Design and use of learning graphs through ASYMPTOTE system

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Abstract. This study investigates the integration of the ASYMPTOTE system into a Mathematics course for pre-service teachers. In particular, the study focuses on pre-service teachers' task design and its impact on their understanding of course topics and exam performance. Pre-service teachers embraced the task design process, producing more tasks than anticipated. They recognized the value of task design for comprehensive topic review and deepening understanding. Collaborative efforts between instructors and students were crucial, resulting in a significant portion of participants passing exams. Initial data show that this direct involvement not only enriches the pre-service teachers' mathematical knowledge but also promises to strengthen their future teaching skills.

Keywords: ASYMPTOTE, pre-service teachers, task design.

MSC 2020: 97B50, 97C70, 97D40, 97D80, 97N80, 97U70.

1. Introduction and theoretical background

It is documented that many pre-service primary school teachers experience difficulties with mathematics, compromising the quality of teaching (e.g. Buxton, 1981; Coppola et al., 2012; Di Martino & Sabena, 2011). To effectively support pre-service teachers (henceforth PT) in overcoming these challenges, one promising approach is integrating the teaching and learning of mathematics with technology (Niess, 2005; Thomas & Hong, 2013). In fact, this combination can offer innovative and accessible methods for enhancing their understanding and delivery of mathematical concepts (Goos & Bennison, 2008). For example, by using mobile phones, students' assimilation of knowledge becomes more accessible, because learning flexibility is fostered, i.e. studying can be done anywhere and anytime (Crompton et al., 2017). Recognising their advantages, students understand that their mobile devices play a crucial role in simplifying and facilitating the teaching-learning process, making it more enjoyable and stimulating. However, using these technological tools in education must be guided by appropriate and well-defined strategies (Mehdi et al., 2020). In this study, the ASYMPTOTE system (https://www.asymptote-project.eu/en/welcome/), which is a modern tool for teaching and learning mathematics remotely from primary education to university level, was considered as a technology to be used to support the study and examination preparation of PT.

From March 2021 to February 2023, the ASYMPTOTE project, which stands for Adaptive Synchronous Mathematics Learning Paths for Online Teaching in Europe, developed and improved the digital system of the same name. It consists of two components, a web portal for teachers and a smartphone application for students (Barlovits et al., 2022). In the web portal, teachers can create tasks or search and choose them from those available in a rich task repository. Tasks must include basic information: a title, the delivery, information about the author who created them; and core information: answer format, sample solution, some hints. The answer format can be: "Exact value", "Interval", "Multiple choice", "Fill in the blanks", "Exact value vector", "Interval vector", "Sets", "Fractions", "Matrices", and "Information station". Teachers can then group the tasks identified in a sequence with multiple levels of difficulty (by specific topic, but not necessarily) generating what is called a Learning Graph (LG): a veritable succession of nodes (the tasks) and arcs (how one moves from one task to the next). Barlovits and colleagues (2022) define a LG as follow: it "is defined as a directed graph, where each vertex represents a [...] task, based on a learning trajectory as the intended and expected way of learning" (p. 14).

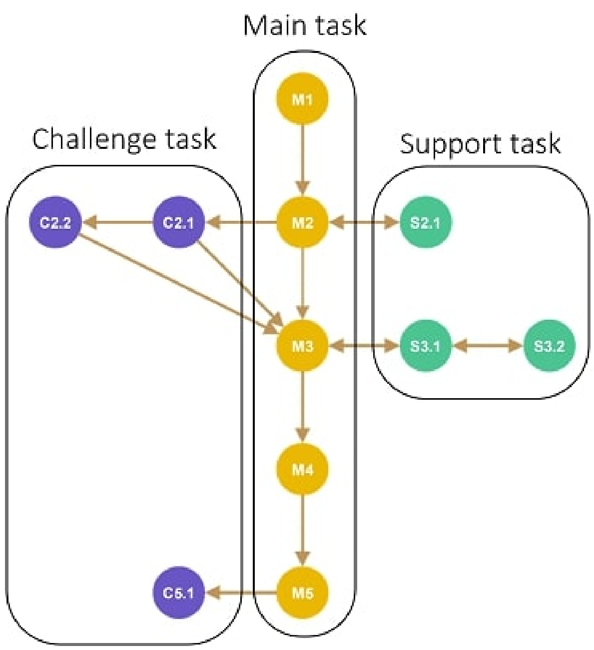

Figure 1: A generic LG

In particular, a LG consists of a sequence of tasks of different typology (Figure 1): the main tasks (yellow), which are mandatory and positioned to form the backbone of the LG; support tasks (green), positioned to the right of a main task and optional, are at a simpler level and allow support to be given to tackle the corresponding main task; challenge tasks (purple), positioned to the left of a main task and optional, are at a higher level and are designed to challenge learners to solve the corresponding main task straight away or to allow them to explore the topic in more depth. This structure ensures that the entire learning process is self-guided and autonomous (Barlovits et al., 2022).

