# On a maximal subgroup $\overline{G} = 5^4{:}((3{\times}2L_2(25)){:}2_2)$ of the Monster $\mathbb{M}$

**David Mwanzia Musyoka**
*Department of Mathematics and Actuarial Science*
*Kenyatta University*
*PO Box 43844-00100, Nairobi*
*Kenya*
*davidmusyoka21@yahoo.com*

**Lydia Nyambura Njuguna**
*Department of Mathematics and Actuarial Science*
*Kenyatta University*
*PO Box 43844-00100, Nairobi*
*Kenya*
*njuguna.lydia@ku.ac.ke*
*lydiahnjuguna@yahoo.com*

**Abraham Love Prins***
*Department of Mathematics and Applied Mathematics*
*Faculty of Science*
*Nelson Mandela University*
*PO Box 77000, Gqeberha, 6031*
*South Africa*
*abraham.prins@mandela.ac.za*
*abrahamprinsie@yahoo.com*

**Lucy Chikamai**
*Department of Mathematics and Actuarial Science*
*Kibabii University*
*PO Box 1699-50200, Bungoma*
*Kenya*
*chikamail@kibu.ac.ke*
*lucychikamai@gmail.com*

**Abstract.** The split extension $\overline{G} = 5^4{:}((3 \times 2L_2(25)){:}2_2)$ is a maximal subgroup of the sporadic Monster group $\mathbb{M}$ of order $58500000 = 2^5.3^2.5^6.13$. The technique of Fischer-Clifford matrices has been applied to numerous examples of split and non-split extensions where the kernels are either elementary abelian 2 or 3-groups but very few examples exist where the kernel is an elementary abelian 5-group. In this paper, the Fischer-Clifford matrices technique is applied to the group $\overline{G} = 5^4{:}((3 \times 2L_2(25)){:}2_2)$, where the kernel $5^4$ of the extension is an elementary abelian 5-group.

**Keywords:** coset analysis, Fischer-Clifford matrices, split extension, inertia factor, character table, fusion map, restriction of characters.

*. Corresponding author

## 1. Introduction

The sporadic Monster group $\mathbb{M}$ has a conjugacy class of maximal 5-local subgroups of the form $5^4{:}((3 \times 2L_2(25)){:}2_2)$ [6]. Obtaining a permutation representation on 625 points for $\overline{G}{=}5^4{:}((3{\times}2L_2(25)){:}2_2)$ from the online ATLAS [23], the group $\overline{G}$ is generated by using the algebra computational system MAGMA [5]. The normal subgroup $N = 5^4$ and subgroup $G = (3 \times 2L_2(25)){:}2_2 \cong SL_2(25){:}S_3$ of $\overline{G}$ are constructed by MAGMA as permutation groups on 625 points. Using the MAGMA commands, "M:=GModule($\overline{G}, N$); and "M:Maximal;", the group $G =< g_1, g_2 >$ is constructed as a matrix group of degree 4 over $GF(5)$ with generators $g_1$ and $g_2$ such that $o(g_1) = 2$, $o(g_2) = 39$ and $o(g_1g_2) = 8$ (see, Figure 1).

$$g_1 = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 2 & 0 & 4 \end{pmatrix}, \; g_2 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 4 & 1 & 4 & 2 \\ 0 & 0 & 0 & 1 \\ 3 & 4 & 2 & 0 \end{pmatrix}$$

Figure 1: Generators of $G$

Considering $N = V_4(5)$ as the vector space of dimension 4 over $GF(5)$, on which the matrix group $G =< g_1, g_2 >$ acts absolutely irreducibly, it was found with aid of GAP [8] that $G$ has two orbits on $N$ of lengths 1 and 624 with corresponding point stabilizers $P_1 = G$ and $P_2 = 5^2{:}S_3$. By Brauer's theorem (see Theorem 5.1.5 in [12]), the action of $G$ on Irr($N$) also has two orbits of lengths 1 and 624 with corresponding inertia factor groups $H_1 = G$ and $H_2 = 5^2{:}S_3$. It is worth noting that the vector space $N$ and its dual space $N^* =$Irr($N$) are isomorphic as 4-dimensional modules over $GF(5)$ for $G$. Having obtained $G$ as a 4-dimensional matrix group over the finite field $GF(5)$ and treating $N$ as the vector space $V_4(5)$ we can apply Fischer-Clifford theory (see, for example, [7] and [14]) to the split extension $\overline{G}$ to construct its ordinary character table. The Fischer-Clifford matrices technique is powerful if the kernel of a suitable split extension group is elementary abelian as it is the case with the group $\overline{G}$. A GAP routine found in [21] which is based on coset analysis technique found in [11] and [14] is used to compute the conjugacy classes of $\overline{G}$. This method is very efficient when the kernel of a split extension is an elementary-abelian $p$-group. The importance of computing conjugacy classes of $\overline{G}$ from a coset $Ng$ is that the centralizer orders of these classes play a role in the computation of the entries of a Fischer-Clifford matrix $M(g)$, where $g$ is a conjugacy class representative of $G$. In the paper [10], Fischer-Clifford technique was applied to a non-split extension $\overline{G_1} = 5^{3\cdot}L_3(5)$, which is a maximal subgroup of the Lyons sporadic simple group $\mathbb{L}y$. Besides our group $\overline{G}$ , $\overline{G_1}$ is one of the few extension groups in the literature with the kernel being an elementary abelian 5-group, where the method of Fischer-Clifford matrices has been applied to.

In the sections that follow, an outline of the Fischer-Clifford matrices technique is going to be given. The conjugacy classes and Fischer-Clifford matrices of $\overline{G}$ are also computed using appropriate GAP routines. In addition, the ordinary character table of $\overline{G}$ is constructed and the fusion of conjugacy classes of $\overline{G}$ into those of the Monster $\mathbb{M}$ is determined. For an update on recent developments around Fischer-Clifford matrices, interested readers are referred to the papers [1], [2], [15] [16], [17], [18] and [19]. Most of the computations in this paper are carried out with computer algebra systems MAGMA and GAP. Notation from the ATLAS [6] is mostly followed.

## 2. Theory of Fischer-Clifford matrices

Since the ordinary character table of $\overline{G} = 5^4{:}((3 \times 2L_2(25)){:}2_2)$ will be constructed by the technique of Fischer-Clifford matrices, an outline of this technique is given for a split extension $\overline{G} = N{:}G$, where $N$ is an elementary abelian $p$-group, see for example, [14] or [22].

Let $\overline{G} = N{:}G$ be a split extension of $N$ by $G$, where $N$ is an elementary abelian $p$-group. The subgroup $\overline{H} = N{:}H = \left\{x \in \overline{G} | \theta^x = \theta\right\}$ of $\overline{G}$ is defined as the inertia group of $\theta \in \mathrm{Irr}(N)$ in $\overline{G}$, with inertia factor $H = \overline{H}/N$. Note that a lifting $\overline{g} \in \overline{G}$ of $g \in G$ into $\overline{G}$ under the natural homomorphism $\eta{:}\overline{G} \longrightarrow G$ is just $g$ itself, since $G \leq \overline{G}$. Let $X(g) = \{x_1, x_2, \cdots, x_{c(g)}\}$ be a set of representatives of the conjugacy classes of $\overline{G}$ from the coset $Ng$ whose images under the natural homomorphism $\eta$ are in the conjugacy class $[g]$ of $G$ where $x_1 = g$. Now let $\theta_1 = 1_N, \theta_2, \cdots, \theta_t$ be representatives of the orbits of $\overline{G}$ on $\mathrm{Irr}(N)$. Since $N$ is elementary abelian, we have by Mackey's Theorem (see Theorem 5.1.15 in [12]) that each $\theta_i$, $1 \leq i \leq t$, extends to a $\psi_i \in \mathrm{Irr}(\overline{H_i})$, i.e. $\psi_i \downarrow_N = \theta_i$. By Theorem 5.1.7, Remark 5.1.8 and Theorem 5.1.19 in [12], an ordinary irreducible character $\chi = (\psi_i\overline{\beta})^{\overline{G}}$ of $\overline{G}$ consists of $\psi_i\overline{\beta} \in \mathrm{Irr}(\overline{H_i})$ which is induced to $\overline{G}$, where $N$ is contained in the kernel $\ker(\overline{\beta})$ of a lifting $\overline{\beta} \in \mathrm{Irr}(\overline{H_i})$ of $\beta \in \mathrm{Irr}(H_i)$ into $\overline{H_i}$. Therefore,

$$\mathrm{Irr}(\overline{G}) = \bigcup_{i=1}^{t}\{(\psi_i\overline{\beta})^{\overline{G}} | \overline{\beta} \in \mathrm{Irr}(\overline{H_i}), N \subseteq \ker(\overline{\beta})\} = \bigcup_{i=1}^{t}\{(\psi_i\overline{\beta})^{\overline{G}} | \beta \in \mathrm{Irr}(H_i)\}.$$

Hence, the set $\mathrm{Irr}(\overline{G})$ are partitioned into $t$ blocks $B_i$ with each block $B_i$ corresponding to an inertia subgroup $\overline{H_i}$ of $\overline{G}$. Observe that $|\mathrm{Irr}(\overline{G})| = |\mathrm{Irr}(H_1)| + ... + |\mathrm{Irr}(H_t)|$.

We take $\overline{H_1} = \overline{G}$ and $H_1 = G$. Choose $y_1, y_2, .., y_r$ to be representatives of the conjugacy classes $[y_k]$, $k = 1, ..., r$, of $H_i$ that fuse to $[g]$ in $G$. We define $R(g) = \{(i, y_k) \,|\, 1 \leq i \leq t, H_i \cap [g] \neq \emptyset, 1 \leq k \leq r\}$ and we observe that $y_k$ runs over representatives of the conjugacy classes $[y_k]$ of $H_i$ which fuse into $[g]$ of $G$. We define $y_{l_k} \in \overline{H_i}$ such that $y_{l_k}$ ranges over all representatives of the conjugacy classes of $\overline{H}_i$ which map to $y_k$ under the homomorphism $\overline{H_i} \longrightarrow H_i$ whose kernel is $N$.

