Strong modular product and complete fuzzy graphs

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Abstract. In this paper, we provide an improvement of the modular product of fuzzy graphs defined by [16] in 2015, which we call strong modular product. We give sufficient conditions for the strong modular product of two fuzzy graphs to be complete and we show that if the strong modular product of two fuzzy graphs is complete, then at least one factor is a complete fuzzy graph. Moreover, we give necessary and sufficient conditions for the strong modular product of two balanced fuzzy graphs to be balanced.
Keywords: fuzzy graph, complete fuzzy graph, strong modular product, balanced fuzzy graph.

1. Introduction

Graph theory applications in system analysis, operations research and economics are very important. Since the appearance of graph problems are somtimes not known beyond doubt, it is nice to deal with them via fuzzy logic. The concept of fuzzy relation was introduced by Zadeh [23] in his landmark paper "Fuzzy sets" in 1965. Fuzzy graph and several fuzzy graph concepts were introduced by Rosenfeld [21] in 1975. Lately, fuzzy graph theory is having more and more applications in real time modeling in which the level of information immanent in the system changes.

Mordeson and Peng [17] defined the concept of complement of fuzzy graph and studied some operations on fuzzy graphs. In [22], modified the definition of complement of a fuzzy graph so that the complement of the complement is the original fuzzy graph, which agrees with the classical graph case. Moreover several properties of self-complementary fuzzy graphs and the complement of some operations of fuzzy graphs that were introduced in [17] were studied. For more on the previous notions and the following ones, one can see [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 17, 18, 19, 20, 21, 22].

A fuzzy subset of a non-empty set V is a function $\sigma : V \to [0,1]$ and a fuzzy relation μ on σ is a fuzzy subset of $V \times V$. All throughout this paper, we assume that V is finite, σ is reflexive and μ is symmetric.

Definition 1.1. *[21] A* fuzzy graph $G : (\sigma, \mu)$ *where* σ *is a fuzzy subset of* V *and* μ *is a fuzzy relation on* σ *such that* $\mu(x, y) \le \sigma(x) \wedge \sigma(y)$ *for all* $x, y \in V$, *where $\wedge$ stands for minimum. The underlying crisp graph of G is denoted by $G^* : (\sigma^*, \mu^*)$ where $\sigma^* = \sup p(\sigma) = \{x \in V : \sigma(x) > 0\}$ and $\mu^* = \sup p(\mu) = \{(x,y) \in V \times V : \mu(x,y) > 0\}$.$H = (\sigma', \mu')$ is a* fuzzy subgraph *of G if there exists $X \subseteq V$ such that, $\sigma' : X \to [0,1]$ is a fuzzy subset and $\mu' : X \times X \to [0,1]$ is a fuzzy relation on σ' such that $\mu(x,y) \leq \sigma(x) \wedge \sigma(y)$ for all $x, y \in X$.*

Definition 1.2 ([20])**.** *A fuzzy graph $G : (\sigma, \mu)$ is* complete *if $\mu(x,y) = \sigma(x) \wedge \sigma(y)$ for all $x, y \in V$.*

Next, we recall the following two results from [22].

Lemma 1.1. *Let $G : (\sigma, \mu)$ be a self-complemetary fuzzy graph. Then $\sum_{x,y \in V} \mu(x,y) = (1/2) \sum_{x,y \in V} (\sigma(x) \wedge \sigma(y))$*

Lemma 1.2. *Let $G : (\sigma, \mu)$ be a fuzzy graph satisfying $\mu(x,y) = (1/2)(\sigma(x) \wedge \sigma(y))$ for all $x, y \in V$.Then G is self-complemetary.*

Definition 1.3 ([15])**.** *Two fuzzy graphs $G_1 : (\sigma_1, \mu_1)$ with crisp graph $G_1^* : (V_1, E_1)$ and $G_2 : (\sigma_2, \mu_2)$ with crisp graph $G_2^* : (V_2, E_2)$ are* isomorphic *if there exists a bijection $h : V_1 \to V_2$ such that $\sigma_1(x) = \sigma_2(h(x))$ and $\mu_1(x,y) = \mu_2(h(x), h(y))$ for all $x, y \in V_1$.*