The students work on the learning graph via the smartphone app: they solve tasks having an immediate validation of the solutions inserted, they get feedback synchronously and systematically and, depending on how well they solve the task, they are directed to the next task, which can be of a simpler, more advanced or equal level to the previous task performed.

This paper will present an exploratory study on the use of the ASYMPTOTE system in the Mathematics for Primary and Pre-school course of the Master of Education degree course at Kore University of Enna (UKE), Italy. The context in which the study was conducted will be presented and the results of how the primary PT carried out the task design and made subsequent use of it will be illustrated. The aim is to share reflections about their experience in relation to the following research questions:

i) What personal perceptions of the PT emerge in the task design process with ASYMPTOTE, in terms of ease and usefulness, and in the enjoyment of the LG created?

ii) What impact does such task design work with ASYMPTOTE have on the examination performance of the PT?

2. Methods

2.1 Research condition and process

At UKE, the Master of Education degree course requires PT to attend third-year lessons in Mathematics for Primary and Pre-school (taught by the paper's author). From the first day of class, in the academic year 2023-2024, I explained the ASYMPTOTE system and proposed to the PT to engage in the design of tasks that would form an LG. The course proceeded by macro-topics: Set Theory, Arithmetic, Geometry, Probability and Statistics. For each macro-topic, the theoretical part is accompanied by a series of exercises. Some are carried out in the classroom: generally, work is done between peers and then a volunteer, at the blackboard, shows the class the exercise. I ask to argue and justify the steps and, if necessary, ask the class if other solving strategies have been considered. Other exercises are left for home. These could be considered by the PT to conceive a task for the LG, taking them as a cue or considering them verbatim. Figure 2 represents the Excel file that was shared with PT. In each line, the topic of the lesson is listed in first column; while the second column shows possible sub-topics for which LG tasks can be designed. The typologies to which each task can belong are the next three columns, coloured purple for challenge tasks, yellow for main tasks and green for support tasks. The PT had to work in groups (min 3 - max 5 people), on the design of specific tasks listed in the table. They had to choose the task to be designed and its typology by writing their names in the cells of the Excel file (the names of colleagues were public for the students; for privacy reasons, they have been blacked out in Figure 2). For example, for the Arithmetic macro-topic, among the proposed topics was 'Writing numbers and numbering systems' - cell A8 in Figure 2. There, I specified two possible sub-topics for generating the task, 'Writing numbers in positional notation' - cell B8 of Figure 2; and 'Basic conversions' - cell B9 of Figure 2. Most of the groups booked to create a yellow task. If others had already booked this, there were two ways: either a group chose another topic and booked for a yellow task or, if they particularly liked the topic or felt comfortable with it, they could propose a green or purple task, comparing themselves with colleagues who had booked the yellow one. Some groups sometimes asked me for support in identifying the task to be designed. In that case, I suggested that they design a specific type of task according to the current presence of yellow/green/purple tasks, to generate an LG as balanced as possible.

	A	B	C	D	E
4	**TASK PER IL LEARNING GRAPH**				
5					
6	**ARITMETICA** (15 task in tutto di cui **6 task principali**)				
7					
8	**Lezione 5 (scrittura dei numeri e sistemi di nuemrazione)**	Scrittura dei numeri in notazione posizionale		[redacted] (DEFINITIVO)_I Sistemi di numerazione e la scrittura dei numeri	
9		Conversioni di base	[redacted] (DEFINITIVO)_Aiutiamo Claudio!		[redacted] (DEFINITIVO)_Conversione dei numeri tra basi differenti
10					
11	**Lezione 6 (i numeri naturali e le operazioni; espressioni; potenze)**	Esercizio 2 (espressioni)	[redacted] (DEFINITIVO)_Espressioni di sfida	[redacted] (DEFINITIVO)_Alla scoperta delle espressioni	[redacted] (DEFINITIVO)_Esercitiamoci con le espressioni
12		Esercizio 3 (errori nella risoluzione delle espressioni)			
13		Esercizio 4 (dal linguaggio naturale all'espressione)		[redacted] (DEFINITIVO)_Espressioni con i numeri naturali	
14		Esercizio 5 (Zio Paperone - come sfida)	[redacted] (DEFINITIVO)_Zio Paperone		
15		Motliplicare e dividere per 100 (come supporto, prendendo spunto dagli esercizi 6, 7, 8, 9, 10, 11)			[redacted] (DEFINITIVO)_Moltiplicare e dividere per potenze di 10

