A nonmonotone damped Gauss-Newton method for nonlinear complementarity problems

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Abstract. The damped Gauss-Newton methods have been successfully applied to solve the nonlinear complementarity problem (NCP). This class of methods is usually designed based on a monotone Armijo line search. In this paper, we propose a damped Gauss-Newton method with a nonmonotone line search to solve the NCP. Without requiring any problem assumptions, we prove that the proposed method is well defined and it is globally convergent. Moreover, under the nonsingularity assumption, we show that the proposed method is locally superlinearly/quadratically convergent. Some numerical results are reported.

Keywords: nonlinear complementarity problem, Gauss-Newton method, nonmonotone line search, quadratic convergence.

1. Introduction

The nonlinear complementarity problem (NCP) is to find $x \in \mathcal{R}^n$ such that

$$x \geq 0,\ F(x) \geq 0,\ x^T F(x) = 0, \tag{1}$$

where $F : \mathcal{R}^n \to \mathcal{R}^n$ is a continuously differentiable function. The NCP has been studied extensively due to its various applications in operations research, economic equilibrium and engineering design.

There has been developed a number of numerical algorithms for solving the NCP. Among them, the Newton-type algorithm is one kind of the most effective algorithms which is designed based on some equation reformulation of the NCP. One class of well-known Newton-type algorithms is the smoothing Newton methods (e.g., [2, 3, 5, 6, 11, 13]). This class of algorithms usually reformulates the NCP as a smooth nonlinear equation and then solves it by Newton method. It is worth pointing out that, in these smoothing Newton methods, to ensure Newton step be feasible, one usually requires that the function F has Cartesian P_0-property, that is, for every x and y in $\mathcal{R}^n$ with $x \neq y$, there is an index $i_0 \in \{1, ..., n\}$ such that $x_{i_0} \neq y_{i_0}$ and $(x_{i_0} - y_{i_0})(F_{i_0}(x) - F_{i_0}(y)) \geq 0$.

Another class of Newton-type algorithms is the damped Gauss-Newton methods (e.g, [4, 9, 10]). Different from smoothing Newton methods, the damped

Gauss-Newton methods usually reformulate the NCP as a nonsmooth nonlinear equation and then solve it. Since the Gauss-Newton equation is always solvable, the damped Gauss-Newton method is well defined without requiring that the function F has Cartesian P_0-property. It is worth pointing out that, in many damped Gauss-Newton methods (e.g., [8, 10]), the nonmonotone line search technique has been used to improve numerical results when the methods are implemented. However, the theoretical analyses are based on the methods with some monotone line search. As is well known, the nonmonotone line search technique can improve the likelihood of finding a global optimal solution and convergence speed in cases where the involving function is highly nonconvex and has a valley in a small neighbourhood of some point (e.g., [1, 14]). Encouraging numerical results have been reported when smoothing Newton methods with nonmonotone line search schemes were applied to solve NCPs (e.g., [2, 7, 11]).

In this paper, we propose a damped Gauss-Newton method to solve the NCP which is designed based on a nonmonotone line search scheme. We prove that the proposed method is well defined and it is globally convergent without requiring any problem assumptions. Moreover, we show that the convergence rate of the proposed method is local superlinear/quadratic under the nonsingularity assumption. We also report some numerical results which indicate that our method is very effective for solving NCPs even though these problems have no Cartesian P_0-property.

2. A nonmonotone damped Gauss-Newton method

2.1 The reformulation of the NCP

In this paper, we consider the following Fischer-Burmeister function:

$$\phi(a,b) := \sqrt{a^2+b^2} - (a+b), \quad \forall (a,b) \in \mathcal{R}^2, \tag{2}$$

which satisfies

$$\phi(a,b) = 0 \Longleftrightarrow a \geq 0,\ b \geq 0,\ ab = 0. \tag{3}$$

By using ϕ, we can reformulate the NCP as the following nonsmooth equation:

$$H(x) := \begin{pmatrix} \phi(x_1, F_1(x)) \\ \vdots \\ \phi(x_n, F_n(x)) \end{pmatrix} = 0. \tag{4}$$

Obviously, x is a solution of the NCP if and only if $H(x) = 0$.

