On soft p_c-regular and soft p_c-normal spaces

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Abstract. The aim of this paper is to introduce two new types of soft separation axioms called soft $p_c$regular and soft p_c-normal spaces by using the concept of soft p_c-open sets in soft topological spaces. We explore several properties and relations of such spaces. Also, we investigate hereditary and soft invariance properties by considering certain soft mappings.

Keywords: soft p_c-open set, soft p_c-regular space, soft p_c-normal space.

1. Introduction

Molodtsov [18] initiated the concept of soft set theory in 1999 as a new mathematical tool to treat many complicated problems related to probability and fuzzy set theory. After that many researchers presented applications of soft set theory in many fields of mathematics such as operations researches, mathematical analysis and algebraic structures. Shabir and Naz [21] in 2011 applied the notion of soft sets to introduce the concept of soft topological spaces which are defined over an initial universe with a fixed set of parameters. They introduced

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almost all the essential classical notions in topology and defined the concept of soft open sets, soft closed sets, soft interior point, soft closure and soft separation axioms. Al-shami et al. [4] and [5], investigated several types of soft separation axioms and studied studied their images ang pre-images under soft mappings.

Husain and Ahmed [13] in 2015 studied the properties of soft interior, soft closure and soft boundary operators and they introduced separation axioms by using ordinary points in the universal set also Georgiou et. al [9] in 2013, studied some soft separation axioms, soft continuity in soft topological spaces using ordinary points of a topological space X. Bayramov et al. in [7], defined the notion of soft points and applied them to discuss the properties of soft interior, soft closure and soft boundary operators. They also defined and introduced soft neighborhoods and soft continuity in soft topological spaces using soft points.

It is noticed that a soft topological space gives a parametrized family of topologies on the initial universe but the converse is not true i.e. if some topologies are given for each parameter, we cannot construct a soft topological space from the given topologies. Consequently we can say that the soft topological spaces are more generalized than the classical topological spaces for more details we refer to [3] and [4].

Recently, Hamko and Ahmed [1] introduced the notion of soft p_c-open sets. They applied this notion to define and discuss the concept of soft p_c-interior, soft soft p_c-closure and soft p_c-boundary operators. Also, they introduced the concept of soft continuity and almost soft continuity by employing soft points and soft p_c-open sets in a soft topological space.

The aim of this paper, is to introduce and discuss a study of soft separation axioms which we call them soft p_c-regular and soft p_c-normal spaces which are defined over an initial universe with a fixed set of parameters. We indicate the relationships between them and present several of their properties.

Throughout the present paper, X will be a nonempty initial universal set and E will be a set of parameters. A pair (F, E) is called a soft set over X, where F is a mapping $F : E \to P(X)$. The collection of soft sets (F, E) over a universal set X with a parameter set E is denoted by $SP(X)_E$. Any logical operation (λ) on soft sets in soft topological spaces are denoted by usual set theoretical operations with symbol $(\widetilde{s}(\lambda))$.

2. Preliminaries

In this section we present the main definitions and results which will be used in the sequel. For some definitions or results which are not mentioned in this section, we refer to [2], [3], [7], [12], [17] and [22].

Definition 2.1 ([21]). *A soft set (F, E) over X is said to be null soft set denoted by $\widetilde{\phi}$ if, for all $e \in E$, $F(e) = \phi$ and (F, E) over X is said to be absolute soft set denoted by $\widetilde{X}$ if, for all $e \in E$, $F(e) = X$.*

Definition 2.2 ([21]). *The complement of a soft set (F,E) is denoted by $(F,E)^c$ or $\tilde{X} \setminus (F,E)$ and is defined by $(F,E)^c = (F^c, E)$ where $F^c : E \to P(X)$ is a mapping given by $F^c(e) = X \setminus F(e)$, for all $e \in E$.*

It is clear that $((F,E)^c)^c = (F,E)$, $\widetilde{\phi}^c = \widetilde{X}$ and $\widetilde{X}^c = \widetilde{\phi}$.

Definition 2.3 ([21]). *For two soft sets (F,E) and (G,B) over a common universe X, we say that (F,E) is a soft subset of (G,B), if*

1. *$E \subseteq B$;*
2. *for all $e \in E$, $F(e) \subseteq G(e)$.*

We write $(F,E) \sqsubseteq (G,B)$.

Definition 2.4 ([21]). *The union of two soft sets of (F,E) and (G,B) over the common universe X is the soft set $(H,C) = (F,E) \sqcup (G,B)$, where $C = E \cup B$ and for all $e \in C$,*

$$H(e) = \begin{cases} F(e), & if \quad e \in E - B, \\ G(e), & if \quad e \in B - E, \\ F(e) \cup G(e), & if \quad e \in E \cap B. \end{cases}$$

In particular, $(F,E) \sqcup (G,E) = F(e) \cup G(e)$, for all $e \in E$.

Definition 2.5 ([21]). *The intersection (H,C) of two soft sets (F,E) and (G,B) over a common universe X, denoted $(F,E) \sqcap (G,B)$, is defined as $C = E \cap B$, and $H(e) = F(e) \cap G(e)$, for all $e \in C$.*

In particular, $(F,E) \sqcap (G,E) = F(e) \cap G(e)$, for all $e \in E$.

Definition 2.6 ([7]). *Let $x \in X$, then (x,E) denotes the soft set over X for which $x(e) = \{x\}$, for all $e \in E$.*

Let (F,E) be a soft set over X and $x \in X$. We say that $x \widetilde{\in} (F,E)$ read as x belongs to the soft set (F,E) whenever $x \in F(e)$, for all $e \in E$.

Definition 2.7 ([7]). *The soft set (F,E) is called a soft point, denoted by (x_e, E) or x_e, if for the element $e \in E$, $F(e) = \{x\}$ and $F(e) = \phi$, for all $e \in E \setminus \{e\}$.*

We say that $x_e \widetilde{\in} (G,E)$ if $x \in G(e)$.

Two soft points x_e and $y_{e'}$ are distinct if either $x \neq y$ or $e \neq e'$.

Remark 2.1. From Definition 2.6 and Definition 2.7, it is clear that:

1. (x,E) is the smallest soft set containing x;
2. if $x \widetilde{\in} (F,E)$ then $x_e \widetilde{\in} (F,E)$;
3. $(F,E) = \sqcup \{(x_e, E) : e \in E\}$.

Definition 2.8 ([21]). *Let $\widetilde{\tau}$ be a collection of soft sets over a universe X with a fixed set E of parameters. Then, $\widetilde{\tau} \subseteq SP(X)_E$ is called a soft topology if*

1. *$\tilde{\phi}$ and $\tilde{X}$ belongs to $\widetilde{\tau}$.*
2. *The union of any number of soft sets in $\widetilde{\tau}$ belongs to $\widetilde{\tau}$.*
3. *The intersection of any two soft sets in $\widetilde{\tau}$ belongs to $\widetilde{\tau}$.*

The triplet $(X, \widetilde{\tau}, E)$ is called a soft topological space over X. The members of $\tilde{\widetilde{\tau}}$ are called soft open sets in $\tilde{X}$ and complements of them are called soft closed sets in $\tilde{X}$ and they are denoted by $SO(\tilde{X})$ and $SC(\tilde{X})$, respectively. Soft interior and soft closure are denoted by $\tilde{s}int$ and $\tilde{s}cl$, respectively.

Definition 2.9 ([21]). *Let $(X, \widetilde{\tau}, E)$ be a soft topological space and let (G, E) be a soft set. Then:*

1. *The soft closure of (G, E) is the soft set $\tilde{s}cl(G, E) = \sqcap\{(K, B)\widetilde{\in}SC(\tilde{X}) : (G, E) \sqsubseteq (K, B)\}$*
2. *The soft interior of (G, E) is the soft set $\tilde{s}int(G, E) = \sqcup\{(H, B)\widetilde{\in}SO(\tilde{X}) : (H, B) \sqsubseteq (G, E)\}$.*

Definition 2.10 ([12]). *Let $(X, \widetilde{\tau}, E)$ be a soft topological space, (G, E) be a soft set over $\tilde{X}$ and $x_e\widetilde{\in}\tilde{X}$. Then, (G, E) is said to be a soft neighborhood of x_e if there exists a soft open set (H, E) such that $x_e\widetilde{\in}(H, E) \sqsubseteq ((G, E)$.*

Proposition 2.1 ([21]). *Let $(Y, \widetilde{\tau}_Y, E)$ be a soft subspace of a soft topological space $(X, \widetilde{\tau}, E)$ and $(F, E)\widetilde{\in}SP(X)_E$. Then:*

1. *If (F, E) is a soft open set in $\widetilde{Y}$ and $\widetilde{Y}\widetilde{\in}\widetilde{\tau}$, then $(F, E)\widetilde{\in}\widetilde{\tau}$.*
2. *(F, E) is a soft open set in $\widetilde{Y}$ if and only if $(F, E) = \widetilde{Y}\sqcap(G, E)$ for some $(G, E)\widetilde{\in}\widetilde{\tau}$.*
3. *(F, E) is a soft closed set in $\widetilde{Y}$ if and only if $(F, E) = \widetilde{Y}\sqcap(H, E)$ for some soft closed (H, E) in $\widetilde{X}$.*