Figure 2: Excel file in which PT choose the topic and the typology of the task they decided to design

The PT did not work on the ASYMPTOTE web portal: I provided them with a design sheet created on a Word file that exactly traced the entries of the task creation template in ASYMPTOTE. Once the task had been designed, the PT's group sent this sheet by email to me and I reviewed it. Once the task was deemed suitable (it took between 1 and 3 revision rounds), I digitized the task on the ASYMPTOTE web portal and created the LG according to the choices made on the Excel file. I then delivered the code associated with the LG to the PT so that they could use it on their smartphones via the app. In order to foster the PT's involvement in this design work, I stated that I would award an extra point in the exam (it consists of a written test) to all those who would develop a task for the LG.

2.2 Participants and data collection

In the academic year 2023-2024, there were 300 PT enrolled in the third year of the Master of Education degree course. 156 PT have attended my Mathematics for Primary and Pre-school course at least once. 83 PT have voluntarily decided to engage in the design of at least one LG task. For the data collection, the research followed a mixed quantitative and qualitative approach. More specifically, at the end of the lessons and after the first exam call, I administered a questionnaire to investigate the satisfaction index of both the design activity carried out and the user experience with the LGs. The questionnaire consisted of multiple-choice, open-ended and Likert scale questions. It was administered via Google Form. It is available, in Italian, at the following link: https://lc.cx/iX9B4d. The data were analyzed using Excel. In particular, inductive coding (Thomas, 2006) was used for the analysis of open-ended questions. The questionnaire was answered by both students who worked on the design and students who only enjoyed the LGs ([1]). In the following, I will focus on the data collected among the students involved in the task design work and who then answered the questionnaire.

3. Results

3.1 Observations drawn from the task creation and review process

The interest shown by the PT in task design for the LG is evidenced by the fact that they produced 50 tasks in all (out of 83 PT, 19 groups were formed and each produced between 1 and 5 tasks). As anticipated, it took between 1 and 3 rounds of review before a task was deemed suitable. The most common errors were: calculation errors or improper use of mathematical symbols; terminology errors (e.g. showing only a few numerical explorations cannot be considered a proof). At other times, it was necessary to better direct the PT to a different response format than the one they had thought of (e.g. to study the proof of a theorem, the PT had thought of using the response format "Information station", the purpose of which is only to read the content of a task. Instead, it was suggested that they use the "Fill in the blanks" response format, which

1. The questionnaire consists of several parts and the questions related to task design are only made available to those who answer *Yes* to the question asking: *'Did you contribute to the creation of the Learning Graphs, i.e. did you design at least one task?*. Those who answer *No* are automatically sent to the questions relevant to the use of the LGs.

works exactly like a cloze, i.e. there are blanks in the steps of the proof and it is up to the user to work out how to fill them in).

At other times, more support was needed in the formulation of hints. In the latter case, in almost all cases, I formulated the hints instead of the PT. The formulation of appropriate hints that guide and support, without suggesting, reasoning towards a resolution strategy is a non-trivial practice and it is necessary to have a deep knowledge of the topics in order to be able to adequately support the resolution processes (Perrenet & Groen, 1993). During the course, I worked in the direction of acquiring mathematical knowledge, I did not focus on metacognitive aspects (which are instead the subject of the course that PT would take the following semester).

