A blended teaching and learning environment for developing attitude towards mathematics

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Abstract. This paper reports on the experimentation of a blended learning approach to mathematics at the University level. A first result regards how the teacher's design exploited the didactical potentialities offered by the facilities of a platform, allowing the conceiving specific task based alternatively, on quiz and assignments. Teacher's interventions aim to make students reflect, but mainly to make students focus and interrelate the three fundamental aspects of mathematical thinking: the intuitive, the algorithmic and the formal.

Keywords: digital resources, blended environment, linear algebra, university mathematics education.

MSC 2020: 97C70, 97D40.

1. Introduction

Within the university context, students and teachers encounter difficulties related to some penalising logistical conditions of the university setting, such as the heterogeneity of students' backgrounds and motivations, the high number of students per teacher, and the consequent impossibility of a strong relationship between the learner and the teacher. This implies that students, especially freshmen, experience both learning difficulties at different levels (cognitive, metacognitive and affective) and psychological obstacles (Di Martino & Gregorio, 2019). These obstacles, to be overcome, require the development of autonomy and responsibility of each learner with respect to his or her own learning, as well as a sense of being valued as a person, against the sense of massification that the university context can induce. A number of studies devoted to describing and

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explaining students' difficulties in the transition to University (Clark & Lovric, 2008; Gueudet, 2008; Tall, 2008). They have highlighted as key element the problematic relationship between a procedural approach, privileging calculations and formulas application, and a conceptual approach, based on definitions and proving theorems, i.e dealing with mathematics subjects from a theoretical perspective (Dorier, 2000, Ufer et al., 2017), and as a consequence, the need for a change in students' attitude towards mathematics and mathematics learning. According to the objective of helping students overcome the predominance of procedural knowledge in favour of developing conceptual knowledge (Hiebert and Lefevre, 1986) purposeful teaching experiments were carried out (Albano & Pierri, 2014; Albano, 2017), from which sprouts the study presented in this paper. The main issues in focus were: finding out which Moodle tools offered to support interactivity and designing their use in order to enable students to develop a new attitude towards mathematics knowledge. Pilot studies carried out with university students attending a regular course of "Linear Algebra", led us to design a teaching and learning environment aimed at engaging students outside the classroom and inside the classroom, on the base of the general assumption about the effectiveness of a *blended* approach (Hrastinsk, 2019) combining traditional face-to-face lectures, student's individual interactive purposeful work on platform and the related collective discussions guided by the teacher. In the following, after presenting an outline of our conceptual background and the key elements of the didactical scenario, we will describe the didactical potentialities of the different tools available in the used learning platform, and the general organisation of their use within the blended teaching and learning environment.

2. Theoretical background

As shortly said in the Introduction, there is consensus among researchers that students attending University courses had difficulties to accomplish the requested move from a rote learning of procedures to mathematics as "a scientific discipline based on explicit definitions, deductive proofs, and formal representations" (Ufer et al. 2017, p. 397). This is especially true for these application study courses where mathematics is considered as a "service" subject, that means mathematics aims to provide students with those mathematical competencies required in study courses and their professional practices (Apers, 2020). Let's clarify our assumptions on the requested change.

