Editorial for the Special Issue on "The role of digital technologies in supporting the teaching of mathematics: experiences of good practices at the university level"

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1. Introduction

This Special Issue inaugurates the opening of the Italian Journal of Pure and Applied Mathematics (IJPAM) to contributions in the field of Mathematics Education. The spirit of the IJPAM, since its foundation by Professor Piergiulio Corsini, has been to welcome original articles that range across all sectors of Mathematics, Pure and Applied. In the past there have been some contributions, even if few, in this area. So now, it will be devoted to original research in the fields of pure and applied mathematics and their teaching and learning. The editorial board has also been enriched with experts in Mathematics Education, which is a young research strand compared to other sciences such as Mathematics. Mathematics Education researchers might take the advantage from the development of mathematics and, conversely, mathematicians might take the advantage from the development of didactics. At an institutional level, mathematicians should take an interest in the teaching of mathematics, both to train future generations of mathematicians and to help students to know, understand, and formalize mathematical content. Mathematics educators, on the other hand, play a crucial role in understanding students' ways of thinking

and, building on mathematical contributions, develop methodologies and teaching strategies to transpose these contents. By creating specific situations of discovery, investigation, and institutionalization, educators stimulate students' curiosity, fostering their learning and appropriation of mathematical concepts. Hence the need to welcome contributions in both directions. This dual perspective, where the evolution of mathematics and its didactics intersect, naturally leads to a reflection on the historical and contemporary role of mathematics education in shaping knowledge, as well as the impact of emerging technologies and pedagogical approaches on the teaching and learning process.

Mathematics Education plays a central role in shaping the cultural and scientific knowledge of humanity. As early as antiquity, mathematicians like Euclid and Pythagoras not only developed foundational concepts but also reflected on the methods of teaching them. During the Middle Ages, with the establishment of universities, mathematics became an academic discipline, often linked to logic and philosophy. The Enlightenment and the Industrial Revolution saw mathematics education evolve to meet the practical demands of science and engineering. In the 20th century, the pedagogical theories of Piaget and Vygotsky emphasized the importance of interaction and the active construction of knowledge by the learner. Over time, mathematics teaching has adapted to cultural and technological contexts, transitioning from methods based on memorization and repetition to approaches that prioritize reasoning and problem-solving. The advent of digital technologies has ushered mathematics education into a new era. Tools like GeoGebra and Desmos, along with online platforms and adaptive tutoring systems, have revolutionized how mathematics is taught and learned. These technologies facilitate the visualization of abstract concepts, allow students to explore mathematical relationships interactively and dynamically, and offer opportunities for deeper engagement through multiple pathways ([3]; [5]). They have also redefined the teacher's role, transforming it from a mere transmitter of knowledge to a facilitator and guide in the learning process. However, as highlighted by Noss and Hoyles ([4]), integrating digital tools into education requires careful consideration: it is essential to maintain a focus on deep mathematical thinking, ensuring that digital tools complement rather than replace the development of abstraction, deduction, and argumentation skills.

Today, the primary goal of Mathematics Education is to prepare students to navigate an increasingly complex and interconnected world. Achieving this requires a balance between tradition and innovation, digital tools, and established practices, while fostering critical and creative thinking. As educational research shows, a continuous dialogue between theory and practice remains the cornerstone for building mathematics education that meets the challenges of the 21st century ([1], [2]). On the above premises, the DIGiMATH group (`www.digimath.it`) was born within the Italian Mathematical Union with the aim of reflecting and understanding on how didactics in presence must be reorganized to integrate and exploit the potential offered by the digital environment,

taking into account that learning processes are strictly dependent on the tools and modes through which they are mediated by the teacher.

The working hypothesis of this group is that specific technological resources can complement face-to-face teaching and help solve the problems identified at the various levels, as long as the integration of digital resources and environments into teaching interventions is carefully designed in relation to the objectives.