**Lemma 2.1.** *With notation as above,*

$$(\psi_i\overline{\beta})^{\overline{G}}(x_j) = \sum_{y_k:(i,y_k)\in R(g)} \left[ \sum_l{}' \frac{|C_{\overline{G}}(x_j)|}{|C_{\overline{H_i}}(y_{l_k})|} \psi_i(y_{l_k}) \right] \beta(y_k).$$

**Proof.** See [22]. □

Then, the Fischer-Clifford matrix $M(g) = (a^j_{(i,y_k)})$ is defined as $(a^j_{(i,y_k)}) = (\sum_l{}' \frac{|C_{\overline{G}}(x_j)|}{|C_{\overline{H_i}}(y_{l_k})|}\psi_i(y_{l_k}))$, with columns indexed by $X(g)$ and rows indexed by $R(g)$ and where $\sum_l{}'$ is the summation over all $l$ for which $y_{l_k} \sim x_j$ in $\overline{G}$. So, we can write Lemma 2.1 as

$$(\psi_i\overline{\beta})^{\overline{G}}(x_j) = \sum_{y_k:(i,y_k)\in R(g)} a^j_{(i,y_k)}\beta(y_k).$$

The Fischer-Clifford $M(g)$ (see, Figure 2) is partitioned row-wise into blocks $M_i(g)$, where each block corresponds to an inertia group $\overline{H}_i$. We write $|C_{\overline{G}}(x_j)|$, for each $x_j \in X(g)$, at the top of the columns of $M(g)$ and at the bottom we write $m_j \in \mathbb{N}$, where we define $m_j = |N|\frac{|C_G(g)|}{|C_{\overline{G}}(x_j)|}$. On the left of each row we write $|C_{H_i}(y_k)|$, where the conjugacy classes $[y_k]$, $k = 1, 2, ..., r$, of an inertia factor $H_i$ fuse into the conjugacy class $[g]$ of $G$.

$$M(g)= \begin{array}{c} \\ |C_G(g)| \\ \hline |C_{H_2}(y_1)| \\ |C_{H_2}(y_2)| \\ \vdots \\ \hline |C_{H_i}(y_1)| \\ |C_{H_i}(y_2)| \\ \vdots \\ \hline |C_{H_t}(y_1)| \\ |C_{H_t}(y_2)| \\ \vdots \\ \hline \\ \end{array} \left( \begin{array}{cccc} |C_{\overline{G}}(x_1)| & |C_{\overline{G}}(x_2)| & \cdots & |C_{\overline{G}}(x_{c(g)})| \\ a^1_{(1,g)} & a^2_{(1,g)} & \cdots & a^{c(g)}_{(1,g)} \\ \hline a^1_{(2,y_1)} & a^2_{(2,y_1)} & \cdots & a^{c(g)}_{(2,y_1)} \\ a^1_{(2,y_2)} & a^2_{(2,y_2)} & \cdots & a^{c(g)}_{(2,y_2)} \\ \vdots & \vdots & \vdots & \vdots \\ \hline a^1_{(i,y_1)} & a^2_{(i,y_1)} & \cdots & a^{c(g)}_{(i,y_1)} \\ a^1_{(i,y_2)} & a^2_{(i,y_2)} & \cdots & a^{c(g)}_{(i,y_2)} \\ \vdots & \vdots & \vdots & \vdots \\ \hline a^1_{(t,y_1)} & a^2_{(t,y_1)} & \cdots & a^{c(g)}_{(t,y_1)} \\ a^1_{(t,y_2)} & a^2_{(t,y_2)} & \cdots & a^{c(g)}_{(t,y_2)} \\ \vdots & \vdots & \vdots & \vdots \\ \hline m_1 & m_2 & \cdots & m_{c(g)} \end{array} \right)$$

Figure 2: The Fischer-Clifford Matrix $M(g)$

In practice it is difficult to compute the elements $y_{l_k}$ or the ordinary irreducible character tables of the inertia groups $\overline{H}_i$, since the sets $\mathrm{Irr}(\overline{H}_i)$ of ordinary irreducible characters of the $\overline{H}_i$'s are in general much larger and more complicated to compute than the one for $\overline{G}$. Instead of using the above formal definition of a Fischer-Clifford matrix $M(g)$, the arithmetical properties of $M(g)$ found in [14] are used to compute the entries of $M(g)$. The matrix $M(g)$ is square where the number of rows is equal to the number of conjugacy classes of the inertia factors $H_i$'s, $1 \leq i \leq t$, which fuse into the class $[g]$ in $G$ and the number of columns is equal to the number $c(g)$ of conjugacy classes of $\overline{G}$ which is obtained from the coset $N\overline{g}$. Then, the partial character table of $\overline{G}$ on the classes $\{x_1, x_2, \cdots, x_{c(g)}\}$ is given by

$$\begin{bmatrix} C_1(g)\, M_1(g) \\ C_2(g)\, M_2(g) \\ \vdots \\ C_t(g)\, M_t(g) \end{bmatrix}$$

with each block $M_i(g)$ of $M(g)$ (see Figure 2) corresponding to an inertia group $\overline{H}_i$ and $C_i(g)$ consists of the columns of the ordinary character table of $H_i$ which correspond to the conjugacy classes of $H_i$ that fuse into the class $[g]$ of $G$. We obtain the characters of $\overline{G}$ by multiplying the relevant columns of the ordinary irreducible characters of $H_i$ by the rows of $M(g)$.

## 3. The conjugacy classes of $\overline{G}$

In this section, a GAP routine (labelled as Programme A in [21]), which is based on the method of coset analysis (see [11], [13] or [14]), is used to compute the conjugacy classes of $\overline{G}$. This GAP routine is written for a split extension $S = p^n{:}Q$ of an elementary abelian $p$-group $p^n$ by a linear matrix group $Q$ of dimension $n$ over the field $GF(p)$. The group $p^n$ (regarded as a vector space $V_n(p)$ of degree $n$ over the finite field $GF(p)$ ($p$ is a prime)) is a $Q$-module where upon the matrix group $Q$ acts naturally. A coset $p^n q$ is considered for each conjugacy class $[q]$ representative $q$ in $Q$ and then consider the action of the stabilizer $C_g = p^n{:}C_Q(q) = \{x \in S | x(p^n q)x^{-1} = p^n q\}$ of the coset $p^n q$ in $S$ by conjugation on the elements of $p^n q$. Since $C_g$ is split extension we will first act $p^n$ on $p^n q$ to form $k$ orbits $Q_1, Q_2, ..., Q_k$, with each orbit $Q_i$ containing $|p^n|/k$ elements. Under the action of the centralizer $C_Q(q)$ of $q$ in $Q$, $f_j$ of the $k$ orbits $Q_i$ fuse together to form an orbit $O_j$. The orbit $O_j$ contains the elements from the coset $p^n q$ which belong to a conjugacy class $[x_j]$ of $S$ with class representative $x_j$. Note that $\sum f_j = k$. The order of the centralizer $|C_S(x_j)|$ of the class representative $x_j$ is then computed by $|C_S(x_j)| = \frac{k|C_Q(q)|}{f_j}$. In this manner, the conjugacy classes of $S$, with class representatives $X(q) = \{x_1, x_2, ..., x_{c(q)}\}$ (see Section 2) coming from the coset $p^n q$, are obtained.

Using similar techniques as in [14], the permutation character $\chi(G|5^4)$ of $G = (3 \times 2L_2(25)){:}2_2$ on the conjugacy classes of $N = 5^4$ is computed as

$$\chi(G|5^4) = \sum_{i=1}^{2} I_{P_i}^{G} = 1aa + 13cd + 25b + 26dd + 52bbdejklmno.$$

Note that $\chi(G|5^4)$ is the sum of the identity characters $I_{P_i}^{G}$, $i = 1, 2$, of the point stabilizers $P_i$ of the orbits of $G$ on $N$, which are induced to $G$. Also, $\chi(G|5^4)$ is written in terms of the ordinary irreducible characters of $G$. For an element $g$ in a conjugacy class $[g]$ of $G$, it is required that $\chi(G|5^4)(g) = 5^n$, for some $n \in \{0, 1, 2, 3, 4\}$. The value $\chi(G|5^4)(g)$ gives the number of elements of $N$ which is fixed by an element $g \in G$ and it is also the number of orbits of $N$ on a coset $Ng$.

In Section 1, the group $G = (3 \times 2L_2(25)){:}2_2 =< g_1, g_2 >$ was computed as a 4-dimensional matrix group over the field $GF(5)$ and with $N = 5^4$ represented as a vector space $V_4(5)$ of dimension 4 over $GF(5)$, we now proceed to compute the conjugacy classes for $\overline{G}$ as described above. The permutation character $\chi(G|5^4)$ is evaluated on each class representative $g \in G$ to determine the number $k = \chi(G|5^4)(g)$ of orbits of $N$ on $Ng$. Programme A in [21] written in GAP is then used to calculate the number $f_j$ of these $k$ orbits which come together as an orbit $O_j$ under the action of $C_G(g)$. With the values of $k$ and the $f_j$'s obtained, the order of the centralizer $|C_{\overline{G}}(d_jg)| = \frac{k|C_G(g)|}{f_j}$ of a class representative $d_jg \in O_j$, where $d_j \in N$ and $g \in G$, is computed (see Table 1). Altogether 70 conjugacy classes are obtained for $\overline{G}$. Using the GAP routine, Programme B in [21], which is based on Theorem 2.7 and Remark 2.8 in [14], the order $o(d_jg)$ of a representative $d_jg$ in the orbit $O_j$, is computed. Let $(d_jg)^{o(g)} = w \in N$. If $w = 1_N$, then $o(d_jg) = o(g)$. Otherwise for $w \neq 1_N$ we have $o(d_jg) = 5o(g)$, since $N$ is an elementary abelian 5-group. Hence the order for each class representative $d_jg$ in a conjugacy class $[d_jg]$ of $\overline{G}$ coming from a coset $Ng$ is determined and is found in Table 1. From Programme A and Programme B in [21] the $p$-power maps, $p$ a prime, are computed for the elements in each conjugacy class $[d_jg]$ of $\overline{G}$ and are listed in Table 1. The values of the parameter, $m_j = \frac{f_j|N|}{k}$, which are useful in determining the entries of a Fischer-Clifford matrix $M(g)$ are also listed in Table 1. We identify $d_jg$ with $x_j$ used in Section 2 and in the beginning of Section 3.