Lemma 1.3 ([18])**.** *Any two isomorphic fuzzy graphs $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ satisfy $\sum_{x \in V_1} \sigma_1(x) = \sum_{x \in V_2} \sigma_2(x)$ and*

$$\sum_{x,y \in V_1} \mu_1(x,y) = \sum_{x,y \in V_2} \mu_2(x,y).$$

Definition 1.4 ([5])**.** *The* density *of a fuzzy graph $G : (\sigma, \mu)$ is*

$$D(G) = 2\Big(\sum_{u,v \in V} \mu(u,v)\Big)/\Big(\sum_{u,v \in V} (\sigma(u) \wedge \sigma(v))\Big).$$

G is balanced *if. $D(H) \leq D(G)$ for all fuzzy non-empty subgraphs H of G.*

Theorem 1.1 ([5])**.** *A complete fuzzy graph is balanced.*

A new operation on fuzzy graphs is next recalled:

Definition 1.5 ([16])**.** *The* modular product *of two fuzzy graphs $G_1 : (\sigma_1, \mu_1)$ with crisp graph $G_1^* : (V_1, E_1)$ and $G_2 : (\sigma_2, \mu_2)$ with crisp graph $G_2^* : (V_2, E_2)$ is defined to be the fuzzy graph $G_1 \circledcirc G_2 : (\sigma_1 \circledcirc \sigma_2, \mu_1 \circledcirc \mu_2)$ with crisp graph $G^* : (V_1 \times V_2, E)$ where*

$$E = \{(u_1, v_1)(u_2, v_2) : u_1u_2 \in E_1, v_1v_2 \in E_2\},$$

$(\sigma_1 \circledcirc \sigma_2)(u,v) = \sigma_1(u) \wedge \sigma_2(v)$, for all $(u,v) \in V_1 \times V_2$ and $(\mu_1 \circledcirc \mu_2)((u_1,v_1)(u_2,v_2)) = \mu_1(u_1u_2) \wedge \mu_2(v_1v_2)$ when $u_1u_2 \in E_1, v_1v_2 \in E_2$, $(\mu_1 \circledcirc \mu_2)((u_1,v_1)(u_2,v_2)) = \sigma_1(u_1) \wedge \sigma_1(u_2) \wedge \sigma_2(v_1) \wedge \sigma_2(v_2)$ when $u_1u_2 \notin E_1, v_1v_2 \notin E_2$.

In [16], it was proved thet the modular product of two strong fuzzy graphs is a strong fuzzy graph. Clearly, the modular product of two complete fuzzy graphs need not be a complete fuzzy graph as $(\mu_1 \circledcirc \mu_2)((u_1,v_1)(u_2,v_2))$ is not defined above for the case $u_1 = u_2$ or $v_1 = v_2$.

In Section 2 of this paper, we provide an improvement of the modular product of fuzzy graphs defined by [16], which we call strong modular product. We give sufficient conditions for the strong modular product of two fuzzy graphs to be complete and we show that if the strong modular product is complete, then at least one factor is a complete fuzzy graph. Section 3 is divoted to give necessary and sufficient conditions for the strong modular product of two fuzzy balanced graphs to be balanced.

2. Strong modular product of fuzzy graphs

It clear that the modular product of two complete fuzzy graphs need not be complete, see the example in Figure 4.1 in [16]. Next, we modify the above definition so that the preceding property holds.

Definition 2.1. *The* strong modular product *of two fuzzy graphs $G_1 : (\sigma_1, \mu_1)$ with crisp graph $G_1^* : (V_1, E_1)$ and $G_2 : (\sigma_2, \mu_2)$ with crisp graph $G_2^* : (V_2, E_2)$ is defined to be the fuzzy graph $G_1 \boxplus G_2 : (\sigma_1 \boxplus \sigma_2, \mu_1 \boxplus \mu_2)$ with crisp graph $G^* : (V_1 \times V_2, E)$ where*

$$E = \{(u_1,v_1)(u_2,v_2) : u_1u_2 \in E_1, v_1v_2 \in E_2\},$$

$(\sigma_1 \boxplus \sigma_2)(u,v) = \sigma_1(u) \wedge \sigma_2(v),$ *for all* $(u,v) \in V_1 \times V_2$ *and*

$$(\mu_1 \boxplus \mu_2)((u_1,v_1)(u_2,v_2)) = \begin{cases} \mu_1(u_1u_2) \wedge \mu_2(v_1v_2), & u_1u_2 \in E_1, v_1v_2 \in E_2 \\ \sigma_1(u_1) \wedge \sigma_1(u_2) \wedge \sigma_2(v_1) \wedge \sigma_2(v_2), & u_1u_2 \notin E_1, v_1v_2 \notin E_2 \\ \sigma_1(u_1) \wedge \mu_2(v_1v_2), & u_1 = u_2, v_1v_2 \in E_2 \\ \sigma_2(v_1) \wedge \mu_1(u_1u_2), & u_1u_2 \in E_1, v_1 = v_2. \end{cases}$$

Next, we show that the above definition is well-defined.