Definition 2.11 ([14]). *A soft subset (F, E) of a soft space $\widetilde{X}$ is said to be soft pre-open if $(F, E) \sqsubseteq \tilde{s}int(\tilde{s}cl(F, E))$. The complement of a soft pre-open set is said to be soft pre-closed. The family of soft pre-open (soft pre-closed) set is denoted by $\tilde{s}PO(X)$ ($\tilde{s}PC(X)$).*

Lemma 2.1 ([14]). *Arbitrary union of soft pre-open sets is a soft pre-open set.*

Lemma 2.2 ([2]). *A subset (F, E) of a soft topological spaces $(X, \widetilde{\tau}, E)$ is soft pre-open if and only if there exists a soft open set (G, E) such that*

$$(F, E)\sqsubseteq(G, E)\sqsubseteq\tilde{s}cl(F, E).$$

Lemma 2.3 ([2]). *Let $(F,E)\sqsubseteq\widetilde{Y}\sqsubseteq\widetilde{X}$, where $(X,\widetilde{\tau},E)$ is a soft topological space and $\widetilde{Y}$ is a soft pre-open subspace of $\widetilde{X}$. Then, $(F,E)\widetilde{\in}\ \widetilde{s}PO(X)$, if and only if $(F,E)\widetilde{\in}\ \widetilde{s}PO(Y)$.*

Theorem 2.1 ([19]). *If (U,E) is soft open and (F,E) is soft pre-open in $(X,\widetilde{\tau},E)$, then $(U,E)\sqcap(F,E)$ is soft pre-open.*

Lemma 2.4 ([1]). *Let $(F,E)\sqsubseteq\widetilde{Y}\sqsubseteq\widetilde{X}$, where $(X,\widetilde{\tau},E)$ is a soft topological space and $\widetilde{Y}$ is a soft subspace of $\widetilde{X}$. If $(F,E)\widetilde{\in}\tilde{s}PO(X)$, then $(F,E)\widetilde{\in}\tilde{s}PO(Y)$.*

Definition 2.12 ([1]). A soft pre-open set (F,E) in a soft topological space $(X,\widetilde{\tau},E)$ is called soft p_c-open if for each $x_e\widetilde{\in}(F,E)$, there exists a soft closed set (K,E) such that $x_e\widetilde{\in}(K,E)\sqsubseteq(F,E)$. The soft complement of each soft p_c-open set is called soft p_c-closed set.

The family of all soft p_c-open (resp., soft p_c-closed) sets in a soft topological space $(X,\widetilde{\tau},E)$ is denoted by $\widetilde{s}P_cO(X,\widetilde{\tau},E)$ (resp., $\widetilde{s}P_cC(X,\widetilde{\tau},E)$) or $\widetilde{s}P_cO(X)$ (resp., $\widetilde{s}P_cC(X)$).

Definition 2.13 ([2]). *Let $(X,\widetilde{\tau},E)$ be a soft topological space and let (G,E) be a soft set. Then:*

1. *The soft pre-closure of (G,E) is the soft set*

$$\tilde{s}pcl(G,E)=\sqcap\{(K,B)\widetilde{\in}\tilde{s}PC(\tilde{X}):(G,E)\sqsubseteq(K,B)\}.$$

2. *The soft pre-interior of (G,E) is the soft set*

$$\tilde{s}pint(G,E)=\sqcup\{(H,B)\widetilde{\in}\tilde{s}PO(\tilde{X}):(H,B)\sqsubseteq(G,E)\}.$$

Definition 2.14 ([11]). *Let $(X,\widetilde{\tau},E)$ be a soft topological space and let (G,E) be a soft set. Then:*

1. *A soft point $x_e\widetilde{\in}\widetilde{X}$ is said to be a soft p_c-limit soft point of a soft set (F,E) if for every soft p_c-open set (G,E) containing x_e, $(G,E)\sqcap[(F,E)\setminus\{x_e\}]\neq\widetilde{\phi}$.*

   The set of all soft p_c-limit soft points of (F,E) is called the soft p_c-derived set of (F,E) and is denoted by $\tilde{s}P_cD(F,E)$.

2. *The soft p_c-closure of (G,E) is the soft set*

$$\tilde{s}p_ccl(G,E)=\sqcap\{(K,B)\widetilde{\in}\tilde{s}P_cC(\tilde{X}):(G,E)\sqsubseteq(K,B)\}.$$

3. *The soft p_c-interior of (G,E) is the soft set*

$$\tilde{s}p_cint(G,E)=\sqcup\{(H,B)\widetilde{\in}\tilde{s}P_cO(\tilde{X}):(H,B)\sqsubseteq(G,E)\}.$$

Lemma 2.5 ([1]). *If $(F,E)\sqsubseteq\widetilde{Y}\sqsubseteq\widetilde{X}$ and $\widetilde{Y}$ is soft clopen. Then, $(F,E)\widetilde{\in}\tilde{s}P_cO(Y)$ if and only if $(F,E)\widetilde{\in}\tilde{s}P_cO(X)$.*

Lemma 2.6 ([1]). *Let $(F,E) \sqsubseteq \widetilde{Y} \sqsubseteq \widetilde{X}$ and $\widetilde{Y}$ be soft clopen. If $(F,E)\widetilde{\in}\tilde{s}P_cO(X)$, then $(F,E)\sqcap\widetilde{Y}\widetilde{\in}\tilde{s}P_cO(Y)$.*

Lemma 2.7 ([11]). *Let $(F,E) \sqsubseteq \widetilde{Y} \sqsubseteq \widetilde{X}$. If $\widetilde{Y}$ is soft clopen, then $\tilde{s}p_ccl_Y(F,E) = \tilde{s}p_ccl_X(F,E)\sqcap\widetilde{Y}$.*

Definition 2.15 ([10]). *A soft topological space $(X,\widetilde{\tau},E)$ is said to be:*

1. *Soft T_0, if for each pair of distinct soft points x, $y\widetilde{\in}X$, there exist soft open sets (F,E) and (G,E) such that either $x\widetilde{\in}(F,E)$ and $y\widetilde{\notin}(F,E)$ or $y\widetilde{\in}(G,E)$ and $x\widetilde{\notin}(G,E)$.*

2. *Soft T_1, if for each pair of distinct soft points x, $y \in X$, there exist two soft open sets (F,E) and (G,E) such that $x\widetilde{\in}(F,E)$ but $y\widetilde{\notin}(F,E)$ and $y\widetilde{\in}(G,E)$ but $x\widetilde{\notin}(G,E)$.*

3. *Soft T_2, if for each pair of distinct soft points x, $y \in X$, there exist two disjoint soft open sets (F,E) and (G,E) containing x and y, respectively.*

In [7], S. Bayramov and C. G. Aras redefined soft T_i-spaces by replacing soft points x_e, $y_{e'}$ instead of the ordinary points x, y in Definition 2.15.

Proposition 2.2 ([7]). *1. Every soft T_2-space $\Rightarrow$ soft T_1-space $\Rightarrow$ soft T_0-space.*
2. A soft topological space $(X,\widetilde{\tau},E)$ is soft T_1 if and only if each soft point is soft closed.

In [21], a soft regular space is defined by using ordinary points as:

Definition 2.16 ([21]). *If for every $x \in X$ and every soft closed set (F,E) not containing x, there exist two soft open sets (G,E) and (H,E) such that $x\widetilde{\in}(G,E)$, $(F,E) \sqsubseteq (H,E)$ and $(G,E)\sqcap(H,E) = \widetilde{\phi}$ then $\widetilde{X}$ is called soft regular.*

In [12] a soft regular space is defined by by replacing soft points x_e instead of the ordinary point x in Definition 2.16.

Definition 2.17 ([15]). *A soft topological space $(X,\widetilde{\tau},E)$ is said to be*

1. *$\tilde{s}p_c$-T_0, if for each pair of distinct soft points x_e, $y_{e'}\widetilde{\in}SP(X)_E$, there exist soft p_c-open sets (F,E) and (G,E) such that $x_e\widetilde{\in}(F,E)$ and $y_{e'}\widetilde{\notin}(F,E)$ or $y_{e'}\widetilde{\in}(G,E)$ and $x_e\widetilde{\notin}(G,E)$.*

2. *$\tilde{s}p_c$-T_1, if for each pair of distinct soft points x_e, $y_{e'}\widetilde{\in}SP(X)_E$, there exist two soft p_c-open sets (F,E) and (G,E) such that $x_e\widetilde{\in}(F,E)$ but $y_{e'}\widetilde{\notin}(F,E)$ and $y_{e'}\widetilde{\in}(G,E)$ but $x_e\widetilde{\notin}(G,E)$.*

3. *$\tilde{s}p_cT_2$, if for each pair of distinct soft points x_e, $y_{e'}\widetilde{\in}SP(X)_E$, there exist two disjoint soft p_c-open sets (F,E) and (G,E) containing x_e and $y_{e'}$, respectively.*

Definition 2.18 ([15]). *A soft topological space $(X, \widetilde{\tau}, E)$ is said to be*