Table 1: The Conjugacy Classes of $\bar{G}$

| $[g]_G$ | $k$ | $f_j$ | $m_j$ | $d_j$ | $w$ | $[d_jg]_{\overline{G}}$ | $\lvert C_{\overline{G}}(d_jg)\rvert$ | 2 | 3 | 5 | 13 | $\mapsto \mathbb{M}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $1A$ | 625 | 1 | 1 | $(0,0,0,0)$ | $(0,0,0,0)$ | $1A$ | 58500000 | | | | | $1A$ |
| | | 624 | 624 | $(0,0,0,1)$ | $(0,0,0,1)$ | $5A$ | 93750 | | | $1A$ | | $5B$ |
| $2A$ | 1 | 1 | 625 | $(0,0,0,0)$ | $(0,0,0,0)$ | $2A$ | 93600 | $1A$ | | | | $2B$ |
| $2B$ | 25 | 1 | 25 | $(0,0,0,0)$ | $(0,0,0,0)$ | $2B$ | 6000 | $1A$ | | | | $2B$ |
| | | 24 | 600 | $(0,0,0,1)$ | $(1,2,3,2)$ | $10A$ | 250 | $5A$ | | $2B$ | | $10E$ |
| $3A$ | 1 | 1 | 625 | $(0,0,0,0)$ | $(0,0,0,0)$ | $3A$ | 46800 | | $1A$ | | | $3B$ |
| $3B$ | 1 | 1 | 625 | $(0,0,0,0)$ | $(0,0,0,0)$ | $3B$ | 144 | | $1A$ | | | $3C$ |
| $3C$ | 25 | 1 | 25 | $(0,0,0,0)$ | $(0,0,0,0)$ | $3C$ | 1800 | | $1A$ | | | $3C$ |
| | | 24 | 600 | $(0,1,2,2)$ | $(1,2,4,4)$ | $15A$ | 75 | | $5A$ | $3C$ | | $15D$ |
| $4A$ | 1 | 1 | 625 | $(0,0,0,0)$ | $(0,0,0,0)$ | $4A$ | 144 | $2A$ | | | | $4D$ |
| $4B$ | 1 | 1 | 625 | $(0,0,0,0)$ | $(0,0,0,0)$ | $4B$ | 240 | $2A$ | | | | $4D$ |
| $5A$ | 25 | 1 | 25 | $(0,0,0,0)$ | $(0,0,0,0)$ | $5B$ | 7500 | | | $1A$ | | $5B$ |
| | | 12 | 300 | $(0,0,2,4)$ | $(0,0,0,0)$ | $5C$ | 625 | | | $1A$ | | $5B$ |
| | | 12 | 300 | $(0,1,0,0)$ | $(0,0,0,0)$ | $5D$ | 625 | | | $1A$ | | $5B$ |
| $5B$ | 25 | 1 | 25 | $(0,0,0,0)$ | $(0,0,0,0)$ | $5E$ | 7500 | | | $1A$ | | $5A$ |
| | | 6 | 150 | $(0,0,0,4)$ | $(0,0,0,0)$ | $5F$ | 1250 | | | $1A$ | | $5B$ |
| | | 6 | 150 | $(0,0,0,2)$ | $(0,0,0,0)$ | $5G$ | 1250 | | | $1A$ | | $5B$ |
| | | 6 | 150 | $(0,1,0,0)$ | $(0,0,0,0)$ | $5H$ | 1250 | | | $1A$ | | $5B$ |
| | | 6 | 150 | $(0,1,0,3)$ | $(0,0,0,0)$ | $5I$ | 1250 | | | $1A$ | | $5B$ |
| $6A$ | 1 | 1 | 625 | $(0,0,0,0)$ | $(0,0,0,0)$ | $6A$ | 46800 | $3A$ | $2A$ | | | $6B$ |
| $6B$ | 1 | 1 | 625 | $(0,0,0,0)$ | $(0,0,0,0)$ | $6B$ | 72 | $3B$ | $2A$ | | | $6F$ |
| $6C$ | 1 | 1 | 625 | $(0,0,0,0)$ | $(0,0,0,0)$ | $6C$ | 144 | $3C$ | $2A$ | | | $6F$ |
| $6D$ | 1 | 1 | 625 | $(0,0,0,0)$ | $(0,0,0,0)$ | $6D$ | 12 | $3B$ | $2B$ | | | $6F$ |
| $8A$ | 1 | 1 | 625 | $(0,0,0,0)$ | $(0,0,0,0)$ | $8A$ | 72 | $4B$ | | | | $8F$ |
| $8B$ | 1 | 1 | 625 | $(0,0,0,0)$ | $(0,0,0,0)$ | $8B$ | 8 | $4B$ | | | | $8F$ |
| $10A$ | 1 | 1 | 625 | $(0,0,0,0)$ | $(0,0,0,0)$ | $10B$ | 300 | $5B$ | | $2A$ | | $10D$ |
| $10B$ | 1 | 1 | 625 | $(0,0,0,0)$ | $(0,0,0,0)$ | $10C$ | 300 | $5E$ | | $2A$ | | $10B$ |
| $10C$ | 5 | 1 | 125 | $(0,0,0,0)$ | $(0,0,0,0)$ | $10D$ | 100 | $5E$ | | $2B$ | | $10B$ |
| | | 2 | 250 | $(0,0,0,1)$ | $(0,0,0,0)$ | $10E$ | 50 | $5H$ | | $2B$ | | $10E$ |
| | | 2 | 250 | $(0,0,0,2)$ | $(0,0,0,0)$ | $10F$ | 50 | $5I$ | | $2B$ | | $10E$ |
| $10D$ | 5 | 1 | 125 | $(0,0,0,0)$ | $(0,0,0,0)$ | $10G$ | 100 | $5E$ | | $2B$ | | $10B$ |
| | | 2 | 250 | $(0,0,2,2)$ | $(0,0,0,0)$ | $10H$ | 50 | $5G$ | | $2B$ | | $10E$ |
| | | 2 | 250 | $(0,0,0,3)$ | $(0,0,0,0)$ | $10I$ | 50 | $5F$ | | $2B$ | | $10E$ |
| $12A$ | 1 | 1 | 625 | $(0,0,0,0)$ | $(0,0,0,0)$ | $12A$ | 72 | $6A$ | $4A$ | | | $12J$ |
| $12B$ | 1 | 1 | 625 | $(0,0,0,0)$ | $(0,0,0,0)$ | $12B$ | 72 | $6A$ | $4B$ | | | $12F$ |
| $12C$ | 1 | 1 | 625 | $(0,0,0,0)$ | $(0,0,0,0)$ | $12C$ | 72 | $6C$ | $4B$ | | | $12J$ |
| $12D$ | 1 | 1 | 625 | $(0,0,0,0)$ | $(0,0,0,0)$ | $12D$ | 72 | $6C$ | $4B$ | | | $12J$ |
| $12E$ | 1 | 1 | 625 | $(0,0,0,0)$ | $(0,0,0,0)$ | $12E$ | 12 | $6B$ | $4B$ | | | $12J$ |
| $13A$ | 1 | 1 | 625 | $(0,0,0,0)$ | $(0,0,0,0)$ | $13A$ | 78 | | | | $1A$ | $13B$ |
| $13B$ | 1 | 1 | 625 | $(0,0,0,0)$ | $(0,0,0,0)$ | $13B$ | 78 | | | | $1A$ | $13B$ |
| $13C$ | 1 | 1 | 625 | $(0,0,0,0)$ | $(0,0,0,0)$ | $13C$ | 78 | | | | $1A$ | $13B$ |
| $15A$ | 1 | 1 | 625 | $(0,0,0,0)$ | $(0,0,0,0)$ | $15B$ | 150 | | $5B$ | $3A$ | | $15C$ |
| $15B$ | 1 | 1 | 625 | $(0,0,0,0)$ | $(0,0,0,0)$ | $15C$ | 150 | | $5E$ | $3A$ | | $15B$ |
| $20A$ | 1 | 1 | 625 | $(0,0,0,0)$ | $(0,0,0,0)$ | $20A$ | 20 | $10B$ | | $4A$ | | $20E$ |
| $20B$ | 1 | 1 | 625 | $(0,0,0,0)$ | $(0,0,0,0)$ | $20B$ | 20 | $10B$ | | $4A$ | | $20E$ |

Table 1. The Conjugacy Classes of $\bar{G}$ (continued)