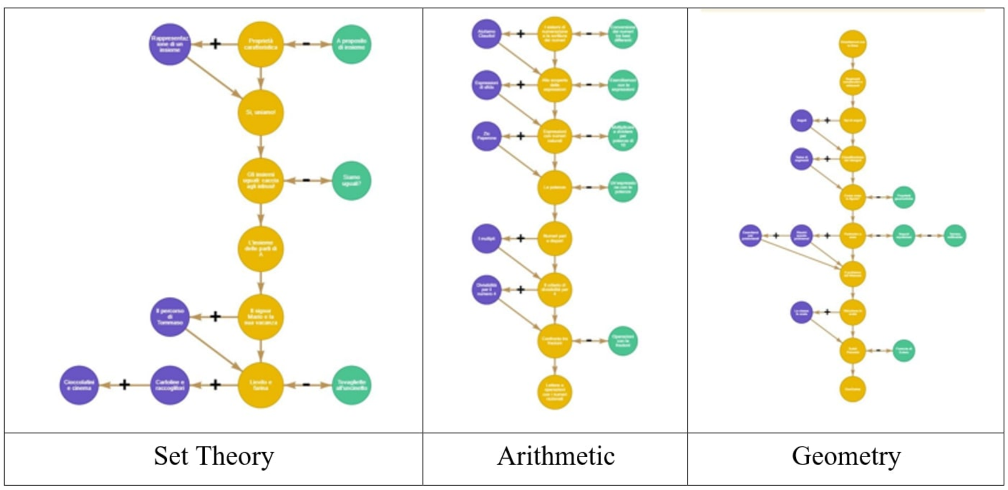

Figure 3: Realized LGs broken down by macro-topics of the course

In the end, the result of their productions culminated in the generation of 3 LGs on the following macro subjects: Set Theory, Arithmetic, Geometry (Figure 3).

3.2 Results drawn from questionnaire

3.2.1 PT's perceptions of task design regarding ease, usefulness and revisions

The questionnaire collected 63 answers, of which 35 were from PT who worked on task design. They were asked, using a Likert scale question (from 1 = absolutely disagree to 4 = absolutely agree), how easy and useful the PT found the task design work, as well as how much they appreciated the revision work on their productions. Considering values 3 and 4 on the scale, it can be observed that ease of use, usefulness and importance of receiving reviews were considered as such by 94%, 92% and 100% respectively (Figure 4).

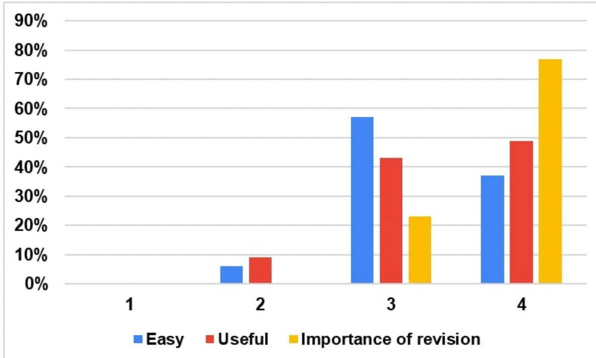

Figure 4: Degree of agreement of PT on the ease, usefulness and importance of receiving reviews

For each of the previous Likert-scale questions, an open question is asked to justify the answer. The Table 1 shows the frequency distribution of the respondents as to why it was perceived as easy or not easy to design tasks for the LG.

Why the task design work was ...			
Easy (#32)		**Not easy (#3)**	
Thanks to the guidance provided by the teacher	47%	Coordinating with colleagues	34%
The templates to fill was clear and easy to use	19%	Not having good pre-knowledge in mathematics	33%
It just takes a little practice	16%	Not able to formulate suggestions	33%
Having good pre-knowledge in mathematics	6%		
Thanks to group work	6%		
The answer format drives the design	3%		
Not Applicable	3%		
Total	100%		100%

Table 1: Why it was/was not easy for me to design a task for the LG?

It can be observed that the teacher guidance emerges as the most significant factor in making the task easy, with nearly half of the respondents (47%) indicating it. For example, someone who believes this has written: *"It was easy because the professor's instructions were very clear and the format was quite easy to understand"*. Other reasons the task design was considered easy include the clarity of the templates (e.g. *"The task planning sheet is very clear in its various steps, [. . .]"*). Still, others felt that task design was easy as soon as one gained some practice (e.g. *"[. . .] at the beginning with the group we had some difficulties to understand certain things well but then with the others we were faster and faster"*). A minority believes that the ease can be attributed to having worked in a group (e.g. *"Thanks to the support among us colleagues [. . .]"*) or to the fact that response formats guide the design (e.g. *"I was able to find the appropriate types of answers to structure any task [...]"*). Coordination with colleagues was the main reason for difficulty, with 34% of respondents citing it. It is a PT who wrote this: *"I do not totally agree because I sometimes had difficulties [...] in coordination with colleagues"*. Mathematical knowledge plays an ambivalent role: while good knowledge helps (6%), the lack of it is one of the main causes of difficulty (33%). In these cases, there were similar responses: *"[it was easy because] I had understood the exercises and their execution"* and *"I often found it difficult to write the solution examples. For me, solving an exercise meant a great achievement. Having to explain it was a double achievement [when I succeeded], because I had internalised the process."*, respectively. In summary, teacher guidance and the clarity of the templates appear to be the most relevant factors in facilitating the task design work, while the major difficulties are related to coordination and prior mathematical knowledge.