1. $\tilde{s}p_cT_0^*$, *if for each pair of distinct points $x, y \in X$, there exist soft p_c-open sets (F, E) and (G, E) such that $x\widetilde{\in}(F, E)$ and $y\widetilde{\notin}(F, E)$ or $y\widetilde{\in}(G, E)$ and $x\widetilde{\notin}(G, E)$.*

2. $\tilde{s}p_c$-T_1^*, *if for each pair of distinct points $x, y \in X$, there exist two soft p_c-open sets (F, E) and (G, E) such that $x\widetilde{\in}(F, E)$ but $y\widetilde{\notin}(F, E)$ and $y\widetilde{\in}(G, E)$ but $x\widetilde{\notin}(G, E)$.*

3. $\tilde{s}p_c$-T_2^*, *if for each pair of distinct soft points $x, y \in X$, there exist two disjoint soft p_c-open sets (F, E) and (G, E) containing x and y, respectively.*

Proposition 2.3 ([15]). *A space $(X, \widetilde{\tau}, E)$ is $\tilde{s}p_c - T_0$ if and only if every soft points $x_e \neq y_{e'}$ implies $\tilde{s}p_c cl\{x_e\} \neq \tilde{s}p_c cl\{y_{e'}\}$.*

Proposition 2.4 ([15]). *A space $(X, \widetilde{\tau}, E)$ is $\tilde{s}p_c - T_1$ if and only if every soft point of the space $(X, \widetilde{\tau}, E)$ is an soft p_c-closed set.*

Proposition 2.5 ([15]). *If a soft topological space $(X, \widetilde{\tau}, E)$ is $\tilde{s}p_c - T_1$, then it is soft $\tilde{s}p_c - T_1^*$.*

Definition 2.19 ([16]). *Let $SP(X)_E$ and $SP(Y)_B$ be families of soft sets. Let $u : X \to Y$ and $p : E \to B$ be mappings. Then, a mapping $f_{pu} : SP(X)_E \to SP(Y)_B$ is defined as:*

1. *Let (F, E) be a soft set in $SP(X)_E$. The image of (F, E) under f_{pu}, written as $f_{pu}(F, E) = (f_{pu}(F), p(E))$, is a soft set in $SP(Y)_B$ such that*

$$f_{pu}(F)(e') = \begin{cases} \bigcup\limits_{e \in p^{-1}(e') \cap E} u(F(e)), & if \quad p^{-1}(e') \cap E \neq \phi \\ \phi, & if \quad p^{-1}(e') \cap E = \phi, \end{cases}$$

   for all $e' \in B$.

2. *Let (G, B) be a soft set in $SP(Y)_B$. Then, the inverse image of (G, B) under f_{pu}, written as $f_{pu}^{-1}(G, B) = (f_{pu}^{-1}(G), p^{-1}(B))$, is a soft set in $SP(X)_E$ such that*

$$f_{pu}^{-1}(G)(e) = \begin{cases} u^{-1}(G(p(e))), & if \quad p(e) \in B \\ \phi, & otherwise, \end{cases}$$

   for all $e \in E$.

The soft function f_{pu} is called surjective if p and u are surjective and it is called injective if p and u are injective.

Definition 2.20 ([22]). *Let $(X, \widetilde{\tau}, E)$ and $(Y, \widetilde{\mu}, B)$ be two soft topological spaces. A soft mapping $f_{pu}:(X, \widetilde{\tau}, E) \to (Y, \widetilde{\mu}, B)$ is called soft continuous if $f_{pu}^{-1}((G, B))\widetilde{\in}\widetilde{\tau}$, for all $(G, B)\widetilde{\in}\widetilde{\mu}$.*

3. Soft p_c-regular spaces

In this section, we introduce some types of soft regular spaces by using soft p_c-open sets. Many characterizations of these spaces are found. Also, some hereditary properties and relations between these spaces are investigated.

Definition 3.1. *A soft space $\widetilde{X}$ is said to be $\tilde{s}p_c$-regular (resp., $\tilde{s}p_c^*$-regular), if for each $x_e\widetilde{\in}\widetilde{X}$ and each soft closed (resp., $\tilde{s}p_c$-closed) set (K,E) such that $x_e\widetilde{\notin}(K,E)$, there exist two disjoint soft p_c-open sets (F,E) and (G,E) such that $x_e\widetilde{\in}(F,E)$ and $(K,E)\sqsubseteq(G,E)$.*

Remark 3.1. In a finite soft space $SP(X)_E$, if (F,E) is any soft p_c-open set, then by definition it is soft pre-open and a union of soft closed sets and hence it is soft closed, so we obtain that (F,E) is both soft open and soft closed.

Equivalently, any soft p_c-closed set is both open and closed.

From the above remark, we get the following result

Proposition 3.1. *If $SP(X)_E$ is finite, then every $\tilde{s}p_c$-regular space is both $\tilde{s}p_c^*$-regular and soft regular.*

The following example shows that an $\tilde{s}p_c$-regular space is not necessary $\tilde{s}p_c-T_i$ for $i=0,1,2$.

Example 3.1. Let $X=\{x,y\}$, $E=\{e_1,e_2\}$ and $\widetilde{X}=\{(e_1,X),(e_2,X)\}$ and let $\widetilde{\tau}=\{\widetilde{X},\widetilde{\phi},(F_1,E),(F_2,E),(F_3,E),(F_4,E),(F_5,E),(F_6,E)\}$, where $(F_1,E)=\{(e_1,\{x\}),(e_2,\phi)\}$, $(F_2,E)=\{(e_1,\{x\}),(e_2,X)\}$, $(F_3,E)=\{(e_1,\{y\}),(e_2,\phi)\}$, $(F_4,E)=\{(e_1,\{y\}),(e_2,X)\}$, $(F_5,E)=\{(e_1,X),(e_2,\phi)\}$, $(F_6,E)=\{(e_1,\phi),(e_2,X)\}$. Then, it can be checked that $\tilde{s}p_cO(X)=\widetilde{\tau}$. Since $x_{e_2}\neq y_{e_2}$ and each soft open set containing one of them contains the other, so it is not $\tilde{s}p_c-T_i$ for $i=0,1,2$. This space is $\tilde{s}p_c-T_i^*$ for $i=0,1$ but it is not $\tilde{s}p_c-T_2^*$. By easy calculation it can be shown that this space is $\tilde{s}p_c$-regular and hence by Proposition 3.1 it is both $\tilde{s}p_c^*$-regular and soft regular.

Example 3.2. Let $X=\{x,y\}$, $E=\{e_1,e_2\}$ and $\widetilde{X}=\{(e_1,X),(e_2,X)\}$ and let $\widetilde{\tau}=\{\widetilde{X},\widetilde{\phi},(F_1,E),(F_2,E),(F_3,E),(F_4,E)\}$, where $(F_1,E)=\{(e_1,X),(e_2,\phi)\}$, $(F_2,E)=\{(e_1,\phi),(e_2,\{x,y\})\}$, $(F_3,E)=\{(e_1,\{y\}),(e_2,X)\}$ and $(F_4,E)=\{(e_1,\{y\}),(e_2,\phi)\}$. Since $y_{e_1}\widetilde{\notin}(F_3,E)^c$ but there are no disjoint soft p_c-open sets containing them. Hence, this space is not $\tilde{s}p_c$-regular and not soft regular but it can be checked that it is $\tilde{s}p_c^*$-regular.

Recall that a soft space $(X,\widetilde{\tau},E)$ is called soft-Alexandroff space [20] if any arbitrary intersection of soft open sets is soft open. Equivalently, any arbitrary union of soft closed sets is soft closed.

Proposition 3.2. *Every soft-Alexandroff space is $\tilde{s}p_c^*$-regular.*

Proof. Similar to Remark 3.1, in a soft-Alexandroff space $(X, \widetilde{\tau}, E)$. If (F, E) is an $\tilde{s}p_c^*$-open set, then it is soft closed and hence (F, E) and its complement are both soft open and soft closed. Therefore, for each $x_e \widetilde{\notin} (F, E)$, we have (F, E) and $(F, E)^c$ are the required disjoint soft $\tilde{s}p_c^*$-open sets. □

If we take $X = \mathbb{R}$ with the usual topology and if E consists only one parameter, then $\mathbb{R}$ is both soft regular and $\tilde{s}p_c^*$-regular but it is not soft-Alexandroff.

