Chat plugin within the framework of the Theory of Objectification, resorting to the notions of joint labor and subjectification ([13]; [14]; [15]; [16]). Activity is defined by the Theory of Objectification as *joint labor* and it is a dynamic system where individuals produce knowledge and, in doing so, co-produce themselves in processes of *subjectification*. Our research questions are as follows. In a prospective teacher education environment,

- (RQ1) how does the Quick Chat plugin mold the dynamics and the development of joint labor?
- (RQ2) with respect to which cultural symbolic superstructures do prospective teachers position themselves in subjectification processes?

In the following sections we describe: the theoretical framework underlying our work, the research method, and the analysis of the data derived from the study. The paper ends with a section devoted to a discussion of the results and conclusions.

2. Theoretical framework

Asenova ([17]) describes two possible lines in Mathematics Education Research. Research dealing with Topic-Specific (TS) issues that produces usually local results is less broad than research projects dealing with Context-Specific (CS) issues that have more general and abstract scopes. Our study is TS research, in that it provides local results regarding the use of technology in teacher training within the lines of investigation accomplished by the DIGIMATH group that aims at context specific results regarding teacher training with a more extensive impact on this field of research. The DIGiMATH group has identified at least two approaches to using technology for mathematics teaching/learning: to mediate mathematical knowledge, and to foster communication and interaction among students during mathematical practice (e.g., [18]). VCBLAs based

on the Quick Chat plugin implementation can be analyzed using both of these approaches. In this study, we focus on the second approach to the use of technology. In order to deepen the meaning of the activities that take place in VCBLA and according to a Vygotskian perspective, we analyze this specific context of practice characterized by the introduction of digital technology, that is, the Quick Chat's potential, within the framework of Objectification Theory, resorting to the notions of *joint labor* and *subjectification*.

Joint labor is one of the pillars of Objectification Theory in which, continuing along the line of Activity Theory ([19]), mathematical knowledge, thinking and learning are the product of the individual-culture-activity dialectic. Activity is not a series of actions (*Aktivität*) but a *Tätigkeit*, that is, a dynamic system where individuals produce knowledge. Activity is thus a form of life. It is a system ([19]) in continuous development, always influenced by the entities to which it relates. Activity as Tätigkeit is defined by the Theory of Objectification as *joint labor*. The notion of *joint labor* allows us to explore the social interactions of individuals engaged in knowledge production in the context of VCBLA. Radford ([13]) states that teachers and students engage in the activity of knowledge production together. Knowledge is produced collectively in *joint labor*. This means that students and teachers work together to propose possible mathematical interpretations related to forms of human action and collaboration shaped by cultural signs and artefacts. These are modes of mathematical enquiry, truth conception, proofs, mathematical argumentation, the use of symbols and the creation of meanings. In the socio-cultural texture of *joint labor*, individuals not only produce knowledge but also co-produce their own identity through processes of *subjectification*. Radford ([14], [15]) defines *subjectification* as the processes in which individuals produce their identity in dialectical interaction with the cultural-historical context by positioning themselves in their social and cultural-historical environment. In the dialogue with the cultural context, through reflexive and mediated activities, the individual is understood as an entity in constant flux, a life project always in progress ([16]). The pillars of the TO are rooted in the dialectical relationship between joint labor and the use of signs and artifacts. In fact, activity bridges the individual to their sociocultural dimension in terms of objectification of knowledge and the ensuing production of subjectivity as the positioning in such a cultural dimension. Subjectification and objectification are two sides of the same coin in our agentic and reflexive encounter with culture as knowing and becoming. As regards the shift of the sociocultural perspective from the realm of the teaching and learning of mathematics to teacher training that we are presenting in this paper, knowing embraces subject and pedagogical content knowledge, becoming involves a positioning with respect to semiotic systems characterizing the school sociocultural environment. What is my positioning towards epistemological stances in mathematics, Platonist or pragmatic? How do I conceive learning in mathematics? How do I use assessment in relation to my conception of the student? How do I use language and languages? What forms of rationality do I accept? In the

subjectification processes teachers express their agentic reaction to the school culture realizing ways of thought and action thereby co-producing in joint labor unique and singular identity.

