Characterization of generalized n-semiderivations of 3-prime near rings and their structure

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Abstract. Let N be a near ring and n be a fixed positive integer. An n-additive (additive in each argument) mapping $F : \underbrace{N\times N\times \ldots \times N}_{n-times} \to N$ is said to be a permuting generalized n-semiderivation on a near ring N if there exists an n-semiderivation $d : \underbrace{N \times N \times ... \times N}_{n-times} \to N$ associated with a map $g : N \to N$ such that the relation $F(x_1x'_1, x_2, \ldots, x_n)=F(x_1, x_2, \ldots, x_n)x'_1+g(x_1)d(x'_1, x_2, \ldots, x_n)=d(x_1, x_2, \ldots, x_n)g(x'_1) +x_1F(x'_1, x_2, \ldots, x_n)$ and $g(F(x_1, x_2, \ldots, x_n)) = F(g(x_1), g(x_2), \ldots, g(x_n))$ hold, for all $x_1, x'_1, x_2, ..., x_n \in N$. The purpose of the present paper is to prove some commutativity theorems in case of a semigroup ideal of a 3-prime near ring admitting a generalized n-semiderivation, thereby extending some known results of derivations, semiderivations and generalized derivations.

Keywords: 3-prime near-rings, n-semiderivations, generalized n-semiderivations, semigroup ideals.

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1. Introduction

A left near ring N is a triplet $(N, +, \cdot)$, where $+$ and $\cdot$ are two binary operations such that (i) $(N, +)$ is a group (not necessarily abelian), (ii) $(N, \cdot)$ is a semigroup, and (iii) $x \cdot (y + z) = x \cdot y + x \cdot z$, for all $x, y, z \in N$. Analogously, if instead of (iii), N satisfies the right distributive law, then N is said to be a right near ring. The most natural example of a non-commutative left near ring is the set of all identity preserving mappings acting from right of an additive group G (not necessarily abelian) into itself with pointwise addition and composition of mappings as multiplication. If these mappings act from left on G, then we get a non-commutative right near ring (For more examples, we can refer Pilz [2]). Throughout the paper, N represents a zero-symmetric left near ring with multiplicative centre $Z(N)$ and for any pair of elements $x, y \in N$, the symbols $[x, y]$ and $(x \circ y)$ denote the Lie Product $xy - yx$ and Jordan product $xy + yx$. A near ring N is called zero-symmetric if $0x = 0$, for all $x \in N$ (recall that left distributivity yields that $x0 = 0$). A near ring N is said to be 3-prime if $xNy = \{0\}$ for $x, y \in N$ implies that $x = 0$ or $y = 0$. A near ring N is called 2-torsion free if $(N, +)$ has no element of order 2. A nonempty subset U of N is called a semigroup right (resp. semigroup left) ideal of N if $UN \subseteq U$ (resp. $NU \subseteq U$) and if U is both a semigroup right ideal and a semigroup left ideal, it is called a semigroup ideal. Let $n \geq 2$ be a fixed positive integer and $N^n = \underbrace{N \times N \times \ldots \times N}_{n-times}$. A map $\Delta : N^n \to N$ is said to be permuting on a near ring N if the relation $\Delta(x_1, x_2, \ldots, x_n) = \Delta(x_{\pi(1)}, x_{\pi(2)}, \ldots, x_{\pi(n)})$ holds, for all $x_i \in N$, $i = 1, 2, \ldots, n$ and for every permutation $\pi \in S_n$, where S_n is the permutation group on $\{1, 2, \ldots, n\}$. An additive mapping $F : N \to N$ is said to be a right (resp. left) generalized derivation with associated derivation d if $F(xy) = F(x)y + xd(y)$ (resp. $F(xy) = d(x)y + xF(y)$), for all $x, y \in N$ and F is said to be a generalized derivation with associated derivation d on N if it is both a right generalized derivation and a left generalized derivation on N with associated derivation d.

Ozturk et. al. [6] and Park et. al. [5] studied bi-derivations and tri-derivations in near rings. A symmetric bi-additive mapping $d : N \times N \to N$ (i.e., additive in both arguments) is said to be a symmetric bi-derivation on N if $d(xy, z) = d(x, z)y + xd(y, z)$ holds, for all $x, y, z \in N$. A permuting tri-additive mapping $d : N \times N \times N \to N$ is said to be a permuting tri-derivation on N if

$$d(xw, y, z) = d(x, y, z)w + xd(w, y, z)$$

is fulfilled, for all $w, x, y, z \in N$. Muthana [7] defined bimultipliers in rings as follows: A biadditive (additive in both arguments) mapping $B : R \times R \to R$ is called a left (resp. right) bimultiplier on a ring R if $B(xy, z) = B(x, z)y$ (resp. $B(xy, z) = xB(y, z)$) holds, for all $x, y, z \in R$. Motivated by this definition we define an n-additive mapping $F : \underbrace{N \times N \times ... \times N}_{n-times} \to N$ is called a left (resp.

right) n-multiplier on a near ring N if $F(x_1x_1', x_2, \ldots, x_n) = F(x_1, x_2, \ldots, x_n)x_1'$ (resp. $F(x_1x_1', x_2, \ldots, x_n) = x_1F(x_1', x_2, \ldots, x_n)$), for all $x_1, x_1', x_2, \ldots, x_n \in N$. Very recently Asma et. al. [1] defined semiderivations in near rings. An additive mapping $d : N \to N$ is said to be a semiderivation on a near ring N if there exists a mapping $g : N \to N$ such that $d(xy) = d(x)g(y) + xd(y) = d(x)y + g(x)d(y)$ and $d(g(x)) = g(d(x))$, for all $x, y \in N$. Let n be a fixed positive integer. An n-additive (i.e., additive in each argument) mapping $d : \underbrace{N \times N \times ... \times N}_{n-times} \to N$ is said to be an n-semiderivation on a near ring N if there exists a mapping $g : N \to N$ such that the relations

$$\begin{aligned}
d(x_1x_1', x_2, \ldots, x_n) &= d(x_1, x_2, \ldots, x_n)g(x_1') + x_1d(x_1', x_2, \ldots, x_n)\\
&= d(x_1, x_2, \ldots, x_n)x_1' + g(x_1)d(x_1', x_2, \ldots, x_n)\\
d(x_1, x_2x_2', \ldots, x_n) &= d(x_1, x_2, \ldots, x_n)g(x_2') + x_2d(x_1, x_2', \ldots, x_n)\\
&= d(x_1, x_2, \ldots, x_n)x_2' + g(x_2)d(x_1, x_2', \ldots, x_n)\\
&\vdots\\
d(x_1, x_2, \ldots, x_nx_n') &= d(x_1, x_2, \ldots, x_n)g(x_n') + x_nd(x_1, x_2, \ldots, x_n')\\
&= d(x_1, x_2, \ldots, x_n)x_n' + g(x_n)d(x_1, x_2, \ldots, x_n')
\end{aligned}$$

and $g(d(x_1, x_2, \ldots, x_n)) = d(g(x_1), g(x_2), \ldots, g(x_n))$ hold, for all $x_i, x_i' \in N$ for $i = 1, 2, \ldots, n$. An n-additive (i.e., additive in each argument) mapping $F : \underbrace{N \times N \times ... \times N}_{n-times} \to N$ is said to be a generalized n-semiderivation on N if there exists an n-semiderivation $d : \underbrace{N \times N \times ... \times N}_{n-times} \to N$ associated with a map $g : N \to N$ such that the relations

$$\begin{aligned}
F(x_1x_1', x_2, \ldots, x_n) &= F(x_1, x_2, \ldots, x_n)x_1' + g(x_1)d(x_1', x_2, \ldots, x_n)\\
&= d(x_1, x_2, \ldots, x_n)g(x_1') + x_1F(x_1', x_2, \ldots, x_n)\\
F(x_1, x_2x_2', \ldots, x_n) &= F(x_1, x_2, \ldots, x_n)x_2' + g(x_2)d(x_1, x_2', \ldots, x_n)\\
&= d(x_1, x_2, \ldots, x_n)g(x_2') + x_2F(x_1, x_2', \ldots, x_n)\\
&\vdots\\
F(x_1, x_2, \ldots, x_nx_n') &= F(x_1, x_2, \ldots, x_n)x_n' + g(x_n)d(x_1, x_2, \ldots, x_n')\\
&= d(x_1, x_2, \ldots, x_n)g(x_n') + x_nF(x_1, x_2, \ldots, x_n')
\end{aligned}$$

and $g(F(x_1, x_2, \ldots, x_n)) = F(g(x_1), g(x_2), \ldots, g(x_n))$ hold, for all $x_i, x_i' \in N$ for $i = 1, 2, \ldots, n$. All n-semiderivations are generalized n-semiderivations. Moreover, if g is the identity map on N, then all generalized n-semiderivations are merely generalized n-derivations, the notion of generalized n-semiderivation generalizes that of generalized n-derivation. Moreover, generalization is not trivial, as the following example shows:

Example 1. Let S be a commutative near ring. Consider

$$N=\left\{\begin{pmatrix} 0 & x & y \\ 0 & 0 & z \\ 0 & 0 & 0 \end{pmatrix} \mid 0,x,y,z\in S\right\}.$$

Then N is a zero-symmetric left near ring with respect to matrix addition and matrix multiplication. Define mappings $F, d : \underbrace{N\times N\times ...\times N}_{n-times}\to N$ by

$$F\left(\begin{pmatrix}0 & x_1 & y_1\\ 0 & 0 & z_1\\ 0 & 0 & 0\end{pmatrix},\begin{pmatrix}0 & x_2 & y_2\\ 0 & 0 & z_2\\ 0 & 0 & 0\end{pmatrix},\dots,\begin{pmatrix}0 & x_n & y_n\\ 0 & 0 & z_n\\ 0 & 0 & 0\end{pmatrix}\right)=\begin{pmatrix}0 & 0 & z_1z_2...z_n\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix},$$

$$d\left(\begin{pmatrix}0 & x_1 & y_1\\ 0 & 0 & z_1\\ 0 & 0 & 0\end{pmatrix},\begin{pmatrix}0 & x_2 & y_2\\ 0 & 0 & z_2\\ 0 & 0 & 0\end{pmatrix},\dots,\begin{pmatrix}0 & x_n & y_n\\ 0 & 0 & z_n\\ 0 & 0 & 0\end{pmatrix}\right)=\begin{pmatrix}0 & 0 & x_1x_2...x_n\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}$$

and a map $g : N\to N$ by

$$g\begin{pmatrix}0 & x & y\\ 0 & 0 & z\\ 0 & 0 & 0\end{pmatrix}=\begin{pmatrix}0 & 0 & z\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}.$$

It can be easily verified that F is a generalized n-semiderivation associated with an n-semiderivation d and a map g associated with d on N.

