A study on intuitionistic fuzzy topological operators

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Abstract. In this paper, new intuitionistic fuzzy topological operators are introduced by considering Marinov and Atanassov's last operators. We show that these operators are also pair of conjugate preinterior-preclosure operators. In addition, some properties of these operators are examined.
Keywords: intuitionistic fuzzy sets, intuitionistic fuzzy pretopological operators, intuitionistic fuzzy topological operators.

1. Introduction

Fuzzy set theory was introduced by Zadeh [18] as an object whose elements have memberships degrees in the $[0,1]$ interval. In following years, many researchers studied on the generalization of the fuzzy set concept. Atanassov introduced the concept of Intuitionistic Fuzzy Sets, form an extension of fuzzy sets by expanding the truth value set to the lattice $[0,1]\times[0,1]$ is defined as following:

Definition 1.1. *Let $L=[0,1]$ then $L^*=\{(x_1,x_2)\in[0,1]^2 : x_1+x_2\leq 1\}$ is a lattice with $(x_1,x_2)\leq(y_1,y_2) :\Longleftrightarrow$ "$x_1\leq y_1$ and $x_2\geq y_2$". For $(x_1,y_1),(x_2,y_2)\in L^*$,the operators $\wedge$ and $\vee$ on $(L^*,\leq)$ are defined as following;*

$$(x_1,y_1)\wedge(x_2,y_2)=(\min(x_1,x_2),\max(y_1,y_2)),$$
$$(x_1,y_1)\vee(x_2,y_2)=(\max(x_1,x_2),\min(y_1,y_2)).$$

For each $J\subseteq L^*$ $\sup J=(\sup\{x:(x,y\in[0,1]),((x,y)\in J)\},\inf\{y:(x,y\in[0,1])((x,y)\in J)\})$ and $\inf J=(\inf\{x:(x,y\in[0,1])((x,y)\in J)\},\sup\{y:(x,y\in[0,1])((x,y)\in J)\})$.

Topology concept is widely used by mathematicians and other scientists in modeling real-world structures and problems. This approach is based on identifying and using the common points of different shapes. The Intuitionistic Fuzzy Topology was defined by Çoker in 1997 ([6]). Expanding the topology theory on intuitionistic fuzzy sets has attracted the attention of many researchers.Various studies were done based on the application and theoretical fields [8, 9, 10, 13].

The application of intuitionistic fuzzy topology on spatial objects was examined firstly by M.R. Malek [11].

Operator theory has an important role in modeling real-world problems. The concept of intuitionistic fuzzy modal operators was defined by K. Atanassov in 1999 and then modal operators were studied extensively in various fields (see [2, 4, 7, 15]). The first intuitionistic fuzzy topological operators were defined by K. Atanassov and in subsequent studies new intuitionistic fuzzy topological operators were introduced (see [4, 8, 15]). Fuzzy and intuitionistic fuzzy pre-topological /topological operators are applied in computing the values of fuzzy relations of spatial objects with uncertainty in determining the boundaries such as forest area, lake, sea, etc. ([5, 13, 14, 16, 17]). Therefore, the defining of new topological operators is important for approaching spatial problems.

In this paper, new intuitionistic fuzzy topological operators are introduced and some properties are examined.

2. Preliminaries

Definition 2.1 ([2]). *An intuitionistic fuzzy set (shortly IFS) on a set X is an object of the form*

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\},$$

where $\mu_A(x), (\mu_A : X \to [0,1])$ is called the "degree of membership of x in A", $\nu_A(x), (\nu_A : X \to [0,1])$ is called the "degree of non- membership of x in A", and where μ_A and ν_A satisfy the following condition:

$$\mu_A(x) + \nu_A(x) \leq 1, \text{ for all } x \in X.$$

The class of intuitionistic fuzzy sets on X is denoted by $IFS(X)$. The hesitation degree of x is defined by $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$.

Definition 2.2 ([2]). *An IFS A is said to be contained in an IFS B (notation $A \sqsubseteq B$) if and only if, for all $x \in X : \mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$.*

It is clear that $A = B$ if and only if $A \sqsubseteq B$ and $B \sqsubseteq A$.

Definition 2.3 ([2]). *Let $A \in IFS$ and let $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$ then the above set is callede the complement of A*

$$A^c = \{\langle x, \nu_A(x), \mu_A(x)\rangle : x \in X\}.$$

The intersection and the union of two IFSs A and B on X is defined by

$$A \sqcap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)\rangle : x \in X\},$$

$$A \sqcup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x)\rangle : x \in X\}.$$

Some special Intuitionistic Fuzzy Sets on X are defined as following;

$$O^* = \{\langle x, 0, 1\rangle : x \in X\},$$

$$X^* = \{< x, 1, 0 >: x \in X\}.$$

Atanassov introduced topological operators, and the extensions of these operators was defined by same author in 2001 as:

Definition 2.4 ([2])**.** *Let X be a set and $A \in IFS(X)$*

$$C(A) = \{\langle x, K, L\rangle : x \in X\},$$

where $K = \sup_{y \in X} \mu_A(y)$, $L = \inf_{y \in X} \nu_A(y)$ and

$$I(A) = \{\langle x, k, l\rangle : x \in X\},$$

where $k = \inf_{y \in X} \mu_A(y), l = \sup_{y \in X} \nu_A(y)$.

Definition 2.5 ([3, 4])**.** *Let X be a set and $A \in IFS(X)$. Let K, L, k and l be as above forms:*

1. $C_\mu(A) = \{\langle x, K, \min(1 - K, \nu_A(x))\rangle : x \in X\};$
2. $C_\nu(A) = \{\langle x, \mu_A(x), L\rangle : x \in X\};$
3. $I_\mu(A) = \{\langle x, k, \nu_A(x)\rangle : x \in X\};$
4. $I_\nu(A) = \{\langle x, \min(1 - l, \mu_A(x)), l\rangle : x \in X\}.$

The characteristic of these operators were investigated in the same study. We will now define some topological concepts that we refer to in this study.