3. Method

The study reported in this paper involved 20 students in the second year of the master's degree in mathematics (hereafter referred to as participants) attending the Mathematics Education course at the University of Campania "L. Vanvitelli". The study was carried out during a lecture in that course and participants were informed about the conduct of the study. The participants - potential prospective teachers - worked in presence on a learning activity designed to make them reflect on teaching practices and methodologies. The methodology was informed by Educational Design Research in which the iterative development of solutions to practical and complex educational problems provides the setting for scientific inquiry ([20]).

The activity involved 4 phases. In phase 1, the participants responded individually to a questionnaire designed to investigate typical misconceptions in mathematics, administered as Google Form. Figure 1 shows some of the questions in the initial questionnaire. The first two questions are related to radicals in the real number field. The third question, taken from the work of Tall and Vinner [21], refers to typical misconceptions regarding tangents at a point to the graph of a function.

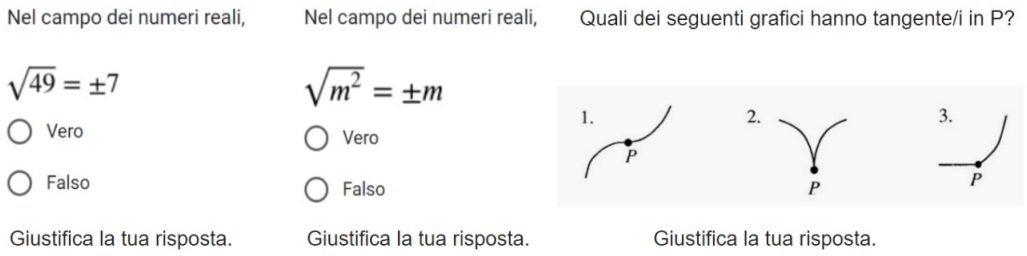

Figure 1: Some questions from the initial questionnaire

[In the field of real numbers; True, False, Justify your answer; In the field of real numbers; True, False, Justify your answer; Which of the following graphs have tangent(s) in P? Justify your answer].

In phase 2, the participants worked in small random groups (3 participants for each group except for one group consisting of 2 participants) to confront each other in the Moodle environment and to communicate through a group chat. The aim of this phase is for participants to arrive at common and shared answers. Each participant worked on their own laptop. The groups were created by the teacher so that the participants in the same group were physically far

away from each other in the classroom so that they could communicate with their group mates only using Moodle chat.

In phase 3, the responses of each group agreed upon in phase 2 were reported by each group's spokesperson in a General chat. In this chat a discussion started mediated by the teacher. This phase aims to foster a discussion among participants in all groups. The Quick Chat plugin allowed participants to view assignments, Group and General chats on the same webpage. In addition, the teacher had real-time monitoring of all conversations through the plugin (Figure 2). At the end of the activity, the teacher facilitated a class discussion.