2.1 Students difficulties and the complexity of mathematical knowledge

The terms procedural and conceptual, referred to mathematical knowledge, have become widespread in the mathematical education literature. A common reference can be the classic definitions given by Hiebert and Lefevre (1986): "*procedural knowledge*" is characterised by managing rules or procedures for solving mathematical problems; at the same time, because many of the procedures consist in chains of prescriptions for manipulating symbols, procedural knowledge also consists in a managing individual symbols and the syntactic conventions of the representation system" (pp.7-8). Still following these authors, *conceptual knowledge* can be thought of as a connected web of knowledge, a networking in which the linking relationships are as prominent as the discrete pieces of information (pp. 3-4). For this relational character conceptual knowledge is also referred to as *relational knowledge*. It is widely recognized the role that both types (modes) of *knowledge* play in teaching and learning Mathematics. A number of research studies highlight a lack of equilibrium between the two modes, and provide evidence showing that in school practice, often may happen that the procedural prevail over the conceptual (Engelbrecht, Harding & Potgieter, 2005; Baroody, Feil & Johnson, 2007). Applying this distinction showed its effectiveness in describing students' difficulties and it can also explain those emerging in the transition from high school to university. However, the specific change of focus that students need to get when entering the University, cannot be fully captured by the simple opposition between a mathematics of procedures and a mathematics of concepts. Specifically, what is meant for 'conceptual knowledge' requires a more elaborated explanation, overcoming the ambiguity intrinsic of the term knowledge; as a matter of fact, 'conceptual knowledge' can be referred both to a formal discipline, organised in a set of properties (axioms, definition, theorems, ...) or to individuals as cognitive agents, for instance, when they solve a mathematical problem. In order to overcome such ambiguity, Fischbein's discussion (1994) provides an interesting perspective that considers mathematics as a human activity, characterised by the combination of three interrelated aspects: the formal, the algorithmic and the intuitive. All these aspects pertain to Mathematics as human activity and as a formal science, each aspect must have an active part in the mathematical reasoning processes and for this actively used by the student. The formal aspect concerns mathematics as a formal science, that is, the hypothetical deductive construction of theories – axioms, definitions, proofs of theorems – and the feeling of coherence and consistency that must accompany any reasoning or solution process. At the same time, the formal aspect must be related both to the algorithmic aspect, i.e. to the memorised procedures to be applied in problem solving, giving such procedures the reason for application, and to intuition, i.e. that kind of cognition that, immediately accepted without asking any kind of justification, can move problems' solution processes. According to Fischbein (ibd.) interaction and sometimes conflicts between the three aspects can explain students' difficulties. Specifically, the difficulty in dealing with mathematics subjects from a theoretical perspective (Gueudet, 2008; Dorier, 2000) can be interpreted as a flaw in the formal aspect, due to its insufficient development at the school level; at the same time a weak or absent relationship between the formal and the algorithmic aspect can explain students' difficulties in moving from the solution of a single problem, to the solution of a theoretical problem that is a problem addressing a class of problems. In summary, in the study that we present in this paper we assume that students not only need to develop the interrelationships between the three components but also that students must get awareness of such interrelationships.

2.2 Teaching and learning blended environment

Though the name *blended environment* is used for different educational settings, a general characterization can be shared of blended environment as a combination of face-to-face with distance delivery systems (Osguthorpe & Graham, 2003): asynchronous/synchronous, face-to-face/distance, paper and pencil/digital, students can be engaged in their personal study time/space inside and outside the classroom. Studies show that the online component of a blended environment reduces distractions that are typical in classrooms or lecture halls, increases time-on-task, and improves student performance (Borba et al., 2016). This definition is sufficiently general for describing the nature of what we decided to implement; the various models of blended environment presented in the literature (see Hrastinski (2019) for an overview differ in the modes of combining and the amount of time devoted to each of the two components, however all of them seem to focus on the students, their personal engagement and the effectiveness of the setting. As anticipated in the Introduction, previous pilot studies make us to develop our fundamental hypothesis concerning the didactical potential offered by combining students online work and traditional face-to-face lectures, as well proposing interactive purposeful work on platform and collective discussions guided by the teacher, focussing on the main difficulties that working on a digital platform might have been evidenced; such hypothesis fittingly resonate with the general hypothesis of combining the effectiveness of a teaching and learning approach that might engage students on both the cognitive and metacognitive level (Schoenfeld,1985). More specifically, students should be invited non only to solve problems but also to discuss the proposed solutions and their theoretical validation: activities should be proposed that invite students to reflect on their own learning trajectory. Differently, our concern in the design of the blended environment focused on the relationship between the two components, on the basis of the key assumption about the crucial mediation role played by the teacher. In the following section, we are going to describe the main principles driving the design of the architecture of an instructional environment engaging students and teacher with specific activities inside and outside the classroom, exploiting the potentialities of the platform for supporting the teaching and learning process in the way of thinking and reflecting on the necessary change of attitude towards mathematics. In particular, we take into account the Three- dimendional Model for Attitude towards mathematics consisting of the three dimensions: the emotional disposition towards mathematics (I like/don't like), the vision of mathematics (the mathematics is....), the perceived competence in mathematics (I can/cannot do it) (Di Martino &

Zan, 2011). In the following we focus on the vision of mathematics varying from procedural to conceptual knowledge, as specified in the Introduction. Thus, our research goal has been to analyse whether and which features of the implemented blended environment can foster the development and the awareness of interrelation among the formal, the algorithmic and the intuitive aspects.