Example 2. Let S be a commutative near ring. Consider

$$N=\left\{\begin{pmatrix} 0 & x & y \\ 0 & 0 & 0 \\ 0 & 0 & z \end{pmatrix} \mid 0,x,y,z\in S\right\}.$$

Then N is a zero-symmetric left near ring with respect to matrix addition and matrix multiplication. Define mappings $F, d : \underbrace{N\times N\times ...\times N}_{n-times}\to N$ by

$$F\left(\begin{pmatrix}0 & x_1 & y_1\\ 0 & 0 & 0\\ 0 & 0 & z_1\end{pmatrix},\begin{pmatrix}0 & x_2 & y_2\\ 0 & 0 & 0\\ 0 & 0 & z_2\end{pmatrix},\dots,\begin{pmatrix}0 & x_n & y_n\\ 0 & 0 & 0\\ 0 & 0 & z_n\end{pmatrix}\right)=\begin{pmatrix}0 & x_1x_2...x_n & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix},$$

$$d\left(\begin{pmatrix}0 & x_1 & y_1\\ 0 & 0 & 0\\ 0 & 0 & z_1\end{pmatrix},\begin{pmatrix}0 & x_2 & y_2\\ 0 & 0 & 0\\ 0 & 0 & z_2\end{pmatrix},\dots,\begin{pmatrix}0 & x_n & y_n\\ 0 & 0 & 0\\ 0 & 0 & z_n\end{pmatrix}\right)=\begin{pmatrix}0 & 0 & y_1z_2...z_n\\ 0 & 0 & 0\\ 0 & 0 & z_1z_2...z_n\end{pmatrix}$$

and a map $g : N \to N$ by

$$g\begin{pmatrix} 0 & x & y \\ 0 & 0 & 0 \\ 0 & 0 & z \end{pmatrix} = \begin{pmatrix} 0 & x & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

It is easy to see that F is a generalized n-semiderivation associated with an n-semiderivation d and a map g associated with d on N. However, F is not a generalized n-derivation on N.

2. Preliminary results

We begin with several Lemmas, most of which have been proved elsewhere.

Lemma 2.1 ([3, Lemma 1.2 and Lemma 1.3])**.** *Let N be 3-prime near ring.*

- *(i) If $z \in Z(N)\backslash\{0\}$, then z is not a zero divisor.*
- *(ii) If $Z(N)\backslash\{0\}$ contains an element z for which $z + z \in Z(N)$, then $(N, +)$ is abelian.*
- *(iii) If $Z(N)\backslash\{0\}$ and x is an element of N for which $xz \in Z(N)$, then $x \in Z(N)$.*

Lemma 2.2 ([3, Lemma 1.3 and Lemma 1.4])**.** *Let N be 3-prime near ring and U be a nonzero semigroup ideal of N.*

- *(i) If $x \in N$ and $xU = \{0\}$ or $Ux = \{0\}$, then $x = 0$.*
- *(ii) If $x, y \in N$ and $xUy = \{0\}$, then $x = 0$ or $y = 0$.*
- *(iii) If $x \in N$ centralizes U, then $x \in Z(N)$.*

Lemma 2.3 ([3, Lemma 1.5])**.** *If N is a 3-prime near ring and $Z(N)$ contains a nonzero semigroup left ideal or a nonzero semigroup right ideal, then N is a commutative ring.*

Lemma 2.4. *Let N be a 3-prime near ring and d be a nonzero n-semiderivation of N associated with a map g. If $U_1, U_2, \ldots, U_n$ are nonzero semigroup ideals of N, then $d(U_1, U_2, \ldots, U_n) \neq \{0\}$.*

Proof. Suppose that $d(U_1, U_2, \ldots, U_n) = \{0\}$. Then

$$(1) \qquad d(x_1, x_2, \ldots, x_n) = 0, \text{ for all } x_1 \in U_1, x_2 \in U_2, \ldots, x_n \in U_n$$

Replacing x_1 by x_1r_1 for $r_1 \in N$ in (1) and using it, we have

$$x_1 d(r_1, x_2, \ldots, x_n) = 0.$$

By Lemma 2.2(i), we obtain

$$d(r_1, x_2, \dots, x_n) = 0. \tag{2}$$

Now, substituting $x_2 r_2$ for x_2, where $r_2 \in N$ in (2), we get $d(r_1, r_2, \dots, x_n) = 0$. Proceeding inductively as above, we conclude that $d(r_1, r_2, \dots, r_n) = 0$, for all $r_1, r_2, \dots, r_n \in N$. This shows that $d(N, N, \dots, N) = \{0\}$, leading to a contradiction as d is a nonzero n-semiderivation. Therefore, $d(U_1, U_2, \dots, U_n) \neq \{0\}$. □

Lemma 2.5. *Let N be a 3-prime near ring. Then F is a generalized n-semiderivation associated with an n-semiderivation d and a map g associated with d of N if and only if*

$$F(x_1 x_1', x_2, \dots, x_n) = g(x_1) d(x_1', x_2, \dots, x_n) + F(x_1, x_2, \dots, x_n) x_1',$$

for all $x_1, x_1', x_2, \dots, x_n \in N$.

Proof. We have

$$\begin{aligned} &F(x_1(x_1' + x_1'), x_2, \dots, x_n) \\ &= F(x_1, x_2, \dots, x_n)(x_1' + x_1') + g(x_1) d(x_1' + x_1', x_2, \dots, x_n) \\ &= F(x_1, x_2, \dots, x_n) x_1' + F(x_1, x_2, \dots, x_n) x_1' \\ &+ g(x_1) d(x_1', x_2, \dots, x_n) + g(x_1) d(x_1', x_2, \dots, x_n) \end{aligned} \tag{3}$$

and

$$\begin{aligned} F(x_1 x_1' + x_1 x_1', x_2, \dots, x_n) &= F(x_1 x_1', x_2, \dots, x_n) + F(x_1 x_1', x_2, \dots, x_n) \\ &= F(x_1, x_2, \dots, x_n) x_1' + g(x_1) d(x_1', x_2, \dots, x_n) \\ &+ F(x_1, x_2, \dots, x_n) x_1' + g(x_1) d(x_1', x_2, \dots, x_n). \end{aligned} \tag{4}$$

Comparing (3) and (4), we get

$$\begin{aligned} &F(x_1, x_2, \dots, x_n) x_1' + g(x_1) d(x_1', x_2, \dots, x_n) \\ &\qquad = g(x_1) d(x_1', x_2, \dots, x_n) + F(x_1, x_2, \dots, x_n) x_1'. \end{aligned}$$

This implies that

$$F(x_1 x_1', x_2, \dots, x_n) = g(x_1) d(x_1', x_2, \dots, x_n) + F(x_1, x_2, \dots, x_n) x_1'.$$

Converse can be proved in a similar way. □

Lemma 2.6. *Let N be a 3-prime near ring and $U_1, U_2, \dots, U_n$ be nonzero semigroup ideals of N. If N admits a generalized n-semiderivation F associated with an n-semiderivation d and a map g associated with d such that $g(U_1) = U_1$ and $U_1 \cap Z(N) \neq \{0\}$, then $F(Z(N), U_2, U_3, \dots, U_n) \subseteq Z(N)$.*

Proof. If $z \in U_1 \cap Z(N)$, then

$$F(zx_1, x_2, \ldots, x_n) = F(x_1 z, x_2, \ldots, x_n), \quad \text{for all } x_i \in U_i \text{ for } i = 1, 2, \ldots, n.$$

Using Lemma 2.5, we have

$$\begin{aligned} g(z)d(x_1, x_2, \ldots, x_n) + F(z, x_2, \ldots, x_n)x_1 = d(x_1, x_2, \ldots, x_n)g(z) \\ + x_1 F(z, x_2, \ldots, x_n). \end{aligned}$$

Since $g(U_1) = U_1$, so replacing $g(z)$ by arbitrary element $z' \in U_1 \cap Z(N)$, we get

$$z'd(x_1, x_2, \ldots, x_n) + F(z, x_2, \ldots, x_n)x_1 = d(x_1, x_2, \ldots, x_n)z' + x_1 F(z, x_2, \ldots, x_n).$$

This implies that $F(z, x_2, \ldots, x_n)x_1 = x_1 F(z, x_2, \ldots, x_n)$, for all $z \in U_1 \cap Z(N), x_i \in U_i$ for $i = 1, 2, \ldots, n$. Now, replacing x_1 by $x_1 r$, where $r \in N$ in the last expression and using it again, we obtain $x_1[F(z, x_2, \ldots, x_n), r] = 0$, for all $x_i \in U_i, r \in N$ for $i = 1, 2, \ldots, n$. By Lemma 2.2(i), we find that $[F(z, x_2, \ldots, x_n), r] = 0$. Hence, $F(Z(N), U_2, U_3, \ldots, U_n) \subseteq Z(N)$. □

Lemma 2.7. *Let N be a 3-prime near ring admitting an n-semiderivation d associated with a map g such that $g(x_1 x_1') = g(x_1)g(x_1')$, for all $x_1, x_1' \in N$, then N satisfies the following partial distributive law:*

$$\begin{aligned} &\{d(x_1, x_2, \ldots, x_n)x_1' + g(x_1)d(x_1', x_2, \ldots, x_n)\}y \\ &\qquad = d(x_1, x_2, \ldots, x_n)x_1'y + g(x_1)d(x_1', x_2, \ldots, x_n)y, \end{aligned}$$

for all $x_1, x_1', x_2, \ldots, x_n, y \in N$.