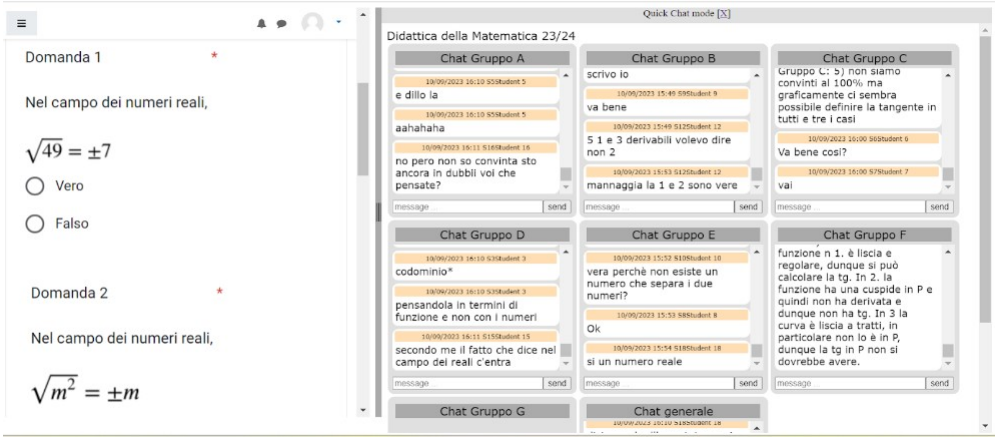

Figure 2: Teacher view of the browser page

In phase 4, an oral discussion (no longer in the Moodle environment) mediated by the lecturer took place.

At the end of the "Mathematics Education" course, participants – prospective teachers – were given a metacognitive task through a Google Form, designed to make them reflect on the whole course and make them relive the most significant moments of the course: *Review the path we took together. Dwell on the moments/activities that were most significant to you and indicate the reasons why these moments/activities were significant.* We collected the following data:

- responses to the initial questionnaire through Google Form;
- discussions in the Group and General chats using the Quick Chat plugin in the Moodle environment;
- the responses to the final metacognitive task using Google Form.

4. Data analysis

In this section, we analyze the collected data from the study in accordance with the theoretical framework described above. In particular, we analyzed discus-

sions in the Group and General chats and the responses to the final metacognitive task. Below, we show a brief excerpt showing a joint labor part related to the use of Quick Chat in the exchange between members of a group that addressed the first two questions of the questionnaire. Specifically, we show the intense relationship between participant P5 and participant P16. We selected the dialogue between P16 and P5 because they are confronted with two meanings of the square root: as an operator to find the number that when squared results in the radicand, and as a real number. In reference to the list of semiotic systems of cultural signification mentioned above, the students' objectification of the notion of square root is entangled with their co-positioning with respect to epistemology and ontology of mathematics, systems of truths and forms of rationality:

> P16: My motivation is that the root of a negative number in R does not exist which by the way is the same answer I also gave to the second question
>
> P5: however to the second one you could put a negative number because there is the square
>
> P16: and then the first one is also a square
>
> P5: yes but of 7 times 7
>
> P16 could be also - 7 squared
>
> P5 that is, yes also -7
>
> P16 but the condition should be that the number under root must be positive and not zero
>
> P5 so what shall we say
>
> P16 so let's say both are true, the first since 49 we can write it as 7 to the quad...
>
> P5 First question: the root is a positive value so the answer is unique, that is 7
>
> P16 is the answer I also gave earlier however now I have got the doubt [...] i.e. what is true for the first is not also true for the second?
>
> P5 First question: the answer is false the root is a positive value so the answer is unique, that is 7
>
> P16 i.e. what is true for the first is not also true for the second?
>
> ...

> P5 First question: the answer is false the root is a positive value so the answer is unique, that is 7. Second question: the answer is true the root is a positive value so the answer is - or + m.

From the excerpt above, we observe a co-positioning of participants with respect to mathematical culture, particularly of the meaning of radicals in the real number field. P16 and P5 are confronted with two meanings of the square root: as an operator to find the number that when squared results in the radical and as a real number. Their co-positioning in *joint labor* expresses the tensions, contradictions, continuous interpretations and reinterpretations that characterize symbolic superstructures and, in this case, the duality between process and object ([22]) that characterizes mathematical concepts. The small group discussion has not overcome the tension between process and object, and the *joint labor* maintains its dynamic and fluid nature in keeping the *subjectification* process open. Indeed, the group concludes that in the first question the answer is false because the root is a positive value so the answer is unique, that is, 7; in the second question the answer is true because the root is a positive value so the answer is - or + m. The contradiction will be overcome in the broader structure of *joint labor* centered on the use of Quick Chat, which systematizes group activity in chat, general discussion also in chat, and in person activities.