| $[g]_G$ | $k$ | $f_j$ | $m_j$ | $d_j$ | $w$ | $[d_jg]_{\overline{G}}$ | $\lvert C_{\overline{G}}(d_jg)\rvert$ | 2 | 3 | 5 | 13 | $\mapsto \mathbb{M}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $24A$ | 1 | 1 | 625 | $(0,0,0,0)$ | $(0,0,0,0)$ | $24A$ | 72 | $12B$ | $8B$ | | | $12F$ |
| $24B$ | 1 | 1 | 625 | $(0,0,0,0)$ | $(0,0,0,0)$ | $24B$ | 72 | $12B$ | $8B$ | | | $24J$ |
| $24C$ | 1 | 1 | 625 | $(0,0,0,0)$ | $(0,0,0,0)$ | $24C$ | 72 | $12D$ | $8B$ | | | $24J$ |
| $24D$ | 1 | 1 | 625 | $(0,0,0,0)$ | $(0,0,0,0)$ | $24D$ | 72 | $12D$ | $8B$ | | | $24J$ |
| $24E$ | 1 | 1 | 625 | $(0,0,0,0)$ | $(0,0,0,0)$ | $24E$ | 72 | $12C$ | $8B$ | | | $24J$ |
| $24F$ | 1 | 1 | 625 | $(0,0,0,0)$ | $(0,0,0,0)$ | $24F$ | 72 | $12C$ | $8B$ | | | $24J$ |
| $24G$ | 1 | 1 | 625 | $(0,0,0,0)$ | $(0,0,0,0)$ | $24G$ | 72 | $12E$ | $8B$ | | | $24J$ |
| $24H$ | 1 | 1 | 625 | $(0,0,0,0)$ | $(0,0,0,0)$ | $24H$ | 72 | $12E$ | $8B$ | | | $24J$ |
| $26A$ | 1 | 1 | 625 | $(0,0,0,0)$ | $(0,0,0,0)$ | $26A$ | 78 | $13C$ | | | $2A$ | $26B$ |
| $26B$ | 1 | 1 | 625 | $(0,0,0,0)$ | $(0,0,0,0)$ | $26B$ | 78 | $13A$ | | | $2A$ | $26B$ |
| $26C$ | 1 | 1 | 625 | $(0,0,0,0)$ | $(0,0,0,0)$ | $26C$ | 78 | $13B$ | | | $2A$ | $26B$ |
| $30A$ | 1 | 1 | 625 | $(0,0,0,0)$ | $(0,0,0,0)$ | $30A$ | 150 | $15A$ | $10B$ | $6A$ | | $30A$ |
| $30B$ | 1 | 1 | 625 | $(0,0,0,0)$ | $(0,0,0,0)$ | $30B$ | 150 | $15B$ | $10C$ | $6A$ | | $30D$ |
| $39A$ | 1 | 1 | 625 | $(0,0,0,0)$ | $(0,0,0,0)$ | $39A$ | 78 | | $13C$ | | $3A$ | $39C$ |
| $39B$ | 1 | 1 | 625 | $(0,0,0,0)$ | $(0,0,0,0)$ | $39B$ | 78 | | $13C$ | | $3A$ | $39D$ |
| $39C$ | 1 | 1 | 625 | $(0,0,0,0)$ | $(0,0,0,0)$ | $39C$ | 78 | | $13A$ | | $3A$ | $39C$ |
| $39D$ | 1 | 1 | 625 | $(0,0,0,0)$ | $(0,0,0,0)$ | $39D$ | 78 | | $13A$ | | $3A$ | $39D$ |
| $39E$ | 1 | 1 | 625 | $(0,0,0,0)$ | $(0,0,0,0)$ | $39E$ | 78 | | $13B$ | | $3A$ | $39C$ |
| $39F$ | 1 | 1 | 625 | $(0,0,0,0)$ | $(0,0,0,0)$ | $39F$ | 78 | | $13B$ | | $3A$ | $39D$ |
| $78A$ | 1 | 1 | 625 | $(0,0,0,0)$ | $(0,0,0,0)$ | $78A$ | 78 | $39E$ | $26C$ | | $6A$ | $78B$ |
| $78B$ | 1 | 1 | 625 | $(0,0,0,0)$ | $(0,0,0,0)$ | $78B$ | 78 | $39F$ | $26C$ | | $6A$ | $78C$ |
| $78C$ | 1 | 1 | 625 | $(0,0,0,0)$ | $(0,0,0,0)$ | $78C$ | 78 | $39A$ | $26A$ | | $6A$ | $78B$ |
| $78D$ | 1 | 1 | 625 | $(0,0,0,0)$ | $(0,0,0,0)$ | $78D$ | 78 | $39B$ | $26A$ | | $6A$ | $78C$ |
| $78E$ | 1 | 1 | 625 | $(0,0,0,0)$ | $(0,0,0,0)$ | $78E$ | 78 | $39C$ | $26B$ | | $6A$ | $78B$ |
| $78F$ | 1 | 1 | 625 | $(0,0,0,0)$ | $(0,0,0,0)$ | $78F$ | 78 | $39D$ | $26B$ | | $6A$ | $78C$ |

## 4. Inertia factor groups of $\overline{G}$

We have already seen in the Introduction of this paper, that the orbit stabilizers (the so-called inertia factors) of the action of $G$ on $\mathrm{Irr}(N)$ are two groups of the form $H_1 = G$ and $H_2 = 5^2{:}S_3$. The inertia factor $H_2 =< \alpha_1, \alpha_2 >$ is generated from elements $\alpha_1 \in 2B$ and $\alpha_2 \in 10C$ (see Figure 3) in the conjugacy classes $2B$ and $10C$ of $G$.

The fusion maps of the conjugacy classes of $H_2$ into $G$ are shown in Table 2 and will be used in the construction of the Fischer-Clifford matrices and ordinary character table of $\overline{G}$.

$$\alpha_1 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 2 & 0 & 4 & 0 \\ 0 & 2 & 0 & 4 \end{pmatrix}, \alpha_2 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 4 & 3 & 2 & 4 \\ 2 & 0 & 4 & 0 \\ 4 & 3 & 1 & 2 \end{pmatrix}$$

Figure 3: Generators of $H_2$

Table 2: The fusion of $H_2$ into $G$

| $[h]_{H_2}$ | $\longrightarrow [g]_{(3\times 2L_2(25)):2_2}$ | $[h]_{H_2}$ | $\longrightarrow [g]_{(3\times 2L_2(25)):2_2}$ |
|---|---|---|---|
| $1A$ | $1A$ | $5E$ | $5B$ |
| $2A$ | $2B$ | $5F$ | $5B$ |
| $3A$ | $3C$ | $10A$ | $10C$ |
| $5A$ | $5A$ | $10B$ | $10D$ |
| $5B$ | $5A$ | $10C$ | $10C$ |
| $5C$ | $5B$ | $10D$ | $10D$ |
| $5D$ | $5B$ | | |

## 5. The Fischer-matrices of $\overline{G}$

In this section, the Fischer-Clifford matrices of the group $\overline{G}$ are going to be obtained by using a GAP routine, Programme D in [3] and [4]. This routine gives a possible candidate for a Fischer-Clifford matrix $M(g)$ and then the properties of Fischer-Clifford matrices (see [2], [14]) are used to rearrange the rows and columns in order to get the unique matrix $M(g)$ corresponding to a class representative $g \in G$. A brief outline of the theory behind the development of Programme D, as found in [9] and [14], is given first.

We restrict our discussion to a split extension $S = p^n{:}Q$, with $p^n$ an elementary abelian $p$-group. For a class representative $q \in Q$, it can be shown that the map $\phi_q{:}p^n \longrightarrow p^n$, defined by $\phi_q(\overline{n}) = \overline{n}q\overline{n}^{-1}q^{-1}$, is an endomorphism of $p^n$. The image $\mathbb{I} = \mathrm{Im}(\phi_q)$ and kernel $\ker(\phi_q)$ are $C_q$-sub-modules of $p^n$, where $C_q = p^n{:}C_Q(q)$ is the stabilizer of the coset $p^nq$. The actions of $p^n$ by conjugation on $p^nq$ and that of $\mathbb{I}$ by left multiplication result in the same number $k$ of orbits. It follows that the action of $C_q$ on the $k$ orbits of $p^n$ on $p^nq$ is the same as the action of $C_q$ on the module $p^n/\mathbb{I} \cong ker(\phi_q)$. Therefore, we can identify the $k$ orbits of the action of $\mathbb{I}$ on $p^nq$ with the $k$ elements of $p^n/\mathbb{I}$. Since $p^n$ is an elementary abelian $p$-group, $\mathbb{I}$ and $\ker(\phi_q)$ are also elementary abelian $p$-groups and it follows that the index of $\mathbb{I}$ in $p^n$ is $[p^n{:}\mathbb{I}] = k$. Instead of acting $C_q$ on the $k$ orbits, the centralizer $C_Q(q)$ of $q$ in $Q$ is used. With the above discussion and notation and more details in [9], the following theorem is formulated.

**Theorem 5.1.** *A Fischer-Clifford matrix $M(q)$ of a split extension $S = p^n{:}Q$, corresponding to a class representative $q \in Q$, is a matrix of orbit sums of*

*$C_q$ acting on the rows of the ordinary character table of $p^n/\mathbb{I}$ with duplicating columns discarded.*

**Corollary 5.1.** *If $q = 1_Q$, then $\mathbb{I} = Im(\phi_q) = 1_{p^n}$ and the Fischer-Clifford matrix $M(1_Q)$ is the matrix of orbit sums of $C_q = S$ acting on the rows of the ordinary character table of $p^n/\mathbb{I} = p^n$ with duplicating columns discarded.*

The following GAP routine, which is based on the above theoretical discussion, is taken from Programme D in [3] and can compute a candidate FM for a Fischer-Clifford matrix $M(q)$ of $S = p^n{:}Q$.

```
C:=List(ConjugacyClasses(G),Representative);; M:=[];;
g:=C[i];; for n in N do
Add(M, n*g*Inverse(n)*Inverse(g));; od;
M:=AsGroup(M);; cent:=Centralizer(G, g);
I:=Irr(N);; IM:=[];; for i in [1..Size(I)] do
if IsSubgroup(Kernel(I[i]), M) then Add(IM,I[i]);
fi; od; oo:=Orbits(cent,IM);; FM:=[];;
for i in [1..Size(oo)] do
Append(FM,[AsList(Sum(oo[i]))]);od;
M1:=TransposedMat(FM);;
M2:=AsDuplicateFreeList(M1);;
FM:=TransposedMat(M2);; Display(FM)
```

As an example, consider the conjugacy class $5B$ of $G$. By making use of Theorem 5.2.4 and property (e) in [12], $M(5B)$ has the following form with corresponding weights attached to the rows and columns,

$$\begin{array}{lc} & \begin{array}{ccccc} |C_{\overline{G}}(5E)| & |C_{\overline{G}}(5F)| & |C_{\overline{G}}(5G)| & |C_{\overline{G}}(5H)| & |C_{\overline{G}}(5I)| \\ 7500 & 1250 & 1250 & 1250 & 1250 \end{array} \\ \begin{array}{lc} |C_{H_1}(5B)|= & 300 \\ |C_{H_2}(5C)|= & 50 \\ |C_{H_2}(5D)|= & 50 \\ |C_{H_2}(5E)|= & 50 \\ |C_{H_2}(5F)|= & 50 \end{array} & \left(\begin{array}{ccccc} 1 & 1 & 1 & 1 & 1 \\ 6 & g & h & i & j \\ 6 & l & m & n & o \\ 6 & q & r & s & t \\ 6 & v & w & x & y \end{array}\right). \\ \qquad\qquad\qquad m_j & \begin{array}{ccccc} 25 & 150 & 150 & 150 & 150 \end{array} \end{array}$$

To determine the unknown entries $M(5B)$, the above GAP routine gives the candidate FM,

$$M(5B) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 6 & A^* & A & B^* & B \\ 6 & A & A^* & B & B^* \\ 6 & B^* & B & A & A^* \\ 6 & B & B^* & A^* & A \end{pmatrix}$$

where $A = 1 - \sqrt{5}$ and $B = (-3 - \sqrt{5})/2$.

