Acceptance sampling plans for truncated lifetime tests under two-parameter Pranav distribution

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Abstract. The single acceptance sampling plans (SASP) are one of the main statistical tools in industry and production fields. Both of the customers and producers are interesting in the product, where the customers want a product of good quality with long life time and the producers want to keep the quality of the products with minimum cost and variation. In this study, it is supposed that the lifetime of the products follows the two parameters Pranav distribution (TPPD) and the mean is taken as a quality parameter. The necessary tables of the minimum sample size, operating characteristic (OC) function and the producer's risk values are obtained for various model parameters. Also, for applicability investigation of the suggested SASP based on TPPD, a real data set of failure times of 20 identical components is analyzed and used. It turns out that the new ASP gives minimum sample sizes and it is recommended for practitioners.

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Keywords: Acceptance sampling plan, two-parameter Pranav distribution, lifetime, producer's risk, truncated tests, operating characteristic function.

1. Introduction

The acceptance sampling plans are one of the most commonly used sampling methods in quality control when the product quality depends on its life time. It is used to find the optimal plan parameters as the minimum sample size and its acceptance number to save the time and cost of testing the lots within the experiment. In such life tests the final decision based on the tested units is to accept or reject the lot.

Several authors have suggested various types of acceptance sampling plans using different distributions. In the past few years, much strength is employed in the studying of acceptance sampling plans under a truncated life test. For illustration, Jose and Sivadas [17] suggested ASP for negative binomial Marshall-Olkin Rayleigh distribution; Al-Omari et al. [5,6] for under two-parameter Quasi Shanker distribution and length-biased weighted Lomax distribution, respectively. Al-Omari et al. [6] for the Akash distribution, Al-Omari et al. (2019) for two parameter quasi Lindley distribution, Singh et al. (2020) for generalized Pareto distribution, Gillariose and Tommy (2020) for extended Birnbaum-Saunders distribution, Hamurkaroglu et al. (2020) for single and double ASP for the compound Weibull-Exponential distribution, Al-Nasser et al. (2018) for the Ishita distribution, Al-Omari (2015, 2018) for generalized inverted exponential and Garima distributions respectively, Lio et al. (2010) for Burr type XII percentiles, Kaviyarasu and Fawaz (2017) for percentiles using Weibull-Poisson distribution, Gadde and Durgamamba (2021) for group ASP for size biased Lomax distribution, Chiang et al. (2018) for group ASP based on the Kumaraswamy Burr XII distribution, Aslam et al. (2009) for group ASP for gamma distribution, Rao et al. (2019) for percentiles for Type-II generalized log logistic distribution, Al-Omari and Zamanzade (2017) offered double ASP for transmuted generalized inverse Weibull distribution.Several authors have suggested various types of acceptance sampling plans using different distributions. In the past few years, much strength is employed in the studying of acceptance sampling plans under a truncated life test. For illustration, Aslam et al. (2009) for group ASP for gamma distribution, Lio et al. (2010) for Burr type XII percentiles, Jose and Sivadas [17] suggested ASP for negative binomial Marshall-Olkin Rayleigh distribution; Al-Omari (2015) for generalized inverted exponential distribution, Kaviyarasu and Fawaz (2017) for percentiles using Weibull-Poisson distribution, Al-Omari and Zamanzade (2017) offered double ASP for transmuted generalized inverse Weibull distribution, Al-Omari (2018) for Garima distribution, Al-Nasser et al. (2018) for Quasi Lindley distribution, Al-Nasser et al. (2018) for Ishita distribution, Al-Omari (2018) for Sushila distribution, Chiang et al. (2018) for group ASP based on the Kumaraswamy Burr XII distribution, Al-Nasser et al. (2018) for the Ishita distribution, Al-Omari et al. (2019) for two parameter quasi Lindley distribution, Rao et al. (2019) for percentiles for Type-II generalized log logistic distribution, Al-Omari et al. (2019) for Rama distribution, Al-Omari et al. (2020) for the Akash distribution, Singh et al. (2020) for generalized Pareto distribution, Gillariose and Tommy (2020) for extended Birnbaum-Saunders distribution, Hamurkaroglu et al. (2020) for single and double ASP for the compound Weibull-Exponential distribution, Singh et al. (2020) for generalized Pareto distribution. Al-Omari et al. (2021a, b) for two-parameter Quasi Shanker distribution and length-biased weighted Lomax distribution, respectively, Gadde and Durgamamba (2021) for group ASP for size biased Lomax distribution, Al-Nasser and ul Haq (2021) for Lomax distribution.

To the best of our knowledge this work is the first one considered the SASP based on the two-parameter Pranav distribution. In this article, the two-parameter Pranav distribution is introduced in Section 2. The proposed acceptance sampling plan with its main parameters and illustrations are given in Section 3. Section 4 deals with the tables of minimum sample sizes, OC values and the minimum ratio of true average life time as well as some illustration examples are introduced. Section 5 exhibits an application of a real data set in industry, and some conclusions and recommendations are presented in Section 6.