Proof. For all $x_1, x_1', x_2, \ldots, x_n, y \in N$, we have

$$\begin{aligned} d((x_1 x_1')y, x_2, \ldots, x_n) &= d(x_1 x_1', x_2, \ldots, x_n)y + g(x_1 x_1')d(y, x_2, \ldots, x_n) \\ &= \{d(x_1, x_2, \ldots, x_n)x_1' + g(x_1)d(x_1', x_2, \ldots, x_n)\}y \\ &\quad + g(x_1)g(x_1')d(y, x_2, \ldots, x_n). \end{aligned} \tag{5}$$

On the other hand

$$\begin{aligned} d(x_1(x_1'y), x_2, \ldots, x_n) &= d(x_1, x_2, \ldots, x_n)x_1'y + g(x_1)d(x_1'y, x_2, \ldots, x_n) \\ &= d(x_1, x_2, \ldots, x_n)x_1'y + g(x_1)\{d(x_1', x_2, \ldots, x_n)y \\ &\quad + g(x_1')d(y, x_2, \ldots, x_n)\}, \\ d(x_1(x_1'y), x_2, \ldots, x_n) &= d(x_1, x_2, \ldots, x_n)x_1'y + g(x_1)d(x_1', x_2, \ldots, x_n)y \\ &\quad + g(x_1)g(x_1')d(y, x_2, \ldots, x_n). \end{aligned} \tag{6}$$

From (5) and (6), we get

$$\begin{aligned} &\{d(x_1, x_2, \ldots, x_n)x_1' + g(x_1)d(x_1', x_2, \ldots, x_n)\}y \\ &\qquad = d(x_1, x_2, \ldots, x_n)x_1'y + g(x_1)d(x_1', x_2, \ldots, x_n)y. \end{aligned}$$ □

Lemma 2.8. *Let N be a 3-prime near ring and $U_1, U_2, \ldots, U_n$ be nonzero semigroup ideals of N. Let d be a nonzero n-semiderivation of N associated with a map g such that $g(x_1x_1') = g(x_1)g(x_1')$, for all $x_1, x_1' \in U_1$. If $x \in N$ and $d(U_1, U_2, \ldots, U_n)x = \{0\}$(or $xd(U_1, U_2, \ldots, U_n) = \{0\}$), then $x = 0$.*

Proof. By hypothesis,

$$d(x_1, x_2, \ldots, x_n)x = 0, \text{ for all } x_i \in U_i; 1 \leq i \leq n, x \in N. \tag{7}$$

Replacing x_1 by r_1x_1 for $r_1 \in N$ in (7), we get

$$\{d(r_1, x_2, \ldots, x_n)x_1 + g(r_1)d(x_1, x_2, \ldots, x_n)\}x = 0.$$

Using Lemma 2.7 and (7), we get $d(r_1, x_2, \ldots, x_n)U_1x = \{0\}$. By Lemma 2.2(ii), we have either $d(r_1, x_2, \ldots, x_n) = 0$ or $x = 0$. If $d(r_1, x_2, \ldots, x_n) = 0$, for all $r_1 \in N$, $x_2 \in U_2, \ldots, x_n \in U_n$, then proceeding as in the proof of Lemma 2.4, we can show that $d(N, N, \ldots, N) = \{0\}$, leading to a contradiction. Therefore, $x = 0$.

A similar argument using above, handles the case $xd(x_1, x_2, \ldots, x_n)=\{0\}$. □

Lemma 2.9. *Let N be a 3-prime near ring admitting a generalized n-semiderivation F associated with an n-semiderivation d and an onto map g associated with d such that $g(x_1x_1') = g(x_1)g(x_1')$, for all $x_1, x_1' \in N$. Then N satisfies the following partial distributive laws:*

$$(i)\{F(x_1, x_2, \ldots, x_n)x_1' + g(x_1)d(x_1', x_2, \ldots, x_n)\}y = F(x_1, x_2, \ldots, x_n)x_1'y + g(x_1)d(x_1', x_2, \ldots, x_n)y.$$

$$(ii)\{d(x_1, x_2, \ldots, x_n)g(x_1') + x_1F(x_1', x_2, \ldots, x_n)\}y = d(x_1, x_2, \ldots, x_n)g(x_1')y + x_1F(x_1', x_2, \ldots, x_n)y,$$

for all $x_1, x_1', x_2, \ldots, x_n, y \in N$.

Proof. For all $x_1, x_1', x_2, \ldots, x_n, y \in N$, we have

$$\begin{aligned} F((x_1x_1')y, x_2, \ldots, x_n) &= F(x_1x_1', x_2, \ldots, x_n)y + g(x_1x_1')d(y, x_2, \ldots, x_n) \\ &= \{F(x_1, x_2, \ldots, x_n)x_1' + g(x_1)d(x_1', x_2, \ldots, x_n)\}y \\ &\quad + g(x_1)g(x_1')d(y, x_2, \ldots, x_n). \end{aligned} \tag{8}$$

On the other hand

$$\begin{aligned} F(x_1(x_1'y), x_2, \ldots, x_n) &= F(x_1, x_2, \ldots, x_n)x_1'y + g(x_1)d(x_1'y, x_2, \ldots, x_n) \\ &= F(x_1, x_2, \ldots, x_n)x_1'y + g(x_1)\{d(x_1', x_2, \ldots, x_n)y \\ &\quad + g(x_1')d(y, x_2, \ldots, x_n)\}, \end{aligned}$$

$$\begin{aligned} F(x_1(x_1'y), x_2, \ldots, x_n) &= F(x_1, x_2, \ldots, x_n)x_1'y + g(x_1)d(x_1', x_2, \ldots, x_n)y \\ &\quad + g(x_1)g(x_1')d(y, x_2, \ldots, x_n). \end{aligned} \tag{9}$$

From (8) and (9), we get

$$\begin{aligned}&\{F(x_1, x_2, \ldots, x_n)x'_1 + g(x_1)d(x'_1, x_2, \ldots, x_n)\}y\\&\qquad = F(x_1, x_2, \ldots, x_n)x'_1 y + g(x_1)d(x'_1, x_2, \ldots, x_n)y,\end{aligned}$$

for all $x_1, x'_1, x_2, \ldots, x_n, y \in N$.

Arguing in the similar manner, we can prove the result for case (ii). □

Lemma 2.10. *Let N be a 3-prime near ring and $U_1, U_2, \ldots, U_n$ be nonzero semigroup ideals of N. If F is a nonzero generalized n-semiderivation on N associated with an n-semiderivation d and a map g associated with d such that $g(U_1) = U_1$, then $F(U_1, U_2, \ldots, U_n) \neq \{0\}$.*

Proof. Suppose that

$$(10) \quad F(x_1, x_2, \ldots, x_n) = 0, \text{ for all } x_1 \in U_1, x_2 \in U_2, \ldots, x_n \in U_n.$$

Substituting $x_1 r_1$ in place of x_1, where $r_1 \in N$ in (10), we have

$$F(x_1, x_2, \ldots, x_n)r_1 + g(x_1)d(r_1, x_2, \ldots, x_n) = 0.$$

Using (10) and since $g(U_1) = U_1$, so replacing $g(x_1)$ by an arbitrary element x'_1, we get

$$x'_1 d(r_1, x_2, \ldots, x_n) = 0, \text{ for all } x'_1 \in U_1, x_2 \in U_2, \ldots, x_n \in U_n, r_1 \in N.$$

It follows by Lemma 2.2(i) that $d(r_1, x_2, \ldots, x_n) = 0$, for all $x_2 \in U_2, \ldots, x_n \in U_n, r_1 \in N$. Arguing in the similar manner as in Lemma 2.4, we obtain $d = 0$. Therefore, we have $F(r_1 x_1, x_2, \ldots, x_n) = F(r_1, x_2, \ldots, x_n)x_1 = 0$, for all $x_1 \in U_1, x_2 \in U_2, \ldots, x_n \in U_n, r_1 \in N$, and another appeal to Lemma 2.2(i) gives $F = 0$, which is a contradiction. □

Lemma 2.11. *Let N be a 3-prime near ring and $U_1, U_2, \ldots, U_n$ be nonzero semigroup ideals of N. If N admits a nonzero generalized n-semiderivation F associated with an n-semiderivation d and a map g associated with d such that $g(U_1) = U_1$ and $g(x_1 x'_1) = g(x_1)g(x'_1)$, for all $x_1, x'_1 \in U_1$. If $a \in N$ and $aF(U_1, U_2, \ldots, U_n) = \{0\}$ (or $F(U_1, U_2, \ldots, U_n)a = \{0\}$), then $a = 0$.*

Proof. Suppose that

$$(11) \quad aF(x_1, x_2, \ldots, x_n) = 0, \quad \text{for all } x_1 \in U_1, x_2 \in U_2, \ldots, x_n \in U_n, a \in N.$$

Replacing x_1 by $x_1 x'_1$ in (11) for $x'_1 \in U_1$, we get

$$aF(x_1, x_2, \ldots, x_n)x'_1 + ag(x_1)d(x'_1, x_2, \ldots, x_n) = 0.$$

This implies that $aU_1 d(x_1, x_2, \ldots, x_n) = \{0\}$. In view of Lemma 2.2(ii), we obtain either $d(U_1, U_2, \ldots, U_n) = \{0\}$ or $a = 0$, for all $a \in N$.