Why the task design work was ...			
Useful (#32)		**Not useful (#3)**	
It allows you to go over all the topics of the course	38%	The difficulties in mathematics possessed became more evident	33%
It allows you to understand what course topics to study in depth	22%	Not Applicable	67%
It allows you to challenge yourself as a future teacher	13%		
It allows you to reflect on how best to write your own resolution strategy	9%		
It allows you to put into practice what you have studied	9%		
Reading activities created by other peers allows comparison	3%		
Not Applicable	6%		
Total	100%		100%

Table 2: Why it was/was not useful for me to design a task for the LG?

Let's see how the PT expressed themselves with regard to the usefulness of task design work (Table 2). Reviewing all course topics emerges as the main reason the task was perceived as useful, with 38% of respondents indicating this. Here are some of the students' justifications in this respect: *"It is an excellent alternative for revising by finding targeted exercises for each topic."*; *"It was useful because I could go over all the topics quickly by doing the exercises"*. Mathematical difficulties, instead, are the main reasons the task was perceived as not useful, with 33% of respondents noting this. It is a PT who wrote this: *"For my previous difficulties"*. Other reasons the task design was considered useful include the ability to understand which topics to study in depth (e.g. *"It helped me to understand some topics more deeply"*); the challenge it presents for their future role as teachers (e.g. *"Yes, because it allows you to get involved as a 'teacher' "* ; *"It is also useful in the future"*); reflection on how to write their own resolution strategy (e.g. *"Useful because it allowed me to pause and* *reflect on the way I was explaining a problem"*), and putting into practice what they have studied (e.g. *"Because I was able to apply the notions learned during the lessons"*). A minority of respondents found it useful to compare their work with activities created by other peers. It is a PT who expresses himself as follows: *"It is useful to create a task because it puts the student in the game. In my opinion, it is much more useful instead to observe activities constructed by others so that you can compare yourself"*. In summary, the design task was mainly considered useful for its ability to provide a comprehensive review of the course topics and to help identify areas that need further study. On the other hand, mathematical difficulties emerged as one of the obstacles to perceiving the task design's usefulness.

For all respondents, it was important to receive revisions to their productions.

Why was it important to receive revisions?	
Understanding how to correct errors	22%
Understanding how to improve the task deisng	10%
To be sure that you have worked appropriately	8%
Unders	5%
Not Applicable	3%
Total	100%

Table 3: Why was it important for me to receive lecture's reviews of the task(s) created for the LG?

In particular, Table 3 shows the motivations that explored PT. The majority (22%) felt that receiving revisions was important to understand and correct errors (e.g. *"It made me realize mistakes in such a way that I would never make them again"*); others to improve the design of the task (e.g. *"I understood how something should be explained and how to help make it understood"*); others to ensure appropriate work (e.g. *"To be sure you have worked correctly"*) and to choose the appropriate response format (e.g. *"To better organize task response according to [system] response formats"*).

Another semi-open question asked "Would you advise a future student who is to start attending/studying the 'Mathematics for Primary and Pre-school' course to try their hand at creating a task for a LG?". 100% answered Yes. And motivations fall into two categories: to review, practice and improve learning (71%) and to put oneself out there for one's future (17%). Examples of motivations that fall into the first category are: *"because it could help during the study of the subject, it could serve both as an exercise, but also as a way to clarify if there are some"*; *"These tasks allow for a deeper understanding of the topics also thanks to the fact that it is we, the students, who carry them out"*; while examples for the second category are: *"Because we will be future teachers and it is important to try our hand at creating these useful learning* *tasks"*; *"Because it is good to be able to get involved and have full knowledge of what we should one day be doing in the classroom with our pupils"*. Only in the answer of one PT (3%) does the presence of a didactic contract echo (Brousseau, 1988), in fact he writes: *"Yes mainly because it is important to the professor"*. The remainder (9%) indicate answers that are not relevant, therefore classified as Not Applicable.