2. The two-parameter Pranav distribution

Shukla (2018) suggested a one parameter lifetime distribution known as the Pranav distribution (PD) with probability density function (pdf) given by

$$f_{PD}(x) = \frac{\theta^4}{6+\theta^4}(\theta + x^3)e^{-\theta x}, \quad x > 0,\ \theta > 0, \tag{1}$$

and cumulative distribution function (cdf) defined by

$$F_{PD}(x) = 1 - \left(1 + \frac{6\theta x + 3\theta^2\ x^2 + \theta^3 x^3}{6+\theta^4}\right)e^{-\theta x}, \quad x > 0,\ \theta > 0. \tag{2}$$

As a modification of the PD, Umeh and Ibenegbu (2019) proposed a new distribution of two parameters called as a two-parameter Pranav distribution (TPPD) with probability density function (pdf) defined as

$$f_{TPPD}(x) = \frac{\theta^4}{6+\alpha\theta^4}(\alpha\theta + x^3)e^{-\theta x}, \quad x > 0,\ \alpha > 0,\ \theta > 0. \tag{3}$$

Figure 1 shows the pdf of the TPPD plots for some selections of model parameters.

The corresponding cumulative distribution function (cdf) of (3) is

$$F_{TPPD}(x) = 1 - \left(1 + \frac{6\theta x + 3\theta^2\ x^2 + \theta^4 x^4}{6+\alpha\theta^4}\right)e^{-\theta x},\ x > 0,\ \alpha > 0,\ \theta > 0. \tag{4}$$

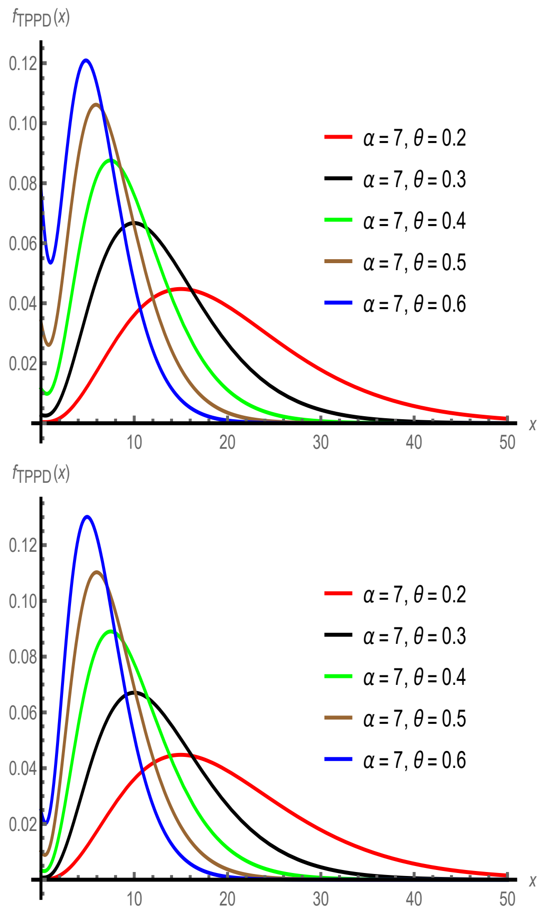

Figure 1: The TPPD pdf plots for some model parameters

The additional parameters to the base PD makes the TPPD more flexible and applicable model more than the Ishita, Akash, Pranav, Shanker, Lindley, Sujatha, and exponential distributions. The flexibility of the TPP distribution is due that is a mixture of two well-known distributions, which are exponential (θ) and gamma $(4, \theta)$ with a mixture factor $A = \frac{\alpha\theta^4}{\alpha\theta^4+6}$. The survival function of the TPPD is given by

$$S_{TPPD}(x) = 1 - F_{TPPD}(x) = \left(1 + \frac{6\theta x + 3\theta^2\ x^2 + \theta^4 x^4}{6 + \alpha\theta^4}\right) e^{-\theta x}, \tag{5}$$

$$x > 0,\ \alpha > 0,\ \theta > 0.$$

Figure 2 presents the survival function of the TPPD for some selected parameters. It can be seen that the survival function plots are decreasing for large values of X.

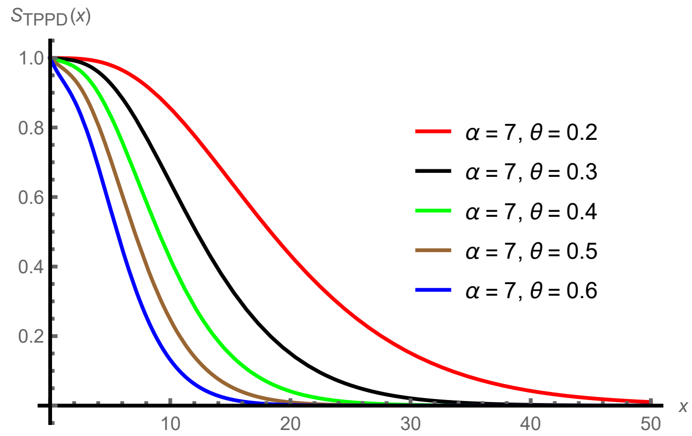

Figure 2: The survival function of the TPPD for some model parameters

The mean, hazard rate and mean residual life functions of the TPPD, respectively, are defined as

$$E(X) = \frac{\alpha\theta^4 + 24}{\theta(\alpha\theta^4 + 6)},$$

$$h_{TPPD}(x) = \frac{f_{TPPD}(x)}{1 - F_{TPPD}(x)} = \frac{\theta^4(\alpha\theta + x^3)}{\theta^3 x^3 + 3\theta^2 x^2 + 6\theta x + \alpha\theta^4 + 6}, \tag{6}$$

and

$$\begin{aligned} m_{TPPD}(x) &= \frac{1}{1 - F_{TPPD}(x)} \int_x^\infty [1 - F_{TPPD}(x)]\, dv \\ &= \frac{\theta^3 x^3 + 6\theta^2 x^2 + 18\theta x + \alpha\theta^4 + 24}{\theta(\theta^3 x^3 + 3\theta^2 x^2 + 6\theta x + \alpha\theta^4 + 6)}. \end{aligned} \tag{7}$$