If $d(U_1, U_2, \ldots, U_n) = \{0\}$, then $aF(x_1x_1', x_2, \ldots, x_n) = ax_1F(x_1', x_2, \ldots, x_n) = 0$, for all $x_1, x_1' \in U_1, x_2 \in U_2, \ldots, x_n \in U_n, a \in N$. Therefore, it follows by Lemma 2.2(ii) and Lemma 2.10 that $a = 0$.

Suppose that $F(U_1, U_2, ...U_n)a = \{0\}$. Then,

$$F(x_1, x_2, \ldots, x_n)a = 0, \text{ for all } x_1 \in U_1, x_2 \in U_2, \ldots, x_n \in U_n, a \in N. \tag{12}$$

Replacing x_1 by x_1x_1' in (12), where $x_1' \in U_1$, we get

$$(d(x_1, x_2, \ldots, x_n)g(x_1') + x_1F(x_1', x_2, \ldots, x_n))a = 0.$$

Using Lemma 2.9(i), we get

$$d(x_1, x_2, \ldots, x_n)g(x_1')a + x_1F(x_1', x_2, \ldots, x_n)a = 0.$$

This implies that $d(x_1, x_2, \ldots, x_n)g(x_1')a = 0$, for all $x_1 \in U_1, x_2 \in U_2, \ldots, x_n \in U_n, a \in N$. Replacing $g(x_1')$ by an arbitrary element $x_1'' \in U_1$ in the last expression and applying Lemma 2.2(ii), we find that $d(U_1, U_2, ...U_n) = \{0\}$ or $a = 0$, for all $a \in N$.

If $d(U_1, U_2, ...U_n) = \{0\}$, then $F(x_1x_1', x_2, \ldots, x_n)a = F(x_1, x_2, \ldots, x_n)x_1'a = 0$, for all $x_1, x_1' \in U_1, x_2 \in U_2, \ldots, x_n \in U_n, a \in N$. Therefore, it follows by Lemma 2.2(ii) and Lemma 2.10 that $a = 0$. □

3. Main results

Theorem 3.1. *Let N be a 3-prime near ring and $U_1, U_2, \ldots, U_n$ are nonzero semigroup ideals of N. Let F_1 and F_2 be any two generalized n-semiderivations associated with n-semiderivations d_1 and d_2 respectively and a map g associated with d_1 and d_2 such that $g(U_1) = U_1$. If $[F_1(U_1, U_2, \ldots, U_n), F_2(U_1, U_2, \ldots, U_n)] = \{0\}$, then at least one of F_1 and F_2 is trivial or $(N, +)$ is an abelian group.*

Proof. Suppose that $x \in N$ is such that

$$[x, F_2(U_1, U_2, \ldots, U_n)] = [x + x, F_2(U_1, U_2, \ldots, U_n)] = 0.$$

For all $x_1, x_1' \in U_1$ such that $x_1 + x_1' \in U_1$,

$$[x + x, F_2(x_1 + x_1', x_2, \ldots, x_n)] = 0.$$

This implies that

$$\begin{aligned}
&(x + x)F_2(x_1 + x_1', x_2, \ldots, x_n) = F_2(x_1 + x_1', x_2, \ldots, x_n)(x + x),\\
&(x + x)F_2(x_1, x_2, \ldots, x_n) + (x + x)F_2(x_1', x_2, \ldots, x_n)\\
&= F_2(x_1 + x_1', x_2, \ldots, x_n)x + F_2(x_1 + x_1', x_2, \ldots, x_n)x,\\
&F_2(x_1, x_2, \ldots, x_n)(x + x) + F_2(x_1', x_2, \ldots, x_n)(x + x)\\
&= xF_2(x_1 + x_1', x_2, \ldots, x_n) + xF_2(x_1 + x_1', x_2, \ldots, x_n),\\
&F_2(x_1, x_2, \ldots, x_n)x + F_2(x_1, x_2, \ldots, x_n)x + F_2(x_1', x_2, \ldots, x_n)x + F_2(x_1', x_2, \ldots, x_n)x\\
&= xF_2(x_1, x_2, \ldots, x_n) + xF_2(x_1', x_2, \ldots, x_n) + xF_2(x_1, x_2, \ldots, x_n)\\
&+ xF_2(x_1', x_2, \ldots, x_n),
\end{aligned}$$

which reduces to $xF_2((x_1,x_1'),x_2,\ldots,x_n)=0$, for all $x_2\in U_2,\ldots,x_n\in U_n, x\in N$, where (x_1,x_1') is the additive commutator $(x_1+x_1'-x_1-x_1')$.

If $r,s\in U_1$, we have $rs\in U_1$ and $rs+rs=r(s+s)\in U_1$ and since $[F_1(U_1,U_2,\ldots,U_n),F_2(U_1,U_2,\ldots,U_n)]=\{0\}$, taking $x=F_1(rs,x_2',\ldots,x_n')$, where $r,s\in U_1, x_2'\in U_2,\ldots,x_n'\in U_n$ gives

$$\begin{aligned}&[F_1(rs,x_2',\ldots,x_n'),F_2(U_1,U_2,\ldots,U_n)]=\{0\}\\ &\quad=[F_1(rs,x_2',\ldots,x_n')+F_1(rs,x_2',\ldots,x_n'),F_2(U_1,U_2,\ldots,U_n)].\end{aligned}$$

Arguing in the similar manner as above, we get

$$F_1(U_1^2,U_2,\ldots,U_n)F_2(x_1+x_1'-x_1-x_1',x_2,\ldots,x_n)=\{0\}.$$

Since U_1^2 is a semigroup ideal, Lemma 2.11 gives

$$F_2(x_1+x_1'-x_1-x_1',x_2,\ldots,x_n)=0, \tag{13}$$

for all $x_1,x_1'\in U_1$ such that $x_1+x_1'\in U_1$. Now, take $x_1=rx'$ and $x_1'=ry'$ for $r\in U_1$ and $x',y'\in N$, so that x_1,x_1' and $x_1+x_1'=rx'+ry'=r(x'+y')\in U_1$. It follows from relation (13) that

$$F_2(rx'+ry'-rx'-ry',x_2,\ldots,x_n)=0,\ \text{ for all } r\in U_1,x',y'\in N.$$

Replacing r by rw, $w\in U_1$ we get $F_2(U_1,U_2,\ldots,U_n)U_1(x'+y'-x'-y')=\{0\}$, for all $x',y'\in N$ and by Lemma 2.2(ii) either $F_2(U_1,U_2,\ldots,U_n)=\{0\}$ or $x'+y'-x'-y'=0$, for all $x',y'\in N$. If $F_2(U_1,U_2,\ldots,U_n)=\{0\}$, then proceeding as in Lemma 2.10, we find $F_2=0$ and the second case implies that $(N,+)$ is an abelian group. Similarly if we consider

$$[F_1(U_1,U_2,\ldots,U_n),x]=[F_1(U_1,U_2,\ldots,U_n),x+x]=0$$

and proceeding as above, we can find either $F_1=0$ or $(N,+)$ is an abelian group. □

Theorem 3.2. *Let N be a 3-prime near ring and $U_1,U_2,\ldots,U_n$ are nonzero semigroup ideals of N. Let F be a generalized n-semiderivation associated with an n-semiderivation d and a map g associated with d such that $g(U_1)=U_1$ and $g(x_1x_1')=g(x_1)g(x_1')$, for all $x_1,x_1'\in U_1$. If $F(U_1,U_2,\ldots,U_n)\subseteq Z(N)$, then $F=0$ or N is a commutative ring.*

Proof. For all $x_1,x_1'\in U_1,x_2\in U_2,\ldots,x_n\in U_n$, we get

$$F(x_1x_1',x_2,\ldots,x_n)=d(x_1,x_2,\ldots,x_n)g(x_1')+x_1F(x_1',x_2,\ldots,x_n)\in Z(N). \tag{14}$$

Now, commuting (14) with the element x_1, we get

$$\begin{aligned}&(d(x_1,x_2,\ldots,x_n)g(x_1')+x_1F(x_1',x_2,\ldots,x_n))x_1\\ &\qquad=x_1(d(x_1,x_2,\ldots,x_n)g(x_1')+x_1F(x_1',x_2,\ldots,x_n)).\end{aligned}$$

Using the hypothesis and Lemma 2.9(ii), we have

$$\begin{aligned} &d(x_1, x_2, \ldots, x_n)g(x_1')x_1 + x_1x_1F(x_1', x_2, \ldots, x_n) \\ &\qquad = x_1d(x_1, x_2, \ldots, x_n)g(x_1') + x_1x_1F(x_1', x_2, \ldots, x_n). \end{aligned}$$

This implies that,

$$d(x_1, x_2, \ldots, x_n)x_1'x_1 = x_1d(x_1, x_2, \ldots, x_n)x_1'. \tag{15}$$

Replacing x_1' by $x_1'r$ for $r \in N$ in (22) and using it again, we get

$$d(x_1, x_2, \ldots, x_n)x_1'[x_1, r] = 0, \text{ for all } x_1, x_1' \in U_1, x_2 \in U_2, \ldots, x_n \in U_n, r \in N.$$