Definition 2.6 ([1])**.** *An pre$-$closure operator $\mathbf{c} : \mathbf{X} \to \mathbf{X}$ is a map which associates to each set $A \in \mathbf{X}$ a set $\mathbf{c}(A)$ such that:*

1. $\mathbf{c}(\emptyset) = \emptyset;$
2. $A \subseteq \mathbf{c}(A);$
3. $\mathbf{c}(A \cup B) = \mathbf{c}(A) \cup \mathbf{c}(B)$, *for all $A, B \subset X$.*

If in addition to above axioms the operator $\mathbf{c}$ is idempotent, that is $\mathbf{c}(A) = \mathbf{c}(\mathbf{c}(A))$ then $\mathbf{c}$ is called closure operator in $\mathbf{X}$. $\mathbf{X}$ can be $\wp(X), FS(X)$ or $IFS(X)$.

Definition 2.7 ([1])**.** *For the pre$-$closure operator $\mathbf{c}$ defined on $\mathbf{X}$ we say that a set $A \in \mathbf{X}$ is closed iff $\mathbf{c}(A) = A$. Also,*

$$\tau^{\mathbf{c}} = \{A : A \in \mathbf{X} \;\&\; \mathbf{c}(A) = A\}$$

is the topology generated by the pre$-$closure operator $\mathbf{c}$. If $\mathbf{X}$ is $\wp(X), FS(X)$ or $IFS(X)$ then τ is called crisp topology, fuzzy topology or intuitionistic fuzzy topology, respectively.

Definition 2.8 ([1]). *An pre−interior operator* $\mathbf{i} : \mathbf{X} \to \mathbf{X}$ *is a map which associates to each set* $A \in \mathbf{X}$ *a set* $\mathbf{i}(A)$ *such that:*

1. $\mathbf{i}(\mathbf{X}) = \mathbf{X}$;
2. $\mathbf{i}(A) \subseteq A$;
3. $\mathbf{i}(A \cap B) = \mathbf{i}(A) \cap \mathbf{i}(B)$, *for all* $A, B \subset X$.

If in addition to above axioms the operator $\mathbf{i}$ is idempotent, that is $\mathbf{i}(A) = \mathbf{i}(\mathbf{i}(A))$ then $\mathbf{i}$ is called interior operator in $\mathbf{X}$. $\mathbf{X}$ can be $\wp(X), FS(X)$ or $IFS(X)$.

Definition 2.9 ([1]). *For the pre−interior operator* $\mathbf{i}$ *defined on* $\mathbf{X}$ *we say that a set* $A \in \mathbf{X}$ *is open iff* $\mathbf{i}(A) = A$. *Also,*

$$\tau_{\mathbf{i}} = \{A : A \in \mathbf{X} \ \& \ \mathbf{i}(A) = A\}$$

is the topology generated by the pre−interior operator $\mathbf{i}$. *If* $\mathbf{X}$ *is* $\wp(X), FS(X)$ *or* $IFS(X)$ *then* τ *is called crisp topology, fuzzy topology or intuitionistic fuzzy topology, respectively.*

Remark 2.1. If $\mathbf{i}$ is (pre)interior operator then $\mathbf{c}(A) = \neg\mathbf{i}(\neg A)$ is its corresponding $pre-$closure. That is $(\mathbf{c}(A), \neg\mathbf{i}(\neg A))$ is a pair of conjugate preclosure-preinterior operators.

Proposition 2.1 ([1]). *If* $\mathbf{i}$ *and* $\mathbf{c}$ *is a conjugate pair of preinterior and preclosure operators in* $\mathbf{X}$, *then*

$$\tau^{\mathbf{c}} = \{\neg A : A \in \tau_{\mathbf{i}}\} \ \text{ and } \tau_{\mathbf{i}} = \{\neg B : B \in \tau^{\mathbf{c}}\}.$$

In [14], Marinov and Atanassov generalized the $pre-$interior and $pre-$closure operators to intuitionistic fuzzy sets and introduced new intuitionistic fuzzy topological operators. In the same paper, they examined topological properties of these operators in detail.

Definition 2.10 ([14]). *Let us denote* $\overline{\alpha} = (\alpha_0, \alpha_1)$ *and* $\overline{\beta} = (\beta_0, \beta_1)$, *where* $\alpha_i, \beta_i \in [0,1]$ *for* $i \in \{0,1\}$ *and* $\alpha_0 \leq \alpha_1, \beta_0 \leq \beta_1$. *For every* $\gamma_{\overline{\alpha}}, \gamma_{\overline{\beta}} \in [0,1]$ *and based on an arbitrary* $A \in IFS(X)$. *The topological operators*

$$I^{\gamma_{\overline{\alpha}},\gamma_{\overline{\beta}}}_{\mu;\overline{\alpha},\overline{\beta}}, C^{\gamma_{\overline{\alpha}},\gamma_{\overline{\beta}}}_{\nu;\overline{\alpha},\overline{\beta}} : IFS(X) \to IFS(X)$$

are defined as follow;

$$\mu_{I^{\gamma_{\overline{\alpha}},\gamma_{\overline{\beta}}}_{\mu;\overline{\alpha},\overline{\beta}}(A)}(x) = \begin{cases} \mu_A(x), & 0 \leq \mu_A(x) < \alpha_0 \\ \alpha_0, & \alpha_0 \leq \mu_A(x) < \alpha_0 + \gamma_{\overline{\alpha}}(\alpha_1 - \alpha_0) \\ \frac{1}{1-\gamma_{\overline{\alpha}}}(\mu_A(x) - \alpha_1) + \alpha_1, & \alpha_0 + \gamma_{\overline{\alpha}}(\alpha_1 - \alpha_0) \leq \mu_A(x) < \alpha_1 \\ \mu_A(x), & \alpha_1 \leq \mu_A(x) \leq 1 \end{cases},$$