Define the merit function $\psi(x) : \mathcal{R}^n \to \mathcal{R}$ as

$$\psi(x) := \frac{1}{2}\|H(x)\|^2 = \frac{1}{2}\sum_{i=1}^{n}(\phi(x_i, F_i(x)))^2. \tag{5}$$

The following lemma gives some useful properties which can be found in [4].

Lemma 2.1. (a) *$H(x)$ defined in (4) is semismooth on $\mathcal{R}^n$ and it is strongly semismooth on $\mathcal{R}^n$ if $F'(x)$ is Lipschitz continuous on $\mathcal{R}^n$.*

(b) *For any $x \in \mathcal{R}^n$ and $V \in \partial H(x)$, V can be represented as follows*

$$V = \mathrm{diag}(a_i)\nabla F(x)^T + \mathrm{diag}(b_i),$$

where $\mathrm{diag}(\alpha_i)$ *denotes a diagonal matrix with the diagonal elements $\alpha_1, ..., \alpha_n$ and $(a_i + 1)^2 + (b_i + 1)^2 \leq 1,\ i = 1, ..., n$.*

(c) *$\psi(x)$ defined by (5) is continuously differentiable on $\mathcal{R}^n$ and its gradient $\nabla\psi(x)$ can be represented as $\nabla\psi(x) = V^T H(x)$ for any $V \in \partial H(x)$.*

2.2 The algorithm

We now describe our nonmonotone damped Gauss-Newton method (NDGNM) as follows.

Algorithm NDGNM

Step 1. Choose $\gamma \in (0, 1/2), \eta \in (0, 1)$ and $x^0 \in \mathcal{R}^n$. Choose a sequence $\{\mu_k\}$ such that $\mu_k > 0$ for all $k \geq 0$. Choose a sequence $\{\tau_k\}$ such that $\tau_k \in (\tau, 1]$ where $\tau > 0$ is a constant. Set $R_0 := \psi(x^0)$. Set $k := 0$.

Step 2. Choose $V_k \in \partial H(x^k)$ and compute $\nabla\psi(x^k) = V_k^T H(x^k)$. If $\nabla\psi(x^k) = 0$, then stop.

Step 3. Let d_k be the solution of the following linear system

$$(V_k^T V_k + \mu_k I)d = -\nabla\psi(x^k). \tag{6}$$

Step 4. Find a step-size $\lambda_k := \eta^{m_k}$, where m_k is the smallest nonnegative integer m satisfying

$$\psi(x^k + \eta^m d_k) \leq R_k + \gamma\eta^m \nabla\psi(x^k)^T d_k. \tag{7}$$

Step 5. Set $x^{k+1} := x^k + \lambda_k d_k$ and

$$R_{k+1} := (1 - \tau_k)R_k + \tau_k \psi(x^{k+1}). \tag{8}$$

Set $k := k + 1$. Go to Step 2.

Theorem 2.1. *Algorithm NDGNM is well defined and its generated sequence $\{x^k\}$ satisfies $\psi(x^k) \leq R_k$ for all $k \geq 0$.*

Proof. Suppose that $\psi(x^k) \leq R_k$ holds for some k. If $\nabla\psi(x^k) = 0$, then Algorithm NDGNM terminates. Now, we suppose that $\nabla\psi(x^k) \neq 0$. Since $\mu_k > 0$, $V_k^T V_k + \mu_k I$ is positive definite and the search direction d_k in Step 3 is well defined. Moreover, since $\nabla\psi(x^k) \neq 0$, we have $d_k \neq 0$ and hence

$$\nabla\psi(x^k)^T d_k = -d_k^T (V_k^T V_k + \mu_k I)d_k < 0. \tag{9}$$

Next we show that there exists at least a nonnegative integer m satisfying (7). On the contrary, we suppose that for any nonnegative integer m,

$$\psi(x^k + \eta^m d_k) > R_k + \gamma\eta^m \nabla\psi(x^k)^T d_k. \tag{10}$$

Since $\psi(x^k) \leq R_k$, by (10), we have

$$\frac{\psi(x^k + \eta^m d_k) - \psi(x^k)}{\eta^m} > \gamma \nabla \psi(x^k)^T d_k.$$