Note that, $f(0) = h(0) = \frac{\alpha\theta^5}{\alpha\theta^4+6}$ and $m(0) = E(X) = \frac{\alpha\theta^4+6}{\alpha\theta^4+24}$. The rth moment and coefficient of variation (C.V) of the TPPD are

$$\mu^r = \frac{r!(\alpha\theta^4 + (r+1)(r+2)(r+3))}{\theta^r(6 + \alpha\theta^4)}, \quad r = 1, 2, \ldots$$

and

$$CV = \frac{\alpha^2\theta^8 + 84\alpha\theta^4 + 144}{\theta(24 + \alpha\theta^4)(6 + \alpha\theta^4)}.$$

3. Designing the SASP

In this section, a new SASP is developed supposing that the lifetime distribution of the products follows the TPPD. A produced lot is considered good if the true mean life time of items, say μ, is not less than a identified value μ_0. And the lot is not good if $\mu < \mu_0$. The test terminates at a pre-specified time t, while the failures number detected on the time interval given by $[0, t]$ are determined. The decision to accept the determined mean depends on the number of failures at the final of the time t that doesn't exceeds the acceptance number c. It is assumed that the lot is large enough so to that the mathematical theory of the binomial distribution can be employed. The rejection of acceptance of the product are same to the rejection or acceptance of the hypothesis $H_0 : \mu \geq \mu_0$. A SASP $(n, c, t/\mu_0)$ consists of (1). The number of items to be tested, say n, (2) the acceptance number c, and (3) the ratio $t/\mu_0 \longrightarrow t$, where μ_0 is the indicated mean lifetime and t is the pre-identified testing time. The producer's risk which is known as the probability of acceptance lot classified as a bad is fixed to be at most $1-p^*$, where p^* is the confidence level in the direction that the probability of rejecting a lot with a mean $\mu < \mu_0$ is p^* at least. At this stage, the researcher want to obtain the minimum sample size (MSS), n holding the inequality

$$\sum_{i=0}^{c} \binom{n}{i} p^i (1-p)^{n-i} \leq 1 - p^*, \tag{8}$$

where $p = F(t, \mu_0)$ is the probability of a failure occurring in time t when the true mean life is μ_0. It depends simply on t/μ_0 and this function is a monotonically increasing in the ratio. Therefore, the experiment requires to determine this ratio. If the number of failures detected is at most equal to c, then from (8) we can assert with probability level p^* that $F(t; \mu) \leq F(t; \mu_0)$, that implies $\mu \geq \mu_0$. Hence, the mean life of the units can be asserted to be at least equal to their determined value with predetermined probability p^*. The minimum values of the sample size favorable (5) are obtained and presented in Table 1 for p^*=0.75, 0.90, 0.95, 0.99, t/μ_0=0.628, 0.942, 1.257, 1.571, 2.356, 3.141, 3.927, 4.712 and $c = 0, 1, 2, ..., 10$ when $\alpha = 83.7$ and $\theta = 0.092$.

The operating characteristic function (OCF) is very important in SASP where it determine the effectiveness of a statistical hypothesis test structured to reject or accept a lot. The OCF of any sampling plan, say $(n, c, t/\mu_0)$ gives the probability of accepting the lot and it is defined as

$$L(P) = \sum_{i=0}^{c} \binom{n}{i} p^i (1-p)^{n-i}, \tag{9}$$

where $p = F(t; \mu)$ is a function of lot quality parameter μ. The OCF is an increasing function in μ; a decreasing function of p while p is a decreasing function of μ. Now, for given probability p^* and ratio t/μ_0, the selection of the MSS n with acceptance number c based on the OCF values. The OCF values for the proposed SASP are presented in Table 2 for $\alpha = 83.7$ and $\theta = 0.092$.

The producer's risk (PR) is the probability of rejecting a lot with $\mu > \mu_0$. For the SASP under investigation and a fixed value of the PR η, the researchers are involved in determining the value of μ/μ_0 that will emphasize the PR is less than or equal to η. Therefore, the probability function is found as

$$p = F\left(\frac{t}{\mu_0}\frac{\mu_0}{\mu}\right). \tag{10}$$

Therefore, μ/μ_0 is the lowest positive number for which p fulfills the inequality

$$\sum_{i=0}^{c}\binom{n}{i}p^i(1-p)^{n-i} \leq \eta. \tag{11}$$

For $\alpha = 83.7$ and $\theta = 0.092$ with a given p^*, the smallest values of the ratio μ/μ_0 satisfying the last inequality (9) are provided in Table 3.