By Lemma 2.2(ii), either $d(x_1, x_2, \ldots, x_n) = 0$, for all $x_1 \in U_1, x_2 \in U_2, \ldots, x_n \in U_n$ or $U_1 \subseteq Z(N)$. If $d(x_1, x_2, \ldots, x_n) = 0$, for all $x_1, \in U_1, x_2 \in U_2, \ldots, x_n \in U_n$, then

$$F(x_1x_1', x_2, \ldots, x_n) = F(x_1, x_2, \ldots, x_n)x_1' \in Z(N).$$

This implies that $F(x_1, x_2, \ldots, x_n)x_1's = sF(x_1, x_2, \ldots, x_n)x_1'$, for all $x_1, x_1' \in U_1, x_2 \in U_2, \ldots, x_n \in U_n$, and $s \in N$. Replacing x_1' by $x_1'x_1''$, for all $x_1'' \in U_1$ in above expression and using it again, we find that

$$F(x_1, x_2, \ldots, x_n)U_1[x_1'', s] = \{0\}.$$

By Lemma 2.2(ii), we have $F(x_1, x_2, \ldots, x_n) = 0$, for all $x_1 \in U_1, x_2 \in U_2, \ldots, x_n \in U_n$ or $U_1 \subseteq Z(N)$. If $F(x_1, x_2, \ldots, x_n) = 0$, for all $x_1 \in U_1, x_2 \in U_2, \ldots, x_n \in U_n$, then proceeding as in Lemma 2.10, we can get $F = 0$ on N. In later case $U_1 \subseteq Z(N)$ implies that N is a commutative ring by Lemma 2.3. □

Theorem 3.3. *Let N be a 2-torsion free 3-prime near ring and $U_1, U_2, \ldots, U_n$ are nonzero semigroup ideals of N. Suppose that N admits a nonzero generalized n-semiderivation F associated with an n-semiderivations d and a map g associated with d such that $g(U_1) = U_1$ and $g(x_1x_1') = g(x_1)g(x_1')$, for all $x_1, x_1' \in U_1$. If $[F(U_1, U_2, \ldots, U_n), F(U_1, U_2, \ldots, U_n)] = \{0\}$, then F maps U^n into $Z(N)$ or F is an n-multiplier on N.*

Proof. By hypothesis, for all $x_1, y_1 \in U_1, x_2, y_2 \in U_2, \ldots, x_n, y_n \in U_n$,

$$F(x_1, x_2, \ldots, x_n)F(y_1, y_2, \ldots, y_n) = F(y_1, y_2, \ldots, y_n)F(x_1, x_2, \ldots, x_n). \tag{16}$$

Replacing y_1 by $F(z_1, z_2, \ldots, z_n)y_1$ in (16), where $z_1 \in U_1, z_2 \in U_2, \ldots, z_n \in U_n$, we get

$$\begin{aligned} &F(x_1, x_2, \ldots, x_n)F(F(z_1, z_2, \ldots, z_n)y_1, y_2, \ldots, y_n) \\ &= F(F(z_1, z_2, \ldots, z_n)y_1, y_2, \ldots, y_n)F(x_1, x_2, \ldots, x_n), \end{aligned}$$

$$\begin{aligned}&F(x_1,x_2,\ldots,x_n)\{d(F(z_1,z_2,\ldots,z_n),y_2,\ldots,y_n)g(y_1)\\&+F(z_1,z_2,\ldots,z_n)F(y_1,y_2,\ldots,y_n)\}\\&=\{d(F(z_1,z_2,\ldots,z_n),y_2,\ldots,y_n)g(y_1)\\&+F(z_1,z_2,\ldots,z_n)F(y_1,y_2,\ldots,y_n)\}F(x_1,x_2,\ldots,x_n).\end{aligned}$$

By Lemma 2.9(ii), we have

$$\begin{aligned}&F(x_1,x_2,\ldots,x_n)d(F(z_1,z_2,\ldots,z_n),y_2,\ldots,y_n)g(y_1)\\&+F(x_1,x_2,\ldots,x_n)F(z_1,z_2,\ldots,z_n)F(y_1,y_2,\ldots,y_n)\\&=d(F(z_1,z_2,\ldots,z_n),y_2,\ldots,y_n)g(y_1)F(x_1,x_2,\ldots,x_n)\\&+F(z_1,z_2,\ldots,z_n)F(y_1,y_2,\ldots,y_n)F(x_1,x_2,\ldots,x_n).\end{aligned}$$

This implies that

$$\begin{aligned}&F(x_1,x_2,\ldots,x_n)d(F(z_1,z_2,\ldots,z_n),y_2,\ldots,y_n)y_1\\&=d(F(z_1,z_2,\ldots,z_n),y_2,\ldots,y_n)y_1F(x_1,x_2,\ldots,x_n).\end{aligned}\tag{17}$$

Replacing y_1 by y_1t, for all $t\in N$ and using (17), we obtain

$$\begin{aligned}&F(x_1,x_2,\ldots,x_n)d(F(z_1,z_2,\ldots,z_n),y_2,\ldots,y_n)y_1t\\&=F(x_1,x_2,\ldots,x_n)y_1td(F(z_1,z_2,\ldots,z_n),y_2,\ldots,y_n),\end{aligned}$$

which reduces to,

$$d(F(z_1,z_2,\ldots,z_n),y_2,\ldots,y_n)U_1[F(x_1,x_2,\ldots,x_n),t]=\{0\}.$$

By Lemma 2.2(ii), we get $[F(x_1,x_2,\ldots,x_n),t]=0$, for all $x_1\in U_1,x_2\in U_2,\ldots,x_n\in U_n,t\in N$ or $d(F(z_1,z_2,\ldots,z_n),y_2,\ldots,y_n)=0$, for all $z_1\in U_1,y_2,z_2\in U_2,\ldots,y_n,z_n\in U_n$. In the first case $F(U_1,U_2,\ldots,U_n)\subseteq Z(N)$ shows that F maps U^n into $Z(N)$, the centre of N. Let us assume that $d(F(U_1,U_2,\ldots,U_n),U_2,\ldots,U_n)=\{0\}$, then

$$\begin{aligned}0&=d(F(y_1y_1',y_2,\ldots,y_n),y_2,\ldots,y_n)\\&=d\{(F(y_1,y_2,\ldots,y_n)y_1'+g(y_1)d(y_1',y_2,\ldots,y_n)),y_2,\ldots,y_n\}\\&=d((F(y_1,y_2,\ldots,y_n)y_1',y_2,\ldots,y_n)+d(y_1d(y_1',y_2,\ldots,y_n),y_2,\ldots,y_n)\\&=F(y_1,y_2,\ldots,y_n)d(y_1',y_2,\ldots,y_n)+d(y_1,y_2,\ldots,y_n)d(y_1',y_2,\ldots,y_n)\\&+y_1d(d(y_1',y_2,\ldots,y_n),y_2,\ldots,y_n)\text{ for all }y_1,y_1'\in U_1,y_2\in U_2,\ldots,y_n\in U_n.\end{aligned}$$

Now, replacing y_1 by y_1z_1, for all $z_1\in U_1$, we have

$$\begin{aligned}&\{d(y_1,y_2,\ldots,y_n)z_1+y_1F(z_1,y_2,\ldots,y_n)\}d(y_1',y_2,\ldots,y_n)\\&\qquad+\{d(y_1,y_2,\ldots,y_n)z_1+y_1d(z_1,y_2,\ldots,y_n)\}d(y_1',y_2,\ldots,y_n)\\&\qquad+y_1z_1d(d(y_1',y_2,\ldots,y_n),y_2,\ldots,y_n)\}=0,\end{aligned}$$

$$2d(y_1,y_2,\ldots,y_n)z_1d(y_1',y_2,\ldots,y_n)+y_1\{F(z_1,y_2,\ldots,y_n)d(y_1',y_2,\ldots,y_n)$$
$$+\,d(z_1,y_2,\ldots,y_n)d(y_1',y_2,\ldots,y_n)+z_1d(d(y_1',y_2,\ldots,y_n),y_2,\ldots,y_n)\}=0,$$

which implies that

$2d(y_1,y_2,\ldots,y_n)z_1d(y_1',y_2,\ldots,y_n)=0$ for all $y_1,y_1',z_1\in U_1,y_2\in U_2,\ldots,y_n\in U_n$.

Since N is 2-torsion free, we get

$d(y_1,y_2,\ldots,y_n)U_1d(y_1',y_2,\ldots,y_n)=\{0\}$ for all $y_1,y_1'\in U_1,y_2\in U_2,\ldots,y_n\in U_n$.

Thus, we obtain $d(U_1,U_2,\ldots,U_n)=\{0\}$. Arguing as above, we conclude that F is an n-multiplier on N. □

Theorem 3.4. *Let N be a 3-prime near ring and $U_1,U_2,\ldots,U_n$ be nonzero semigroup ideals of N. Suppose that N admits a generalized n-semiderivation F associated with an n-semiderivation d and an additive map g associated with d such that $g(U_1)=U_1$ and $g(x_1x_1')=g(x_1)g(x_1')$, for all $x_1,x_1'\in U_1$. If $F([x,y],x_2,\ldots,x_n)=\pm[x,y]$, for all $x,y\in U_1,x_2\in U_2,\ldots,x_n\in U_n$, then F is an n-multiplier or N is a commutative ring.*

Proof. By hypothesis

$$(18)\quad F([x,y],x_2,\ldots,x_n)=\pm[x,y],\ \text{ for all } x,y\in U_1,x_2\in U_2,\ldots,x_n\in U_n.$$

Replacing y by xy in (18) and using $[x,xy]=x[x,y]$, we get

$$F(x[x,y],x_2,\ldots,x_n)=\pm x[x,y],$$
$$d(x,x_2,\ldots,x_n)g([x,y])+xF([x,y],x_2,\ldots,x_n)=\pm x[x,y].$$

Using (18), we get

$$(19)\quad d(x,x_2,\ldots,x_n)g([x,y])=0,\ \text{ for all } x,y\in U_1,x_2\in U_2,\ldots,x_n\in U_n.$$