Since ψ is continuously differentiable at x^k, by letting $m \to \infty$ in the above inequality, we have $\nabla \psi(x^k)^T d_k \geq \gamma \nabla \psi(x^k)^T d_k$. This contradicts (9) and $\gamma \in (0, 1/2)$. Hence, we can find a step-size λ_k in Step 4 and get the $(k+1)$-th iteration $x^{k+1} = x^k + \lambda_k d_k$. Moreover, from (7) and (9) we have

$$\psi(x^{k+1}) \leq R_k + \gamma \lambda_k \nabla \psi(x^k)^T d_k \leq R_k. \tag{11}$$

Using this fact, we obtain from (8) that

$$\psi(x^{k+1}) = (1 - \tau_k)\psi(x^{k+1}) + \tau_k \psi(x^{k+1}) \leq (1 - \tau_k)R_k + \tau_k \psi(x^{k+1}) = R_{k+1}.$$

Hence, we can conclude that if $\psi(x^k) \leq R_k$, then x^{k+1} can be generated by Algorithm NDGNM and it satisfies $\psi(x^{k+1}) \leq R_{k+1}$. Since $\psi(x^0) = R_0$, by the mathematical induction, we prove the theorem. The proof is completed. □

3. Convergence analysis

In this section, we assume that Algorithm NDGNM does not terminate in finitely many steps, i.e., $\nabla \psi(x^k) \neq 0$ for all $k \geq 0$. To establish the global convergence of Algorithm NDGNM, we need the following result.

Lemma 3.1 ([12], Corollary 1)**.** *Let $\{x^k\} \subset \mathcal{R}^n$ be a sequence converging to x. Let $\{V_k\}$ be a sequence such that $V_k \in \partial H(x^k)$ for all $k \geq 0$. Then $\{V_k\}$ is bounded. Moreover, if $\{V_k\}$ converges to V, then $V \in \partial H(x)$.*

Theorem 3.1 (Global convergence)**.** *Assume that x^* is an accumulation point of $\{x^k\}$ generated by Algorithm NDGNM. Then x^* is a stationary point of the merit function $\psi(x)$ if any one of the following conditions holds:*

(i) both $\{\mu_k\}$ and $\{d_k\}$ are bounded.

(ii) $\tilde{\mu} < \mu_k < \bar{\mu}$ for some $\bar{\mu} > \tilde{\mu} > 0$.

(iii) $\mu_k = \alpha \|H(x)\|^\beta$ for any $\alpha, \beta > 0$.

Moreover, x^* is a solution of the NCP if there exists a nonsingular element in $\partial H(x^*)$.

Proof. By (11), we have $\psi(x^{k+1}) \leq R_k$ for all $k \geq 0$. Then, it follows from (8) that for all $k \geq 0$

$$R_{k+1} = (1 - \tau_k)R_k + \tau_k \psi(x^{k+1}) \leq (1 - \tau_k)R_k + \tau_k R_k = R_k. \tag{12}$$

Thus, there exists a constant $R^* \geq 0$ such that $\lim_{k\to\infty} R_k = R^*$. By (8), we have for all $k \geq 1$

$$\psi(x^k) = R_{k-1} + \frac{R_k - R_{k-1}}{\tau_{k-1}}.$$

Since $\tau_k \geq \tau > 0$, we have $\lim_{k\to\infty} \psi(x^k) = R^*$. Now, without loss of generality, we assume that $\lim_{(K\ni)k\to\infty} x^k = x^*$ where K is a subsequence of $\{0, 1, ...\}$.