LG	**# downolad to 20 December 2023 (first exam call)**	**# download to 16 January 2024 (second exam call)**
Set Theory	141	153
Arithmetic	95	106
Geometry	66	70

Table 4: Number of downloads by PT of the 3 LGs created

3.2.2 The PT's use of LG

As I created the LGs (as shown in Figure 3), I shared the code associated with each LG on the Google Classroom created for the 'Mathematics for Primary and Pre-school' course. The Set Theory one was completed in early November 2023; the Arithmetic one in mid-November 2023; the Geometry one in early December 2023. In this way, any of the PT were in a position to download them onto their smartphones via the ASYMPTOTE app and enjoy them. Table 4 shows the data on the downloads made by the PT of the 3 LGs created ([2]). Downloads made by the date of the first exam call in December 2023 and downloads made by the date of the second call in January 2024 are recorded. Although the downloads varied in number, depending on the macro-topic related to the LG, this shows that the LGs also benefited other students, not only those who engaged in the task design work.

Although it would be interesting to investigate how LGs were used by those who did not work on the task design, this will not be the subject of this contribution. In the following, we will always focus on the answers of the 35 PT who worked on the task design. The questionnaire asked them to specify which LGs were downloaded. The LG on Set Theory was downloaded by 30 out of 35 PT, the one on Arithmetic by 28 out of 35 PT and the one on Geometry by 26 out of 35 PT. There were 2 PT who did not download any. The reasons why not all downloads were made were not investigated. The 33 PT who downloaded the LGs were asked the following open question: "Have you regularly studied the subject with the use of LG or have you used them only in the run-up to exam?". Forty-eight per cent stated that they had used them in the run-up to examina-

2. This data is available from the web portal: each user who creates an LG also sees how many downloads associated with it have taken place on the app.

tions, writing for example: *"I used it to revise and practise for the exam"*; *"[...] in the run-up to the exam to compare the exercises I did with those done by other colleagues and to see alternative ways of solving them to mine"*; *"I used it close to the exam because I preferred to first fix the theoretical part accompanied by the exercises proposed by the professor in class and use the LGs as a final revision"*. In contrast, 42% stated that they regularly accompanied their study with the use of LGs, writing for example: *"I used the Learning Graph to assess my preparation as soon as it was possible to do so, having studied the subject throughout the semester and not just in the run-up to the exam"*; *"I regularly accompanied my study of the subject with the use of Learning Graphs because it gradually helped me to understand some aspect that was unclear and served as practice"*; *"Throughout the course I studied the subject matter with the constant use of Learning Graphs, they were very helpful even when I didn't quite understand topics"*. The remaining 9% did not report relevant reasons and were classified as Not Applicable.

3.3 The PT's exam performance

As far as exam performance is concerned, we can observe that at the first exam call out of 225 students, 78 were among those who had worked on the design of at least one LG task. Of these, only 9 failed the examination. At the second exam call, out of 96 students there were 13 PT who had worked on the design of at least one task for a LG and of these only 1 failed the examination.

For both of these exam calls, a chi-square test was performed to examine the relationship between having done the task design for the LGs and having passed the exam. The relationship between these variables was significant (first exam call: $\chi^2(2, N = 225) = 17.49$, $p < .05$; second exam call: $\chi^2(2, N = 96) = 3.62$, $p < .10$. Those who performed at least one task design for an LG were more likely to pass the examination.

4. Discussion and conclusion

This paper intended to answer the two research questions indicated at the end of Section 1. Answers to each of them are given below.

Involving PT in the design of tasks for LGs was a challenge. It is well known, at least in Italy, that in the Educational Sciences courses, mathematics is neither the preferred subject nor a main subject of the curriculum. However, as it can be seen from the data, the attitude of the PT towards this proposed activity was surprisingly positive. The engagement demonstrated exceeded my expectations and I do not consider this to be related to having promised 1 extra point in the examination. Those who worked on the task design did so seriously and showed that they were involved in the design. Designing turned out to be easy for most of them ([3]), given the instructions they received from me, the opportunity to