4. Description of tables

Assuming that the life distribution follows the TPPD and Table 1 displayed the MSS needed to assert that μ is greater than μ_0 with probability at least p^* with c as an acceptance number. For illustration, when $p^* = 0.95$, $c = 2$ and $t/\mu_0 = 0.942$, the corresponding entry table is $n = 10$. Hence, if out of the 10 items, less than or equal to two fail before time t, then the decision is the lot can be accepted with a probability of 0.95. This means that out of the 10 items, if there are two items fail previous the time t, then a 95% upper confidence interval for μ is $(t/0.942, \infty)$. Table 2 devoted to the OCF values for the suggested ASP, and for the plan $(n = 10, c = 2, t/\mu_0 = 0.942)$ with $p^* = 0.95$ the OCF values are:

μ/μ_0	2	4	6	8	10	12
$L(P)$	0.884749	0.999532	0.999990	0.999999	1	1
PR	0.115251	0.000468	0.0001	0.000001	0	0

From this OCF values, it is found that if the real mean life time is twice the identified mean life, then the producer's risk is approximately 0.115251 and zero for big values of μ/μ_0.
Table 3 includes the values of the minimum ratio of the true average life to the identified mean lifetime (μ/μ_0) for different choices of c and t/μ_0 provided that the producer's risk not than 0.05. Hence, for the $(n = 10, c = 2, t/\mu_0 = 0.942)$, the value of μ/μ_0 is 2.276. This displays that the product must have a mean life of 2.276 times the determined mean life 1000 hours accept the lot with probability of at least 0.90.

5. Application of real data

We take a dataset that is already investigated by Murthy (2004). The dataset represents the failure times of 20 identical components. The observations are:

15.32, 8.29, 8.09, 11.89, 11.03, 10.54, 4.51, 1.79, 7.93, 6.29, 5.46, 2.87, 11.12, 11.23, 3.58, 9.74, 8.45, 2.99, 3.14, 1.80.

Figure 3 displays the fitted density and cdf for the dataset. Figure 4 provides the Total test time (TTT) curve and box plot of the estimates for the data set based on the TPPD, for more details about the TTT see Aarset (1987).

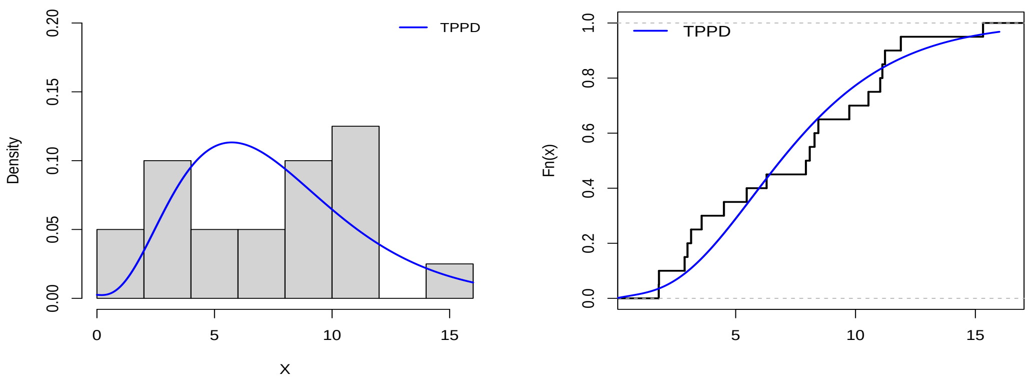

Figure 3: Fitted pdf for failure times of 20 data and the estimated cumulative distribution function for the TPPD

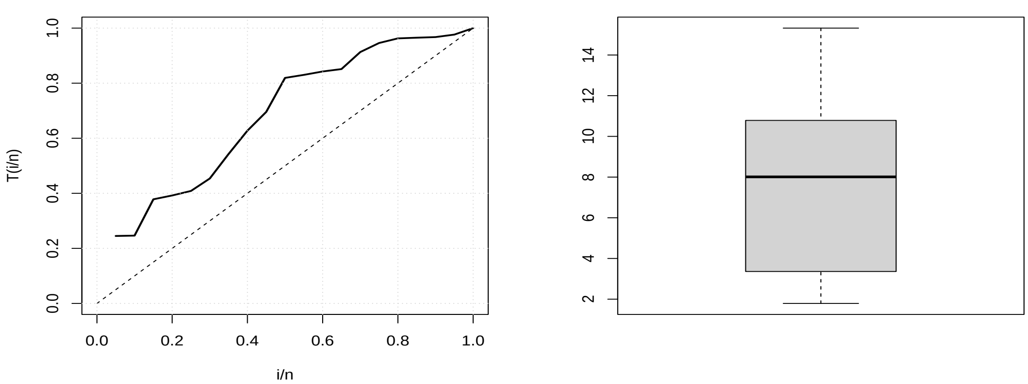

Figure 4: TTT curve and box plot of the estimates for the data set based on the TPPD

First we test whether the TPPD can be used or not. The maximum likelihood estimation method (MLE) is used to estimatethe unknown TPPD parameters. The following criteria consist of the Akaike Information criterion (AIC), consistent Akaike information criterion (CAIC), Hannan-Quinn information criterion (HQIC), Bayesian information criterion (BIC), are presented. Also, the Kolmogorov-Smirnov (KS), Anderson-Darling (A) and Cramer-von-Mises (W) are obtained and the results are presented in Table 4.