3. Didactical scenario

The design of the blended environment, above illustrated, sees the involvement of about 250 Computer Engineering freshmen at the University of Salerno, attending a course of Linear Algebra, at the beginning of the second term of the first year. The course develops along 12 weeks, with 3 face-to-face 2-hours classes (both lectures and exercises sessions) per week. The course is mandatory for all students, but participation in online activities is voluntary although strongly recommended. We set a blended instructional environment consisting of traditional face-to-face lectures and a dedicated course space on the e-learning platform Moodle. The Moodle course has been populated by the teacher with didactical material and resources. The didactical material consists of videos showing how to prove some main theorems or how to proceed for solving some typical tasks, books, screenshots of the digital boards used during the lectures, worked-out exercises, slides. The resources consisted of tasks, quizzes, FAQ forum, for reviewing macro-sections of course content, personal wiki. The use of the e-learning platform allows students to have all the course materials available, to use a known environment, and to access their registers of used educational resources. The teacher also has a global view of each student's history. Before each lesson, the students have the theoretical material (a .pdf file). From lecture to lecture he makes available the slides of the lecture he are going to do with the intent that students take only the necessary notes (already having the slides in front of them), not being absorbed in copying the slides students can concentrate on take notes on what the teacher's comments. According to the theoretical framework, we designed the following didactical sequence:

- frontal lecture: introduction of theoretical concepts and results or solving methods, through use of slides or live whiteboard;
- student involvement in interactive activities: the students are required to carry out a quiz or an assignment. The activity is individual in the sense that each student has to hand in his or her own quiz or assignment, but in the classroom they are left free to collaborate (which happens naturally) as well as to consult any teaching materials (from those on the platform, to their own notes, to what is on the web and even to chatGPT);
- classroom discussion: the quiz was made to end with the time break so that the teacher during the break could choose the most meaningful answers for fruitful discussion also based on the a priori analysis that led to the definition of the quiz questions. So, the teacher engineers a collec-

tive discussion, providing formative feedback (Hattie & Timperley, 2007) on possible errors occurred, making constant references to the processes and mathematical theoretical tools underlying the resolution and choices made, supporting the differentiation of the discussion focus by means of different colours. A trace left on on the whiteboard by the development of a discussion and of its interlaced focuses is displayed below (see Figure1).