Theorem 2.1. *The strong modular product of two fuzzy graphs is a fuzzy graph.*

Proof. Let $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ be two fuzzy graphs with underlying graphs $G_1^* : (V_1, E_1)$ and $G_2^* : (V_2, E_2)$, respectively. Since Case 1 and Case 2 are proved in [16] and as Case 3 is similar to Case 4, we only prove Case 3.
Case 3. If $u_1 = u_2, v_1v_2 \in E_2$, then as G_2 is a fuzzy graph

$$\begin{aligned}(\mu_1 \boxplus \mu_2)((u_1,v_1)(u_2,v_2)) &= \sigma_1(u_1) \wedge \mu_2(v_1v_2) \\ &\leq \sigma_1(u_1) \wedge \sigma_2(v_1) \wedge \sigma_2(v_2).\end{aligned}$$

Thus

$$\begin{aligned}(\mu_1 \boxplus \mu_2)((u_1,v_1)(u_2,v_2)) &\leq \sigma_1(u_1) \wedge \sigma_1(u_2) \wedge \sigma_2(v_1) \wedge \sigma_2(v_2)\\ &\quad ((\sigma_1 \boxplus \sigma_2)(u_1,v_1)) \wedge ((\sigma_1 \boxplus \sigma_2)(u_2,v_2)). \quad \square\end{aligned}$$

Next, we show that the strong modular product of two complete fuzzy graphs are again a complete fuzzy graph.

Theorem 2.2. *If $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ are complete fuzzy graphs, then $G_1 \boxplus G_2$ is a complete fuzzy graph.*

Proof. If $(u_1, v_1)(u_2, v_2) \in E$, then we have the following cases:
Case 1. $u_1u_2 \in E_1, v_1v_2 \in E_2$.
Case 2. $u_1u_2 \notin E_1, v_1v_2 \notin E_2$.
Case 3. $u_1 = u_2, v_1v_2 \in E_2$.
Case 4. $u_1u_2 \in E_1, v_1 = v_2$.

Cases 1 and 2 follow from the proof of Theorem 4.2 in [16]. Case 3 and Case 4 are similar, so we only prove Case 3.
Case 3. Since G_2 is complete,

$$\begin{aligned}(\mu_1 \boxplus \mu_2)((u_1,v_1)(u_2,v_2)) &= \sigma_1(u_1) \wedge \mu_2(v_1v_2)\\ &= \sigma_1(u_1) \wedge \sigma_2(v_1) \wedge \sigma_2(v_2)\\ &= \sigma_1(u_1) \wedge \sigma_1(u_2) \wedge \sigma_2(v_1) \wedge \sigma_2(v_2)\\ &= (\sigma_1 \boxplus \sigma_2)((u_1,v_1)) \wedge (\sigma_1 \boxplus \sigma_2)((u_2,v_2)).\end{aligned}$$

Hence, $G_1 \boxplus G_2$ is complete. $\square$

Corollary 2.1. *If $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ are complete (strong) fuzzy graphs, then $G_1 \boxplus G_2$ is a strong fuzzy graph.*

An interesting property of complement is given next.

Theorem 2.3. *If $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ are complete fuzzy graphs, then $\overline{G_1 \boxplus G_2} \simeq \overline{G_1} \boxplus \overline{G_2}$.*