From the $p$-power maps of $\overline{G}$ in Table 1, we have that $(10I)^2 = 5F$, $(10H)^2 = 5G$, $(10E)^2 = 5H$ and $(10F)^2 = 5I$. Thus, for any $\chi \in \mathrm{Irr}(\overline{G})$, the congruent relations $\chi(5F) \equiv \chi(10I) \pmod 2$, $\chi(5G) \equiv \chi(10H) \pmod 2$, $\chi(5H) \equiv \chi(10E) \pmod 2$ and $\chi(5I) \equiv \chi(10F) \pmod 2$ must be satisfied. Checking the validity of these relations for the parts of the ordinary character tables of $\overline{G}$ corresponding to $M(10C)$, $M(10D)$ and the candidate $FM$ for $M(5B)$, the rows of FM are rearranged to find the desired Fischer-Clifford matrix $M(5B)$ of $\overline{G}$ (see Figure 4).

$$M(5B) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 6 & A^* & A & B^* & B \\ 6 & B & B^* & A^* & A \\ 6 & A & A^* & B & B^* \\ 6 & B^* & B & A & A^* \end{pmatrix}$$

Figure 4: Fischer-Clifford matrix $M(5B)$

Only the Fischer-matrices $M(5B)$, $M(10C)$ and $M(10D)$ were computed with the aid of the above GAP routine. The rest of the Fischer-Clifford matrices of $\overline{G}$ were computed manually. The above GAP routine comes in very handy when some entries of the Fischer-Clifford matrices are algebraic integers which are not integers. If there are considerately many inertia factors $H_i$ for the action of a split extension $S = p^n{:}Q$ on $\mathrm{Irr}(p^n)$, the Fischer-Clifford matrices can become very large. Consequently, to compute the desired Fischer-Clifford matrices of $S$, it is necessary also to use other techniques such as restriction of ordinary characters of the parent group of $S$ to the ordinary irreducible characters of $S$ together with the GAP routine. However, when the group $S$ becomes too large, the computational power to use the GAP routine becomes difficult. We have then to resort to other methods, if possible, to compute the Fischer-Clifford matrices. The Fischer-Clifford matrices of $\overline{G}$ have sizes ranging from 1 to 5 and are contained in Table 3.

Table 3: The Fischer-Clifford Matrices of $\overline{G}$

| $M(g)$ | $M(g)$ |
|---|---|
| $M(1A) = \begin{pmatrix} 1 & 1 \\ 624 & -1 \end{pmatrix}$ | $M(2B) = \begin{pmatrix} 1 & 1 \\ 24 & -1 \end{pmatrix}$ |
| $M(3C) = \begin{pmatrix} 1 & 1 \\ 24 & -1 \end{pmatrix}$ | $M(5A) = \begin{pmatrix} 1 & 1 & 1 \\ 12 & -3 & 2 \\ 12 & 2 & -3 \end{pmatrix}$ |
| $M(5B) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 6 & A^* & A & B^* & B \\ 6 & B & B^* & A^* & A \\ 6 & A & A^* & B & B^* \\ 6 & B^* & B & A & A^* \end{pmatrix}$ | $M(10C) = \begin{pmatrix} 1 & 1 & 1 \\ 2 & C & C^* \\ 2 & C^* & C \end{pmatrix}$ |
| $M(10D) = \begin{pmatrix} 1 & 1 & 1 \\ 2 & C & C^* \\ 2 & C^* & C \end{pmatrix}$ | $M(g_i) = (1), \forall g_i \notin \{1A, 2B, 3C, 5A, 5B, 10C, 10D\}$ |

where $A = 1 - \sqrt{5}$, $B = (-3 - \sqrt{5})/2$, $C = (-1 - \sqrt{5})/2$

## 6. The character table of $\overline{G}$ and fusion into the Monster $\mathbb{M}$

With all the necessary information obtained in the previous sections, the ordinary character table of $\overline{G}$ can now be constructed by the technique of Fischer-Clifford matrices as discussed in Section 2. The character table (see Table 4) is a $70 \times 70$ $\mathbb{C}-$valued matrix partitioned row-wise into two blocks $\triangle_1 = \{\chi_i | 1 \leq i \leq 57\}$ and $\triangle_2 = \{\chi_i | 58 \leq i \leq 70\}$, where $\chi_i \in \mathrm{Irr}(\overline{G}) = \cup_{i=1}^{2} \triangle_i$. Note that each block corresponds to an inertia group $\overline{H}_i = 5^4{:}H_i$. Checks for consistency and accuracy of the character table obtained have been carried out with the GAP routine, Programme C [20].

Unique $p$-power maps for the elements of $\overline{G}$ are obtained for our Table 4 using Programme C, which coincide with the $p$-power maps in Table 1. Using the power maps of $\overline{G}$ and $\mathbb{M}$, the permutation character $\chi(\mathbb{M}|\overline{G})$ of $\mathbb{M}$ on the classes of $\overline{G}$ which was computed directly by GAP, we obtained partial fusion from the classes of $\overline{G}$ into $\mathbb{M}$. To complete the fusion map from $\overline{G}$ to $\mathbb{M}$, the technique of set intersections [14] was used to restrict ordinary irreducible characters of $\mathbb{M}$ of small degrees to $\overline{G}$. For example, the character $196883a \in \mathrm{Irr}(\mathbb{M})$ will restrict to $\overline{G}$ as $(196883a)_{\overline{G}} = 13c + 24a + 26cef + 52acjk + 624a + 4(624b) + 5(1248a) + 5(1872a) + 7(1872b) + 5(1872c) + 7(1872d) + 5(1872e) + 7(1872f) + 5(1872g) + 7(1872h) + 13(3744a) + 13(3744b)$. The fusion map of the classes of $\overline{G}$ into the classes of $\mathbb{M}$ is found in the last column of Table 1.