As mentioned in the previous section, at the end of the entire course, a final questionnaire was proposed with the following question: Review the path we took together. Dwell on the moments/activities that were most significant to you and indicate the reasons why these moments/activities were significant. The participants' statements show the processes of *subjectification* in which they establish themselves not only as mathematical subjects in mathematical practice but also as mathematics teachers in the training activity of the "Mathematics Education" course. An example is the following response from a participant:

I found the course very interesting because it completely changed my thinking about how to teach math in school. I realized the importance of creating activities and methods to derive mathematical definitions and theorems. The activity on misconceptions was crucial in allowing me to understand what "mistakes" students can be led to make because of explanations that use particular examples. I found the initial activity on grading students and the mistakes we future teachers might make in grading very interesting. It is important to give weight to the process and not to the outcome of the problem. In this course, I also found the group activities very interesting, which then later enabled me to understand how groups should be formed in the classroom. Creating seminars is important for our personal growth and for trying to impart knowledge to our peers. This course really allowed the learning of fundamental notions for us aspiring teachers, and I think we would also need its continuation in order to constantly improve. One thing that I think is very positive is that the lecturer was always available for discussion on all our ideas even if sometimes wrong.

We observe how the symbolic superstructures against which prospective teachers' co-positionings occur are broader and more specific than those with which students are usually confronted: mathematical objects and ways of representing them ("my thinking about how to teach math in school"), systems of truth, forms of rationality; misconceptions and obstacles ("misconceptions was crucial in allowing me to understand what "mistakes" students can be led to make because of explanations that use particular examples"), cognition and metacognition, teaching methodologies ("I also found the group activities very interesting, which then later enabled me to understand how groups should be formed in the classroom"), forms of action and behavior, conceptions of teaching and learning ("It is important to give weight to the process and not to the outcome of the problem"), assessment, affect, communication ("One thing that I think is very positive is that the lecturer was always available for discussion").

5. Discussion and conclusions

The educational problem of mathematics is to provide opportunities for students to encounter mathematical knowledge in rich and meaningful ways, to enter into conversation with culture, and to critically co-position themselves toward mathematical knowledge in and through its practice. Co-positioning refers to the conscious movement of students in knowledge production activities. It refers to the way in which students establish themselves as subjects in the practice of mathematics but, in this context, as future mathematics teachers in training activities. In this paper, we asked how Quick Chat plugin shapes the dynamics and development of *joint labor* in a teacher professional development environment (RQ1), and with respect to which cultural symbolic superstructures future teachers position themselves in *subjectification* processes (RQ2). Concerning RQ1, the Quick Chat plugin fosters the emergence of dynamic and fluid systems of activity that, in the interaction between online and in presence activities, allow for new forms of *joint labor* and the resulting co-positionings of prospective teachers toward mathematics and mathematics teaching/learning. The teacher monitors in real time the performance of the task by the various groups and can intervene in the discussion of individual groups. She also monitors and intervenes in the general discussion that takes place simultaneously with the activity of the individual groups. The student can switch from Group Chat to General Chat when she feels it is appropriate for the performance of the task, both in terms of the mathematical aspects and the emotional and collaborative support offered by the *joint labor*. Student communication and interactions occur at multiple levels and in parallel: at the Group and General chat levels. We also note that in the interaction between oral communication and online activities via Quick Chat, students activate the oral verbal channel during the general oral discussion (in phase 4) and the written channel in Quick Chat communication (in phases 2 and 3). This dual channel can have interesting effects both at the level of objectification of knowledge and *subjectification*. The

introduction of Quick Chat modifies the classroom activity schema introduced by the Theory of Objectification, intensifying the systemic, fluidity and dynamic aspects typical of *joint labor*. Indeed, the relationships between the elements that make up the scheme of activity, take on a higher level of complexity in spatial and temporal terms (see Figure 3). The left side of Figure 3 shows the classroom activity according to the TO based on in presence activity based on subsequent general discussion, group work and group work with the teacher. The right side of Figure 3 shows the enlarged structure of the Classroom activity resulting from the implementation of Quick Chat plugin that blends in a synchronous mode and in presence different channels of communication and activity with a general control on the part of the teacher (on the different chats, via oral discussion, in the big group or in subgroups).