Theorem 3.1. *The following statements about a space $\tilde{X}$ are equivalent:*

1. *$\tilde{X}$ is $\tilde{s}p_c^*$-regular (resp., $\tilde{s}p_c$-regular) space.*
2. *For each $x_e \widetilde{\in} \widetilde{X}$ and each soft p_c-open (resp., soft open) set (F, E) containing x_e, there exist soft p_c-open set (G, E) containing x_e such that $x_e \widetilde{\in} (G, E) \sqsubseteq \tilde{s}p_c cl(G, E) \sqsubseteq (F, E)$.*
3. *Each element of X has an $\tilde{s}p_c$- neighborhood (resp., soft neighborhood) base consisting of soft p_c-closed sets.*
4. *Every soft p_c-closed (resp., soft closed) set (K, E) is the intersection of all soft p_c-closed neighborhoods of (K, E).*
5. *For every non-empty soft subset (F, E) of $\tilde{X}$ and every soft p_c-open (resp., soft open) subset (G, E) of $\tilde{X}$ such that $(F, E) \sqcap (G, E) \neq \widetilde{\phi}$, there exist $\tilde{s}p_c$- open subset (W, E) of $\tilde{X}$ such that $(F, E) \sqcap (W, E) \neq \widetilde{\phi}$, and $\tilde{s}p_c cl(W, E) \sqsubseteq (G, E)$.*
6. *For every non-empty soft subset (F, E) of $\tilde{X}$ and every soft p_c-closed (resp., soft closed) subset (K, E) of $\tilde{X}$ such that $(F, E) \sqcap (K, E) = \widetilde{\phi}$, there exist two soft p_c-open subset (G, E) and (W, E) such that $(F, E) \sqcap (G, E) \neq \widetilde{\phi}$, $(W, E) \sqcap (G, E) = \widetilde{\phi}$ and $(K, E) \sqsubseteq (W, E)$.*

Proof. We only prove the $\tilde{s}p_c^*$-regular case. Since the other case can be proved similarly.

(1) → (2). Let (F, E) be soft p_c-open set and $x_e \widetilde{\in} (F, E)$. Then, $\widetilde{X} \setminus (F, E)$ is a soft p_c-closed set such that $x_e \widetilde{\notin} \widetilde{X} \setminus (F, E)$. By $\tilde{s}p_c^*$-regularity of X, there are soft p_c-open sets (G, E), (H, E) such that $x_e \widetilde{\in} (G, E)$, $\widetilde{X} \setminus (F, E) \sqsubseteq (H, E)$ and $(H, E) \sqcap (G, E) = \widetilde{\phi}$. Therefore, $x_e \widetilde{\in} (G, E) \sqsubseteq \widetilde{X} \setminus (H, E) \sqsubseteq (F, E)$, Hence, $x_e \widetilde{\in} (G, E) \sqsubseteq \tilde{s}p_c cl(G, E) \sqsubseteq \tilde{s}p_c cl(\widetilde{X} \setminus (H, E)) = \widetilde{X} \setminus (H, E) \sqsubseteq (F, E)$. This gives $\tilde{s}p_c cl(G, E) \sqsubseteq \widetilde{X} \setminus (H, E) \sqsubseteq (F, E)$. Consequently, $x_e \widetilde{\in} (G, E)$ and $\tilde{s}p_c cl(G, E) \sqsubseteq (F, E)$.

(2) → (3). Let $y_{e'} \widetilde{\in} \widetilde{X}$. Then, for every soft p_c-open set (G, E) such that $y_{e'} \widetilde{\in} (G, E)$, $\tilde{s}p_c cl(G, E) \sqsubseteq (F, E)$. Thus, for each $y_{e'} \widetilde{\in} \widetilde{X}$, the sets $\tilde{s}p_c cl(G, E)$ form an $\tilde{s}p_c$- neighborhood base consisting of soft p_c-closed sets of $\widetilde{X}$. This proves (3).

(3) → (1). Let (K, E) be soft p_c-closed set which does not contain x_e. Then, $\widetilde{X} \setminus (K, E)$ is soft p_c-open, so it is $\tilde{s}p_c$- neighborhood of x_e. By (3), there is

soft p_c-closed set (L,E) which contains x_e and it is an $\tilde{s}p_c$- neighborhood of x_e with $(L,E) \sqsubseteq \widetilde{X} \setminus (K,E)$. Consider the sets (L,E) and $\widetilde{X} \setminus (L,E)$. Then, $x_e \widetilde{\in} (L,E)$, $(K,E) \sqsubseteq \widetilde{X} \setminus (L,E) = (G,E)$ and $(K,E) \sqcap (L,E) = \widetilde{\phi}$. Therefore, $\widetilde{X}$ is $\tilde{s}p_c^*$-regular.

(2) $\to$ (4). Let (K,E) be soft p_c-closed and $x_e \widetilde{\notin} (K,E)$. Then, $x_e \widetilde{\in} \widetilde{X} \setminus (K,E)$ and $\widetilde{X} \setminus (K,E)$ is $\tilde{s}p_c$- open subset of $\widetilde{X}$. Using the hypothesis, there exists an soft p_c-open set (F,E) such that $x_e \widetilde{\in} (F,E) \sqsubseteq \tilde{s}p_c cl(F,E) \sqsubseteq \widetilde{X} \setminus (K,E)$. Hence, $(K,E) \sqsubseteq \widetilde{X} \setminus \tilde{s}p_c cl(F,E) \sqsubseteq \widetilde{X} \setminus (F,E)$. Consequently $\widetilde{X} \setminus (F,E)$ is soft p_c-closed neighborhood of (K,E) to which x_e does not belong. This proves (4).

(4) $\to$ (5). Let $\phi \neq (F,E) \sqsubseteq \tilde{X}$ and (G,E) be a soft p_c-open subset of $\widetilde{X}$ such that $(F,E) \sqcap (G,E) \neq \widetilde{\phi}$. Let $x_e \widetilde{\in} (F, \ E) \sqcap (G, \ E)$. Since $x_e \widetilde{\notin} \widetilde{X} \setminus (G,E)$ and $\widetilde{X} \setminus (G,E)$ is soft p_c-closed, so there exists an soft p_c-closed neighborhood of $\widetilde{X} \setminus (G,E)$ say (E,E), such that $x_e \widetilde{\notin} (E,E)$. Let $\widetilde{X} \setminus (G,E) \sqsubseteq (D,E) \sqsubseteq (E,E)$ where (D,E) is soft p_c-open set. Then, $(W,E) = \widetilde{X} \setminus (E,E)$ is soft p_c-open set, $x_e \widetilde{\in} (W, \ E)$ and $(F,E) \sqcap (W,E) \neq \widetilde{\phi}$. Also, $\widetilde{X} \setminus (D,E)$ being soft p_c-closed. $\tilde{s}p_c cl(W,E) = \tilde{s}p_c cl(\widetilde{X} \setminus (E,E)) \sqsubseteq \widetilde{X} \setminus (D,E) \sqsubseteq (G,E)$.

(5) $\to$ (6). Let $\phi \neq (F,E) \sqsubseteq \tilde{X}$ and (K,E) be soft p_c-closed subset of $\widetilde{X}$ such that $(K,E) \sqcap (F,E) = \widetilde{\phi}$, then $\widetilde{X} \setminus (K,E) \sqcap (F,E) \neq \widetilde{\phi}$, and $\widetilde{X} \setminus (K,E)$ is soft p_c-open. Using (5), there exists an soft p_c-open subset $G,E)$ of $\widetilde{X}$ such that $(G,E) \sqcap (F,E) \neq \widetilde{\phi}$ and $(G,E) \sqsubseteq \tilde{s}p_c cl(G,E) \sqsubseteq \widetilde{X} \setminus (K,E)$. Putting $(W,E) = \widetilde{X} \setminus \tilde{s}p_c cl(G,E)$, then $(K,E) \sqsubseteq (W,E) \sqsubseteq \widetilde{X} \setminus (G,E)$, and (W,E) is soft p_c-open. Hence the proof.

(6) $\to$ (1). Let $x_e \widetilde{\notin} (K,E)$, where (K,E) is soft p_c-closed, and let $(F,E) = \{x_e\} \neq \phi$, Then, $(K,E) \sqcap (F,E) = \widetilde{\phi}$ and hence, using (6) there exist two soft p_c-open sets (G,E), and (W,E) such that $(W,E) \sqcap (G,E) = \widetilde{\phi}$, $(G,E) \sqcap (F,E) \neq \widetilde{\phi}$ and $(K,E) \sqsubseteq (W,E)$, which implies that $\widetilde{X}$ is $\tilde{s}p_c^*$-regular. □

Theorem 3.2. *A topological space $(X, \widetilde{\tau}, E)$ is $\tilde{s}p_c^*$-regular (resp., $\tilde{s}p_c$-regular) if and only if for each $x_e \widetilde{\in} \widetilde{X}$ and soft p_c-closed (resp., soft closed) set (K,E) such that $x_e \widetilde{\notin} (K,E)$, there exist soft p_c-open sets (G,E), (H,E) such that $x_e \widetilde{\in} (G,E)$, $(K,E) \sqsubseteq (H,E)$ and $\tilde{s}p_c cl(G,E) \sqcap \tilde{s}p_c cl(H,E) = \widetilde{\phi}$.*

Proof. We only prove the $\tilde{s}p_c^*$- regular case because the other case can be proved similarly.