$$\nu_{I^{\gamma_{\overline{\alpha}},\gamma_{\overline{\beta}}}_{\mu;\overline{\alpha},\overline{\beta}}(A)}(x)=\begin{cases}\nu_A(x), & 0\le \nu_A(x)<\beta_0\\ \min\left\{\begin{array}{c}\left(1-\gamma_{\overline{\beta}}\right)\nu_A(x)+\beta_1\gamma_{\overline{\beta}},\\ 1-\mu_{I^{\gamma_{\overline{\alpha}},\gamma_{\overline{\beta}}}_{\mu;\overline{\alpha},\overline{\beta}}(A)}(x)\end{array}\right\}, & \beta_0\le \nu_A(x)<\beta_1\\ \nu_A(x), & \beta_1\le \nu_A(x)\le 1\end{cases}$$

and

$$\mu_{C^{\gamma_{\overline{\alpha}},\gamma_{\overline{\beta}}}_{\nu;\overline{\alpha},\overline{\beta}}(A)}(x)=\begin{cases}\mu_A(x), & 0\le \mu_A(x)<\beta_0\\ \min\left\{\begin{array}{c}\left(1-\gamma_{\overline{\beta}}\right)\mu_A(x)+\beta_1\gamma_{\overline{\beta}},\\ 1-\nu_{C^{\gamma_{\overline{\alpha}},\gamma_{\overline{\beta}}}_{\nu;\overline{\alpha},\overline{\beta}}(A)}(x)\end{array}\right\}, & \beta_0\le \mu_A(x)<\beta_1\ ,\\ \mu_A(x), & \beta_1\le \mu_A(x)\le 1\end{cases}$$

$$\nu_{C^{\gamma_{\overline{\alpha}},\gamma_{\overline{\beta}}}_{\nu;\overline{\alpha},\overline{\beta}}(A)}(x)=\begin{cases}\nu_A(x), & 0\le \nu_A(x)<\alpha_0\\ \alpha_0, & \alpha_0\le \nu_A(x)<\alpha_0+\gamma_{\overline{\alpha}}(\alpha_1-\alpha_0)\\ \frac{1}{1-\gamma_{\overline{\alpha}}}(\nu_A(x)-\alpha_1)+\alpha_1, & \alpha_0+\gamma_{\overline{\alpha}}(\alpha_1-\alpha_0)\le \nu_A(x)<\alpha_1\\ \nu_A(x), & \alpha_1\le \nu_A(x)\le 1\end{cases}$$

3. Main results

In this study, new intuitionistic fuzzy topological operators are defined by considering the operators defined by Marinov and Atanassov in [14]. The variation of *pre*−closure operator, *pre*−interior operator and boundary value according to varying α,β,γ and ω values is examined with an example.

Definition 3.1. *Let X be a set and $A\in IFS(X)$. For $\alpha,\beta,\gamma,\omega\in[0,1]$, the topological operator $I^{\gamma,\omega}_{\alpha,\beta}$ is defined as follow;*

$$I^{\gamma,\omega}_{\alpha,\beta}: IFS(X)\to IFS(X)$$

such that

$$\mu_{I^{\gamma,\omega}_{\alpha,\beta}(A)}(x)=\begin{cases}\inf \mu_A(x), & 0\le \mu_A(x)<\alpha\gamma(1-\beta)\\ (1-\beta)\mu_A(x), & \alpha\gamma(1-\beta)\le \mu_A(x)<\alpha\gamma\\ \frac{1}{1-\gamma}(\mu_A(x)-\alpha)+\alpha, & \alpha\gamma\le \mu_A(x)<\alpha\\ \mu_A(x), & \alpha\le \mu_A(x)\le 1\end{cases}$$

and

$$\nu_{I^{\gamma,\omega}_{\alpha,\beta}(A)}(x)=\begin{cases}\nu_A(x), & 0\le \nu_A(x)<\beta\omega\\ \min\left\{(1-\omega)\nu_A(x)+\beta\omega, 1-\mu_{I^{\gamma,\omega}_{\alpha,\beta}(A)}(x)\right\}, & \beta\omega\le \nu_A(x)<\beta\\ \nu_A(x), & \beta\le \nu_A(x)\le 1\end{cases}$$

Proposition 3.1. *Let X be a set and $A \in IFS(X)$. For $\alpha, \beta, \gamma, \omega \in [0,1]$, the topological operator $I_{\alpha,\beta}^{\gamma,\omega}(A)$ is an intuitionistic fuzzy set.*

Proof. Suppose that $0 \leq \nu_A(x) < \beta\omega$ or $\beta \leq \nu_A(x) \leq 1$ then $\nu_{I_{\alpha,\beta}^{\gamma,\omega}(A)}(x) = \nu_A(x)$ and it is clear that $\mu_{I_{\alpha,\beta}^{\gamma,\omega}(A)}(x) \leq \mu_A(x)$, we obtain that $\mu_{I_{\alpha,\beta}^{\gamma,\omega}(A)}(x) + \nu_{I_{\alpha,\beta}^{\gamma,\omega}(A)}(x) \leq \mu_A(x) + \nu_A(x) \leq 1$.

On the other hand, if $\beta\omega \leq \nu_A(x) < \beta$ then

$$\begin{aligned}
&\mu_{I_{\alpha,\beta}^{\gamma,\omega}(A)}(x) + \min\{(1-\omega)\nu_A(x) + \beta\omega, 1 - \mu_{I_{\alpha,\beta}^{\gamma,\omega}(A)}(x)\} \\
&\leq \mu_{I_{\alpha,\beta}^{\gamma,\omega}(A)}(x) + 1 - \mu_{I_{\alpha,\beta}^{\gamma,\omega}(A)}(x) = 1 \\
&\Rightarrow \mu_{I_{\alpha,\beta}^{\gamma,\omega}(A)}(x) + \nu_{I_{\alpha,\beta}^{\gamma,\omega}(A)}(x) \leq 1.
\end{aligned}$$

□

Proposition 3.2. *Let X be a set and $A \in IFS(X)$. For $\alpha, \beta, \gamma, \omega \in [0,1]$, the operator $I_{\alpha,\beta}^{\gamma,\omega}(A)$ is a pre$-$interior operator in $IFS(X)$.*

Proof. (i) Let $A = X^*$ then $\mu_{I_{\alpha,\beta}^{\gamma,\omega}(A)}(x) = \mu_{X^*(A)}(x)$ and $\nu_{I_{\alpha,\beta}^{\gamma,\omega}(A)}(x) = \nu_{X^*(A)}(x)$ for all $x \in X^*$. $I_{\alpha,\beta}^{\gamma,\omega}(X^*) = X^*$

(ii) Let's examine the $I_{\alpha,\beta}^{\gamma,\omega}(A)$ under the given conditions.