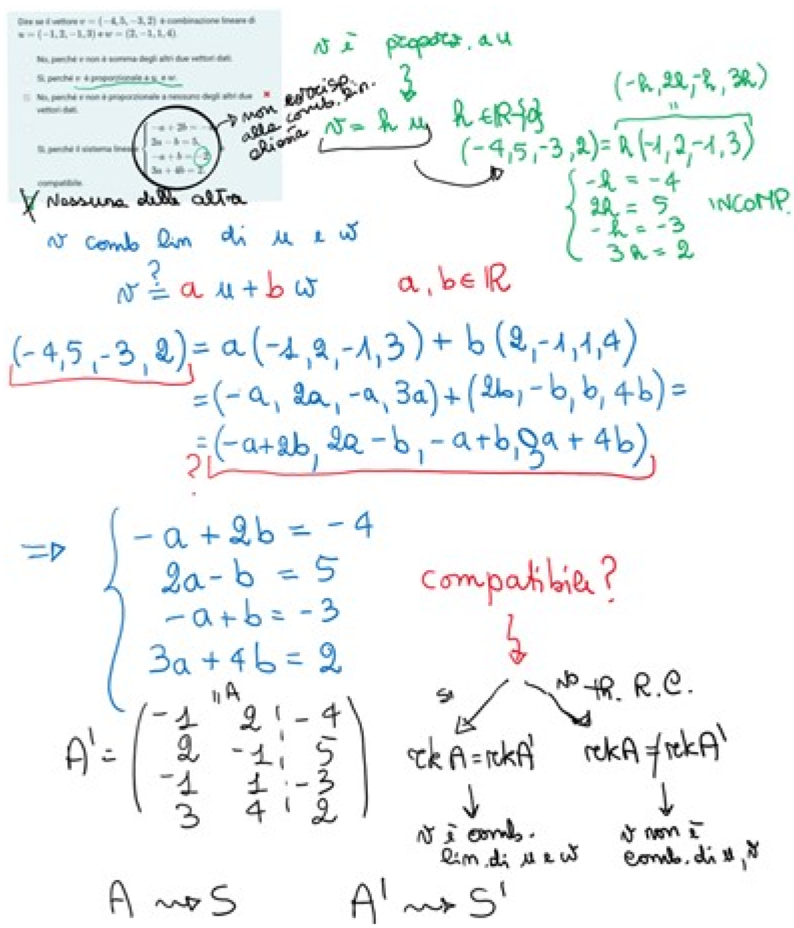

Figure 1: A trace of a collective discussion through a shared whiteboard (e.g. how linearly dependence of vectors are related to resolution of linear systems.)

- the students are required to come back to their performance to reflect on what did or did not work in their solution and why, writing their reflections on their learning portfolio (Albano, et al., 2024), that is implemented as a personal wiki.

4. Moodle resources and their crafting

The core of the teaching approach, as previously mentioned, is based on the interlacement of different types of activities, orchestrated by the teacher; the nature of the activities is related to using different devices designed by purposefully 'crafting' different resources made available by the Moodle platform. The main type of activities proposed by the teacher are:

- individual work activities, based on platform resources such as quizzes, assignments,...: these are activities that involve student interaction with a designed teaching resource, and produces for each student an experienced teaching resource (see Figure 2); such experienced teaching resource can be shared among peers, either autonomously, after the suggestion of the teacher.

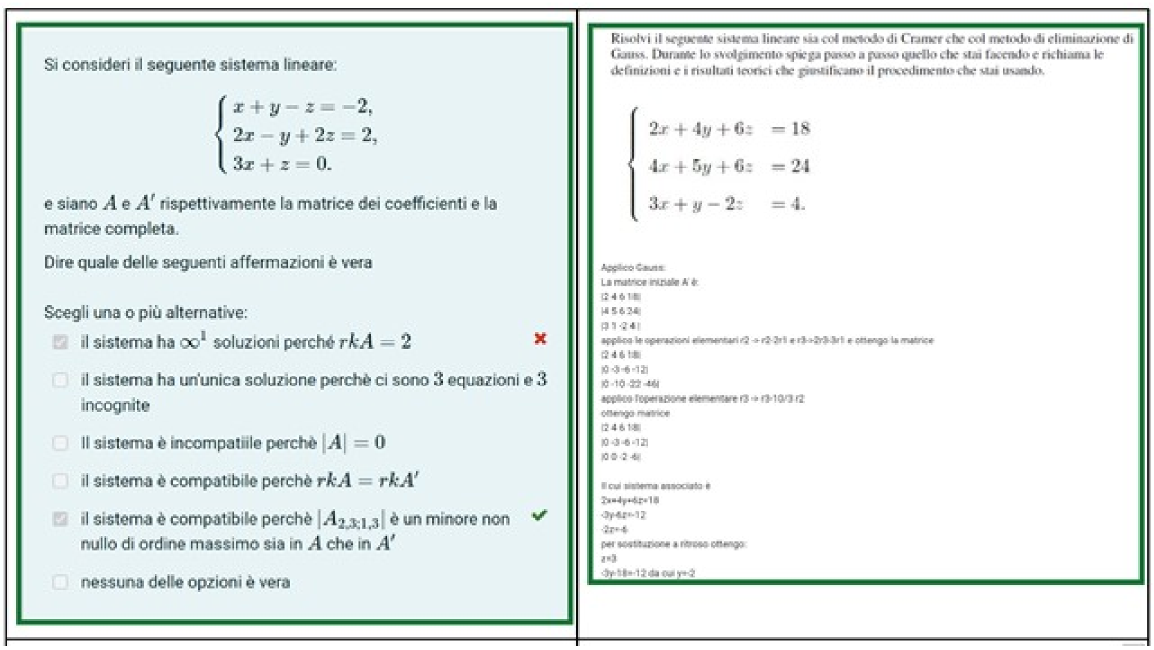

Figure 2: A Sample of experienced quiz (on the left) and experienced assignment (on the right)

- collective activities, led by the teacher: students are involved in a collective discussion, taking place in the classroom and starting with the experienced teaching resources (see Figure 1). Students are directly challenged by the teacher with questions aiming at soliciting students to come back to the task, rethink their own solution process, and compare it with peers' solutions. Examples of the teacher's questions are: "why?" but also "could we have done - or has anyone done - in another way?", "was there a possibility of answering without doing any calculations but resorting only to theoretical results?", "how can we check that what we got is correct?".