First, we consider the condition (i). Since $\{V_k\}_{k\in K}$ is bounded by Lemma 3.1, and $\{\mu_k\}_{k\in K}$ and $\{d^k\}_{k\in K}$ are bounded by the condition (i), by passing to the subsequence, we may assume that

$$\lim_{(K\ni)k\to\infty} V_k = V^*, \quad \lim_{(K\ni)k\to\infty} \mu_k = \mu^*, \quad \lim_{(K\ni)k\to\infty} d^k = d^*.$$

Moreover, by Lemma 3.1 we have $V^* \in \partial H(x^*)$. Thus, from Lemma 2.1 (c) it follows that $\nabla\psi(x^*) = (V^*)^T H(x^*)$ and

$$\lim_{(K\ni)k\to\infty} \nabla\psi(x^k) = \lim_{(K\ni)k\to\infty} V_k^T H(x^k) = (V^*)^T H(x^*) = \nabla\psi(x^*). \tag{13}$$

Now, we prove that $\nabla\psi(x^*)^T d^* = 0$. We divide the proof into the following two parts:

Part 1. $\lambda_k \geq c > 0$ for all $k \in K$ where c is a fixed constant. In this case, it follows from (7) and (9) that for all $k \in K$,

$$0 \leq -\gamma c \nabla\psi(x^k)^T d_k \leq -\gamma\lambda_k \nabla\psi(x^k)^T d_k \leq R_k - \psi(x^{k+1}). \tag{14}$$

Since $\lim_{k\to\infty} R_k = \lim_{k\to\infty} \psi(x^k) = R^*$, by letting $k \to \infty$ with $k \in K$ in (14), we have $\nabla\psi(x^*)^T d^* = 0$.

Part 2. $\{\lambda_k\}_{k\in K}$ has a subsequence converging to zero. We may pass to the subsequence and assume that $\lim_{(K\ni)k\to\infty} \lambda_k = 0$. From the line search (7), we get that for all sufficiently large $k \in K$,

$$\psi(x^k + \eta^{-1}\lambda_k d_k) > R_k + \gamma\eta^{-1}\lambda_k \nabla\psi(x^k)^T d_k.$$

Since $\psi(x^k) \leq R_k$ for all $k \geq 0$, it follows that

$$\psi(x^k + \eta^{-1}\lambda_k d_k) - \psi(x^k) \geq \gamma\eta^{-1}\lambda_k \nabla\psi(x^k)^T d_k,$$

i.e.,

$$\frac{\psi(x^k + \eta^{-1}\lambda_k d_k) - \psi(x^k)}{\eta^{-1}\lambda_k} \geq \gamma\nabla\psi(x^k)^T d_k. \tag{15}$$

Since ψ is continuously differentiable at x^*, by letting $k \to \infty$ with $k \in K$ in (15), we have

$$\nabla\psi(x^*)^T d^* \geq \gamma\nabla\psi(x^*)^T d^*. \tag{16}$$

On the other hand, since $\nabla\psi(x^k)^T d_k < 0$ for all $k \geq 0$ by (9), we have

$$\nabla\psi(x^*)^T d^* \leq 0. \tag{17}$$

Since $\gamma \in (0,1)$, we obtain from (16) and (17) that $\nabla\psi(x^*)^T d^* = 0$.

By Part 1 and Part 2, we can conclude that $\nabla\psi(x^*)^T d^* = 0$. Moreover, from (6) we have

$$\nabla\psi(x^*)^T d^* + (d^*)^T((V^*)^T V^* + \mu^* I)d^* = 0,$$

which gives

$$(d^*)^T((V^*)^T V^* + \mu^* I)d^* = 0. \tag{18}$$

If $\mu^* > 0$, then the matrix $(V^*)^T V^* + \mu^* I$ is positive definite. By (18), we have $d^* = 0$ which together with (6) gives $\nabla\psi(x^*) = -((V^*)^T V^* + \mu^* I)d^* = 0$. If $\mu^* = 0$, then by (18) we have $V^* d^* = 0$. Using (6) again, we have $\nabla\psi(x^*) = -(V^*)^T V^* d^* = 0$. This proves that x^* is a stationary point of ψ.

Next, we consider the condition (ii). Since $0 < \tilde{\mu} < \mu_k < \bar{\mu}$, the matrices $\{V_k^T V_k + \mu_k I\}$ are uniformly positive definite for all k. It follows from (6) that

$$\begin{aligned}
\|d_k\| &= \|(V_k^T V_k + \mu_k I)^{-1}\nabla\psi(x^k)\| \\
&\leq \|(V_k^T V_k + \mu_k I)^{-1}\|\|\nabla\psi(x^k)\| \\
&\leq \frac{1}{\mu_k}\|\nabla\psi(x^k)\| \\
&\leq \frac{1}{\tilde{\mu}}\|\nabla\psi(x^k)\|.
\end{aligned}$$

Since $\{\|\nabla\psi(x^k)\|\}_{k\in K}$ is bounded, $\{d_k\}_{k\in K}$ is bounded. So, by following from (i), we obtain the desired result.