First, $\inf \mu_A(x) \leq \mu_A(x)$ and $(1-\beta)\mu_A(x) \leq \mu_A(x), \beta \in [0,1]$ for all $x \in X$. Now, let $\mu_A(x) < \alpha$, $x \in X$ then for $\gamma \in [0,1]$,

$$\begin{aligned}
\gamma\mu_A(x) &< \gamma\alpha \Rightarrow \mu_A(x) - \gamma\alpha < \mu_A(x) - \gamma\mu_A(x) \\
&\Rightarrow \mu_A(x) > \frac{\mu_A(x) - \alpha}{1-\gamma} + \alpha \\
&\Rightarrow \mu_A(x) > \frac{1}{1-\gamma}(\mu_A(x) - \alpha) + \alpha
\end{aligned}$$

also, let $\nu_A(x) < \beta$, $x \in X$ then for $\omega \in [0,1]$,$\omega\nu_A(x) < \omega\beta \Rightarrow \nu_A(x) < (1-\omega)\nu_A(x) + \omega\beta$ and $\nu_A(x) < 1 - \mu_{I_{\alpha,\beta}^{\gamma,\omega}(A)}(x)$ for all $x \in X$. $I_{\alpha,\beta}^{\gamma,\omega}(A) \sqsubseteq A$.

(iii) Let $A, B \in IFS(X)$.

$$\begin{aligned}
&\mu_{I_{\alpha,\beta}^{\gamma,\omega}(A \sqcap B)}(x) \\
&= \begin{cases}
\inf(\mu_A(x) \wedge \mu_B(x)), & 0 \leq \mu_A(x) \wedge \mu_B(x) < \alpha\gamma(1-\beta) \\
(1-\beta)(\mu_A(x) \wedge \mu_B(x)), & \alpha\gamma(1-\beta) \leq \mu_A(x) \wedge \mu_B(x) < \alpha\gamma \\
\frac{1}{1-\gamma}((\mu_A(x) \wedge \mu_B(x)) - \alpha) + \alpha, & \alpha\gamma \leq \mu_A(x) \wedge \mu_B(x) < \alpha \\
(\mu_A(x) \wedge \mu_B(x)), & \alpha \leq \mu_A(x) \wedge \mu_B(x) \leq 1
\end{cases}
\end{aligned}$$

$$= \begin{cases} \inf \mu_A(x) \wedge \inf \mu_B(x), & 0 \le \mu_A(x) \wedge \mu_B(x) < \alpha\gamma(1-\beta) \\ (1-\beta)\mu_A(x) \wedge (1-\beta)\mu_B(x), & \alpha\gamma(1-\beta) \le \mu_A(x) \wedge \mu_B(x) < \alpha\gamma \\ \begin{array}{l} \left(\frac{1}{1-\gamma}(\mu_A(x) - \alpha) + \alpha\right) \wedge \\ \left(\frac{1}{1-\gamma}(\mu_B(x) - \alpha) + \alpha\right), \end{array} & \alpha\gamma \le \mu_A(x) \wedge \mu_B(x) < \alpha \\ (\mu_A(x) \wedge \mu_B(x)), & \alpha \le \mu_A(x) \wedge \mu_B(x) \le 1 \end{cases}$$

$$= \begin{cases} \inf \mu_A(x), & 0 \le \mu_A(x) < \alpha\gamma(1-\beta) \\ (1-\beta)\mu_A(x), & \alpha\gamma(1-\beta) \le \mu_A(x) < \alpha\gamma \\ \frac{1}{1-\gamma}(\mu_A(x) - \alpha) + \alpha, & \alpha\gamma \le \mu_A(x) < \alpha \\ \mu_A(x), & \alpha \le \mu_A(x) \le 1 \end{cases}$$

$$\wedge \begin{cases} \inf \mu_B(x), & 0 \le \mu_A(x) < \alpha\gamma(1-\beta) \\ (1-\beta)\mu_B(x), & \alpha\gamma(1-\beta) \le \mu_A(x) < \alpha\gamma \\ \frac{1}{1-\gamma}(\mu_B(x) - \alpha) + \alpha, & \alpha\gamma \le \mu_A(x) < \alpha \\ \mu_B(x), & \alpha \le \mu_A(x) \le 1 \end{cases}$$

$$= \mu_{I^{\gamma,\omega}_{\alpha,\beta}(A)}(x) \wedge \mu_{I^{\gamma,\omega}_{\alpha,\beta}(B)}(x)$$

and

$$\nu_{I^{\gamma,\omega}_{\alpha,\beta}(A \sqcap B)}(x)$$

$$= \begin{cases} \nu_A(x) \vee \nu_B(x), & 0 \le \nu_A(x) \vee \nu_B(x) < \beta\omega \\ \min \left\{ \begin{array}{c} (1-\omega)(\nu_A(x) \vee \nu_B(x)) + \beta\omega, \\ 1 - (\mu_{I^{\gamma,\omega}_{\alpha,\beta}(A)}(x) \wedge \mu_{I^{\gamma,\omega}_{\alpha,\beta}(B)}(x)) \end{array} \right\}, & \beta\omega \le \nu_A(x) \vee \nu_B(x) < \beta \\ \nu_A(x) \vee \nu_B(x), & \beta \le \nu_A(x) \vee \nu_B(x) \le 1 \end{cases}$$