2. The role of digital technologies in supporting the teaching of mathematics: experiences of good practices at the university level

This Special Session takes its cue from the workshop of the same name organized in Rome on February 8, 2024 as part of the activities of the DIGiMATH. As a reminder, the purpose of the workshop was to compare and discuss the experiences already underway in Italian universities of practices and research on the use of digital technologies in mathematics education and to promote the development of collaborations between researchers in Mathematics Education and in other areas of Mathematics for the definition of joint practices and research. The themes that emerged, in line with those already identified by DIGiMATH and declined along the three contexts of school/university transition, basic mathematics courses and teacher education courses, were as follows:

- *The use of digital technologies to stimulate meta-cognition*
- *The use of digital technologies to do mathematics*
- *The use of digital technologies to stimulate collaborative processes and foster participation*
- *The use of digital technologies for assessment*

Building on the themes discussed during the workshop, the following sections present a synthesis of the contributions for each of the areas related to the use of digital technologies in mathematics education, offering insights into the practical applications and research findings that emerged in relation to these key aspects.

2.1 The use of digital technologies to stimulate meta-cognition

Cusi and **Telloni** focus their paper on the individualization processes that can be fostered by using digital resources properly designed. They investigate the integration of the expert's scaffolding into the scaffolding provided by the digital resources, with the goal of making this scaffolding more responsive, and the effects of this integration. The presented analysis highlights the usefulness of the role of the expert scaffolding in integrating automatic scaffolding at metacognitive level, both for low-achieving students and for the more competent ones.

The paper of **Albano**, **Mariotti** and **Pierri** describes an experiment in blended learning for university-level mathematics, using a digital platform to

combine quizzes and assignments. The goal is to help students shift from a procedural to a conceptual approach in solving mathematical problems, focusing on the intuitive, algorithmic, and formal aspects of mathematical thinking. The teacher plays a key role in guiding this transition.

2.2 The use of digital technologies to do mathematics

Lepellere, in her article examines, using paper and pencil productions and the answers to a non-anonymous questionnaire, the errors and strategies used in solving three homework tasks regarding visualization and manipulative abilities of boundary surfaces of a three-dimensional region and boundary curves of surfaces. On the basis of the results obtained, four GeoGebra Applets were designed and used during the classroom lessons and the improvements that such use produced were examined. The findings confirm the initial difficulties of the students in drawing sets, expressed through inequalities, in the Cartesian space both for their previous knowledge and for the difficulty in manipulating three-dimensional objects. It shed light on GeoGebra's potential in helping students visualise geometric situations, improve their spatial reasoning, and stimulate the activation of their mental constructions, which are the prelude to knowledge and advanced mathematical thinking.

In her paper, **Taranto** examines the use of the ASYMPTOTE system in a Mathematics course for pre-service teachers, focusing on their involvement in task design. This activity improved both their understanding of mathematical content and exam performance while fostering key teaching skills. Students exceeded expectations by producing more tasks and recognizing task design as a valuable tool for reviewing and deepening course topics. Supported by structured feedback, the process strengthened their disciplinary knowledge and pedagogical abilities. The findings highlight the importance of innovative practices in teacher education to enhance mathematics teaching.

2.3 The use of digital technologies to stimulate collaborative processes and foster participation

In **Dello Iacono** and **Santi** paper, it is described a study involving master's students in a Mathematics Education course, where future teachers used the Quick Chat plugin for group discussions about mathematical misconceptions. The study examines how this tool promotes collaboration and reflection on teaching practices. It shows how Quick Chat helps future teachers deepen their understanding of both mathematical content and teaching methods, integrating online and in-person activities to enhance their professional development.

The paper of **Fiorentino** and **Montone** proposes a new training mode, designed in order to introduce Pre-Service primary Teachers to the pedagogical model of Mathematical Discussion, both from a theoretical and practical point of view. It is based on the combination of a didactical experience lived by the teachers as students in participating to a Mathematical Discussion, and the re-

flective experience involving the same teachers in a new discussion reflecting on their previous lived experience, identifying the key theoretical aspects characterising the experienced pedagogical model of the Mathematical Discussion. The technological environment has been exploited in order to foster collaborative processes and active participation of the teachers.

2.4 The use of digital technologies for assessment

Alessio and **Telloni** propose a sequence of digital tasks involving the representation of subsets of the plane as normal domains in both Cartesian and polar form, starting point for the computation of several categories of double integrals. The tasks contain teacher-driven scaffolding elements, that is immediate and response-specific facilitative feedback, and student-driven scaffolding elements, that are hints of different kinds. Using the combined analysis of video recordings of the students' group interaction with the digital tasks and students' individual answers to a survey concerning their use of the provided scaffolding elements, they identify three categories of behaviors, corresponding to different levels of awareness and ways of grasping the scaffolding.

References

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