3. Excluding the fact that hardly anyone was able to draw up appropriate hints.

work among peers and the willingness to get involved. Sixteen per cent admit that it takes a little practice to understand how to design tasks appropriately. The fact that they produced more than the tasks I expected is also part of the fact that the PT grasped the usefulness of doing such design work. Indeed, they recognised that working on the task design allowed them to make a "general review of the topic" and/or "deepening of certain aspects of it". Moreover, to be able to write the sample solution was for them a confirmation that they understood the exercise's solution process and were able to explain it. This also emerges from their comments on the importance of having been able to benefit from the reviews, which accompanied and supported their design work step by step. It was a team effort between me and them, which required dedication and time on my part, but it seems to have paid off. Almost everyone who worked on the task design took the exam in the first session and passed it. Even though the chi-square test revealed statistical significance, I am well aware of the fact that working on the task design is not the guarantee of passing the exam. It is important to attend classes (which are not compulsory) and above all to study.

The preliminary results of this study are in line with similar research that has seen the use of ASYMPTOTE in both school classrooms (Fesakis et al., 2023) and university courses (Caldeira et al., 2023) as well as in-service and PT training courses (Taranto et al., 2023a; 2023b), in which the system was either used to design tasks or to benefit from them or to do both. Indeed, these studies all agree, as does the present paper, in highlighting how the ease of use, usefulness and acceptance of ASYMPTOTE have a positive impact on teaching and high student engagement. The preliminary study conducted here brings out other aspects, not dealt with in previous studies, which I outline below.

The integration of future teachers in the design of mathematical tasks through the ASYMPTOTE system offer an important indication: direct involvement in the creation of teaching materials can have a significant impact on their understanding and mastery of mathematical content. This process, which goes beyond the traditional study of mathematical concepts, requires PT to apply their knowledge in a practical and reflective manner, thereby improving their competence and confidence in teaching mathematics. When PT engage in task design, they are forced to examine concepts from different perspectives, to reflect on how to present material in a way that is accessible and stimulating to students, and to anticipate possible difficulties that students might encounter. This process not only deepens their understanding of mathematical content, but also develops crucial skills such as analytical skills, creativity in problem solving, and the ability to communicate complex concepts clearly and effectively (Kieran et al., 2015). Furthermore, task design offers PT the opportunity to receive constructive feedback on their materials, allowing them to refine their teaching and assessment techniques. This cycle of design, feedback and revision serves not only to improve the quality of the tasks created but also to consolidate the PT's pedagogical and disciplinary knowledge. In summary, the PT's involvement in the design of mathematical tasks not only improves their understanding of the content (e.g., as we have already seen - *"It helped me to understand some topics more deeply"*) but also prepares them to become more effective educators, capable of stimulating interest and understanding of mathematics in their future students (e.g., as we have already seen - *"Because we will be future teachers and it is important to try our hand at creating these useful learning tasks"*). These preliminary results underline the importance of integrating innovative pedagogical practices into teacher education, with the potential to significantly transform mathematics education.

5. Research limitations and future research perspectives

A potential limitation of this study is that the data analyzed pertain to a course I personally taught; therefore, the analysis may be influenced by my own subjectivity and perception of the course. As well as, the questionnaire - as far as it was sent to all PT who attended the course - was filled out voluntarily. This may have biased the results in the sense that the respondents were PT motivated to want to answer.

Additionally, I deprived the students of the opportunity to work on the web portal because a basic knowledge of the LaTeX language was required to write formulas correctly, and this was not a skill possessed by the PT. For the future, I plan to prepare a booklet with the most frequent LaTeX commands for them to work on the web portal. Furthermore, as anticipated, I often drafted the task hints for the students. The aim of another course, the Mathematics Education course (which I always teach in the following semester), is to get the PT to think about metacognitive processes as well. That will be the opportunity to return to these aspects.

For future research, this preliminary study offers different and diverse insights. From the point of view of the ASYMPTOTE system, since the PT had difficulty creating suggestions of their own, it would be worth investigating how the system could be adapted to help users create appropriate suggestions. From the perspective of mathematics education research, on the other hand, it would be interesting to investigate how the task productions evolved over the various revision rounds; how much and how the practice of writing the sample solution was internalised by the PT and how this practice is reflected in the way the examination questions are carried out. Equally interesting would be to investigate the other questions in the questionnaire, which are more relevant to the use of the LG, especially in PT who did not participate in the task design. Last but not least, PT could be involved in designing not tasks but LGs. It would be worth investigating how they choose main and supporting tasks, e.g. for a specific topic. The design of LGs by PT is an interesting task to give us information about their needs, difficulties and abilities in learning design.

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