$$= \begin{cases} \nu_A(x) \vee \nu_B(x), & 0 \le \nu_A(x) \vee \nu_B(x) < \beta\omega \\ \min \left\{ \begin{array}{c} ((1-\omega)\nu_A(x) + \beta\omega) \vee \\ ((1-\omega)\nu_B(x) + \beta\omega), \\ (1 - \mu_{I^{\gamma,\omega}_{\alpha,\beta}(A)}(x)) \vee (1 - \mu_{I^{\gamma,\omega}_{\alpha,\beta}(B)}(x)) \end{array} \right\}, & \beta\omega \le \nu_A(x) \vee \nu_B(x) < \beta \\ \nu_A(x) \vee \nu_B(x), & \beta \le \nu_A(x) \vee \nu_B(x) \le 1 \end{cases}$$

$$= \begin{cases} \nu_A(x), & 0 \le \nu_A(x) < \beta\omega \\ \min\{(1-\omega)\nu_A(x) + \beta\omega, 1 - \mu_{I^{\gamma,\omega}_{\alpha,\beta}(A)}(x)\}, & \beta\omega \le \nu_A(x) < \beta \\ \nu_A(x), & \beta \le \nu_A(x) \le 1 \end{cases}$$

$$\vee \begin{cases} \nu_B(x), & 0 \le \nu_A(x) < \beta\omega \\ \min\{(1-\omega)\nu_B(x) + \beta\omega, 1 - \mu_{I^{\gamma,\omega}_{\alpha,\beta}(B)}(x)\}, & \beta\omega \le \nu_A(x) < \beta \\ \nu_B(x), & \beta \le \nu_A(x) \le 1 \end{cases}$$

$$= \nu_{I^{\gamma,\omega}_{\alpha,\beta}(A)}(x) \vee \nu_{I^{\gamma,\omega}_{\alpha,\beta}(B)}(x)$$

This completes the proof. $\square$

Definition 3.2. *Let X be a set and $A \in IFS(X)$. For $\alpha, \beta, \gamma, \omega \in [0,1]$, the topological operator $C^{\gamma,\omega}_{\nu;\alpha,\beta}$ is defined as follow;*

$$C^{\gamma,\omega}_{\alpha,\beta} : IFS(X) \to IFS(X)$$

such that

$$\mu_{C^{\gamma,\omega}_{\alpha,\beta}(A)}(x) = \begin{cases} \mu_A(x), & 0 \le \mu_A(x) < \beta\omega \\ \min\left\{(1-\omega)\mu_A(x) + \beta\omega, 1 - \nu_{C^{\gamma,\omega}_{\alpha,\beta}(A)}(x)\right\}, & \beta\omega \le \mu_A(x) < \beta \\ \mu_A(x), & \beta \le \mu_A(x) \le 1 \end{cases}$$

and

$$\nu_{C^{\gamma,\omega}_{\alpha,\beta}(A)}(x) = \begin{cases} \inf \nu_A(x), & 0 \le \nu_A(x) < \alpha\gamma(1-\beta) \\ (1-\beta)\nu_A(x), & \alpha\gamma(1-\beta) \le \nu_A(x) < \alpha\gamma \\ \frac{1}{1-\gamma}(\nu_A(x) - \alpha) + \alpha, & \alpha\gamma \le \nu_A(x) < \alpha \\ \nu_A(x), & \alpha \le \nu_A(x) \le 1 \end{cases}$$

Proposition 3.3. *Let X be a set and $A \in IFS(X)$. For $\alpha, \beta, \gamma, \omega \in [0,1]$, the topological operator $C^{\gamma,\omega}_{\alpha,\beta}(A)$ is an intuitionistic fuzzy set.*

Proof. It can be proved similarly to Proposition 3.1. □

Proposition 3.4. *Let X be a set and $A \in IFS(X)$. For $\alpha, \beta, \gamma, \omega \in [0,1]$, the operator $C^{\gamma,\omega}_{\alpha,\beta}(A)$ is a pre$-$closure operator in $IFS(X)$.*

Proof. (i) Let $A = O^*$ then $\mu_{C^{\gamma,\omega}_{\alpha,\beta}(A)}(x) = \mu_{O^*(A)}(x)$ and $\nu_{C^{\gamma,\omega}_{\alpha,\beta}(A)}(x) = \nu_{O^*(A)}(x)$ for all $x \in O^*$. $C^{\gamma,\omega}_{\alpha,\beta}(O^*) = O^*$

(ii) It is clear that $\inf \nu_A(x) \le \nu_A(x)$ and $(1-\beta)\nu_A(x) \le \nu_A(x), \beta \in [0,1]$ for all $x \in X$.

Now, let $\nu_A(x) < \alpha$, $x \in X$ then for $\gamma \in [0,1]$,

$$\begin{aligned} \gamma\nu_A(x) &< \gamma\alpha \Rightarrow \nu_A(x) - \gamma\alpha < \nu_A(x) - \gamma\nu_A(x) \\ &\Rightarrow \nu_A(x) > \frac{\nu_A(x) - \alpha}{1-\gamma} + \alpha \\ &\Rightarrow \nu_A(x) > \frac{1}{1-\gamma}(\nu_A(x) - \alpha) + \alpha \end{aligned}$$

and also, let $\mu_A(x) < \beta$, $x \in X$ then for $\omega \in [0,1]$,

$$\omega\mu_A(x) < \omega\beta \Rightarrow \mu_A(x) < (1-\omega)\mu_A(x) + \omega\beta$$

and $\mu_A(x) < 1 - \nu_{C^{\gamma,\omega}_{\alpha,\beta}(A)}(x)$ for all $x \in X$. So, $A \sqsubseteq C^{\gamma,\omega}_{\alpha,\beta}(A)$.