Proof. Let $G : (\sigma, \overline{\mu}) = \overline{G_1 \boxplus G_2}$, $\overline{\mu} = \overline{\mu_1 \boxplus \mu_2}$, $\overline{G^*} = (V, \overline{E})$, $\overline{G_1} : (\sigma_1, \overline{\mu_1})$, $\overline{G_1^*} = (V_1, \overline{E_1})$, $\overline{G_2} : (\sigma_2, \overline{\mu_2})$, $\overline{G_2^*} = (V_2, \overline{E_2})$ and $\overline{G_1} \boxplus \overline{G_2} : (\sigma_1 \boxplus \sigma_2, \overline{\mu_1} \boxplus \overline{\mu_2})$. We only need to show $\overline{\mu_1 \boxplus \mu_2} = \overline{\mu_1} \boxplus \overline{\mu_2}$. For any arc e joining nodes of V,we have the following cases:
Case 1. If $u_1u_2 \in E_1, v_1v_2 \in E_2$, then as G is complete by Theorem 2.2, $\overline{\mu}(e) = 0$. On the other hand, $(\overline{\mu_1} \boxplus \overline{\mu_2})(e) = 0$ since $u_1u_2 \notin \overline{E_1}$ and $v_1v_2 \notin \overline{E_2}$.
Case 2. If $u_1u_2 \notin E_1, v_1v_2 \notin E_2$, then is case is not possible to occur as both G_1 and G_2 are complete.
Case 3. $e = (u, v_1)(u, v_2)$ where $v_1v_2 \in E_2$.Then as G is complete by Theorem 2.2, $\overline{\mu}(e) = 0$. On the other hand, $(\overline{\mu_1} \boxplus \overline{\mu_2})(e) = 0$ since $v_1v_2 \notin \overline{E_2}$.
Case 4. Similar proof to Case 3.

In all cases $\overline{\mu_1 \boxplus \mu_2} = \overline{\mu_1} \boxplus \overline{\mu_2}$ and therefore, $\overline{G_1 \boxplus G_2} \simeq \overline{G_1} \boxplus \overline{G_2}$. $\square$

Next, we show that if the strong modular product of two fuzzy graphs is complete, then at least one of the two fuzzy graphs must be complete.

Theorem 2.4. *If $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ are fuzzy graphs such that $G_1 \boxplus G_2$ is complete, then at least G_1 or G_2 must be complete.*

Proof. Suppose to the contrary that both G_1 and G_2 are not complete. Then there exists at least one $u_1, u_2 \in V_1$ and $v_1, v_2 \in V_2$ such that $\mu_1(u_1u_2) < \sigma_1(u_1) \wedge \sigma_1(u_2))$ and $\mu_2(v_1v_2) < \sigma_2(v_1) \wedge \sigma_2(v_2))$ then, we have the following cases:
Case 1. If $u_1u_2 \in E_1, v_1v_2 \in E_2$, then $(\mu_1 \boxplus \mu_2)((u_1, v_1)(u_2, v_2)) = \mu_1(u_1u_2) \wedge \mu_2(v_1v_2)$ and as $G_1 \boxplus G_2$ is complete,

$$\begin{aligned}(\mu_1 \boxplus \mu_2)((u_1, v_1)(u_2, v_2)) &= (\sigma_1 \boxplus \sigma_2)((u_1, v_1)) \wedge (\sigma_1 \boxplus \sigma_2)((u_2, v_2))\\ &= \sigma_1(u_1) \wedge \sigma_1(u_2) \wedge \sigma_2(v_1) \wedge \sigma_2(v_2)\\ &> \mu_1(u_1u_2) \wedge \mu_2(v_1v_2),\end{aligned}$$

which is a contradiction.
Case 2. If $u_1u_2 \notin E_1, v_1v_2 \notin E_2$, then $(\mu_1 \boxplus \mu_2)((u_1, v_1)(u_2, v_2)) = \sigma_1(u_1) \wedge \sigma_1(u_2) \wedge \sigma_2(v_1) \wedge \sigma_2(v_2)$ and as $G_1 \boxplus G_2$ is complete,

$$\begin{aligned}(\mu_1 \boxplus \mu_2)((u_1, v_1)(u_2, v_2)) &= (\sigma_1 \boxplus \sigma_2)((u_1, v_1)) \wedge (\sigma_1 \boxplus \sigma_2)((u_2, v_2))\\ &= \sigma_1(u_1) \wedge \sigma_1(u_2) \wedge \sigma_2(v_1) \wedge \sigma_2(v_2)\\ &= \mu_1(u_1u_2) \wedge \mu_2(v_1v_2),\end{aligned}$$

which is a contradiction.
Case 3. If $u_1 = u_2, v_1v_2 \in E_2$, then $(\mu_1 \boxplus \mu_2)((u_1, v_1)(u_2, v_2)) = \sigma_1(u_1) \wedge \mu_2(v_1v_2)$ and as $G_1 \boxplus G_2$ is complete,

$$\begin{aligned}(\mu_1 \boxplus \mu_2)((u_1, v_1)(u_2, v_2)) &= (\sigma_1 \boxplus \sigma_2)((u_1, v_1)) \wedge (\sigma_1 \boxplus \sigma_2)((u_2, v_2))\\ &= \sigma_1(u_1) \wedge \sigma_2(v_1) \wedge \sigma_2(v_2)\\ &> \mu_1(u_1u_2) \wedge \mu_2(v_1v_2),\end{aligned}$$

thus $G_1 \boxplus G_2$ is not complete.
Case 4. If $u_1u_2 \in E_1, v_1 = v_2$, the proof is similar to Case 3. □