This implies that

$$d(x,x_2,\ldots,x_n)g(x)g(y)=d(x,x_2,\ldots,x_n)g(y)g(x).$$

Replacing y by yz in the above expression and using it again, we arrive at

$$d(x,x_2,\ldots,x_n)g(y)[g(x),g(z)]=0.$$

Since $g(U_1)=U_1$, substituting arbitrary elements x',y' and z' of U_1 in place of $g(x),g(y)$ and $g(z)$ respectively, we obtain

$$d(x,x_2,\ldots,u_n)U_1[x',z']=\{0\},\ \text{ for all } x,x',z'\in U_1,x_2\in U_2,\ldots,x_n\in U_n.$$

By Lemma 2.2(ii), we have either $d(x, x_2, \ldots, x_n) = 0$ or $[x', z'] = 0$, for all $x, x', z' \in U_1, x_2 \in U_2, \ldots, x_n \in U_n$. If $d(x, x_2, \ldots, x_n) = 0$, then proceeding as in Lemma 2.4, we can find $d = 0$ on N. Therefore,

$$F(x_1 x_1', x_2, \ldots, x_n) = x_1 F(x_1', x_2, \ldots, x_n) = F(x_1, x_2, \ldots, x_n) x_1',$$

for all $x_1, x_1', x_2, \ldots, x_n \in N$ and hence F is an n-multiplier on N. In later case, we have $[x', z'] = 0$, i.e., $x'z' = z'x'$. Replacing z' by $z'r$ and using it again, we find that $z'[x', r] = 0$, i.e., $U_1[x', r] = \{0\}$, for all $x' \in U_1, r \in N$. By an application of Lemma 2.2(i) and Lemma 2.3, N is a commutative ring. □

Theorem 3.5. *Let N be a 2-torsion free 3-prime near ring and $U_1, U_2, \ldots, U_n$ be nonzero semigroup ideals of N. Suppose that N admits a generalized n-semiderivation F associated with an n-semiderivation d and an additive map g associated with d such that $g(U_1) = U_1$ and $g(x_1 x_1') = g(x_1)g(x_1')$, for all $x_1, x_1' \in U_1$. If $F(x \circ y, x_2, \ldots, x_n) = 0$, for all $x, y \in U_1, x_2 \in U_2, \ldots, x_n \in U_n$, then $F = 0$.*

Proof. By hypothesis

$$F(x \circ y, x_2, \ldots, x_n) = 0, \text{ for all } x, y \in U_1, x_2 \in U_2, \ldots, x_n \in U_n. \tag{20}$$

Replacing y by xy in (20), we get

$$d(x, x_2, \ldots, x_n) g(x \circ y) + x F(x \circ y, x_2, \ldots, x_n) = 0.$$

Using (20), we get

$$d(x, x_2, \ldots, x_n) g(x \circ y) = 0, \text{ for all } x, y \in U_1, x_2 \in U_2, \ldots, x_n \in U_n. \tag{21}$$

Since g is additive and $g(x_1 x_1') = g(x_1)g(x_1')$, for all $x_1, x_1' \in U_1$, then (21) can be written as

$$d(x, x_2, \ldots, x_n) g(x) g(y) = -d(x, x_2, \ldots, x_n) g(y) g(x).$$

Replacing y by yz in the above expression and using it again, we arrive at

$$d(x, x_2, \ldots, x_n) g(y) g(-x) g(z) = d(x, x_2, \ldots, x_n) g(y) g(z) g(-x),$$

which implies that

$$d(x, x_2, \ldots, x_n) g(y) [g(-x), g(z)] = 0, \text{ for all } x, y, z \in U_1, x_2 \in U_2, \ldots, x_n \in U_n.$$

Putting $-x$ in place of x in the last expression, we obtain

$$d(-x, x_2, \ldots, x_n) g(y) [g(x), g(z)] = 0, \text{ for all } x, y, z \in U_1, x_2 \in U_2, \ldots, x_n \in U_n.$$

Now, replacing $g(x)$, $g(y)$ and $g(z)$ by arbitrary elements x', y' and z' of U_1 and applying Lemma 2.2(ii), we get either $d(-x, x_2, \ldots, x_n) = 0$ or $[x', z'] =$

0, for all $x, y, z \in U_1, x_2 \in U_2, \dots, x_n \in U_n$. Since d is n-additive, then $d(-x, x_2, \dots, x_n) = 0$ implies that $d(x, x_2, \dots, x_n) = 0$. Hence, we have either $d(x, x_2, \dots, x_n) = 0$ or $[x', z'] = 0$, for all $x, y, z \in U_1, x_2 \in U_2, \dots, x_n \in U_n$. Arguing in the similar manner as in Theorem 3.4, we get F is an n-multiplier or N is commutative.

If N is commutative, then the hypothesis becomes

$$0 = F(x \circ y, x_2, \dots, x_n) = 2F(xy, x_2, \dots, x_n).$$

Since N is 2-torsion free, we get

$$F(xy, x_2, \dots, x_n) = 0. \tag{22}$$

Replacing y by yz in (22), we obtain

$$\begin{aligned} F(xy, x_2, \dots, x_n)z + g(xy)d(z, x_2, \dots, x_n) &= 0, \\ g(x)g(y)d(z, x_2, \dots, x_n) &= 0. \end{aligned}$$

Since $g(U_1) = U_1$, then by Lemma 2.2(ii), we have $d(z, x_2, \dots, x_n) = 0$, so Lemma 2.4 forces that $d = 0$, thus F is an n-multiplier and (22) becomes $F(x, x_2, \dots, x_n)y = 0$ and Lemma 2.10 forces that $F = 0$.

If F is an n-multiplier, then replacing y by xy in (20), we obtain

$$F(x, x_2, \dots, x_n)(x \circ y) = 0.$$

By using same argument as above, we get

$$F(x, x_2, \dots, x_n)U_1[x, z] = 0.$$

By Lemma 2.2(ii), we get $x \in Z(N)$ or $F(x, x_2, \dots, x_n) = 0$. If $x \in Z(N)$, then the hypothesis becomes $2F(xy, u_2, u_3, \dots, u_n) = 0$. By 2-torsion freeness of N, we find that $F(x, x_2, ...u_n)y = 0$, thus in all the cases we arrive at $F(x, x_2, \dots, x_n) = 0$ and Lemma 2.10 forces that $F = 0$. □

Theorem 3.6. *Let N be a 2-torsion free 3-prime near ring; $U_1, U_2, \dots, U_n$ are nonzero semigroup ideals of N and an additive map g such that $g(U_1) = U_1$ and $g(x_1x'_1) = g(x_1)g(x'_1)$, for all $x_1, x'_1 \in U_1$. There is no generalized n-semiderivation F associated with an n-semiderivation d and g such that $F(x \circ y, x_2, \dots, x_n) = \pm(x \circ y)$, for all $x, y \in U_1, x_2 \in U_2, \dots, x_n \in U_n$.*

Proof. Suppose that there exists F such that

$$F(x \circ y, x_2, \dots, x_n) = \pm(x \circ y) \text{ for all } x, y \in U_1, x_2 \in U_2, \dots, x_n \in U_n. \tag{23}$$

Substituting xy for y in (23), we get

$$F(x(x \circ y), x_2, \dots, x_n) = \pm x(x \circ y).$$

This implies that

$$d(x, x_2, \dots, x_n)g(x \circ y) + xF((x \circ y), x_2, \dots, x_n) = \pm x(x \circ y).$$

Using (23), we get $d(x, x_2, \dots, x_n)g(x \circ y) = 0$. Arguing in the similar manner as in Theorem 3.4 and Theorem 3.5, we get N is commutative or F is an n-multiplier.

If N is commutative, then the hypothesis becomes $2F(xy, x_2, \dots, x_n) = 2xy$ that is $F(xy, x_2, \dots, x_n) = xy$ this yields that $d = 0$ and replacing x_2 by $x_2x'_2$ and $x_2x''_2$, where $x'_2 \neq x''_2$ and comparing the result, we arrive at

$$(x'_2 - x''_2)(x \circ y) = 0$$

This leads to $N = (0)$, a contradiction.

If F is an n-multiplier, then reasoning as above we arrive at $N = (0)$, a contradiction, so we obtain the required result. □

Theorem 3.7. *Let N be a prime near ring and $U_1, U_2, \dots, U_n$ be nonzero semigroup ideals of N. Suppose that N admits a generalized n-semiderivation F associated with a map $d : \underbrace{N \times N \times ... \times N}_{n-times} \to N$ and a map g such that $g(U_1) = U_1$ and $U_1 \cap Z(N) \neq \{0\}$. If $F([x_1, y_1], x_2, \dots, x_n) = \pm[F(x_1, x_2, \dots, x_n), y_1]$, for all $x_1, y_1 \in U_1, x_2 \in U_2, \dots, x_n \in U_n$, then F is commuting on U_1.*

Proof. By hypothesis

$$F([x_1, y_1], x_2, \dots, x_n) = [F(x_1, x_2, \dots, x_n), y_1] \tag{24}$$

Replacing y_1 by x_1y_1 in (24), we have

$$\begin{aligned} &d(x_1, x_2, ...x_n)g([x_1, y_1]) + x_1F([x_1, y_1], x_2, \dots, x_n) = [F(x_1, x_2, \dots, x_n), x_1y_1], \\ &d(x_1, x_2, ...x_n)g([x_1, y_1]) + x_1[F(x_1, x_2, \dots, x_n), y_1] = [F(x_1, x_2, \dots, x_n), x_1y_1], \\ &d(x_1, x_2, ...x_n)g([x_1, y_1]) + x_1F(x_1, x_2, \dots, x_n)y_1 - x_1y_1F(x_1, x_2, \dots, x_n) \\ &\qquad\qquad = F(x_1, x_2, \dots, x_n)x_1y_1 - x_1y_1F(x_1, x_2, \dots, x_n). \end{aligned}$$