(iii) Let $A, B \in IFS(X)$.

$$
\begin{aligned}
&\nu_{C^{\gamma,\omega}_{\alpha,\beta}(A\sqcup B)}(x)\\
&=\begin{cases}
\inf\left(\nu_A(x)\wedge\nu_B(x)\right), & 0\le \nu_A(x)\wedge\nu_B(x)<\alpha\gamma(1-\beta)\\
(1-\beta)\left(\nu_A(x)\wedge\nu_B(x)\right), & \alpha\gamma(1-\beta)\le \nu_A(x)\wedge\nu_B(x)<\alpha\gamma\\
\frac{1}{1-\gamma}\left(\left(\nu_A(x)\wedge\nu_B(x)\right)-\alpha\right)+\alpha, & \alpha\gamma\le \nu_A(x)\wedge\nu_B(x)<\alpha\\
\left(\nu_A(x)\wedge\nu_B(x)\right), & \alpha\le \nu_A(x)\wedge\nu_B(x)\le 1
\end{cases}\\
&=\begin{cases}
\inf\nu_A(x)\wedge\inf\nu_B(x), & 0\le \nu_A(x)\wedge\nu_B(x)<\alpha\gamma(1-\beta)\\
(1-\beta)\nu_A(x)\wedge(1-\beta)\nu_B(x), & \alpha\gamma(1-\beta)\le \nu_A(x)\wedge\nu_B(x)<\alpha\gamma\\
\begin{array}{l}\left(\frac{1}{1-\gamma}\left(\nu_A(x)-\alpha\right)+\alpha\right)\wedge\\ \left(\frac{1}{1-\gamma}\left(\nu_B(x)-\alpha\right)+\alpha\right),\end{array} & \alpha\gamma\le \nu_A(x)\wedge\nu_B(x)<\alpha\\
\left(\nu_A(x)\wedge\nu_B(x)\right), & \alpha\le \nu_A(x)\wedge\nu_B(x)\le 1
\end{cases}\\
&=\begin{cases}
\inf\nu_A(x), & 0\le\nu_A(x)<\alpha\gamma(1-\beta)\\
(1-\beta)\nu_A(x), & \alpha\gamma(1-\beta)\le\nu_A(x)<\alpha\gamma\\
\frac{1}{1-\gamma}\left(\nu_A(x)-\alpha\right)+\alpha, & \alpha\gamma\le\nu_A(x)<\alpha\\
\nu_A(x), & \alpha\le\nu_A(x)\le 1
\end{cases}\\
&\wedge\begin{cases}
\inf\nu_B(x), & 0\le\nu_B(x)<\alpha\gamma(1-\beta)\\
(1-\beta)\nu_B(x), & \alpha\gamma(1-\beta)\le\nu_B(x)<\alpha\gamma\\
\frac{1}{1-\gamma}\left(\nu_B(x)-\alpha\right)+\alpha, & \alpha\gamma\le\nu_B(x)<\alpha\\
\nu_B(x), & \alpha\le\nu_B(x)\le 1
\end{cases}\\
&=\nu_{C^{\gamma,\omega}_{\alpha,\beta}(A)}(x)\wedge\nu_{C^{\gamma,\omega}_{\alpha,\beta}(B)}(x)
\end{aligned}
$$

and

$$
\begin{aligned}
&\mu_{C^{\gamma,\omega}_{\alpha,\beta}(A\sqcup B)}(x)\\
&=\begin{cases}
\mu_A(x)\vee\mu_B(x), & 0\le\mu_A(x)\vee\mu_B(x)<\beta\omega\\
\min\left\{\begin{array}{c}(1-\omega)(\mu_A(x)\vee\mu_B(x))+\beta\omega,\\ 1-(\nu_{C^{\gamma,\omega}_{\alpha,\beta}(A)}(x)\wedge\nu_{C^{\gamma,\omega}_{\alpha,\beta}(B)}(x))\end{array}\right\}, & \beta\omega\le\mu_A(x)\vee\mu_B(x)<\beta\\
\nu_A(x)\vee\nu_B(x), & \beta\le\mu_A(x)\vee\mu_B(x)\le 1
\end{cases}\\
&=\begin{cases}
\mu_A(x)\vee\mu_B(x), & 0\le\mu_A(x)\vee\mu_B(x)<\beta\omega\\
\min\left\{\begin{array}{c}((1-\omega)\mu_A(x)+\beta\omega)\vee\\ ((1-\omega)\mu_B(x)+\beta\omega),\\ (1-\nu_{C^{\gamma,\omega}_{\alpha,\beta}(A)}(x))\vee(1-\nu_{C^{\gamma,\omega}_{\alpha,\beta}(B)}(x))\end{array}\right\}, & \beta\omega\le\mu_A(x)\vee\mu_B(x)<\beta\\
\mu_A(x)\vee\mu_B(x), & \beta\le\mu_A(x)\vee\mu_B(x)\le 1
\end{cases}
\end{aligned}
$$

$$= \begin{cases} \mu_A(x), & 0 \leq \mu_A(x) < \beta\omega \\ \min\{(1-\omega)\mu_A(x) + \beta\omega, 1 - \nu_{C^{\gamma,\omega}_{\alpha,\beta}(A)}(x)\}, & \beta\omega \leq \mu_A(x) < \beta \\ \mu_A(x), & \beta \leq \mu_A(x) \leq 1 \end{cases}$$

$$\vee \begin{cases} \mu_B(x), & 0 \leq \mu_A(x) < \beta\omega \\ \min\{(1-\omega)\mu_B(x) + \beta\omega, 1 - \nu_{C^{\gamma,\omega}_{\alpha,\beta}(B)}(x)\}, & \beta\omega \leq \mu_A(x) < \beta \\ \mu_B(x), & \beta \leq \mu_A(x) \leq 1 \end{cases}$$

$$= \mu_{C^{\gamma,\omega}_{\alpha,\beta}(A)}(x) \vee \mu_{C^{\gamma,\omega}_{\alpha,\beta}(B)}(x)$$

Hence, proof completed. □

Proposition 3.5. *The operator $I^{\gamma,\omega}_{\alpha,\beta}$ is generalization of the operator I_μ and the operator $C^{\gamma,\omega}_{\alpha,\beta}$ is generalization of the operator C_ν.*