Table 4: Fitting criteria values for the real dataset

Model	W	A	AIC	CAIC	BIC	HQIC	Statistic	p-value
TPPD	0.09219	0.56157	115.7915	116.4974	117.7829	116.1802	0.15758	0.647

The KS is the distance between the fitted and observed distribution functions is 0.15758 with p-value of 0.647. Thus, the TPPD showed a very good fit. For this data, it is found that the MLEs of the distribution parameters are $\hat{\theta} = 0.5226901$, $\hat{\alpha} = 2.8294078$ and hence $\hat{E}(X) = \frac{(\hat{\alpha}\hat{\theta}^4+24)}{\hat{\theta}(\hat{\alpha}\hat{\theta}^4+6)} = 7.45757$.
Assume that the specified mean lifetime is $\mu_0 = 7.45757$ and the time test is $t_0 = 4.6834$. Then, from Table 6 with $p^* = 0.75$, the acceptance sampling plan is $(n = 20, c = 8, t/\mu_0 = 0.628)$. Thus, for the suggested SASP if more than 8 failures obtained before the time 4.6834 the lot is rejected. Since there are only 7 failures $(4.51, 1.79, 2.87, 3.58, 2.99, 3.14, 1.80)$ before 4.6834, then we accept the lot.

6. Conclusions

In this paper, a new truncated life single acceptance sampling plan has been introduced when the life time of the test units follows the TPP distribution. The required tables are presented for the minimum sample size, operating characteristic function values and the minimum ratio for the suggested sampling plan. An application of a real data for the suggested SASP is presented using the failure times of 20 identical components data and can be employed excellently in analyzing the data. The TTPD might entices various applications in reliability and one can use it for other types of ASP or by using the ranked set sampling methods (Haq et al. 2013, 2014a,b).

Table 1: MSS for a given μ_0 with p^* for c with $\alpha = 83.7, \theta = 0.092$ in the TPPD

		t/μ_0							
p^*	c	0.628	0.942	1.257	1.571	2.356	3.141	3.927	4.712
0.75	0	5	2	2	1	1	1	1	1
	1	11	5	3	2	2	2	2	2
	2	15	7	5	4	3	3	3	3
	3	20	9	6	5	4	4	4	4
	4	25	11	8	6	5	5	5	5
	5	29	13	9	7	6	6	6	6
	6	34	16	11	9	7	7	7	7
	7	39	18	12	10	8	8	8	8
	8	43	20	13	11	9	9	9	9
	9	48	22	15	12	10	10	10	10
	10	52	24	16	13	11	11	11	11
0.90	0	9	4	2	2	1	1	1	1
	1	15	6	4	3	2	2	2	2
	2	20	9	6	4	3	3	3	3
	3	26	11	7	6	4	4	4	4
	4	31	14	9	7	5	5	5	5
	5	36	16	10	8	6	6	6	6
	6	41	18	12	9	8	7	7	7
	7	46	20	13	11	9	8	8	8
	8	51	23	15	12	10	9	9	9
	9	56	25	16	13	11	10	10	10
	10	60	27	18	14	12	11	11	11
0.95	0	11	5	3	2	1	1	1	1
	1	18	8	5	3	2	2	2	2
	2	24	10	6	5	3	3	3	3
	3	30	13	8	6	5	4	4	4
	4	35	15	10	7	6	5	5	5
	5	40	18	11	9	7	6	6	6
	6	46	20	13	10	8	7	7	7
	7	51	22	14	11	9	8	8	8
	8	56	25	16	13	10	9	9	9
	9	61	27	18	14	11	10	10	10
	10	66	29	19	15	12	11	11	11
0.99	0	17	7	4	3	2	1	1	1
	1	25	10	6	4	3	2	2	2
	2	31	13	8	6	4	3	3	3
	3	38	16	10	7	5	4	4	4
	4	44	19	11	9	6	5	5	5
	5	50	21	13	10	7	6	6	6
	6	55	24	15	11	8	8	7	7
	7	61	26	17	13	9	9	8	8
	8	66	29	18	14	11	10	9	9
	9	72	31	20	15	12	11	10	10
	10	77	33	21	17	13	12	11	11

Table 2: OCF values of the sampling plan $(n, c = 2, t/\mu_0)$ with $\alpha = 83.7, \theta = 0.092$ in the TPPD