Suppose that $\widetilde{X}$ is $\tilde{s}p_c^*$-regular, then for each $x_e \widetilde{\in} \widetilde{X}$ and soft p_c-closed set (K,E) such that $x_e \widetilde{\notin} (K,E)$, there exist two soft p_c-open sets (U,E) and (V,E) such that $x_e \widetilde{\in} (U,E)$, $(K,E) \sqsubseteq (V,E)$ and $(U,E) \sqcap (V,E) = \phi$. Which implies that $x_e \widetilde{\in} (U,E) \sqsubseteq \widetilde{X} \setminus (V,E) \sqsubseteq \widetilde{X} \setminus (K,E)$. That is $x_e \widetilde{\in} (U,E) \sqsubseteq \tilde{s}p_c cl(U,E) \sqsubseteq \widetilde{X} \setminus (V,E) \sqsubseteq \widetilde{X} \setminus (K,E)$. Using Theorem 3.1(2) and the fact that $x_e \widetilde{\in} (U,E)$, where (U,E) is soft p_c-open set there exist soft p_c-open (G,E) containing x_e such that $x_e \widetilde{\in} (G,E) \sqsubseteq \tilde{s}p_c cl(G,E) \sqsubseteq (U,E))$. Therefore, $(K,E) \sqsubseteq (V,E) \sqsubseteq \widetilde{X} \setminus \tilde{s}p_c cl(U,E) \sqsubseteq \widetilde{X} \setminus (U,E) \sqsubseteq \widetilde{X} \setminus \tilde{s}p_c cl(G,E)$ and $(K,E) \sqsubseteq (V,E) \sqsubseteq \tilde{s}p_c cl(V,E) \sqsubseteq \widetilde{X} \setminus (U,E)$, Now take $(H,E) = (V,E)$, we get $x_e \widetilde{\in} (G,E)$, $(K,E) \sqsubseteq (H,E)$ and

$\tilde{s}p_ccl(G,E) \sqcap \tilde{s}p_ccl(H,E) = \phi$. This proves the necessity part. The proof of sufficiency follows directly. □

Lemma 3.1. *Every soft clopen subspace of an $\tilde{s}p_c$-regular space $\widetilde{X}$ is $\tilde{s}p_c$-regular.*

Proof. Let $\widetilde{Y}$ be a soft clopen subspace of $\tilde{s}p_c$-regular space $\widetilde{X}$. Suppose that (H,E) is soft p_c-closed set in $\widetilde{Y}$ and $y_{e'}\widetilde{\in}\widetilde{Y}$ such that $y_{e'}\widetilde{\notin}(H,E)$. Then, $(H,E) = (G,E) \sqcap Y$, where (G,E) is soft p_c-closed in $\widetilde{X}$. Then, $y_{e'}\widetilde{\notin}(G,E)$. Since $\widetilde{X}$ is $\tilde{s}p_c$-regular, there exist disjoint soft p_c-open sets (U,E), (V,E) in $\widetilde{X}$ such that $y_{e'}\widetilde{\in}(U,E)$, $(H,E) \sqsubseteq (V,E)$. Then, $(U,E) \sqcap Y$ and $(V,E) \sqcap Y$ are disjoint soft p_c-open sets in $\widetilde{Y}$ containing $y_{e'}$ and (H,E), respectively. This completes the proof. □

Remark 3.2. If the soft space $\widetilde{X}$ is finite, then by Remark 3.1, every soft p_c-open set is both closed and open and hence we obtain that Lemma 3.1 is true foe every subspace. Lemma 3.1 is true because the intersection of an soft p_c-open set in $\widetilde{X}$ with a soft clopen subspace remains an soft p_c-open set in the subspace but still we ask the following question.

Every soft subspace of an $\tilde{s}p_c$-regular space $\widetilde{X}$ is $\tilde{s}p_c$-regular or not ?.

Theorem 3.3. *Every $\tilde{s}p_c$-regular and $\tilde{s}p_c - T_0$ space $\widetilde{X}$ is an $\tilde{s}p_c{-}T_2$ space.*

Proof. Let $x_e,\ y_{e'}\widetilde{\in}\widetilde{X}$ such that $x_e \neq y_{e'}$. Since $\widetilde{X}$ is $\tilde{s}p_c - T_0$, then there exists an soft p_c-open set (U,E) containing x_e but not $y_{e'}$. Using the hypothesis that $\widetilde{X}$ is $\tilde{s}p_c$-regular and since $x_e\widetilde{\in}(U,E)$, so there is an soft p_c-open set (V,E), such that $x_e\widetilde{\in}(V,E) \sqsubseteq \tilde{s}p_ccl(V,E) \sqsubseteq (U,E)$. But $y_{e'}\widetilde{\notin}(U,E)$ implies that $y_{e'}\widetilde{\notin}\tilde{s}p_ccl(V,E)$, then we get $y_{e'}\widetilde{\in}\widetilde{X} \setminus \tilde{s}p_ccl(V,E)$. Therefore, we have (U,E) and $\widetilde{X} \setminus \tilde{s}p_ccl(V,E)$ are soft p_c-open sets such that $x_e\widetilde{\in}(U,E)$, $y_{e'}\widetilde{\in}\widetilde{X} \setminus \tilde{s}p_ccl(V,E)$ and $\widetilde{X} \setminus \tilde{s}p_ccl(V,E) \sqcap (U,E) = \widetilde{\phi}$. Hence, the result follows. □

The proof of the following lemma is obvious.

Lemma 3.2. *Let $(X,\widetilde{\tau},E)$ be an $\tilde{s}p_c$-regular (resp., an $\tilde{s}p_c^*$-regular) space and let (H,E) be a soft closed (resp., soft p_c-closed) set such that $x_e\widetilde{\notin}(H,E)$, then there exists an soft p_c-open set (F,E) such that $x_e\widetilde{\in}(F,E)$ and $(F,E) \sqcap (H,E) = \widetilde{\phi}$.*

Proposition 3.3. *A soft topological space is $\tilde{s}p_c$-regular (resp., an $\tilde{s}p_c^*$-regular) if and only if for each soft point $x_e\widetilde{\in}SP(X)_E$ and for each soft open (resp., soft p_c-open) set (F,E) containing x_e, there exists an soft p_c-open set (U,E) of x_e such that $\tilde{s}p_ccl(U,E)\sqsubseteq(F,E)$.*

Proof. Let $(X,\widetilde{\tau},E)$ be $\tilde{s}p_c$-regular space. Let $x_e\widetilde{\in}\widetilde{X}$ and (F,E) is an soft p_c-open set containing x_e. Then, $X \setminus (F,E)$ is an soft p_c-closed set such that $x_e\widetilde{\notin}\widetilde{X} \setminus (F,E)$. Since $(X,\widetilde{\tau},E)$ is an $\tilde{s}p_c$-regular, so there exist soft p_c-open sets (V,E) and (U,E) such that $x_e\widetilde{\in}(U,E)$, $X\setminus(F,E)\sqsubseteq(V,E)$ and $(U,E)\sqcap(V,E) = \widetilde{\phi}$. Thus, $(U,E)\sqsubseteq X \setminus (V,E)$ and hence $\tilde{s}p_ccl(U,E)\sqsubseteq X \setminus (V,E)\sqsubseteq(F,E)$.

Conversely, let $x_e \widetilde{\in} \widetilde{X}$ and (H,E) be an soft p_c-closed set such that $x_e \widetilde{\notin} (H,E)$. Then, $X \setminus (H,E)$ is an soft p_c-open set containing x_e. So, by hypothesis there exist an soft p_c-open set (U,E) of x_e such that $\tilde{s}p_c cl(U,E) \sqsubseteq X \setminus (H,E)$. Thus, $(H,E) \sqsubseteq X \setminus \tilde{s}p_c cl(U,E)$ and $(U,E) \sqcap X \setminus \tilde{s}p_c cl(U,E) = \widetilde{\phi}$. Therefore, $(X,\widetilde{\tau},E)$ is $\tilde{s}p_c$-regular.

The proof when $(X,\widetilde{\tau},E)$ is $\tilde{s}p_c$-regular is analogues. $\square$

Definition 3.2. *A soft topological space $(X,\widetilde{\tau},E)$ is said to be strongly $\tilde{s}p_c^*$-regular (resp., strongly $\tilde{s}p_c$-regular), if for every soft p_c-closed (resp., soft closed) set (H,E) and every point $x \widetilde{\notin} (H,E)$, there exists disjoint soft p_c-open sets (F,E) and (G,E) such that $x \widetilde{\in} (F,E)$ and $(H,E) \sqsubseteq (G,E)$.*

Example 3.3. Let $X = \{x,y\}$, $E = \{e_1,e_2\}$ and $\widetilde{X} = \{(e_1,X),(e_2,X)\}$ and let $\widetilde{\tau} = \{\widetilde{X},\widetilde{\phi},(F_1,E),(F_2,E)\}$, where $(F_1,E) = \{(e_1,\{x\}),(e_2,\{x\})\}$, $(F_2,E) = \{(e_1,\{y\}),(e_2,\{y\})\}$. Then, it is not difficult to check that $(X,\widetilde{\tau},E)$ is both strongly $\tilde{s}p_c^*$-regular and strongly $\tilde{s}p_c$-regular.

The following result is obvious.

Proposition 3.4. *Every strongly $\tilde{s}p_c^*$-regular (resp., strongly $\tilde{s}p_c$-regular) space is $\tilde{s}p_c^*$-regular (resp., $\tilde{s}p_c$-regular).*

The converse of Proposition 3.4 is not true in general. The space in Example 3.1, is $\tilde{s}p_c^*$-regular and $\tilde{s}p_c$-regular but it is neither strongly $\tilde{s}p_c^*$-regular nor strongly $\tilde{s}p_c$-regular.

We shall prove all the results related to strongly $\tilde{s}p_c^*$-regular spaces and the proof of the results related to strongly $\tilde{s}p_c$-regular can be done in a similar way.