3. Blanced notion virsus strong modular product

We begin this section by proving the following lemma that we use to give necessary and sufficient conditions for the strong modular product of two balanced fuzzy graphs to be balanced.

Lemma 3.1. *Let G_1 and G_2 be fuzzy graphs. Then $D(G_i) \leq D(G_1 \boxplus G_2)$ for $i = 1, 2$ if and only if $D(G_1) = D(G_2) = D(G_1 \boxplus G_2)$.*

Proof. If $D(G_i) \leq D(G_1 \boxplus G_2)$ for $i = 1, 2$, then

$$D(G_1) = 2(\sum_{u_1,u_2\in V_1} \mu_1(u_1u_2))/(\sum_{u_1,u_2\in V_1} (\sigma_1(u_1) \wedge \sigma_1(u_2)))$$
$$\geq 2(\sum_{\substack{u_1,u_2\in V_1\\ v_1,v_2\in V_2}} \mu_1(u_1u_2)\wedge\sigma_2(v_1)\wedge\sigma_2(v_2))/(\sum_{\substack{u_1,u_2\in V_1\\ v_1,v_2\in V_2}} (\sigma_1(u_1)\wedge\sigma_1(u_2)\wedge\sigma_2(v_1)\wedge\sigma_2(v_2)))$$
$$= 2(\sum_{\substack{u_1,u_2\in V_1\\ v_1,v_2\in V_2}} \mu_1(u_1u_2) \wedge \mu_2(v_1v_2))/(\sum_{\substack{u_1,u_2\in V_1\\ v_1,v_2\in V_2}} (\sigma_1(u_1) \wedge \sigma_1(u_2) \wedge \sigma_2(v_1) \wedge \sigma_2(v_2)))$$
$$= 2(\sum_{\substack{u_1,u_2\in V_1\\ v_1,v_2\in V_2}} \mu_1 \boxplus \mu_2((u_1, v_1)(u_2, v_2))/(\sum_{\substack{u_1,u_2\in V_1\\ v_1,v_2\in V_2}} (\sigma_1 \boxplus \sigma_2((u_1, v_1)(u_2, v_2)))$$
$$= D(G_1 \boxplus G_2).$$

Hence, in all cases $D(G_1) \geq D(G_1 \boxplus G_2)$ and thus $D(G_1) = D(G_1 \boxplus G_2)$. Similarly, $D(G_2) = D(G_1 \boxplus G_2)$. Therefore, $D(G_1) = D(G_2) = D(G_1 \boxplus G_2)$. □

Theorem 3.1. *Let G_1 and G_2 be balanced fuzzy graphs. Then $G_1 \boxplus G_2$ is balanced if and only if $D(G_1) = D(G_2) = D(G_1 \boxplus G_2)$.*

Proof. If $G_1 \boxplus G_2$ is balanced, then $D(G_i) \leq D(G_1 \boxplus G_2)$ for $i = 1, 2$ and by Lemma 3.1, $D(G_1) = D(G_2) = D(G_1 \boxplus G_2)$.

Conversely, if $D(G_1) = D(G_2) = D(G_1 \boxplus G_2)$ and H is a fuzzy subgraph of $G_1 \boxplus G_2$, then there exist fuzzy subgraphs H_1 of G_1 and H_2 of G_2. As G_1 and G_2 are balanced and $D(G_1) = D(G_2) = n_1/r_1$, then $D(H_1) = a_1/b_1 \leq n_1/r_1$ and $D(H_2) = a_2/b_2 \leq n_1/r_1$. Thus $a_1r_1 + a_2r_1 \leq b_1n_1 + b_2n_1$ and hence $D(H) \leq (a_1 + a_2)/(b_1 + b_2) \leq n_1/r_1 = D(G_1 \boxplus G_2)$. Therefore, $G_1 \boxplus G_2$ is balanced. □

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