			μ/μ_0					
p^*	m	t/μ_0	2	4	6	8	10	12
0.75	15	0.628	0.980479	0.999963	0.999999	1	1	1
	7	0.942	0.955535	0.999859	0.999997	1	1	1
	5	1.257	0.900978	0.999419	0.999986	0.999999	1	1
	4	1.571	0.838090	0.998381	0.999955	0.999997	1	1
	3	2.356	0.668703	0.990410	0.999572	0.999966	0.999996	0.999999
	3	3.141	0.336935	0.943213	0.995854	0.999572	0.999939	0.999989
	3	3.927	0.134187	0.831819	0.980719	0.997439	0.999571	0.999911
	3	4.712	0.046533	0.668703	0.943180	0.990410	0.998130	0.999572
0.90	20	0.628	0.957714	0.999909	0.999998	1	1	1
	9	0.942	0.911507	0.999669	0.999993	1	1	1
	6	1.257	0.837326	0.998873	0.999973	0.999998	1	1
	4	1.571	0.838090	0.998381	0.999955	0.999997	1	1
	3	2.356	0.668703	0.990410	0.999572	0.999966	0.999996	0.999999
	3	3.141	0.336935	0.943213	0.995854	0.999572	0.999939	0.999989
	3	3.927	0.134187	0.831819	0.980719	0.997439	0.999571	0.999911
	3	4.712	0.046533	0.668703	0.943180	0.990410	0.998130	0.999572
0.95	24	0.628	0.933093	0.999840	0.999996	1	1	1
	10	0.942	0.884749	0.999532	0.999990	0.999999	1	1
	6	1.257	0.837326	0.998873	0.999973	0.999998	1	1
	5	1.571	0.708825	0.996180	0.999888	0.999993	0.999999	1
	3	2.356	0.668703	0.990410	0.999572	0.999966	0.999996	0.999999
	3	3.141	0.336935	0.943213	0.995854	0.999572	0.999939	0.999989
	3	3.927	0.134187	0.831819	0.980719	0.997439	0.999571	0.999911
	3	4.712	0.046533	0.668703	0.943180	0.990410	0.998130	0.999572
0.99	31	0.628	0.878208	0.999652	0.999991	0.999999	1	1
	13	0.942	0.790899	0.998925	0.999977	0.999998	1	1
	8	1.257	0.689250	0.997026	0.999926	0.999995	0.999999	1
	6	1.571	0.576240	0.992788	0.999781	0.999986	0.999998	1
	4	2.356	0.362531	0.967752	0.998384	0.999866	0.999983	0.999997
	3	3.141	0.336935	0.943213	0.995854	0.999572	0.999939	0.999989
	3	3.927	0.134187	0.831819	0.980719	0.997439	0.999571	0.999911
	3	4.712	0.046533	0.668703	0.943180	0.990410	0.998130	0.999572

Table 3: Minimum ratio of the true mean life to the specified mean lifetime for suitability of a lot with PR of 0.05 and $\alpha = 83.7, \theta = 0.092$

		t/μ_0							
p^*	c	0.628	0.942	1.257	1.571	2.356	3.141	3.927	4.712
0.75	0	3.081	3.469	4.629	4.618	6.925	9.232	11.543	13.85
	1	2.115	2.388	2.572	2.608	3.911	5.215	6.519	7.822
	2	1.767	1.966	2.244	2.487	3.071	4.094	5.118	6.141
	3	1.631	1.764	1.910	2.128	2.659	3.545	4.432	5.318
	4	1.550	1.644	1.853	1.909	2.409	3.211	4.014	4.817
	5	1.476	1.563	1.697	1.758	2.237	2.982	3.728	4.473
6	1.439	1.555	1.684	1.827	2.110	2.813	3.517	4.220	
	7	1.410	1.504	1.587	1.727	2.012	2.682	3.354	4.024
	8	1.375	1.464	1.508	1.647	1.933	2.577	3.222	3.866
	9	1.357	1.432	1.518	1.581	1.868	2.490	3.113	3.736
10	1.332	1.405	1.460	1.525	1.813	2.417	3.022	3.625	
0.90	0	3.694	4.313	4.629	5.785	6.925	9.232	11.543	13.85
	1	2.346	2.559	2.914	3.214	3.911	5.215	6.519	7.822
	2	1.956	2.183	2.449	2.487	3.071	4.094	5.118	6.141
	3	1.795	1.929	2.080	2.387	2.659	3.545	4.432	5.318
	4	1.679	1.837	1.980	2.132	2.409	3.211	4.014	4.817
	5	1.601	1.726	1.810	1.957	2.237	2.982	3.728	4.473
	6	1.545	1.646	1.776	1.827	2.471	2.813	3.517	4.220
	7	1.503	1.585	1.672	1.864	2.343	2.682	3.354	4.024
	8	1.469	1.570	1.660	1.774	2.240	2.577	3.222	3.866
	9	1.442	1.528	1.586	1.700	2.155	2.490	3.113	3.736
	10	1.410	1.493	1.584	1.637	2.084	2.417	3.022	3.625
0.95	0	3.930	4.621	5.261	5.785	6.925	9.232	11.543	13.85
	1	2.490	2.841	3.186	3.214	3.911	5.215	6.519	7.822
	2	2.082	2.276	2.449	2.804	3.071	4.094	5.118	6.141
	3	1.888	2.069	2.225	2.387	3.191	3.545	4.432	5.318
	4	1.754	1.893	2.092	2.132	2.862	3.211	4.014	4.817
5	1.665	1.820	1.911	2.120	2.637	2.982	3.728	4.473	
	6	1.613	1.728	1.860	1.976	2.471	2.813	3.517	4.220
	7	1.562	1.658	1.749	1.864	2.343	2.682	3.354	4.024
	8	1.522	1.634	1.727	1.885	2.240	2.577	3.222	3.866
	9	1.490	1.586	1.708	1.804	2.155	2.490	3.113	3.736
	10	1.463	1.547	1.639	1.736	2.084	2.417	3.022	3.625
0.99	0	4.503	5.127	5.755	6.575	8.676	9.232	11.543	13.85
	1	2.767	3.071	3.414	3.642	4.820	5.215	6.519	7.822
	2	2.268	2.515	2.776	3.060	3.729	4.094	5.118	6.141
	3	2.047	2.248	2.469	2.600	3.191	3.545	4.432	5.318
	4	1.901	2.089	2.193	2.474	2.862	3.211	4.014	4.817
	5	1.803	1.944	2.086	2.262	2.637	2.982	3.728	4.473
	6	1.721	1.872	2.008	2.105	2.471	3.294	3.517	4.220
	7	1.669	1.788	1.949	2.089	2.343	3.123	3.354	4.024
	8	1.618	1.748	1.847	1.984	2.469	2.986	3.222	3.866
	9	1.585	1.691	1.815	1.897	2.370	2.873	3.113	3.736
	10	1.551	1.644	1.741	1.905	2.286	2.778	3.022	3.625