Table 4: The Character Table of $\overline{G}$

| $[g]_G$ | 1A | | 2A | 2B | | 3A | 3B | 3C | | 4A | 4B | 5A | | | 5B | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $[x]_{\overline{G}}$ | 1A | 5A | 2A | 2B | 10A | 3A | 3B | 3C | 15A | 4A | 4B | 5B | 5C | 5D | 5E | 5F | 5G | 5H | 5I |
| $\chi_1$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| $\chi_2$ | 1 | 1 | 1 | -1 | -1 | 1 | 1 | 1 | 1 | -1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| $\chi_3$ | 2 | 2 | 2 | 0 | 0 | -1 | 2 | -1 | -1 | 0 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| $\chi_4$ | 12 | 12 | -12 | 0 | 0 | 12 | 0 | 0 | 0 | 0 | 0 | -3 | -3 | -3 | 2 | 2 | 2 | 2 | 2 |
| $\chi_5$ | 12 | 12 | -12 | 0 | 0 | 12 | 0 | 0 | 0 | 0 | 0 | -3 | -3 | -3 | 2 | 2 | 2 | 2 | 2 |
| $\chi_6$ | 12 | 12 | -12 | 0 | 0 | 12 | 0 | 0 | 0 | 0 | 0 | 2 | 2 | 2 | -3 | -3 | -3 | -3 | -3 |
| $\chi_7$ | 12 | 12 | -12 | 0 | 0 | 12 | 0 | 0 | 0 | 0 | 0 | 2 | 2 | 2 | -3 | -3 | -3 | -3 | -3 |
| $\chi_8$ | 13 | 13 | 13 | -1 | -1 | 13 | 1 | 1 | 1 | -5 | 1 | -2 | -2 | -2 | 3 | 3 | 3 | 3 | 3 |
| $\chi_9$ | 13 | 13 | 13 | -5 | -5 | 13 | 1 | 1 | 1 | -1 | 1 | 3 | 3 | 3 | -2 | -2 | -2 | -2 | -2 |
| $\chi_{10}$ | 13 | 13 | 13 | 5 | 5 | 13 | 1 | 1 | 1 | 1 | 1 | 3 | 3 | 3 | -2 | -2 | -2 | -2 | -2 |
| $\chi_{11}$ | 13 | 13 | 13 | 1 | 1 | 13 | 1 | 1 | 1 | 5 | 1 | -2 | -2 | -2 | 3 | 3 | 3 | 3 | 3 |
| $\chi_{12}$ | 24 | 24 | -24 | 0 | 0 | -12 | 0 | 0 | 0 | 0 | 0 | -6 | -6 | -6 | 4 | 4 | 4 | 4 | 4 |
| $\chi_{13}$ | 24 | 24 | -24 | 0 | 0 | -12 | 0 | 0 | 0 | 0 | 0 | 4 | 4 | 4 | -6 | -6 | -6 | -6 | -6 |
| $\chi_{14}$ | 25 | 25 | 25 | -5 | -5 | 25 | 1 | 1 | 1 | -5 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{15}$ | 25 | 25 | 25 | 5 | 5 | 25 | 1 | 1 | 1 | 5 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{16}$ | 26 | 26 | 26 | 0 | 0 | -13 | 2 | -1 | -1 | 0 | 2 | 6 | 6 | 6 | -4 | -4 | -4 | -4 | -4 |
| $\chi_{17}$ | 26 | 26 | 26 | 0 | 0 | -13 | 2 | -1 | -1 | 0 | 2 | -4 | -4 | -4 | 6 | 6 | 6 | 6 | 6 |
| $\chi_{18}$ | 26 | 26 | 26 | 6 | 6 | 26 | 2 | 2 | 2 | -6 | -2 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| $\chi_{19}$ | 26 | 26 | 26 | -6 | -6 | 26 | 2 | 2 | 2 | 6 | -2 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| $\chi_{20}$ | 26 | 26 | 26 | -4 | -4 | 26 | -1 | -1 | -1 | -4 | 2 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| $\chi_{21}$ | 26 | 26 | 26 | 4 | 4 | 26 | -1 | -1 | -1 | -4 | 2 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| $\chi_{22}$ | 26 | 26 | 26 | -4 | -4 | 26 | -1 | -1 | -1 | 4 | 2 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| $\chi_{23}$ | 26 | 26 | 26 | 4 | 4 | 26 | -1 | -1 | -1 | 4 | 2 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| $\chi_{24}$ | 48 | 48 | 48 | 0 | 0 | 48 | 0 | 0 | 0 | 0 | 0 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 |
| $\chi_{25}$ | 48 | 48 | 48 | 0 | 0 | 48 | 0 | 0 | 0 | 0 | 0 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 |
| $\chi_{26}$ | 48 | 48 | 48 | 0 | 0 | 48 | 0 | 0 | 0 | 0 | 0 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 |
| $\chi_{27}$ | 48 | 48 | -48 | 0 | 0 | 48 | 0 | 0 | 0 | 0 | 0 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 |
| $\chi_{28}$ | 48 | 48 | -48 | 0 | 0 | 48 | 0 | 0 | 0 | 0 | 0 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 |
| $\chi_{29}$ | 48 | 48 | -48 | 0 | 0 | 48 | 0 | 0 | 0 | 0 | 0 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 |
| $\chi_{30}$ | 48 | 48 | -48 | 0 | 0 | -24 | 0 | 0 | 0 | 0 | 0 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 |
| $\chi_{31}$ | 48 | 48 | -48 | 0 | 0 | -24 | 0 | 0 | 0 | 0 | 0 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 |
| $\chi_{32}$ | 48 | 48 | -48 | 0 | 0 | -24 | 0 | 0 | 0 | 0 | 0 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 |
| $\chi_{33}$ | 48 | 48 | -48 | 0 | 0 | -24 | 0 | 0 | 0 | 0 | 0 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 |
| $\chi_{34}$ | 48 | 48 | -48 | 0 | 0 | -24 | 0 | 0 | 0 | 0 | 0 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 |
| $\chi_{35}$ | 48 | 48 | -48 | 0 | 0 | -24 | 0 | 0 | 0 | 0 | 0 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 |
| $\chi_{36}$ | 48 | 48 | 48 | 0 | 0 | -24 | 0 | 0 | 0 | 0 | 0 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 |
| $\chi_{37}$ | 48 | 48 | 48 | 0 | 0 | -24 | 0 | 0 | 0 | 0 | 0 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 |
| $\chi_{38}$ | 48 | 48 | 48 | 0 | 0 | -24 | 0 | 0 | 0 | 0 | 0 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 |
| $\chi_{39}$ | 48 | 48 | 48 | 0 | 0 | -24 | 0 | 0 | 0 | 0 | 0 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 |
| $\chi_{40}$ | 48 | 48 | 48 | 0 | 0 | -24 | 0 | 0 | 0 | 0 | 0 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 |
| $\chi_{41}$ | 48 | 48 | 48 | 0 | 0 | -24 | 0 | 0 | 0 | 0 | 0 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 |
| $\chi_{42}$ | 50 | 50 | 50 | 0 | 0 | -25 | 2 | -1 | -1 | 0 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{43}$ | 52 | 52 | 52 | 0 | 0 | -26 | 4 | -2 | -2 | 0 | -4 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| $\chi_{44}$ | 52 | 52 | -52 | 0 | 0 | 52 | 4 | 4 | 4 | 0 | 0 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| $\chi_{45}$ | 52 | 52 | 52 | 0 | 0 | 52 | -2 | -2 | -2 | 0 | -4 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| $\chi_{46}$ | 52 | 52 | 52 | 0 | 0 | -26 | -2 | 1 | 1 | 0 | 4 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| $\chi_{47}$ | 52 | 52 | 52 | 0 | 0 | -26 | -2 | 1 | 1 | 0 | 4 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| $\chi_{48}$ | 52 | 52 | -52 | 0 | 0 | -26 | 4 | -2 | -2 | 0 | 0 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| $\chi_{49}$ | 52 | 52 | -52 | 0 | 0 | -26 | 4 | -2 | -2 | 0 | 0 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| $\chi_{50}$ | 52 | 52 | -52 | 0 | 0 | 52 | -2 | -2 | -2 | 0 | 0 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| $\chi_{51}$ | 52 | 52 | -52 | 0 | 0 | 52 | -2 | -2 | -2 | 0 | 0 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| $\chi_{52}$ | 52 | 52 | 52 | 0 | 0 | -26 | -2 | 1 | 1 | 0 | -4 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| $\chi_{53}$ | 52 | 52 | 52 | 0 | 0 | -26 | -2 | 1 | 1 | 0 | -4 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| $\chi_{54}$ | 52 | 52 | -52 | 0 | 0 | -26 | -2 | 1 | 1 | 0 | 0 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| $\chi_{55}$ | 52 | 52 | -52 | 0 | 0 | -26 | -2 | 1 | 1 | 0 | 0 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| $\chi_{56}$ | 52 | 52 | -52 | 0 | 0 | -26 | -2 | 1 | 1 | 0 | 0 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| $\chi_{57}$ | 52 | 52 | -52 | 0 | 0 | -26 | -2 | 1 | 1 | 0 | 0 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| $\chi_{58}$ | 624 | -1 | 0 | 24 | -1 | 0 | 0 | 24 | -1 | 0 | 0 | 24 | -1 | -1 | 24 | -1 | -1 | -1 | -1 |
| $\chi_{59}$ | 624 | -1 | 0 | -24 | 1 | 0 | 0 | 24 | -1 | 0 | 0 | 24 | -1 | -1 | 24 | -1 | -1 | -1 | -1 |
| $\chi_{60}$ | 1248 | -2 | 0 | 0 | 0 | 0 | 0 | -24 | 1 | 0 | 0 | 48 | -2 | -2 | 48 | -2 | -2 | -2 | -2 |
| $\chi_{61}$ | 1872 | -3 | 0 | -24 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 12 | A | *A | -18 | C | $\overline{C}$ | $\overline{D}$ | D |
| $\chi_{62}$ | 1872 | -3 | 0 | -24 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 12 | *A | A | -18 | D | $\overline{D}$ | C | $\overline{C}$ |
| $\chi_{63}$ | 1872 | -3 | 0 | -24 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 12 | A | *A | -18 | $\overline{C}$ | C | D | $\overline{D}$ |
| $\chi_{64}$ | 1872 | -3 | 0 | -24 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 12 | *A | A | -18 | $\overline{D}$ | D | $\overline{C}$ | C |
| $\chi_{65}$ | 1872 | -3 | 0 | 24 | -1 | 0 | 0 | 0 | 0 | 0 | 0 | 12 | A | *A | -18 | C | $\overline{C}$ | $\overline{D}$ | D |
| $\chi_{66}$ | 1872 | -3 | 0 | 24 | -1 | 0 | 0 | 0 | 0 | 0 | 0 | 12 | *A | A | -18 | D | $\overline{D}$ | C | $\overline{C}$ |
| $\chi_{67}$ | 1872 | -3 | 0 | 24 | -1 | 0 | 0 | 0 | 0 | 0 | 0 | 12 | A | *A | -18 | $\overline{C}$ | C | D | $\overline{D}$ |
| $\chi_{68}$ | 1872 | -3 | 0 | 24 | -1 | 0 | 0 | 0 | 0 | 0 | 0 | 12 | *A | A | -18 | $\overline{D}$ | D | $\overline{C}$ | C |
| $\chi_{69}$ | 3744 | -6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -36 | B | *B | 24 | E | E | *E | *E |
| $\chi_{70}$ | 3744 | -6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -36 | -*B | B | 24 | *E | *E | E | E |

where A=$\frac{-1-5\sqrt{5}}{2}$, B=$\frac{3+5\sqrt{5}}{2}$, C=$-7E(5)-2E(5)^2+3E(5)^3+3E(5)^4$,
D=$3E(5)-7E(5)^2+3E(5)^3-2E(5)^4$, E=$-1-5\sqrt{5}$

Table 4: The Character Table of $\overline{G}$ (continued)