Lemma 3.3. *If $(X,\widetilde{\tau},E)$ is strongly $\tilde{s}p_c^*$-regular (resp., strongly $\tilde{s}p_c$-regular) space and (H,E) is an soft p_c-closed (resp., soft closed) set such that $x \widetilde{\notin} (H,E)$, then there exists an soft p_c-open set (F,E) such that $x \widetilde{\in} (F,E)$ and $(F,E) \sqcap (H,E) = \widetilde{\phi}$.*

Proposition 3.5. *A soft topological space $(X,\widetilde{\tau},E)$ is strongly $\tilde{s}p_c^*$-regular (resp., strongly $\tilde{s}p_c$-regular) if and only if for each point $x \in X$ and for each soft p_c-open (resp., soft open) set (F,E) containing x, there exists an soft p_c-open set (U,E) containing x such that $\tilde{s}p_c cl(U,E) \sqsubseteq (F,E)$.*

Proof. Let $(X,\widetilde{\tau},E)$ be a strongly $\tilde{s}p_c^*$-regular space. Let $x \in X$ and (F,E) be an soft p_c-open set containing x. Then, $X \setminus (F,E)$ is an soft p_c-closed set such that $x \widetilde{\notin} \widetilde{X} \setminus (F,E)$. Since $(X,\widetilde{\tau},E)$ is $\tilde{s}p_c^*$-regular, then there exist soft p_c-open sets (V,E) and (U,E) such that $x \widetilde{\in} (U,E)$, $X \setminus (F,E) \sqsubseteq (V,E)$ and $(U,E) \sqcap (V,E) = \widetilde{\phi}$. Thus, $(U,E) \sqsubseteq X \setminus (V,E)$ and hence $\tilde{s}p_c cl(U,E) \sqsubseteq X \setminus (V,E) \sqsubseteq (F,E)$.

Conversely, let $x \in X$ and (H,E) be an soft p_c-closed set such that $x \widetilde{\notin} (H,E)$. Then, $X \setminus (H,E)$ is an soft p_c-open set containing x. So, by hypothesis there exists an soft p_c-open set (U,E) containing x such that $\tilde{s}p_c cl(U,E) \sqsubseteq X \setminus (H,E)$. Thus, $(H,E) \sqsubseteq X \setminus \tilde{s}p_c cl(U,E)$ and $(U,E) \sqcap X \setminus \tilde{s}p_c cl(U,E) = \widetilde{\phi}$. Therefore, $(X,\widetilde{\tau},E)$ is strongly $\tilde{s}p_c^*$-regular. $\square$

Proposition 3.6. *Let $(X, \widetilde{\tau}, E)$ be a soft topological space and $x \in X$. If $(X, \widetilde{\tau}, E)$ is a strongly $\tilde{s}p_c^*$-regular (resp., strongly $\tilde{s}p_c$-regular) space, then the following statements are true:*

1. *$x \widetilde{\notin}(H, E)$ if and only if $(x, E) \sqcap (H, E) = \widetilde{\phi}$ for every soft p_c-closed (resp., soft closed) set (H, E).*

2. *$x \widetilde{\notin}(F, E)$ if and only if $(x, E) \sqcap (F, E) = \widetilde{\phi}$ for every soft p_c-open (resp., soft open) set (F, E).*

Proof. (1) Let $x \widetilde{\notin}(H, E)$, then by Lemma 3.3, there exists an $\tilde{s}p_c$- open set (F, E) such that $x \widetilde{\in}(F, E)$ and $(F, E) \sqcap (H, E) = \widetilde{\phi}$. Since $(x, E) \sqsubseteq (F, E)$, we have $(x, E) \sqcap (H, E) = \widetilde{\phi}$.

Conversely, straightforward.

(2) Let $x \widetilde{\notin}(F, E)$. Then, we have two cases:

(i) $x \notin F(\alpha)$, for all $e \in E$, it is obvious that $(x, E) \sqcap (F, E) = \widetilde{\phi}$.

(ii) $x \notin F(\alpha)$ and $x \in F(\beta)$ for some α, $\beta \in E$, then we have $x \in X \setminus F(\alpha)$ and $x \widetilde{\notin} \widetilde{X} \setminus F(\beta)$ for some α, $\beta \in E$ and so $\widetilde{X} \setminus (F, E)$ is an soft p_c-closed set such that $x \widetilde{\notin} \widetilde{X} \setminus (F, E)$, by (1), $(x, E) \sqcap \widetilde{X} \setminus (F, E) = \widetilde{\phi}$. So, $(x, E) \sqsubseteq (F, E)$ but this contradicts that $x \widetilde{\notin} F(\alpha)$ for some $\alpha \in E$. Consequently, we have $(x, E) \sqcap (F, E) = \widetilde{\phi}$.

The converse part is obvious. □

Proposition 3.7. *Let $(X, \widetilde{\tau}, E)$ be a soft topological space and $x \in X$. Then, the following statements are equivalent:*

1. *$(X, \widetilde{\tau}, E)$ is a strongly $\tilde{s}p_c^*$-regular (resp., strongly $\tilde{s}p_c$-regular) space,*

2. *For each soft p_c-closed (resp., soft closed) set (H, E) such that $(x, E) \sqcap (H, E) = \widetilde{\phi}$, there exist soft p_c-open sets (F, E) and (G, E) such that such that $(x, E) \sqsubseteq (F, E)$, $(H, E) \sqsubseteq (G, E)$ and $(F, E) \sqcap (G, E) = \widetilde{\phi}$.*

Proof. Follows from Proposition 3.6(1) and Lemma 3.3. □

Proposition 3.8. *Let $(X, \widetilde{\tau}, E)$ be a soft topological space and $x \in X$. If $(X, \widetilde{\tau}, E)$ is a strongly $\tilde{s}p_c^*$-regular (resp., strongly $\tilde{s}p_c$-regular), then the following statements are true:*

1. *For an soft p_c-open (resp., soft open) set (F, E), $x \widetilde{\in}(F, E)$ if and only if $x \in F(\alpha)$ for some $\alpha \in E$.*

2. *For an soft p_c-open (resp., soft open) set (F, E), $(F, E) = \sqcup\{(x, E) : x \in F(\alpha)$ for some $\alpha \in E\}$.*

Proof. (1). Suppose that $x \in F(\alpha)$ and $x \widetilde{\notin}(F, E)$ for some $\alpha \in E$. Then, by Proposition 3.7(2), $(x, E) \sqcap (F, E) = \widetilde{\phi}$. By our assumption, this is a contradiction and so $x \in (F, E)$. The Converse is obvious.

(2). Follows from part (1) and Remark 2.1. □

Proposition 3.9. *Let $(X, \widetilde{\tau}, E)$ be a soft topological space and $x \in X$. If $(X, \widetilde{\tau}, E)$ is strongly $\tilde{s}p_c^*$-regular, then the following statements are equivalent:*

1. *$(X, \widetilde{\tau}, E)$ is a $\tilde{s}p_c$-T_1^* space,*
2. *For $x, y \in X$ with $x \neq y$, there exist soft p_c-open sets (F, E) and (G, E) such that $(x, E) \sqsubseteq (F, E)$ and $(y, E) \sqcap (F, E) = \widetilde{\phi}$, $(y, E) \sqsubseteq (G, E)$ and $(x, E) \sqcap (G, E) = \widetilde{\phi}$.*

Proof. It is clear that $x \widetilde{\in} (F, E)$ if and only if $(x, E) \sqsubseteq (F, E)$, and by Proposition 3.8(2), $x \widetilde{\notin} (F, E)$ if and only if $(x, E) \sqcap (F, E) = \widetilde{\phi}$. Hence, statements (1) and (2) are equivalent. $\square$

4. Soft p_c-normal spaces

In this section, we define $\tilde{s}p_c$-normal spaces and derive many of its properties. The relationship to other soft spaces and its image under $\tilde{s}p_c$-continuous functions are discussed.

Definition 4.1. *A soft space $\widetilde{X}$ is said to be $\tilde{s}p_c$-normal (resp., $\tilde{s}p_c^*$-normal) space, if for any disjoint soft closed (resp., $\tilde{s}p_c^*$-closed) sets (K, E) and (L, E) of $\widetilde{X}$, there exist soft p_c-open sets (U, E), (V, E) such that $(K, E) \sqsubseteq (U, E)$, $(L, E) \sqsubseteq (V, E)$ and $(V, E) \sqcap (U, E) = \widetilde{\phi}$.*

Example 4.1. Let $X = \{x_1, x_2, x_3\}$, $E = \{e_1, e_2\}$ and let $\widetilde{\tau} = \{\widetilde{X}, \widetilde{\phi}, (F_1, E), (F_2, E), (F_3, E), (F_4, E)\}$, where $(F_1, E) = \{(e_1, \{x_1, x_2\}), (e_2, \{x_3\})\}$, $(F_2, E) = \{(e_1, \{x_3\}), (e_2, \{x_1, x_2\})\}$, $(F_3, E) = \{(e_1, \{x_1\}), (e_2, \phi)\}$, $(F_4, E) = \{(e_1, \{x_1, x_3\}), (e_2, \{x_1, x_2\})\}$. Then, this space is both $\tilde{s}p_c$-normal and $\tilde{s}p_c^*$-normal but it is not $\tilde{s}p_c$-regular.