Table 5: MSS of the sampling plans with $\theta = 0.13$ for the real data

		t/μ_0							
p^*	c	0.628	0.942	1.257	1.571	2.356	3.141	3.927	4.712
0.75	0	2	2	2	1	1	1	1	1
	1	5	4	3	3	2	2	2	2
	2	7	6	5	4	4	3	3	3
	3	9	7	6	6	5	4	4	4
	4	12	9	8	7	6	6	5	5
	5	14	11	9	8	7	7	6	6
	6	16	12	11	10	8	8	7	7
	7	18	14	12	11	10	9	8	8
	8	20	16	14	12	11	10	9	9
	9	22	17	15	14	12	11	11	10
	10	24	19	16	15	13	12	12	11
0.90	0	4	3	2	2	2	1	1	1
	1	7	5	4	4	3	3	2	2
	2	9	7	6	5	4	4	3	3
	3	12	9	7	7	5	5	5	4
	4	14	11	9	8	7	6	6	5
	5	16	12	11	9	8	7	7	6
	6	19	14	12	11	9	8	8	7
	7	21	16	14	12	10	9	9	9
	8	23	18	15	14	12	11	10	10
	9	25	20	17	15	13	12	11	11
	10	28	21	18	16	14	13	12	12
0.95	0	5	3	3	2	2	2	1	1
	1	8	6	5	4	3	3	3	2
	2	10	8	6	6	5	4	4	3
	3	13	10	8	7	6	5	5	5
	4	16	12	10	9	7	6	6	6
	5	18	14	11	10	9	8	7	7
	6	20	15	13	12	10	9	8	8
	7	23	17	15	13	11	10	9	9
	8	25	19	16	15	12	11	10	10
	9	27	21	18	16	13	12	11	11
	10	30	23	19	17	15	13	12	12
0.99	0	7	5	4	3	3	2	2	2
	1	10	8	6	5	4	4	3	3
	2	13	10	8	7	6	5	4	4
	3	16	12	10	9	7	6	5	5
	4	19	14	12	10	8	7	7	6
	5	22	16	13	12	10	8	8	7
	6	24	18	15	13	11	10	9	8
	7	27	20	17	15	12	11	10	9
	8	29	22	18	16	14	12	11	10
	9	32	24	20	18	15	13	12	12
	10	34	26	22	19	16	14	13	13

Table 6: OCF values of the sampling plan $(n, c = 2, t/\mu_0)$ with $\theta = 0.13$ for the real data

			μ/μ_0					
p^*	m	t/μ_0	2	4	6	8	10	12
0.75	10	0.628	0.621215	0.894689	0.958488	0.97973	0.988658	0.993033
	7 0.942	0.499465	0.836512	0.931149	0.965168	0.980076	0.987577	
	5	1.257	0.480227	0.821326	0.922917	0.960475	0.977198	0.985698
	4	1.571	0.555292	0.853886	0.937922	0.968412	0.981861	0.988658
	3	2.356	0.330815	0.702311	0.853947	0.919411	0.951274	0.968429
	3	3.141	0.503321	0.802751	0.906758	0.949491	0.969798	0.980575
	3	3.927	0.389740	0.720743	0.856556	0.918268	0.949470	0.966729
	3	4.712	0.298140	0.642183	0.802717	0.882333	0.924960	0.949480
0.90	13	0.628	0.432917	0.805974	0.916977	0.957664	0.975668	0.984780
	8	0.942	0.373656	0.764232	0.894689	0.945013	0.967924	0.979730
	6	1.257	0.324185	0.722659	0.870911	0.931024	0.959169	0.973936
	5	1.571	0.348608	0.733419	0.875575	0.933365	0.960491	0.974748
	4	2.356	0.330815	0.702311	0.853947	0.919411	0.951274	0.968429
	3	3.141	0.193303	0.555466	0.753876	0.853977	0.907408	0.937967
	3	3.927	0.389740	0.720743	0.856556	0.918268	0.949470	0.966729
	3	4.712	0.298140	0.642183	0.802717	0.882333	0.924960	0.94948
0.95	15	0.628	0.353110	0.756532	0.891615	0.943547	0.967125	0.979251
	10	0.942	0.271793	0.687954	0.852489	0.920566	0.952760	0.969751
	7	1.257	0.324185	0.722659	0.870911	0.931024	0.959169	0.973936
	6	1.571	0.204691	0.606875	0.799603	0.887273	0.931049	0.954978
	4	2.356	0.150183	0.518697	0.733514	0.842380	0.900347	0.933397
	3	3.141	0.193303	0.555466	0.753876	0.853977	0.907408	0.937967
	3	3.927	0.111852	0.430568	0.651491	0.779480	0.853923	0.899019
	3	4.712	0.298140	0.642183	0.802717	0.882333	0.924960	0.949480
0.99	19	0.628	0.178801	0.602600	0.801994	0.890316	0.933615	0.956982
	12	0.942	0.134838	0.536994	0.756532	0.860865	0.914086	0.943547
	9	1.257	0.131122	0.522558	0.744357	0.852250	0.908078	0.939277
	7	1.571	0.114501	0.487240	0.716287	0.832715	0.894544	0.929686
	5	2.356	0.062859	0.362634	0.606994	0.751976	0.836375	0.887323
	4	3.141	0.064298	0.348790	0.586764	0.733561	0.821488	0.875656
	3	3.927	0.111852	0.430568	0.651491	0.779480	0.853923	0.899019
	3	4.712	0.063767	0.330815	0.555408	0.702311	0.794793	0.853947