At last, we consider the condition (iii). For all $k \geq 0$, since $\nabla\psi(x^k) \neq 0$, we have $H(x^k) \neq 0$ and hence $\mu_k = \alpha\|H(x)\|^\beta > 0$. Suppose that $\nabla\psi(x^*) \neq 0$. Then $\|H(x^*)\| > 0$. Since $\lim_{k\to\infty}\psi(x^k) = R^*$, by (5) and the continuity of H, we have

$$\lim_{k\to\infty}\mu_k = \lim_{k\to\infty}\alpha\left(\sqrt{2\psi(x^k)}\right)^\beta = \alpha(\sqrt{2R^*})^\beta = \alpha\|H(x^*)\|^\beta > 0.$$

So, there exists $\bar{\mu} > \tilde{\mu} > 0$ such that $\tilde{\mu} < \mu_k < \bar{\mu}$. By (ii), x^* must be a stationary point of $\psi(x)$. It is a contradiction. Thus, x^* is a stationary point of $\psi(x)$.

The second part of the theorem follows from Lemma 2.1 (c).

We complete the proof. □

In a similar way as those in [10, Theorem 7.2], we can obtain the local superlinear/quadratic convergence of Algorithm NDGNM as follows.

Theorem 3.2 (Local superlinear/quadratic convergence)**.** *Assume that x^* is an accumulation point of $\{x^k\}$ generated by Algorithm NDGNM. Let $\mu_k = \alpha\|H(x)\|^\beta$ for some $\alpha, \beta > 0$. If all $V \in \partial H(x^*)$ are nonsingular, then the whole sequence $\{x^k\}$ converges to x^* superlinearly. Furthermore, if F' is Lipschitz continuous around x^* and $\beta \geq 1$, then the convergence rate is quadratic.*

In Theorem 3.1 and Theorem 3.2, we assume that the sequence $\{x^k\}$ generated by Algorithm NDGNM has one accumulation point x^* and all $V \in \partial H(x^*)$ are nonsingular. In the following, we show that this assumption is satisfied when F in the NCP is a uniform P-function. For this purpose, we need the following lemma.

Lemma 3.2 ([4], Lemma 4.1)**.** *Suppose that F is a uniform P-function, i.e., there exists a positive scalar $c > 0$ such that*

$$\max_{1\leq i\leq n}(x_i - y_i)(F_i(x) - F_i(y)) \geq c\|x - y\|^2, \quad \forall\ x, y \in \mathcal{R}^n.$$

Then, the following results hold:

(*i*) *The NCP has a unique solution.*

(*ii*) *For any $x \in \mathcal{R}^n$ and any $V \in \partial H(x)$, V is nonsingular.*

(*iii*) *The level set $L(x^0) := \{x \in \mathcal{R}^n : \psi(x) \leq \psi(x^0)\}$ is bounded for any $x^0 \in \mathcal{R}^n$.*

Theorem 3.3. *If F is a uniform P-function, then the sequence $\{x^k\}$ generated by Algorithm NDGNM has at least one accumulation point x^* and all $V \in \partial H(x^*)$ are nonsingular.*

Proof. By Theorem 2.1 and (12), we have $\psi(x^k) \leq R_k \leq R_0 = \psi(x^0)$ for all $k \geq 0$. This together with Lemma 3.2 (iii) implies that $\{x^k\}$ is bounded and it has at least one accumulation point x^*. The second result holds by Lemma 3.2 (ii). □

By Theorems 3.1–3.3 and Lemma 3.2 (i), we can directly have the following result

Theorem 3.4. *If F is a uniform P-function, then the sequence $\{x^k\}$ generated by Algorithm NDGNM converges to the unique solution of the NCP locally superlinearly/quadratically.*

4. Numerical results

In this section, we report some numerical results of Algorithm NDGNM. All experiments are carried on a PC with CPU of Inter(R) Core(TM)i7-7700 CPU @ 3.60 GHz and RAM of 8.00GB. The codes are written in MATLAB and run

in MATLAB R2018a environment. The parameters used in Algorithm NDGNM are chosen as $\gamma = 0.1,\ \eta = 0.8, \tau_k = \frac{2^k+1}{2^{k+1}}$.