Proof. Let X be a set and $A \in IFS(X)$. It is clear that to take $\gamma = \alpha = 1, \omega = 0$ and $\beta = 1 - \inf \mu_A(x)$, i.e. $I_\mu = I^{1,0}_{1,(1-k)}$ is provides the definition. On the other hand, if $\gamma{=}\alpha{=}1, \omega{=}0$ and $\beta{=}1 - \inf \nu_A(x)$ then $C_\nu{=}C^{1,0}_{1,(1-L)}$. □

Theorem 1. *Let X be a set and $A \in IFS(X)$ then $C^{\gamma,\omega}_{\alpha,\beta}(A) = \neg\, I^{\gamma,\omega}_{\alpha,\beta}(\neg A)$, i.e $I^{\gamma,\omega}_{\alpha,\beta}$ and $C^{\gamma,\omega}_{\alpha,\beta}$ is a conjugate pair of pre$-$interior and pre$-$closure operators. They define the same topology $\tau_{I^{\gamma,\omega}_{\alpha,\beta}} = \{\neg B : B \in \tau^{C^{\gamma,\omega}_{\alpha,\beta}}\}$.*

Proof. Let X be a set and $A \in IFS(X)$.

$$\mu_{I^{\gamma,\omega}_{\alpha,\beta}(\neg A)}(x) = \begin{cases} \inf \nu_A(x), & 0 \leq \nu_A(x) < \alpha\gamma(1-\beta) \\ (1-\beta)\nu_A(x), & \alpha\gamma(1-\beta) \leq \nu_A(x) < \alpha\gamma \\ \frac{1}{1-\gamma}(\nu_A(x) - \alpha) + \alpha, & \alpha\gamma \leq \nu_A(x) < \alpha \\ \nu_A(x), & \alpha \leq \nu_A(x) \leq 1 \end{cases} = \nu_{C^{\gamma,\omega}_{\alpha,\beta}(A)}(x)$$

and

$$\nu_{I^{\gamma,\omega}_{\alpha,\beta}(\neg A)}(x) = \begin{cases} \mu_A(x), & 0 \leq \mu_A(x) < \beta\omega \\ \min\left\{(1-\omega)\mu_A(x) + \beta\omega, 1 - \nu_{C^{\gamma,\omega}_{\alpha,\beta}(A)}\right\}, & \beta\omega \leq \mu_A(x) < \beta \\ \mu_A(x), & \beta \leq \mu_A(x) \leq 1 \end{cases}$$

$$= \mu_{C^{\gamma,\omega}_{\alpha,\beta}(A)}(x)$$

So, $C^{\gamma,\omega}_{\alpha,\beta}(A) = \neg\, I^{\gamma,\omega}_{\alpha,\beta}(\neg A)$. From Proposition 2.1, we obtain that $\tau_{I^{\gamma,\omega}_{\alpha,\beta}} = \{\neg B : B \in \tau^{C^{\gamma,\omega}_{\alpha,\beta}}\}$. □

Definition 3.3. *The boundary of set A in the intuitionistic fuzzy topology defined by these pre$-$interior and pre$-$closure operators is $\partial A = C^{\gamma,\omega}_{\alpha,\beta}(A) \cap (\neg\, I^{\gamma,\omega}_{\alpha,\beta}(A))$ according to the IF boundary definition given by Malek [11].*

In the following example, $I_{\alpha,\beta}^{\gamma,\omega}$, $C_{\alpha,\beta}^{\gamma,\omega}$ and boundary value of an intuitionistic fuzzy set A are examined, for different α, β, γ and ω.

Example 3.1. Let the universal X and $A \in IFS(X)$ be given in the table below.