During the discussion the teacher's interventions aim to make students reflect, but mainly to make students shift the focus from the algorithmic aspects to the intuitive and formal aspects: the objective is to ground the algorithmic aspect on the intuitive meaning but also on its formalisation, to make students connect different macro-topics, to highlight the plurality of possible methods and outcomes, and to set in place control processes.

- intrapersonal activities: these consist of those activities suggested for students to do outside the classroom, such as reviewing slides, whiteboards, videos, working with designed instructional resources, but most importantly they can reflect on what they are learning by drafting, when explicitly requested, their own learning portfolios.

5. Samples of designed resources

In order to illustrate what has been described above, in this section we present individual work activities, based on platform resources such as quizzes and assignments. That in order to show the potential of some specific resources offered by the platform in relation to the goal of shifting the perspective of learners centered on the execution of procedures. We point out the different educational value between quiz and assignment: in the former, the student must understand the solving strategy behind the quiz items; in the latter, the student is free to deploy the solving strategy he or she prefers. In the second part of the section we provide sample of designed resources.

5.1 Quiz vs assignment

Quiz and assignment allow teachers to engage students in individual activities that are the basis of a collective discussion, considered as pivotal to move students towards the development and the awareness of interrelation among the formal, the algorithmic and the intuitive aspects. Let us discuss the differences between the two kinds of activities. While quizzes involve the student selecting options predetermined by the teacher, assignments are questions that the student can solve in the way they see fit. The main difference between the two individual work activities is that quizzes need to manage and coordinate the formal, the intuitive and the algorithmic aspects - often answer items are based on the meaning of the objects or mathematical terms involved in the activity, as distinct from its formalisation in a definition or theorem and from the related algorithms - and are intended to shift students' attention to the theory or why of calculus/algorithms rather than the calculus/algorithms themselves. Quiz and assignement have a different educational value:

- the assignment is more "traditional", the student is free to choose his or her resolution strategy (see Figure 2, on the right); he or she takes responsibility for a resolution strategy in which the intuitive and algorithmic aspects intervene, usually well harmonised with each other, circumventing the control of the formal aspect that is not directly called upon by the question;

- the quiz includes questions that present a situation (e.g., a linear system, a vector subspace, etc.) against which scenarios are proposed (through items) that can be true or false (see Figure 2, on the left). The student must identify the true ones. To establish truth, it is not always sufficient to refer to one's intuitions, much less to the application of procedures. Rather, the student must put in practice some control strategies that refer to theory. The required justifications cannot be based only on the resolution strategy the student would implement if he or she were in the situation presented in the quiz; to answer correctly, the student must understand what the resolution strategy presented is and evaluate its correctness or not.

In the assignment, the student's knowledge (sometimes only algorithmic) is activated, based on which he or she chooses a solution strategy, implements it, and finds an outcome. In the quiz, solving processes are put in place that are different from those of the task, more complex and mobilizing all three aspects we are interested in. In fact, the student must disengage from what he or she would do and must emphasize with what someone else has done and must check its correctness. Formal control is unavoidable, but even before that, the contribution of the intuitive aspect that guides the interpretation of the proposed situation is essential; the student who moves only with mnemonic procedures is unlikely to be able to govern the quiz. In the architecture of the course, the alternation between assignment and quiz is aimed precisely at giving the student different experiences; on the one hand, the experience of having freedom to move, choosing among one's own knowledge, those most appropriate for solving a task (working by relating the intuitive and procedural aspects), and on the other hand, to move the student away from resting on having learned only one "way". Indeed, the request in the quiz to interpret and check the proposed answer is intended to challenge the security of the procedural alone and to relate the intuitive aspect, based on the personal meanings constructed by the learner, and the formal aspect, based on the theory that formalizes them.

5.2 Samples of assignment

In the following we show a sample of assignment and of quiz concerning linear systems. Looking at Figures 3 and 4, we analyse the similarities/differences between the two types of individual work activities.