| $[g]_G$ | 6A | 6B | 6C | 6D | 8A | 8B | 10A | 10B | 10C | | | 10D | | | 12A | 12B | 12C | 12D | 12E |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $[x]_{\overline{G}}$ | 6A | 6B | 6C | 6D | 8A | 8B | 10B | 10C | 10D | 10E | 10F | 10G | 10H | 10I | 12A | 12B | 12C | 12D | 12E |
| $\chi_1$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| $\chi_2$ | 1 | 1 | 1 | -1 | -1 | 1 | 1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | 1 | 1 | 1 |
| $\chi_3$ | -1 | 2 | -1 | 0 | 0 | 2 | 2 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | -1 | -1 | 2 |
| $\chi_4$ | -12 | 0 | 0 | 0 | 0 | 0 | 3 | -2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_5$ | -12 | 0 | 0 | 0 | 0 | 0 | 3 | -2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_6$ | -12 | 0 | 0 | 0 | 0 | 0 | -2 | 3 | F | F | F | -F | -F | -F | 0 | 0 | 0 | 0 | 0 |
| $\chi_7$ | -12 | 0 | 0 | 0 | 0 | 0 | -2 | 3 | -F | -F | -F | F | F | F | 0 | 0 | 0 | 0 | 0 |
| $\chi_8$ | 13 | 1 | 1 | -1 | 1 | -1 | -2 | 3 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | 1 | 1 | 1 | 1 |
| $\chi_9$ | 13 | 1 | 1 | 1 | 1 | -1 | 3 | -2 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | 1 | 1 | 1 | 1 |
| $\chi_{10}$ | 13 | 1 | 1 | -1 | -1 | -1 | 3 | -2 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 |
| $\chi_{11}$ | 13 | 1 | 1 | 1 | -1 | -1 | -2 | 3 | 1 | 1 | 1 | 1 | 1 | 1 | -1 | 1 | 1 | 1 | 1 |
| $\chi_{12}$ | 12 | 0 | 0 | 0 | 0 | 0 | 6 | -4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{13}$ | 12 | 0 | 0 | 0 | 0 | 0 | -4 | 6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{14}$ | 25 | 1 | 1 | 1 | -1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 |
| $\chi_{15}$ | 25 | 1 | 1 | -1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | 1 | 1 | 1 | 1 |
| $\chi_{16}$ | -13 | 2 | -1 | 0 | 0 | -2 | 6 | -4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | -1 | -1 | 2 |
| $\chi_{17}$ | -13 | 2 | -1 | 0 | 0 | -2 | -4 | 6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | -1 | -1 | 2 |
| $\chi_{18}$ | 26 | 2 | 2 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | -2 | -2 | -2 | -2 |
| $\chi_{19}$ | 26 | 2 | 2 | 0 | 0 | 0 | 1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | 0 | -2 | -2 | -2 | -2 |
| $\chi_{20}$ | 26 | -1 | -1 | -1 | 0 | 2 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | -1 | 2 | -1 | -1 | -1 |
| $\chi_{21}$ | 26 | -1 | -1 | 1 | 0 | -2 | 1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 2 | -1 | -1 | -1 |
| $\chi_{22}$ | 26 | -1 | -1 | -1 | 0 | -2 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 2 | -1 | -1 | -1 |
| $\chi_{23}$ | 26 | -1 | -1 | 1 | 0 | 2 | 1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | 2 | -1 | -1 | -1 |
| $\chi_{24}$ | 48 | 0 | 0 | 0 | 0 | 0 | -2 | -2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{25}$ | 48 | 0 | 0 | 0 | 0 | 0 | -2 | -2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{26}$ | 48 | 0 | 0 | 0 | 0 | 0 | -2 | -2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{27}$ | -48 | 0 | 0 | 0 | 0 | 0 | 2 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{28}$ | -48 | 0 | 0 | 0 | 0 | 0 | 2 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{29}$ | -48 | 0 | 0 | 0 | 0 | 0 | 2 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{30}$ | 24 | 0 | 0 | 0 | 0 | 0 | 2 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{31}$ | 24 | 0 | 0 | 0 | 0 | 0 | 2 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{32}$ | 24 | 0 | 0 | 0 | 0 | 0 | 2 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{33}$ | 24 | 0 | 0 | 0 | 0 | 0 | 2 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{34}$ | 24 | 0 | 0 | 0 | 0 | 0 | 2 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{35}$ | 24 | 0 | 0 | 0 | 0 | 0 | 2 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{36}$ | -24 | 0 | 0 | 0 | 0 | 0 | -2 | -2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{37}$ | -24 | 0 | 0 | 0 | 0 | 0 | -2 | -2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{38}$ | -24 | 0 | 0 | 0 | 0 | 0 | -2 | -2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{39}$ | -24 | 0 | 0 | 0 | 0 | 0 | -2 | -2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{40}$ | -24 | 0 | 0 | 0 | 0 | 0 | -2 | -2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{41}$ | -24 | 0 | 0 | 0 | 0 | 0 | -2 | -2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{42}$ | -25 | 2 | -1 | 0 | 0 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | -1 | -1 | 2 |
| $\chi_{43}$ | -26 | 4 | -2 | 0 | 0 | 0 | 2 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 2 | 2 | -4 |
| $\chi_{44}$ | -52 | -4 | -4 | 0 | 0 | 0 | -2 | -2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{45}$ | 52 | -2 | -2 | 0 | 0 | 0 | 2 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -4 | 2 | 2 | 2 |
| $\chi_{46}$ | -26 | -2 | 1 | 0 | 0 | -4 | 2 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -2 | 1 | 1 | -2 |
| $\chi_{47}$ | -26 | -2 | 1 | 0 | 0 | 4 | 2 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -2 | 1 | 1 | -2 |
| $\chi_{48}$ | 26 | -4 | 2 | 0 | 0 | 0 | -2 | -2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{49}$ | 26 | -4 | 2 | 0 | 0 | 0 | -2 | -2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{50}$ | -52 | 2 | 2 | 0 | 0 | 0 | -2 | -2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{51}$ | -52 | 2 | 2 | 0 | 0 | 0 | -2 | -2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{52}$ | 26 | -2 | 1 | 0 | 0 | 0 | 2 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | -1 | -1 | 2 |
| $\chi_{53}$ | -26 | -2 | 1 | 0 | 0 | 0 | 2 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | -1 | -1 | 2 |
| $\chi_{54}$ | 26 | 2 | -1 | 0 | 0 | 0 | -2 | -2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | J | -J | 0 |
| $\chi_{55}$ | 26 | 2 | -1 | 0 | 0 | 0 | -2 | -2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | J | -J | 0 |
| $\chi_{56}$ | 26 | 2 | -1 | 0 | 0 | 0 | -2 | -2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -J | J | 0 |
| $\chi_{57}$ | 26 | 2 | -1 | 0 | 0 | 0 | -2 | -2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -J | J | 0 |
| $\chi_{58}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 4 | -1 | -1 | 4 | -1 | -1 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{59}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -4 | 1 | 1 | -4 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{60}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{61}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | G | H | $\overline{H}$ | *G | $\bar{I}$ | I | 0 | 0 | 0 | 0 | 0 |
| $\chi_{62}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | *G | I | $\bar{I}$ | G | H | $\overline{H}$ | 0 | 0 | 0 | 0 | 0 |
| $\chi_{63}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | G | $\overline{H}$ | H | *G | I | $\bar{I}$ | 0 | 0 | 0 | 0 | 0 |
| $\chi_{64}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | *G | $\bar{I}$ | I | G | $\overline{H}$ | H | 0 | 0 | 0 | 0 | 0 |
| $\chi_{65}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -G | -H | -$\overline{H}$ | -*G | -$\bar{I}$ | -I | 0 | 0 | 0 | 0 | 0 |
| $\chi_{66}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -*G | -I | -$\bar{I}$ | -G | -H | -$\overline{H}$ | 0 | 0 | 0 | 0 | 0 |
| $\chi_{67}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -G | -$\overline{H}$ | -H | -*G | -I | -$\bar{I}$ | 0 | 0 | 0 | 0 | 0 |
| $\chi_{68}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -*G | -$\bar{I}$ | -I | -G | -$\overline{H}$ | -H | 0 | 0 | 0 | 0 | 0 |
| $\chi_{69}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{70}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

where F$=-\sqrt{5}$, G$=1+\sqrt{5}$, H$=-E(5)+E(5)^2+E(5)^4$,
I$=-E(5)^2+E(5)^3+E(5)^4$, J $=-3E(4)$

Table 4: The Character Table of $\overline{G}$ (continued)

| $[g]_G$ | 13A | 13B | 13C | 15A | 15B | 20A | 20B | 24A | 24B | 24C | 24D | 24E | 24F | 24G | 24H |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $[x]_{\overline{G}}$ | 13A | 13B | 13C | 15A | 15B | 20A | 20B | 24A | 24B | 24C | 24D | 24E | 24F | 24G | 24H |
| $\chi_1$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| $\chi_2$ | 1 | 1 | 1 | 1 | 1 | -1 | -1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| $\chi_3$ | 2 | 2 | 2 | -1 | -1 | 0 | 0 | -1 | -1 | -1 | -1 | -1 | -1 | 2 | 2 |
| $\chi_4$ | -1 | -1 | -1 | -3 | 2 | N | -N | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_5$ | -1 | -1 | -1 | -3 | 2 | -N | N | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_6$ | -1 | -1 | -1 | 2 | -3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_7$ | -1 | -1 | -1 | 2 | -3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_8$ | 0 | 0 | 0 | -2 | 3 | 0 | 0 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 |
| $\chi_9$ | 0 | 0 | 0 | 3 | -2 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 |
| $\chi_{10}$ | 0 | 0 | 0 | 3 | -2 | 1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 |
| $\chi_{11}$ | 0 | 0 | 0 | -2 | 3 | 0 | 0 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 |
| $\chi_{12}$ | -2 | -2 | -2 | 3 | -2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{13}$ | -2 | -2 | -2 | -2 | 3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{14}$ | -1 | -1 | -1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| $\chi_{15}$ | -1 | -1 | -1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| $\chi_{16}$ | 0 | 0 | 0 | -3 | 2 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | -2 | -2 |
| $\chi_{17}$ | 0 | 0 | 0 | 2 | -3 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | -2 | -2 |
| $\chi_{18}$ | 0 | 0 | 0 | 1 | 1 | -1 | -1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{19}$ | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{20}$ | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 2 | 2 | -1 | -1 | -1 | -1 | -1 | -1 |
| $\chi_{21}$ | 0 | 0 | 0 | 1 | 1 | 1 | 1 | -2 | -2 | 1 | 1 | 1 | 1 | 1 | 1 |
| $\chi_{22}$ | 0 | 0 | 0 | 1 | 1 | -1 | -1 | -2 | -2 | 1 | 1 | 1 | 1 | 1 | 1 |
| $\chi_{23}$ | 0 | 0 | 0 | 1 | 1 | -1 | -1 | 2 | 2 | -1 | -1 | -1 | -1 | -1 | -1 |
| $\chi_{24}$ | K | M | L | -2 | -2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{25}$ | L | K | M | -2 | -2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{26}$ | M | L | K | -2 | -2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{27}$ | K | M | L | -2 | -2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{28}$ | L | K | M | -2 | -2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{29}$ | M | L | K | -2 | -2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{30}$ | K | M | L | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{31}$ | L | K | M | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{32}$ | M | L | K | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{33}$ | M | L | K | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{34}$ | K | M | L | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{35}$ | L | K | M | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{36}$ | K | M | L | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{37}$ | L | K | M | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{38}$ | M | L | K | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{39}$ | M | L | K | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{40}$ | K | M | L | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{41}$ | L | K | M | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{42}$ | -2 | -2 | -2 | 0 | 0 | 0 | 0 | -1 | -1 | -1 | -1 | -1 | -1 | 2 | 2 |
| $\chi_{43}$ | 0 | 0 | 0 | -1 | -1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{44}$ | 0 | 0 | 0 | 2 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{45}$ | 0 | 0 | 0 | 2 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{46}$ | 0 | 0 | 0 | -1 | -1 | 0 | 0 | 2 | 2 | -1 | -1 | -1 | -1 | 2 | 2 |
| $\chi_{47}$ | 0 | 0 | 0 | -1 | -1 | 0 | 0 | -2 | -2 | 1 | 1 | 1 | 1 | -2 | -2 |
| $\chi_{48}$ | 0 | 0 | 0 | -1 | -1 | 0 | 0 | R | -R | R | -R | R | -R | 0 | 0 |
| $\chi_{49}$ | 0 | 0 | 0 | -1 | -1 | 0 | 0 | -R | R | -R | R | -R | R | 0 | 0 |
| $\chi_{50}$ | 0 | 0 | 0 | 2 | 2 | 0 | 0 | 0 | 0 | S | -S | S | -S | S | -S |
| $\chi_{51}$ | 0 | 0 | 0 | 2 | 2 | 0 | 0 | 0 | 0 | -S | S | -S | S | -S | S |
| $\chi_{52}$ | 0 | 0 | 0 | -1 | -1 | 0 | 0 | 0 | 0 | J | J | -J | -J | 0 | 0 |
| $\chi_{53}$ | 0 | 0 | 0 | -1 | -1 | 0 | 0 | 0 | 0 | -J | -J | J | J | 0 | 0 |
| $\chi_{54}$ | 0 | 0 | 0 | -1 | -1 | 0 | 0 | R | -R | T | -T | $\overline{\text{T}}$ | -$\overline{\text{T}}$ | S | -S |
| $\chi_{55}$ | 0 | 0 | 0 | -1 | -1 | 0 | 0 | -R | R | -T | T | -$\overline{\text{T}}$ | $\overline{\text{T}}$ | -S | S |
| $\chi_{56}$ | 0 | 0 | 0 | -1 | -1 | 0 | 0 | R | -R | -$\overline{\text{T}}$ | $\overline{\text{T}}$ | -T | T | -S | S |
| $\chi_{57}$ | 0 | 0 | 0 | -1 | -1 | 0 | 0 | -R | R | $\overline{\text{T}}$ | -$\overline{\text{T}}$ | T | -T | S | -S |
| $\chi_{58}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{59}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{60}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{61}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{62}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{63}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{64}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{65}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{66}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{67}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{68}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{69}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{70}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