If we choose $y_1 \in U_1 \cap Z(N)$, then above relation yields that $x_1F(x_1, x_2, \dots, x_n)y_1 = F(x_1, x_2, \dots, x_n)x_1y_1$. This implies that $y_1[F(x_1, x_2, \dots, x_n), x_1] = 0$ and by Lemma 2.2(i), we find $[F(x_1, x_2, \dots, x_n), x_1] = 0$. Hence, F is commuting on U_1. In the similar manner we can prove the result for $F([x_1, y_1], x_2, \dots, x_n) = -[F(x_1, x_2, \dots, x_n), y_1]$, for all $x_1, y_1 \in U_1, x_2 \in U_2, \dots, x_n \in U_n$. □

Theorem 3.8. *Let N be a 3-prime near ring and $U_1, U_2, \dots, U_n$ are nonzero semigroup ideals of N. Suppose that N admits a generalized n-semiderivation F associated with a map $d : \underbrace{N \times N \times ... \times N}_{n-times} \to N$ and a map g such that $g(U_1) = U_1$ and $U_1 \cap Z(N) \neq \{0\}$. If $F([x_1, y_1], x_2, \dots, x_n) = \pm[x_1, F(y_1, x_2, \dots, x_n)]$, for all $x_1, y_1 \in U_1, x_2 \in U_2, \dots, x_n \in U_n$, then F is commuting on U_1.*

Proof. By hypothesis

$$F([x_1,y_1],x_2,\dots,x_n)=[x_1,F(y_1,x_2,\dots,x_n)] \tag{25}$$

Replacing x_1 by y_1x_1 in (25), we get

$$\begin{aligned}
&d(y_1,x_2,...x_n)g([x_1,y_1])+y_1F([x_1,y_1],x_2,\dots,x_n)=[y_1x_1,F(x_1,x_2,\dots,x_n)],\\
&d(y_1,x_2,...x_n)g([x_1,y_1])+y_1[x_1,F(y_1,x_2,\dots,x_n)]=[y_1x_1,F(x_1,x_2,\dots,x_n)],\\
&d(y_1,x_2,...x_n)g([x_1,y_1])+y_1x_1F(y_1,x_2,\dots,x_n)-y_1F(y_1,x_2,\dots,x_n)x_1\\
&\qquad\qquad=y_1x_1F(x_1,x_2,\dots,x_n)-F(x_1,x_2,\dots,x_n)y_1x_1
\end{aligned}$$

If we choose $x_1\in U_1\cap Z(N)$, then above relation yields that $y_1F(y_1,x_2,\dots,x_n)x_1 = F(x_1,x_2,\dots,x_n)y_1x_1$. This implies that $x_1[F(y_1,x_2,\dots,x_n),y_1]=0$ and by Lemma 2.2(i), we find $[F(y_1,x_2,\dots,x_n),y_1]=0$. Hence F is commuting on U_1. In the similar manner we can prove the result for $F([x_1,y_1],x_2,\dots,x_n) = -[x_1,F(y_1,x_2,\dots,x_n)]$, for all $x_1,y_1\in U_1,x_2\in U_2,\dots,x_n\in U_n$, then F is commuting on U_1. □

Theorem 3.9. *Let N be a 3-prime near ring and $U_1,U_2,\dots,U_n$ be nonzero semigroup ideals of N. Suppose that N admits a nonzero generalized n-semiderivation F associated with an n-semiderivation d on N and a map g such that $g(U_1)=U_1$ and $d(Z(N),U_2,\dots,U_n)\neq\{0\}$. If $[F(x_1,x_2,\dots,x_n),F(y_1,y_2,\dots,y_n)] = 0$, for all $x_1,y_1\in U_1,x_2\in U_2,\dots,x_n\in U_n$, then N is a commutative ring.*

Proof. Let $z\in Z(N)$ and $d(z,y_2,\dots,y_n)\neq 0$. Then by hypothesis

$$\begin{aligned}
F(x_1,x_2,\dots,x_n)F(y_1z,y_2,\dots,y_n)&=F(y_1z,y_2,\dots,y_n)F(x_1,x_2,\dots,x_n),\\
F(x_1,x_2,\dots,x_n)F(y_1,y_2,\dots,y_n)z&+F(x_1,x_2,\dots,x_n)g(y_1)d(z,y_2,\dots,y_n)\\
&=F(y_1,y_2,\dots,y_n)zF(x_1,x_2,\dots,x_n)\\
&+g(y_1)d(z,y_2,\dots,y_n)F(x_1,x_2,\dots,x_n).
\end{aligned}$$

This implies that,

$$F(x_1,x_2,\dots,x_n)g(y_1)d(z,y_2,\dots,y_n)=g(y_1)d(z,y_2,\dots,y_n)F(x_1,x_2,\dots,x_n).$$

By hypothesis, we find $d(z,y_2,\dots,y_n)[F(x_1,x_2,\dots,x_n),g(y_1)]=0$. By Lemma 2.1(i), we get $[F(x_1,x_2,\dots,x_n),y_1]=0$. Replacing y_1 by y_1r for $r\in N$, we have

$$y_1[F(x_1,x_2,\dots,x_n),r]=0.$$

By Lemma 2.2(ii), we obtain

$$[F(x_1,x_2,\dots,x_n),r]=0,\ \text{ for all }\ x_1\in U_1,x_2\in U_2,\dots,x_n\in U_n,r\in N.$$

Therefore, $F(U_1,U_2,\dots,U_n)\subseteq Z(N)$ and hence N is a commutative ring by Theorem 3.2. □

Theorem 3.10. *Suppose that* N *is a prime near ring;* $U_1, U_2, \ldots, U_n$ *are nonzero semigroup ideals of* N *and* $V_1, V_2, \ldots, V_n$ *are nonempty subsets of* N.

If F *is a generalized* n*-semiderivation acts as a left multiplier such that* $F(x_1y_1, x_2, \ldots, x_n) = F(y_1x_1, x_2, \ldots, x_n)$*, for all* $y_1 \in V_1, x_1 \in U_1, x_2 \in U_2 ..., x_n \in U_n$*, then* $F(V_1, V_2, \ldots, V_n) = \{0\}$ *or* $V_1 \subseteq Z(N)$.

Proof. By hypothesis, for all $y_1 \in V_1, x_1 \in U_1, x_2 \in U_2, \ldots, x_n \in U_n$,

$$F(x_1y_1, x_2, \ldots, x_n) = F(y_1x_1, x_2, \ldots, x_n). \tag{26}$$

Replacing x_1 by y_1x_1 in (26), we get

$$F(y_1, x_2, \ldots, x_n)x_1y_1 = F(y_1, x_2, \ldots, x_n)y_1x_1. \tag{27}$$

Replacing x_1 by x_1x_1', for all $x_1' \in U_1$ in (27), we have

$$F(y_1, x_2, \ldots, x_n)x_1x_1'y_1 = F(y_1, x_2, \ldots, x_n)x_1y_1x_1',$$

which implies that,

$$F(y_1, x_2, \ldots, x_n)U_1[x_1', y_1] = \{0\}.$$

By Lemma 2.2(ii), we have $F(y_1, x_2, \ldots, x_n)$=0, for all $y_1 \in V_1, x_2 \in U_2, \ldots, x_n \in U_n$ or y_1 centralizes U_1. In first case, replacing x_2 by y_2x_2, for all $y_2 \in V_2$, we find that $F(y_1, y_2, \ldots, x_n)x_2 = 0$ and again by Lemma 2.2(i), we get $F(y_1, y_2, \ldots, x_n) = 0$. Proceeding inductively, we obtain $F(y_1, y_2, \ldots, y_n) = 0$, for all $y_1 \in V_1, y_2 \in V_2, \ldots, y_n \in V_n$, which completes the proof. □

Theorem 3.11. *Let* N *be a 3-prime near ring and* $U_1, U_2, \ldots, U_n$ *are nonempty subsets of* N *and* $V_1, V_2, \ldots, V_n$ *are nonzero semigroup ideals of* N*. Suppose that* N *admits a generalized* n*- semiderivation* F *associated with an* n*- semiderivation* d *and an additive map* g *such that* $g(V_1) = V_1$*. If* $F(x_1y_1, y_2, \ldots, y_n) = F(y_1x_1, y_2, \ldots, y_n)$*, for all* $x_1 \in U_1, y_1 \in V_1, y_2 \in V_2, \ldots, y_n \in V_n$*, then* $D(U_1, U_2, \ldots, U_n) = \{0\}$ *or* $U_1 \subseteq Z(N)$.