Stabilisci se il seguente sistema lineare è compatibile o meno; utilizza tutti gli strumenti che conosci per verificare il risultato percorrendo strade diverse. In caso di compatibilità, calcola le soluzioni, utilizzando sia il teorema di Cramer (applicato all'opportuno sistema minimale ridotto equivalente a quello dato) sia il metodo di eliminazione di Gauss. Durante lo svolgimento spiega passo a passo quello che stai facendo e richiama le definizioni e i risultati teorici che giustificano il procedimento che stai usando.

$$\begin{cases} x_1 + 2x_2 - 3x_3 + x_4 & = 3 \\ 2x_1 - x_2 + 2x_3 - x_4 & = 1 \\ -3x_1 + 4x_2 - 7x_3 + 3x_4 & = 1, \end{cases}$$

Figure 3: Sample of assignment on linear system

Let us consider the following linear system:

$$\begin{cases} x + 2y + z + t = 2, \\ 3x + 4y + z + 2t = 5, \\ 2x + 2y + t = 3. \end{cases}$$

Indicate which one(s) of the following statements is true.

- ☐ (2,1,1,-2) is a solution of the linear system
- ☐ $\forall y, z \in \mathbb{R}$ (z+1,y,z,-2y-2z+1) is a solution of the linear system
- ☐ none of the other items is true
- ☐ the system is incompatible
- ☐ the solutions of the linear system are (x,y,x-1,-2x-2y+3) $\forall x, y \in \mathbb{R}$

Figure 4: Sample of quiz on linear system

In the assignment (Figure 3) the student is asked to check compatibility, and to check for correctness with all the ways she knows; compatibility also appears among the items in the quiz question: implicitly where 4-uplet solutions are given; explicitly in item 4. However, the strategies the student implements in the assignment and in the quiz change. In the assignment, the student can use the Rouché-Capelli theorem, then calculate the ranks of coefficient matrix A and of augmented matrix A' and can do so in various ways, top-down (by using rank definition as size of largest non-vanishing minor) and, if she is lucky, find a non-zero minor of order 3 in A which also gives information about the rank of A'; or bottom-up (by using Kronecker theorem) and even in this case it may happen that the student, starting from a minor of order 2, finds a bordered matrix in A of order 3 that gives information on rkA'. The student may however also decide to reduce the complete matrix of the system in its echelon form so as to check compatibility or otherwise by looking at what the last non-zero row looks like. In the case of the quiz, the student could use the same strategy as in the assignment if she starts with item 4, but could instead go and check whether one of the quaterns of items 1, 2, 5 is a solution (using the definition of solution, then substituting the 4-uplet in the system and checking the equalities) and if so, she would get the information that item 4 is false for free.

Furthermore, the assignment requires the students to compute, if they exist, the solutions of the system and to do so using two procedures, the first of which assumes the arbitrariness of the choice of the minor that gives the rank on which the reduced minimal system that is solved then depends, and then the description of the solutions. In the case of the quiz, the request is only to verify that certain 4-uplet are solutions, which only implies putting the meaning of system solution into play. Moreover, noting that the numerical quatern is not obtained as a special case of any of the generic 4-uplet provided, in case that one of them is found to be a solution, then one could immediately say that the 4-uplet in item 1 is not a solution. In the case of the quiz, no procedures for computing the solutions of a linear system are required. Figure 5 shows a picture of the whiteboard constructed during the collective discussion related to the quiz of Figure 4.