where K=$-E(13)^4 - E(13)^6 - E(13)^7 - E(13)^9$, L=$-E(13) - E(13)^5 - E(13)^8 - E(13)^{12}$, M=$-E(13)^2 - E(13)^3 - E(13)^{10} - E(13)^{11}$, R=$-\sqrt{6}i$, S=$-\sqrt{6}$, T=$E(24) - E(24)^{17}$

Table 4: The Character Table of $\overline{G}$ (continued)

| $[g]_G$ | 26A | 26B | 26C | 30A | 30B | 39A | 39B | 39C | 39D | 39E | 39F | 78A | 78B | 78C | 78D | 78E | 78F |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $[x]_{\overline{G}}$ | 26A | 26B | 26C | 30A | 30B | 39A | 39B | 39C | 39D | 39E | 39F | 78A | 78B | 78C | 78D | 78E | 78F |
| $\chi_1$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| $\chi_2$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| $\chi_3$ | 2 | 2 | 2 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 |
| $\chi_4$ | 1 | 1 | 1 | 3 | -2 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | 1 | 1 | 1 | 1 | 1 |
| $\chi_5$ | 1 | 1 | 1 | 3 | -2 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | 1 | 1 | 1 | 1 | 1 |
| $\chi_6$ | 1 | 1 | 1 | -2 | 3 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | 1 | 1 | 1 | 1 | 1 |
| $\chi_7$ | 1 | 1 | 1 | -2 | 3 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | 1 | 1 | 1 | 1 | 1 |
| $\chi_8$ | 0 | 0 | 0 | -2 | 3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_9$ | 0 | 0 | 0 | 3 | -2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{10}$ | 0 | 0 | 0 | 3 | -2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{11}$ | 0 | 0 | 0 | -2 | 3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{12}$ | 2 | 2 | 2 | -3 | 2 | 1 | 1 | 1 | 1 | 1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 |
| $\chi_{13}$ | 2 | 2 | 2 | 2 | -3 | 1 | 1 | 1 | 1 | 1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 |
| $\chi_{14}$ | -1 | -1 | -1 | 0 | 0 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 |
| $\chi_{15}$ | -1 | -1 | -1 | 0 | 0 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 |
| $\chi_{16}$ | 0 | 0 | 0 | -3 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{17}$ | 0 | 0 | 0 | 2 | -3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{18}$ | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{19}$ | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{20}$ | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{21}$ | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{22}$ | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{23}$ | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{24}$ | K | M | L | -2 | -2 | K | K | M | M | L | L | K | K | M | M | L | L |
| $\chi_{25}$ | L | K | M | -2 | -2 | L | L | K | K | M | M | L | L | K | K | M | M |
| $\chi_{26}$ | M | L | K | -2 | -2 | M | M | L | L | K | K | M | M | L | L | K | K |
| $\chi_{27}$ | -K | -M | -L | 2 | 2 | K | K | M | M | L | L | -K | -K | -M | -M | -L | -L |
| $\chi_{28}$ | -L | -K | -M | 2 | 2 | L | L | K | K | M | M | -L | -L | -K | -K | -M | -M |
| $\chi_{29}$ | -M | -L | -K | 2 | 2 | M | M | L | L | K | K | -M | -M | -L | -L | -K | -K |
| $\chi_{30}$ | -K | -M | -L | -1 | -1 | O | $\overline{O}$ | Q | $\overline{Q}$ | P | $\overline{P}$ | -O | -$\overline{O}$ | -Q | -$\overline{Q}$ | -P | -$\overline{P}$ |
| $\chi_{31}$ | -L | -K | -M | -1 | -1 | P | $\overline{P}$ | O | $\overline{O}$ | Q | $\overline{Q}$ | -P | -$\overline{P}$ | -O | -$\overline{O}$ | -Q | -$\overline{Q}$ |
| $\chi_{32}$ | -M | -L | -K | -1 | -1 | Q | $\overline{Q}$ | P | $\overline{P}$ | O | $\overline{O}$ | -Q | -$\overline{Q}$ | -P | -$\overline{P}$ | -O | -$\overline{O}$ |
| $\chi_{33}$ | -M | -L | -K | -1 | -1 | $\overline{Q}$ | Q | $\overline{P}$ | P | $\overline{O}$ | O | -$\overline{Q}$ | -Q | -$\overline{P}$ | -P | -$\overline{O}$ | -O |
| $\chi_{34}$ | -K | -M | -L | -1 | -1 | $\overline{O}$ | O | $\overline{Q}$ | Q | $\overline{P}$ | P | -$\overline{O}$ | -O | -$\overline{Q}$ | -Q | -$\overline{P}$ | -P |
| $\chi_{35}$ | -L | -K | -M | -1 | -1 | $\overline{P}$ | P | $\overline{O}$ | O | $\overline{Q}$ | Q | -$\overline{P}$ | -P | -$\overline{O}$ | -O | -$\overline{Q}$ | -Q |
| $\chi_{36}$ | K | M | L | 1 | 1 | O | $\overline{O}$ | Q | $\overline{Q}$ | P | $\overline{P}$ | O | $\overline{O}$ | Q | $\overline{Q}$ | P | $\overline{P}$ |
| $\chi_{37}$ | L | K | M | 1 | 1 | P | $\overline{P}$ | O | $\overline{O}$ | Q | $\overline{Q}$ | P | $\overline{P}$ | O | $\overline{O}$ | Q | $\overline{Q}$ |
| $\chi_{38}$ | M | L | K | 1 | 1 | Q | $\overline{Q}$ | P | $\overline{P}$ | O | $\overline{O}$ | Q | $\overline{Q}$ | P | $\overline{P}$ | O | $\overline{O}$ |
| $\chi_{39}$ | M | L | K | 1 | 1 | $\overline{Q}$ | Q | $\overline{P}$ | P | $\overline{O}$ | O | $\overline{Q}$ | Q | $\overline{P}$ | P | $\overline{O}$ | O |
| $\chi_{40}$ | K | M | L | 1 | 1 | $\overline{O}$ | O | $\overline{Q}$ | Q | $\overline{P}$ | P | $\overline{O}$ | O | $\overline{Q}$ | Q | $\overline{P}$ | P |
| $\chi_{41}$ | L | K | M | 1 | 1 | $\overline{P}$ | P | $\overline{O}$ | O | $\overline{Q}$ | Q | $\overline{P}$ | P | $\overline{O}$ | O | $\overline{Q}$ | Q |
| $\chi_{42}$ | -2 | -2 | -2 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| $\chi_{43}$ | 0 | 0 | 0 | -1 | -1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{44}$ | 0 | 0 | 0 | -2 | -2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{45}$ | 0 | 0 | 0 | 2 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{46}$ | 0 | 0 | 0 | -1 | -1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{47}$ | 0 | 0 | 0 | -1 | -1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{48}$ | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{49}$ | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{50}$ | 0 | 0 | 0 | -2 | -2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{51}$ | 0 | 0 | 0 | -2 | -2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{52}$ | 0 | 0 | 0 | -1 | -1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{53}$ | 0 | 0 | 0 | -1 | -1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{54}$ | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{55}$ | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{56}$ | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{57}$ | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{58}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{59}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{60}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{61}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{62}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{63}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{64}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{65}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{66}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{67}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{68}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{69}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{70}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

where K$=-E(13)^4 - E(13)^6 - E(13)^7 - E(13)^9$, L$=-E(13) - E(13)^5 - E(13)^8 - E(13)^{12}$, M$=-E(13)^2 - E(13)^3 - E(13)^{10} - E(13)^{11}$, O$=-E(39) - E(39)^5 - E(39)^8 - E(39)^{25}$, P$=-E(39)^2 - E(39)^{10} - E(39)^{11} - E(39)^{16}$,Q$=-E(39)^4 - E(39)^{20} - E(39)^{22} - E(39)^{32}$

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