Proof. By hypothesis, for all $x_1 \in U_1, y_1 \in V_1, y_2 \in V_2, \ldots, y_n \in V_n$,

$$F(x_1y_1, y_2, \ldots, y_n) = F(y_1x_1, y_2, \ldots, y_n). \tag{28}$$

Replacing y_1 by x_1y_1 in (28), we have

$$\begin{aligned} & d(x_1, y_2, \ldots, y_n)g(x_1y_1) + x_1F(x_1y_1, y_2, \ldots, y_n) \\ & \qquad = d(x_1, y_2, \ldots, y_n)g(y_1x_1) + x_1F(y_1x_1, y_2, \ldots, y_n), \\ & d(x_1, y_2, \ldots, y_n)g(x_1y_1) + x_1F(x_1y_1, y_2, \ldots, y_n) \\ & \qquad = d(x_1, y_2, \ldots, y_n)g(y_1x_1) + x_1F(x_1y_1, y_2, \ldots, y_n). \end{aligned}$$

This implies that,

$$d(x_1, y_2, \ldots, y_n)g(x_1y_1 - y_1x_1) = 0.$$

Since g is additive and $g(V_1) = V_1$, we have

$$d(x_1, y_2, \dots, y_n)[x_1, y_1]{=}0, \text{ for all } x_1{\in}U_1, y_1{\in}V_1, y_2{\in}V_2, \dots, y_n \in V_n. \tag{29}$$

Replacing y_1 by y_1r, for all $r \in N$ in (29) and using (29), we find

$$d(x_1, y_2, \dots, y_n)y_1[x_1, r] = 0.$$

By Lemma 2.2(ii), we get $d(x_1, y_2, \dots, y_n) = 0$, for all $x_1 \in U_1, y_2 \in V_2, \dots, y_n \in V_n$ or $U_1 \subseteq Z(N)$. In first case, replacing y_2 by x_2y_2, for all $x_2 \in U_2$, we conclude that

$$d(x_1, x_2, \dots, y_n)y_2 + g(x_2)d(x_1, y_2, \dots, y_n) = 0.$$

The last expression yields that $d(x_1, x_2, \dots, y_n) = 0$, for all $x_1 \in U_1, x_2 \in U_2, \dots, y_n \in V_n$. Proceeding inductively, we obtain $d(x_1, x_2, \dots, x_n) = 0$, for all $x_1 \in U_1, x_2 \in U_2, \dots, x_n \in U_n$. Hence, $d(U_1, U_2, \dots, U_n) = \{0\}$ or $U_1 \subseteq Z(N)$. □

The following example demonstrates that the primeness hypothesis in Theorems 3.2, 3.4 to 3.11 is not superfluous.

Example 3. Let S be a commutative near ring. Consider

$$N = \left\{ \begin{pmatrix} 0 & x & y \\ 0 & 0 & 0 \\ 0 & z & 0 \end{pmatrix} \mid 0, x, y, z \in S \right\} \text{ and } U = \left\{ \begin{pmatrix} 0 & x & y \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \mid 0, x, y \in S \right\}.$$

It can be easily seen that N is a non prime zero-symmetric left near ring with respect to matrix addition and matrix multiplication and U is a nonzero semigroup ideal of N. Define mappings $F, d : \underbrace{N \times N \times ... \times N}_{n-times} \to N$ by

$$F\left(\begin{pmatrix} 0 & x_1 & y_1 \\ 0 & 0 & 0 \\ 0 & z_1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & x_2 & y_2 \\ 0 & 0 & 0 \\ 0 & z_2 & 0 \end{pmatrix}, \dots, \begin{pmatrix} 0 & x_n & y_n \\ 0 & 0 & 0 \\ 0 & z_n & 0 \end{pmatrix} \right) = \begin{pmatrix} 0 & z_1z_2...z_n & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$d\left(\begin{pmatrix} 0 & x_1 & y_1 \\ 0 & 0 & 0 \\ 0 & z_1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & x_2 & y_2 \\ 0 & 0 & 0 \\ 0 & z_2 & 0 \end{pmatrix}, \dots, \begin{pmatrix} 0 & x_n & y_n \\ 0 & 0 & 0 \\ 0 & z_n & 0 \end{pmatrix} \right) = \begin{pmatrix} 0 & y_1y_2...y_n & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

Define a map $g : N \to N$ by

$$g \begin{pmatrix} ccc0 & x & y \\ 0 & 0 & 0 \\ 0 & z & 0 \end{pmatrix} = \begin{pmatrix} 0 & z & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

If we choose $U_1 = U_2 = \cdots = U_n = U$, then it is easy to check that F is a nonzero generalized n-semiderivation associated with a nonzero n-semiderivation d and a map g associated with d on N satisfying the following conditions:

(i) $F(U_1, U_2, \ldots, U_n) \subseteq Z(N)$, (ii) $F([x_1, y_1], x_2, \ldots, x_n) = \pm[x_1, y_1]$,

(iii) $F(x_1 \circ y_1, x_2, \ldots, x_n) = 0$, (iv) $F(x_1 \circ y_1, x_2, \ldots, x_n) = \pm(x_1 \circ y_1)$,

(v) $F([x_1, y_1], x_2, \ldots, x_n) = \pm[F(x_1, x_2, \ldots, x_n), y_1]$,

(vi) $F([x_1, y_1], x_2, \ldots, x_n) = \pm[x_1, F(y_1, x_2, \ldots, x_n)]$,

(vii) $[F(x_1, x_2, \ldots, x_n), F(y_1, y_2, \ldots, y_n)] = 0$,

for all $x_1, y_1 \in U_1, x_2, y_2 \in U_2, \ldots, x_n, y_n \in U_n$. However, N is not commutative.

Example 4. Let $N_1 = (\mathbb{C}, +, \cdot)$ be the ring of complex numbers with respect to the usual addition and multiplication of complex numbers and $N_2 = (\mathbb{C}, +, \star)$, where $\mathbb{C}$ is the set of complex numbers, $+$ is the usual addition of complex numbers and $\star$ is defined by $x \star y =\mid x \mid \cdot y$, for all $x, y \in \mathbb{C}$. Then it is easy to see that N_2 is a zero-symmetric left near ring. Now, consider the set $S = N_1 \times N_2$, which is a non-commutative zero-symmetric left near ring with respect to the componentwise addition and multiplication. Suppose that

$$N = \left\{ \begin{pmatrix} (0,0) & (x,x') & (y,y') \\ (0,0) & (0,0) & (0,0) \\ (0,0) & (z,z') & (0,0) \end{pmatrix} \mid (x,x'), (y,y'), (z,z'), (0,0) \in S \right\}.$$

Then N is a zero-symmetric left near ring with respect to matrix addition and matrix multiplication but N is not 3-prime. Let

$$U = \left\{ \begin{pmatrix} (0,0) & (x,x') & (y,y') \\ (0,0) & (0,0) & (0,0) \\ (0,0) & (0,0) & (0,0) \end{pmatrix} \mid (x,x'), (y,y'), (0,0) \in S \right\},$$

which is a nonzero semigroup ideal of N.

Define mappings $F, d : \underbrace{N \times N \times \ldots \times N}_{n-times} \to N$ by

$$F\left(\begin{pmatrix} (0,0) & (x_1,x_1') & (y_1,y_1') \\ (0,0) & (0,0) & (0,0) \\ (0,0) & (z_1,z_1') & (0,0) \end{pmatrix}, \begin{pmatrix} (0,0) & (x_2,x_2') & (y_2,y_2') \\ (0,0) & (0,0) & (0,0) \\ (0,0) & (z_2,z_2') & (0,0) \end{pmatrix}, \ldots, \begin{pmatrix} (0,0) & (x_n,x_n') & (y_n,y_n') \\ (0,0) & (0,0) & (0,0) \\ (0,0) & (z_n,z_n') & (0,0) \end{pmatrix} \right) = \begin{pmatrix} (0,0) & (\bar{y_1}\bar{y_2}\ldots\bar{y_n}, 0) & (0,0) \\ (0,0) & (0,0) & (0,0) \\ (0,0) & (0,0) & (0,0) \end{pmatrix},$$

$$d\left(\begin{pmatrix} (0,0) & (x_1,x_1') & (y_1,y_1') \\ (0,0) & (0,0) & (0,0) \\ (0,0) & (z_1,z_1') & (0,0) \end{pmatrix}, \begin{pmatrix} (0,0) & (x_2,x_2') & (y_2,y_2') \\ (0,0) & (0,0) & (0,0) \\ (0,0) & (z_2,z_2') & (0,0) \end{pmatrix}, \ldots,$$

$$\begin{pmatrix}(0,0) & (x_n,x'_n) & (y_n,y'_n)\\ (0,0) & (0,0) & (0,0)\\ (0,0) & (z_n,z'_n) & (0,0)\end{pmatrix}\Bigg)\Bigg) = \begin{pmatrix}(0,0) & (y_1y_2...y_n,0) & (0,0)\\ (0,0) & (0,0) & (0,0)\\ (0,0) & (0,0) & (0,0)\end{pmatrix}$$

and a map $g : N \to N$ by

$$g\begin{pmatrix}(0,0) & (x,x') & (y,y')\\ (0,0) & (0,0) & (0,0)\\ (0,0) & (z,z') & (0,0)\end{pmatrix} = \begin{pmatrix}(0,0) & (x,x') & (y,y')\\ (0,0) & (0,0) & (0,0)\\ (0,0) & (0,0') & (0,0)\end{pmatrix},$$

where $\bar{y_1}, \bar{y_2}, \ldots, \bar{y_n}$ are the complex conjugates of $y_1, y_2, \ldots, y_n$ respectively. If we choose $U_1 = U_2 = \cdots = U_n = U$, then it is verified that F is a generalized n-semiderivation associated with an n-semiderivation d and a map g associated with d on N satisfying the following conditions:

(i) $F(U_1, U_2, \ldots, U_n) \subseteq Z(N)$, (ii) $F([x_1, y_1], x_2, \ldots, x_n) = \pm[x_1, y_1]$,

(iii) $F(x_1 \circ y_1, x_2, \ldots, x_n) = 0$, (iv) $F(x_1 \circ y_1, x_2, \ldots, x_n) = \pm(x_1 \circ y_1)$,

(v) $F([x_1, y_1], x_2, \ldots, x_n) = \pm[F(x_1, x_2, \ldots, x_n), y_1]$,

(vi) $F([x_1, y_1], x_2, \ldots, x_n) = \pm[x_1, F(y_1, x_2, \ldots, x_n)]$,

(vii) $[F(x_1, x_2, \ldots, x_n), F(y_1, y_2, \ldots, y_n)] = 0$,

for all $x_1, y_1 \in U_1, x_2, y_2 \in U_2, \ldots, x_n, y_n \in U_n$.

But, N is not commutative.

Open problem

(i) However, one can construct a natural example of a non-commutative near ring satisfying the hypothesis of the above theorems. (ii) Our hypothesis are dealt with the prime near rings. For further research, one can discuss the commutativity of semiprime near rings which is an interesting work in future.

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