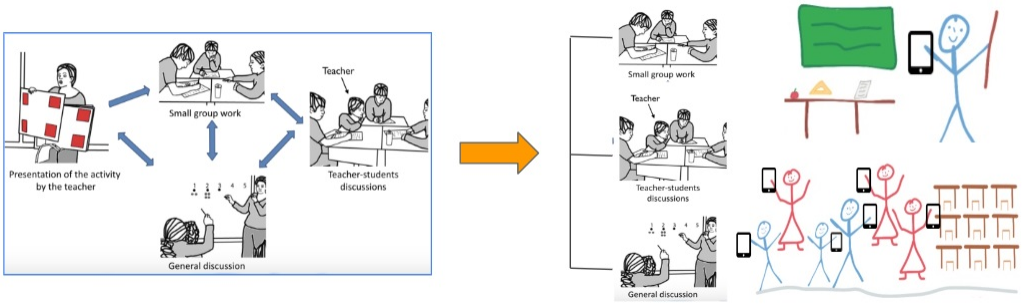

Figure 3: Classroom activity schema modified by Quick Chat plugin

The Quick Chat plugin changes the socio-cultural structure in which teaching-learning processes take place and the specific ways in which knowledge is produced. In this context, the focus shifts from forms of collaboration between teachers and learners in developmental age to those between trainer and trainee teachers. Likewise, the focus shifts from the production of mathematical knowledge to the production of the specialized knowledge of the mathematics teacher ([23]). In the context of teacher education, *subjectification* processes thus enable prospective teachers to position themselves in the cultural and social context of the school characterized by its symbolic superstructures: typically conceptions of the student, conceptions of teaching, conceptions of mathematics related to teaching-learning processes etc. ([11]).

Concerning RQ2, in our study, the effect of the broadened structure and dynamics of *joint labor*, accomplished by the introduction of Quick Chat, was mainly on the prospective teachers' positioning with respect to mathematics and its teaching-learning processes. The individual work on the questionnaire and the subsequent discussion on Quick Chat characterized by the back and forth between the group chat and the general chat triggered a new understanding of the mathematical topics with a more metacognitive outlook that in turn fostered their positioning with respect to the teaching and learning of mathematics as

derives from the answers to the question of the Google Form: *The activity on misconceptions was crucial in allowing me to understand what "mistakes" students can be led to make because of explanations that use particular examples; It is important to give weight to the process and not to the outcome of the problem; In this course, I also found the group activities very interesting, which then later enabled me to understand how groups should be formed in the classroom; This course really allowed the learning of fundamental notions for us aspiring teachers.*

This paper presents a new approach to teacher training that enlarges the understanding of professional development usually marked by subject matter and pedagogical content knowledge ([24]). We take into account the production of new identities as individuals establish a reflexive relationship with the sociocultural context of mathematics teaching and learning. This means that in their training trajectory prospective teachers in addition to being affected by their cultural-historical context they react agentically to such a context thereby producing a new identity as a mathematics teacher. Their professional development is related to the production and transformation of subjectivities and to the continuous co-production of singular and unique individuals against the backdrop of their cultural-historical domain. It allows prospective teachers to critically position themselves in the cultural-historical domain of mathematics teaching-learning practices, realizing new ways of thought and action. We have analyzed the role of Quick Chat plugin in molding the structure of reflexive activity that accomplishes objectification and subjectification processes. The development of our research can be implemented in teacher's professional development programs where Quick Chat acts as a tool for teacher collaboration in training activities that can be implemented as a tool from teacher collaboration in classroom teaching-learning activities ([25]). A similar approach based on information and communication technology has been implemented in mathematics education courses for prospective primary school teachers at the faculties of Education ([11]).

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