X	α	β	γ	ω	(μ_A, ν_A)	$I_{\alpha,\beta}^{\gamma,\omega}(A)$	$C_{\alpha,\beta}^{\gamma,\omega}(A)$	$\partial(A)$
a_0	0.4	0.2	0.7	0.6	(1, 0)	(1, 0)	(1, 0)	(0, 1)
a_1	0.4	0.2	0.7	0.6	(0.65, 0.15)	(0.65, 0.18)	(0.65, 0.01)	(0.18, 0.65)
a_2	0.4	0.2	0.7	0.6	(0.3, 0.4)	(0.066667, 0.4)	(0.3, 0.4)	(0.3, 0.4)
a_3	0.4	0.2	0.7	0.6	(0.25, 0.45)	(0.2, 0.45)	(0.25, 0.45)	(0.25, 0.45)
a_4	0.4	0.2	0.7	0.6	(0.8, 0.07)	(0.8, 0.07)	(0.8, 0.01)	(0.07, 0.8)
a_5	0.4	0.2	0.7	0.6	(0.1, 0.2)	(0.08, 0.2)	(0.1, 0.01)	(0.1, 0.08)
a_6	0.4	0.2	0.7	0.6	(0.2, 0.7)	(0.08, 0.7)	(0.2, 0.7)	(0.2, 0.7)
a_7	0.4	0.2	0.7	0.6	(0.72, 0.24)	(0.72, 0.24)	(0.72, 0.192)	(0.24, 0.72)
a_8	0.4	0.2	0.7	0.6	(0.2, 0.7)	(0.08, 0.7)	(0.2, 0.7)	(0.2, 0.7)
a_9	0.4	0.2	0.7	0.6	(0.35, 0.53)	(0.233333, 0.53)	(0.35, 0.53)	(0.35, 0.53)
a_{10}	0.4	0.2	0.7	0.6	(0.82, 0.1)	(0.82, 0.1)	(0.82, 0.01)	(0.1, 0.82)
a_{11}	0.4	0.2	0.7	0.6	(0.97, 0.01)	(0.97, 0.01)	(0.97, 0.01)	(0.01, 0.97)
a_{12}	0.4	0.2	0.7	0.6	(0.48, 0.32)	(0.48, 0.32)	(0.48, 0. 133333)	(0.32, 0.48)
a_{13}	0.4	0.2	0.7	0.6	(0.15, 0.6)	(0.08, 0.6)	(0.18, 0.6)	(0.18, 0.6)
a_{14}	0.4	0.2	0.7	0.6	(0.55, 0.24)	(0.55, 0.24)	(0.55, 0.192)	(0.24, 0.55)
a_{15}	0.4	0.2	0.7	0.6	(0.74, 0.2)	(0.74, 0.2)	(0.74, 0.01)	(0.2, 0.74)
a_0	0,3	0,8	0,4	0,5	(1, 0)	(1, 0)	(1, 0)	(0, 1)
a_1	0,3	0,8	0,4	0,5	(0.65, 0.15)	(0.65, 0.15)	(0.725, 0.05)	(0.15, 0.65)
a_2	0,3	0,8	0,4	0,5	(0.3, 0.4)	(0.3, 0.6)	(0.3, 0.4)	(0.3, 0.4)
a_3	0,3	0,8	0,4	0,5	(0.25, 0.45)	(0.216667, 0.625)	(0.25, 0.45)	(0.25, 0.45)
a_4	0,3	0,8	0,4	0,5	(0.8, 0.07)	(0.8, 0.07)	(0.8, 0.014)	(0.07, 0.8)
a_5	0,3	0,8	0,4	0,5	(0.1, 0.2)	(0.02, 0.2)	(0.1, 0.13333)	(0.1, 0.1333)
a_6	0,3	0,8	0,4	0,5	(0.2, 0.7)	(0.133333, 0.75)	(0.2, 0.7)	(0.2, 0,7)
a_7	0,3	0,8	0,4	0,5	(0.72, 0.24)	(0.72, 0.24)	(0.76, 0.2)	(0.24, 0.72)
a_8	0,3	0,8	0,4	0,5	(0.2, 0.7)	(0.133333, 0.75)	(0.2, 0.7)	(0.2, 0.7)
a_9	0,3	0,8	0,4	0,5	(0.35, 0.53)	(0.35, 0.65)	(0.35, 0.53)	(0.35, 0.53)
a_{10}	0,3	0,8	0,4	0,5	(0.82, 0.1)	(0.82, 0.1)	(0.82, 0.02)	(0.1, 0.82)
a_{11}	0,3	0,8	0,4	0,5	(0.97, 0.01)	(0.97, 0.01)	(0.97, 0.01)	(0.01, 0.97)
a_{12}	0,3	0,8	0,4	0,5	(0.48, 0.32)	(0.48, 0.32)	(0.64, 0.32)	(0.32, 0.48)
a_{13}	0,3	0,8	0,4	0,5	(0.15, 0.6)	(0.05, 0.7)	(0.15, 0.6)	(0.15, 0.6)
a_{14}	0,3	0,8	0,4	0,5	(0.55, 0.24)	(0.55, 0.24)	(0.675, 0.2)	(0.24, 0.55)
a_{15}	0,3	0,8	0,4	0,5	(0.74, 0.2)	(0.74, 0.2)	(0.77, 0.13333)	(0.2, 0.74)
a_0	0,6	0,7	0,8	0,65	(1, 0)	(1, 0)	(1, 0)	(0, 1)
a_1	0,6	0,7	0,8	0,65	(0.65, 0.15)	(0.65, 0.15)	(0.6825, 0.045)	(0.15, 0.65)
a_2	0,6	0,7	0,8	0,65	(0.3, 0.4)	(0.09, 0.4)	(0.3, 0.12)	(0.3, 0.12)
a_3	0,6	0,7	0,8	0,65	(0.25, 0.45)	(0.075, 0.45)	(0.25, 0.135)	(0.25, 0.135)
a_4	0,6	0,7	0,8	0,65	(0.8, 0.07)	(0.8, 0.07)	(0.8, 0.01)	(0.07, 0.8)
a_5	0,6	0,7	0,8	0,65	(0.1, 0.2)	(0.1, 0.2)	(0.1, 0.06)	(0.1, 0.1)
a_6	0,6	0,7	0,8	0,65	(0.2, 0.7)	(0.06, 0.7)	(0.2, 0.7)	(0.2, 0.7)
a_7	0,6	0,7	0,8	0,65	(0.72, 0.24)	(0.72, 0.24)	(0.72, 0.072)	(0.24, 0.72)
a_8	0,6	0,7	0,8	0,65	(0.2, 0.7)	(0.06, 0.7)	(0.2, 0.7)	(0.2, 0.7)
a_9	0,6	0,7	0,8	0,65	(0.35, 0.53)	(0.105, 0.6405)	(0.35, 0.25)	(0.35, 0.25)
a_{10}	0,6	0,7	0,8	0,65	(0.82, 0.1)	(0.82, 0.1)	(0.82, 0.01)	(0.1, 0.82)
a_{11}	0,6	0,7	0,8	0,65	(0.97, 0.01)	(0.97, 0.01)	(0.97, 0.01)	(0.01, 0.97)
a_{12}	0,6	0,7	0,8	0,65	(0.48, 0.32)	(0, 0.32)	(0.623, 0.096)	(0.32, 0.096)
a_{13}	0,6	0,7	0,8	0,65	(0.15, 0.6)	(0.045, 0.665)	(0.15, 0.6)	(0.15, 0.6)
a_{14}	0,6	0,7	0,8	0,65	(0.55, 0.24)	(0.35, 0.24)	(0.6475, 0.072)	(0.24, 0.35)
a_{15}	0,6	0,7	0,8	0,65	(0.74, 0.2)	(0.74, 0.2)	(0.74, 0.06)	(0.2, 0.74)

Figure 1: Table

As can be seen from the tables, since there are no conditions limiting α, β, γ and ω values, this diversity provides wide application for problems studied using topological operators.

4. Conclusion

In this study, new topological operators are defined on intuitionistic fuzzy sets and their theoretical properties are examined. It is obvious that these defined operators will contribute to the modeling of real-world problems with uncertainty in determining the boundaries.

Acknowledgements

This study has been supported by the Research Fund of Tarsus University in Turkey with Project Number: MF.20.006.

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Accepted: February 17, 2022