Table 7: Minimum ratio of the true mean life to the specified mean lifetime for suitability of a lot with PR of 0.05 and $\theta = 0.13$ for the real data

		t/μ_0							
p^*	c	0.628	0.942	1.257	1.571	2.356	3.141	3.927	4.712
0.75	0	30.161	45.242	60.37	37.636	56.442	75.248	94.078	112.883
	1	9.679	11.215	10.525	13.154	11.209	14.943	18.683	22.417
	2	5.553	6.886	7.248	6.595	9.890	8.037	10.048	12.056
	3	4.121	4.442	4.748	5.934	6.636	5.634	7.044	8.452
	4	3.808	3.903	4.390	4.448	5.062	6.748	5.537	6.644
	5	3.285	3.560	3.509	3.589	4.140	5.519	4.630	5.556
	6	2.941	2.954	3.445	3.677	3.535	4.713	4.023	4.827
	7	2.696	2.841	2.968	3.183	3.962	4.142	3.587	4.304
	8	2.514	2.749	2.973	2.820	3.528	3.717	3.258	3.909
	9	2.373	2.450	2.661	2.938	3.195	3.387	4.234	3.600
	10	2.260	2.413	2.416	2.674	2.929	3.123	3.904	3.351
0.90	0	60.392	67.915	60.37	75.45	113.151	75.248	94.078	112.883
	1	14.064	14.518	14.966	18.704	19.727	26.299	18.683	22.417
	2	7.466	8.329	9.188	9.058	9.890	13.185	10.048	12.056
	3	5.843	6.182	5.927	7.408	6.636	8.847	11.061	8.452
	4	4.605	5.112	5.208	5.487	6.670	6.748	8.437	6.644
	5	3.887	4.018	4.750	4.386	5.382	5.519	6.900	5.556
	6	3.660	3.686	3.941	4.306	4.547	4.713	5.892	4.827
	7	3.291	3.446	3.790	3.710	3.962	4.142	5.179	6.214
	8	3.019	3.263	3.323	3.715	4.229	4.704	4.647	5.575
	9	2.810	3.118	3.269	3.326	3.811	4.259	4.234	5.080
	10	2.773	2.806	2.955	3.019	3.480	3.905	3.904	4.684
0.95	0	75.508	67.915	90.625	75.45	113.151	150.852	94.078	112.883
	1	16.252	17.810	19.373	18.704	19.727	26.299	32.88	22.417
	2	8.420	9.766	9.188	11.483	13.584	13.185	16.484	12.056
	3	6.414	7.045	7.092	7.408	8.900	8.847	11.061	13.272
	4	5.399	5.712	6.017	6.508	6.670	6.748	8.437	10.123
	5	4.487	4.927	4.750	5.166	6.577	7.175	6.900	8.279
	6	3.899	4.049	4.432	4.925	5.514	6.062	5.892	7.070
	7	3.686	3.746	4.195	4.226	4.774	5.281	5.179	6.214
	8	3.354	3.517	3.669	4.152	4.229	4.704	4.647	5.575
	9	3.101	3.339	3.568	3.708	3.811	4.259	4.234	5.080
	10	3.028	3.196	3.220	3.358	4.010	3.905	3.904	4.684
0.99	0	105.739	113.261	120.88	113.263	169.859	150.852	188.601	226.302
	1	20.625	24.378	23.766	24.212	28.050	37.395	32.880	39.453
	2	11.276	12.63	13.031	13.890	17.221	18.110	16.484	19.78
	3	8.125	8.764	9.400	10.309	11.109	11.865	11.061	13.272
	4	6.587	6.908	7.623	7.520	8.228	8.892	11.117	10.123
	5	5.683	5.831	5.970	6.701	7.748	7.175	8.971	8.279
	6	4.851	5.131	5.403	5.538	6.457	7.351	7.579	7.070
	7	4.472	4.640	4.998	5.243	5.563	6.364	6.603	6.214
	8	4.022	4.277	4.353	4.585	5.572	5.638	5.881	5.575
	9	3.824	3.997	4.161	4.459	4.988	5.081	5.325	6.389
	10	3.537	3.775	4.005	4.024	4.528	4.639	4.882	5.858

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