Theorem 4.1. *A space $\widetilde{X}$ is an $\tilde{s}p_c^*$-normal space, if for each pair of soft p_c-open sets (U, E) and (V, E) in $\widetilde{X}$ such that $\widetilde{X} = (U, E) \sqcup (V, E)$, there are soft p_c-closed sets (G, E) and (H, E) which are contained in (U, E) and (V, E), respectively and $\tilde{X} = (G, E) \sqcup (H, E)$.*

Proof. Straightforward. $\square$

Theorem 4.2. *If $\widetilde{X}$ is any soft space, then the following statements are equivalent:*

1. *$\widetilde{X}$ is $\tilde{s}p_c^*$-normal,*
2. *For each $\tilde{s}p_c$- closed set (F_1, E) in $\widetilde{X}$ and soft p_c-open set (G, E) contains (F_1, E), there is an soft p_c-open set (U, E) such that $(F_1, E) \sqsubseteq (U, E) \sqsubseteq \tilde{s}p_c cl(U, E) \sqsubseteq (G, E)$,*
3. *For each $\tilde{s}p_c$- closed set (F_1, E) in $\widetilde{X}$ and soft p_c-open set (G, E) containing (F_1, E), there are soft p_c-open sets (U_n, E) for $n \in N$, such that $(F_1, E) \sqsubseteq \bigsqcup_{n \in N} (U_n, E)$ and $\tilde{s}p_c cl(U_n, E) \sqsubseteq (G, E)$, for each $n \in N$.*

Proof. (1) $\to$ (2). Since (G,E) is soft p_c-open set containing (F_1,E), then $\widetilde{X}\setminus(G,E)$ and (F_1,E) are disjoint soft p_c-closed sets in $\widetilde{X}$. Since $\widetilde{X}$ is $\tilde{s}p_c^*$-normal, so there exist soft p_c-open sets (U,E) and (V,E) such that $(F_1,E) \sqsubseteq (U,E)$, $\widetilde{X} \setminus (G,E) \sqsubseteq (V,E)$ and $(V,E) \sqcap (U,E) = \widetilde{\phi}$. Hence, $(F_1,E) \sqsubseteq (U,E) \sqsubseteq \tilde{s}p_c cl(U,E) \sqsubseteq \tilde{s}p_c cl(\widetilde{X} \setminus (V,E)) = \widetilde{X} \setminus (V,E) \sqsubseteq (G,E)$, or $(F_1,E) \sqsubseteq (U,E) \sqsubseteq \tilde{s}p_c cl(U,E) \sqsubseteq (G,E)$.

(2) $\to$ (3). Let (F_1,E) be an soft p_c-closed set and (G,E) be an soft p_c-open set in an $\tilde{s}p_c^*$-normal space $\widetilde{X}$ such that $(F_1,E) \sqsubseteq (G,E)$. So, by hypothesis, there is an soft p_c-open set (U,E) such that $(F_1,E) \sqsubseteq (U,E) \sqsubseteq \tilde{s}p_c cl(U,E) \sqsubseteq (G,E)$. If we put $(U_n,E) = (U,E)$, for all $n \in N$, the proof follows.

(3) $\to$ (1). Let (F_1,E) and (F_2,E) be a pair of disjoint soft p_c-closed set in the space $\widetilde{X}$, then $\widetilde{X} \setminus (F_2,E)$ is an soft p_c-open set in $\widetilde{X}$ containing (F_1,E). So, by hypothesis, there are soft p_c-open sets (U_n,E) for $n \in N$ such that

$$(F_1,E) \sqsubseteq \bigsqcup_{n\in N} (U_n,E)$$

and $\tilde{s}p_c cl(U_n,E) \sqsubseteq \widetilde{X} \setminus (F_2,E)$ for each $n \in N$. Since $\widetilde{X} \setminus (F_1,E)$ is an soft p_c-open subset of $\widetilde{X}$ containing the soft p_c-closed set (F_2,E), then by applying the condition of the theorem again, we get soft p_c-open sets (V_n,E) for $n \in N$, such that

$$(F_2,E) \sqsubseteq \bigsqcup_{n\in N} (V_n,E)$$

and $\tilde{s}p_c cl(V_n,E) \sqsubseteq \widetilde{X} \setminus (F_1,E)$ for each $n \in N$. Thus, $\tilde{s}p_c cl(U_n,E) \sqcap (F_2,E) = \widetilde{\phi}$ and $\tilde{s}p_c cl(V_n,E) \sqcap (F_1,E) = \widetilde{\phi}$ for each $n \in N$. Setting

$$(G_n,E) = (U_n,E) \setminus \bigsqcup_{n\in N} \tilde{s}p_c cl(V_n,E)$$

and

$$(H_n,E) = (V_n,E) \setminus \bigsqcup_{n\in N} \tilde{s}p_c cl(U_n,E).$$

Then

$$(U,E) = \bigsqcup_{n\in N} (G_n,E)$$

and

$$(V,E) = \bigsqcup_{n\in N} (H_n,E)$$

are disjoint soft p_c-open sets in $\widetilde{X}$ containing (F_1,E) and (F_2,E), respectively. Hence, $\widetilde{X}$ is $\tilde{s}p_c^*$-normal. □

Theorem 4.3. *A soft topological space $\widetilde{X}$ is $\tilde{s}p_c$-normal if and only if for each soft closed set (F_1,E) in $\widetilde{X}$ and soft open set (G,E) contains (F_1,E), there is an soft p_c-open set (U,E) such that $(F_1,E) \sqsubseteq (U,E) \sqsubseteq \tilde{s}p_c cl(U,E) \sqsubseteq (G,E)$.*

Proof. Let (F_1, E) be any soft close subset in an $\tilde{s}p_c$-normal space $\widetilde{X}$ and (G, E) be any soft open subset of $\widetilde{X}$ containing (F_1, E). Then, $\widetilde{X} \setminus (G, E)$ is closed and $\widetilde{X} \setminus (G, E) \sqcap (F_1, E) = \widetilde{\phi}$. Hence, by hypothesis, there exist two disjoint soft p_c-open sets (U, E) and (V, E) such that $(F_1, E) \sqsubseteq (U, E)$, $\widetilde{X} \setminus (G, E) \sqsubseteq (V, E)$ and $(V, E) \sqcap (U, E) = \widetilde{\phi}$. Since $(V, E) \sqcap (U, E) = \widetilde{\phi}$, then $(U, E) \sqsubseteq \widetilde{X} \setminus (V, E)$. But $\widetilde{X} \setminus (G, E) \sqsubseteq (V, E)$, then $\widetilde{X} \setminus (V, E) \sqsubseteq (G, E)$ and so $(U, E) \sqsubseteq (G, E)$. And since (U, E) and (V, E) are soft p_c-open sets, then $\widetilde{X} \setminus (V, E)$ and $\widetilde{X} \setminus (U, E)$ are soft p_c-closed sets and so $\tilde{s}p_c cl(\widetilde{X} \setminus (V, E)) = \widetilde{X} \setminus (V, E)$ and $\tilde{s}p_c cl(\widetilde{X} \setminus (U, E)) = \widetilde{X} \setminus (U, E)$ and then $(F_1, E) \sqsubseteq (U, E) \sqsubseteq \tilde{s}p_c cl(U, E) \sqsubseteq \tilde{s}p_c cl(\widetilde{X} \setminus (V, E)) = \widetilde{X} \setminus (V, E) \sqsubseteq (G, E)$. Thus, $(F_1, E) \sqsubseteq (U, E) \sqsubseteq \tilde{s}p_c cl(U, E) \sqsubseteq (G, E)$.

Conversely, let the condition be satisfied and let (F_1, E), (F_2, E) be two disjoint soft closed subsets of $\widetilde{X}$. Then, $(F_1, E) \sqsubseteq \widetilde{X} \setminus (F_2, E)$ and since (F_2, E) is soft closed then $\widetilde{X} \setminus (F_2, E)$ is a soft open subset containing (F_1, E). So, by hypothesis, there exist soft p_c-open sets (U, E) such that $(F_1, E) \sqsubseteq (U, E) \sqsubseteq \tilde{s}p_c cl(U, E) \sqsubseteq \widetilde{X} \setminus (F_2, E)$. Putting $(V, E) = \widetilde{X} \setminus \tilde{s}p_c cl(U, E)$, then there exist two disjoint soft p_c-open sets (U, E) and (V, E) such that $(F_1, E) \sqsubseteq (U, E)$ and $(F_2, E) \sqsubseteq (V, E)$. Therefore, $\widetilde{X}$ is $\tilde{s}p_c$-normal. $\square$

Theorem 4.4. *Every soft T_1, $\tilde{s}p_c$-normal space $\widetilde{X}$ is $\tilde{s}p_c$-regular.*