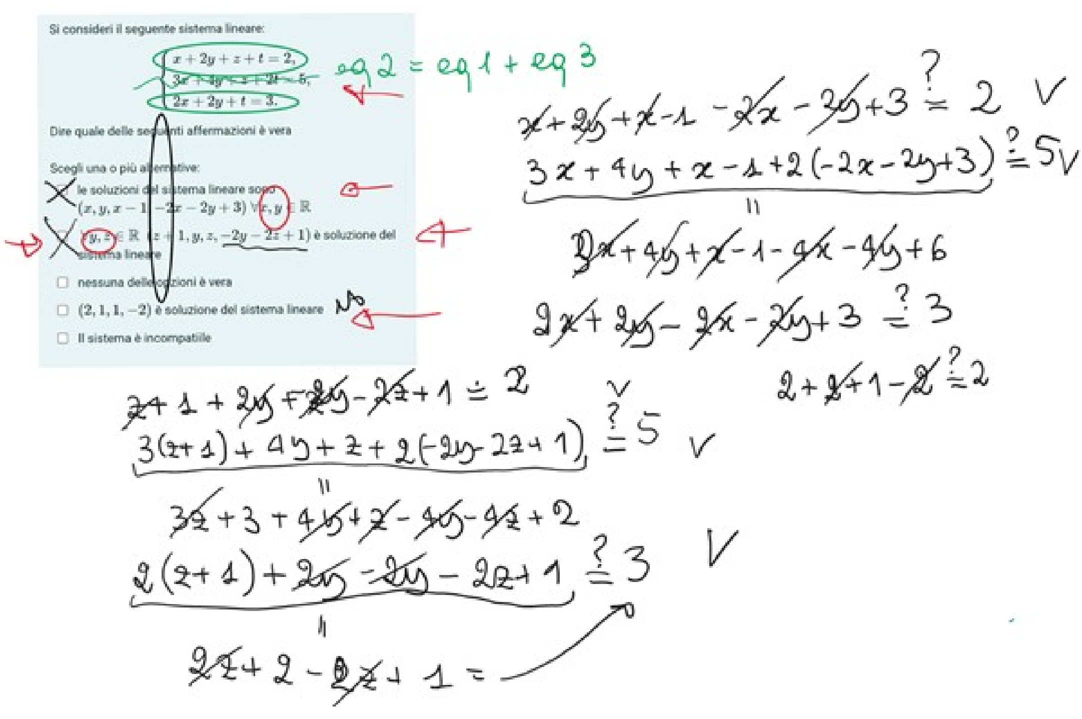

Figure 5: Picture of a whiteboard

Note that both the 4-uplets of items 2 and 4 describe the solutions of the given linear, but most of the students missed both of them or at least one. The teacher started the discussion asking the students how they established the correctness or not of the items. Almost immediately it turned out that the students solved the linear system using Gauss's method and thus they got different solutions, that is the 4-uplet depends on the parameters z and t. After looking for students having acted differently, the whole class arrived at the meaning of solution and thus to the correct algorithm of substituting the 4-uplet in the system for checking the validity of the equalities (Albano et al., 2022).

6. Conclusion

The general issue addressed in our project concerns one of the difficulties, often highlighted in the literature, and emerging at the very beginning of the University courses: overcoming the fixity of a procedural approach and developing a conceptual approach in the solution of mathematical problems. In order to foster in students' change of attitude towards mathematics and mathematics learning, some teaching experiments were carried out, such experiences led to develop a conceptual framework within which to design a complex and articulated structure for a blended learning environment interlacing students' individual work on purposeful online resources and in presence activities, referred to that work and carefully managed by the teacher. As discussed above, the didactical potentialities of a specific blended environment were elaborated on two key assumptions: on the one hand the need of developing the interrelationship between the procedural, the intuitive and the formal aspects that represent the core of mathematical thinking (Fischbein, 1994); on the other hand, the crucial mediating role to be played by the teacher in the development of such interrelationship. Such general principles of design have been consistently articulated in the design of a specific instructional environment experimented with Computer Engineering freshmen at the University of Salerno, attending a course of Linear Algebra, at the beginning of the second term of the first year. What is presented in the previous sections is the discussion of how the didactical potentialities offered by some of the Moodle platform resources can be exploited allowing the conceiving of specific tasks and can be implemented in a specific instructional intervention based alternatively, on quiz or assignments to be accomplished by students, and on collective sessions orchestrated by the teacher. The choice between different types of resources, for instance quiz rather than assignment, is determined by the specific structure of the resource and the educational objective of changing students 'attitude towards mathematical knowledge: moving their perspective from just applying a procedure to the theoretical meaning of the answer given by the procedural solution. In particular, in the case of the quiz when it is required to interpret and check the proposed answer, the student is challenged to relate the three aspects: not only the intuitive the procedural but also the intuitive aspect related to the personal meanings and the formal aspect related to the validity of a statement. During the collective discussion, going back to the reasons that led him/her to answering the quiz, each student has the opportunity to reflect and gain a new point of view. As said, in this 'reflection' activities the delicate and fundamental role of the teacher emerges. The analysis developed above illustrates the possible functioning of this dialectics in the specific case of the teaching experiment of a Linear Algebra course. The teaching experiments are still in progress, the first results of students achievements are very encouraging in supporting the arguments presented in this paper. We hope to share these results soon, in a new paper.

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