We consider the following linear complementarity problem (LCP):

$$x \geq 0,\ y \geq 0,\ y = Mx + q,\ x^T y = 0,$$

in which $M \in \mathcal{R}^{n\times n}$ and $q \in \mathcal{R}^n$. By using the Fischer-Burmeister function ϕ, we have a nonsmooth equation reformulation of the LCP:

$$H(x,y) := \begin{pmatrix} y - Mx - q \\ \phi(x_1, y_1) \\ \vdots \\ \phi(x_n, y_n) \end{pmatrix} = 0,$$

namely, (x, y) is a solution of the LCP if and only if $H(x, y) = 0$.

We apply Algorithm NDGNM to solve $H(x, y) = 0$ and use $\|H(x^k, y^k)\| \leq 10^{-6}$ as the stopping criterion. In our experiments, we investigate the following two LCPs:

(I) Let M be the block diagonal matrix with $M_1, ..., M_4$ as block diagonals, i.e., $M = \text{diag}(M_1, ..., M_4)$, in which $M_i = \frac{N_i^T N_i}{\|N_i^T N_i\|}$ with $N_i = \text{rand}(\frac{n}{4}, \frac{n}{4})$ for $i = 1, ..., 4$. Take $q = \text{rand}(n, 1)$. In this case, the function $F(x) = Mx + q$ has the Cartesian P_0-property.

(II) Let $M = \text{diag}(M_1, ..., M_4)$, in which $M_i = \frac{N_i}{\|N_i\|} - \text{eye}(n/4)$ with $N_i = \text{rand}(\frac{n}{4}, \frac{n}{4})$ for $i = 1, ..., 4$. Take $q = \text{rand}(n, 1)$. In this case, the function $F(x) = Mx + q$ may have no Cartesian P_0-property.

In the experiments, we generate 10 problem instances for each size of n. We use the following two starting points: (1) $x^0 = (1, 0, ..., 0)^T$, $y^0 = (1, 1, ..., 1)^T$; (2) $x^0 = (1, 0, ..., 0)^T$, $y^0 = Mx^0 + q$. Numerical results are listed in Table 1 where **SP** denotes the starting point, **aIT** denotes the average value of the iteration numbers, **aCPU** denotes the average value of the CPU time in seconds and **aHK** denotes the average value of $\|H(x^k, y^k)\|$ when Algorithm NDGNM terminates among the 10 testing. From Table 1, we can see that Algorithm NDGNM is very effective for solving LCPs even though these problems have no Cartesian P_0-property.

Table 1 Numerical results of Algorithm NDGNM

LCP	SP	n	aIT	aCPU	aHK
(I)	(1)	1000	4.9	1.83	2.1706e-07
		1500	5.5	5.68	8.2870e-08
		2000	5.5	14.50	2.6248e-07
		2500	5.4	22.64	1.9886e-07
		3000	6.2	53.75	2.2750e-07
	(2)	1000	4.2	1.54	1.2565e-07
		1500	4.5	4.67	2.2360e-07
		2000	4.8	10.69	1.1695e-07
		2500	4.2	18.02	1.2727e-07
		3000	4.7	34.96	2.3348e-07
(II)	(1)	1000	4.1	1.47	1.2654e-07
		1500	4.0	9.07	2.3691e-07
		2000	4.2	17.62	1.1274e-07
		2500	4.2	30.95	3.7991e-07
		3000	4.4	41.55	1.1590e-07
	(2)	1000	3.2	1.13	1.1403e-07
		1500	3.4	3.47	8.1548e-08
		2000	3.6	8.18	2.1214e-08
		2500	3.2	13.17	4.4488e-08
		3000	3.5	24.83	1.4597e-07

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