Proof. Let (F_1, E) be any soft closed subset in an $\tilde{s}p_c$-normal space $\widetilde{X}$ and $x_e \widetilde{\in} \widetilde{X}$ such that $x_e \widetilde{\notin} (F_1, E)$. Since $\widetilde{X}$ is soft T_1 space, then $\{x_e\}$ is soft closed subset in $\widetilde{X}$ with $\{x_e\} \sqcap (F_1, E) = \widetilde{\phi}$. By $\tilde{s}p_c$-normality of $\widetilde{X}$, there exist two disjoint soft p_c-open sets (U, E) and (V, E) of $\widetilde{X}$ such that $\{x_e\} \sqsubseteq (U, E)$, so $x_e \widetilde{\in} (U, E)$, $(F_1, E) \sqsubseteq (V, E)$ and $(U, E) \sqcap (V, E) = \widetilde{\phi}$. Thus, $\widetilde{X}$ is an $\tilde{s}p_c$-regular space. $\square$

Theorem 4.5. *If $\widetilde{Y}$ is a soft clopen subspace of an $\tilde{s}p_c$-normal (resp., $\tilde{s}p_c^*$-normal) space $\widetilde{X}$, then $\widetilde{Y}$ is $\tilde{s}p_c$-normal (resp., $\tilde{s}p_c^*$-normal).*

Proof. Let $\widetilde{X}$ be an $\tilde{s}p_c^*$-normal space and $\widetilde{Y}$ be a soft clopen subspace of $\widetilde{X}$. Let (K_1, E) and (K_2, E) be two disjoint soft p_c-closed subsets of $\widetilde{Y}$, then By Lemma 2.7, (K_1, E) and (K_2, E) are two disjoint soft p_c-closed subsets of $\widetilde{X}$. By $\tilde{s}p_c^*$ -normality of $\widetilde{X}$, there exist two soft p_c-open sets (F_1, E) and (F_2, E) such that $(K_1, E) \sqsubseteq (F_1, E)$, $(K_2, E) \sqsubseteq (F_2, E)$ and $(F_1, E) \sqcap (F_2, E) = \phi$, then $(K_1, E) \sqsubseteq (F_1, E) \sqcap \widetilde{Y}$ and $(K_2, E) \sqsubseteq (F_2, E) \sqcap \widetilde{Y}$. It follows from, $(F_1, E) \sqcap (F_2, E) = \widetilde{\phi}$, that $((F_1, E) \sqcap \widetilde{Y}) \sqcap ((F_2, E) \sqcap \widetilde{Y}) = \widetilde{\phi}$ and By Lemma 2.5, we have $((F_1, E) \sqcap \widetilde{Y})$ and $((F_2, E) \sqcap \widetilde{Y})$ are soft p_c-open subsets of $\widetilde{Y}$. Hence, $\widetilde{Y}$ is $\tilde{s}p_c^*$ -normal. $\square$

The following example shows that Theorem 4.5, is not true when $\widetilde{Y}$ is soft open or soft closed.

Example 4.2. Let $X = \{x, y, z\}$, $E = \{e_1, e_2\}$ and $\widetilde{X} = \{(e_1, X), (e_2, X)\}$, let $\widetilde{\tau} = \{\widetilde{X}, \widetilde{\phi}, (F_1, E), (F_2, E), (F_3, E), (F_4, E)\}$, where $(F_1, E) = \{(e_1, \{x\}), (e_2, X)\}$, $(F_2, E) = \{(e_1, \{y\}), (e_2, \{y\})\}$, $(F_3, E) = \{(e_1, \phi), (e_2, \{y\})\}$, $(F_4, E) = \{(e_1, \{x, y\}), (e_2, X)\}$. Then, $(X, \widetilde{\tau}, E)$ is both $\tilde{s}p_c^*$-normal and $\tilde{s}p_c$-normal space and the soft open set (F_4, E) is not $\tilde{s}p_c$-normal. Also, $(X, \widetilde{\tau}^c, E)$ is both $\tilde{s}p_c^*$-normal and $\tilde{s}p_c$-normal space and the soft closed set (F_4, E) is not $\tilde{s}p_c$-normal.

Theorem 4.6. *Every $\tilde{s}p_c^*$-normal $\tilde{s}p_c - T_2$ space $\widetilde{X}$ is $\tilde{s}p_c^*$-regular.*

Proof. Suppose that (F_1, E) is an soft p_c-closed set and $x_e \widetilde{\notin} (F_1, E)$ for each $x_e \widetilde{\in} \widetilde{X}$. Since $\widetilde{X}$ is an $\tilde{s}p_c - T_2$ space. Therefore, by Theorem 2.4, each $\{x_e\}$ is soft p_c-closed in $\widetilde{X}$. Since $\widetilde{X}$ is $\tilde{s}p_c^*$-normal, so there exist soft p_c-open sets (U, E), (V, E) such that $\{x_e\} \sqsubseteq (U, E)$, $(F_1, E) \sqsubseteq (V, E)$ and $(U, E) \sqcap (V, E) = \widetilde{\phi}$, this implies that $\widetilde{X}$ is $\tilde{s}p_c^*$-regular. □

Definition 4.2. *A soft mapping $f_{pu} : (X, \widetilde{\tau}, E) \to (Y, \widetilde{\mu}, B)$ is called an soft p_c-open mapping if and only if the image of every soft p_c-open set in $\widetilde{X}$ is an soft p_c-open in $\widetilde{Y}$.*

Proposition 4.1. *Let $(X, \widetilde{\tau}, E)$ and $(Y, \widetilde{\mu}, B)$ be soft topological spaces and $f_{pu} : SP(X)_E \to SP(Y)_B$ be a soft bijective and soft p_c-open mapping. If $(X, \widetilde{\tau}, E)$ is $\tilde{s}p_c$-T_i, then $(Y, \widetilde{\mu}, B)$ is $\tilde{s}p_c$-T_i spaces $(i = 0, 1, 2)$.*

Proof. We prove only the case for $\tilde{s}p_c$-T_0 space and the proof of the other are similar. Let $y_{\beta 1}$, $y_{\beta 2} \widetilde{\in} SP(Y)_B$ be two distinct soft points. Since f_{pu} is bijective, there exist distinct soft points x_{e1}, $x_{e2} \widetilde{\in} \widetilde{X}$ such that $f_{pu}(x_{e1}) = y_{\beta 1}$, $f_{pu}(x_{e2}) = y_{\beta 2}$. Since $(X, \widetilde{\tau}, E)$ is an $\tilde{s}p_c$-T_0 space, there exist soft p_c-open sets (F, E), (G, E) such that $x_{e1} \widetilde{\in} (F, E)$ and $x_{e2} \widetilde{\notin} (F, E)$ or $x_{e2} \widetilde{\in} (G, E)$ and $x_{e1} \widetilde{\notin} (G, E)$. As f_{pu} is an soft p_c-open mapping, then $f_{pu}(F, E)$, $f_{pu}(G, E)$ are soft p_c-open sets such that $y_{\beta 1} \widetilde{\in} f_{pu}(F, E)$ and $y_{\beta 2} \widetilde{\notin} f_{pu}(F, E)$ or $y_{\beta 2} \widetilde{\in} f_{pu}(G, E)$ and $y_{\beta 1} \widetilde{\notin} f_{pu}(G, E)$. This implies that, $(Y, \widetilde{\mu}, B)$ is $\tilde{s}p_c$-T_0. □

Definition 4.3. *A function $f_{pu} : (X, \widetilde{\tau}, E) \to (Y, \widetilde{\mu}, B)$ is injective soft point $\tilde{s}p_c$-closure if and only if for every $x_e, y_{e'} \widetilde{\in} \widetilde{X}$ such that $\tilde{s}p_c cl(\{x_e\}) \neq \tilde{s}p_c cl(\{y_{e'}\})$, then $\tilde{s}p_c cl(\{f(x_e)\}) \neq \tilde{s}p_c cl(\{f(y_{e'})\})$.*

It is clear that the identity function from any soft topological space onto itself is a function which satisfies Definition 4.3.

Theorem 4.7. *If a function $f_{pu} : (X, \widetilde{\tau}, E) \to (Y, \widetilde{\mu}, B)$ is injective soft point $\tilde{s}p_c$-closure and $\widetilde{X}$ is an $\tilde{s}p_c$-T_0 space, then f_{pu} is soft injective.*

Proof. Let $x_e, y_{e'} \widetilde{\in} \widetilde{X}$ with $x_e \neq y_{e'}$. Since $\widetilde{X}$ is $\tilde{s}p_c$-T_0, therefore by Proposition 2.3, $\tilde{s}p_c cl(\{x_e\}) \neq \tilde{s}p_c cl(\{y_{e'}\})$. But f_{pu} is $(1-1)$ soft point $\tilde{s}p_c$-closure, implies that $\tilde{s}p_c cl(\{f(x_e)\}) \neq \tilde{s}p_c cl(\{f(y_{e'})\})$. Hence, $f_{pu}(x_e)\}) \neq f_{pu}(y_{e'})$. Thus, f_{pu} is soft injective. □

5. Conclusion

Many topological notions are extended to the soft topology after introducing the concept of soft topological spaces. Several classes of soft sets are defined and applied to present many notions in soft topology. In this paper, we employ the notion of soft p_c-open set to introduce some types of soft regular and soft normal spaces and give many properties of these spaces. Also, we discuss relations between these spaces, hereditary properties and their images under soft p_c-continuous mappings.

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