Global stability analysis and persistence for an ecological food web-model

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Abstract. The ecological food web problems and their impact on the environment play vital role for balancing of some environments in our daily life. In the present work, the analytic results of an ecological food web-model are rigorously examined and analyzed. The model includes interactions and natural variables occur in different organisms of the species that influence by the competition and refuge as two basic conditions. The persistence of variant species for the resources competition is also analyzed. The global asymptotic stability of the positive equilibrium points is investigated numerically based on the Runge-Kutta predictor-corrector algorithm. Finally, the effects of the variation of each parameter on the proposed model are inspected numerically.

Keywords: food web-model, global stability, persistence, period dynamic, stage-structure.

1. Introduction

Our external environment suffers from many problems, including environmental, economic, social, ..., etc, as well as, the spread of epidemics and infectious diseases of all kinds. However, with the current technological universe and the increase of population, many scientists are motivating to orient their interest

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for studying such natural phenomena, through mathematical modeling to be analyzed deeply [1, 2, 3, 4, 5, 6, 7, 8].

With the time, and the arrival of the age of technology accompanied by the increase in the population, these problems have become more complex and more difficult. Therefore, it has become necessary to use modern technologies to help us in diagnosing and analyzing the scientific results that obtained in theory. A comprehensive number of studies have been developed to solve such difficulty and complexity. Marcus R. [9] presented two finite-difference algorithms for studying the dynamics of spatially extended predator-prey interactions with the Holling type II functional response and logistic growth of the prey. Naji,and Hussien, [10] proposed an epidemic model that describes the dynamics of the spread of two different types of infectious diseases that spread through both horizontal and vertical transmission in the host population. Whereas, Li, Hongli, et al. [11] investigated a three-species food chain model in a patchy environment, where the prey species, mid-level predator species, and top predator species can disperse among n different patches ($n \geq 0$).

The environmental-model that deals with endangered species (lemur animals) and two types of hunters (the black panthers and hyenas animals) that are link together by a food web is studied and analyzed theoretically by Al-Jubouri, et al. in [12].

The essential contribution of this study lies in demonstrating the theoretical aspect of model (2) given in [12]. New criteria are introduced to study the global stability of its unique equilibrium points, as well as, their existence. The simulations results substantiate the feasibility of the analytical findings.

2. Mathematical formulation

The idea of the proposed ecological-model is based on three-types of different species link together by a food web model. A high dimensional prey- predator model proposed in [12] is shown in Figure 1, and expressed mathematically in equation (1).

This model will be represented by the following nonlinear autonomous differential equations,

$$
\begin{aligned}
\frac{dI_1}{dT} &= rI_2 - I_1\left(\frac{\rho_1(1-m)}{b_1+(1-m)I_1}P_1 + a_1(1-m)P_2 + \beta + d_1 + c_1 I_1\right),\\
\frac{dI_2}{dT} &= \beta I_1 - I_2\left(\frac{\rho_2(1-m)}{b_2+(1-m)I_2}P_1 + a_2(1-m)P_2 + d_2 + c_2 I_2\right),\\
\frac{dP_1}{dT} &= P_1\left(\frac{e_1\rho_1(1-m)I_1}{b_1+(1-m)I_1} + \frac{e_2\rho_2(1-m)I_2}{b_2+(1-m)I_2} - d_3 - c_3 P_2\right),\\
\frac{dP_2}{dT} &= P_2\left(e_3 a_1(1-m)I_1 + e_4 a_2(1-m)I_2 - d_4 - c_4 P_1\right).
\end{aligned}
\tag{2.1}
$$

This model consists of a two stage-structure of prey species (Lemur animals), which is an immature $I_1(t)$, and a mature $I_2(t)$, with a mid-level predator

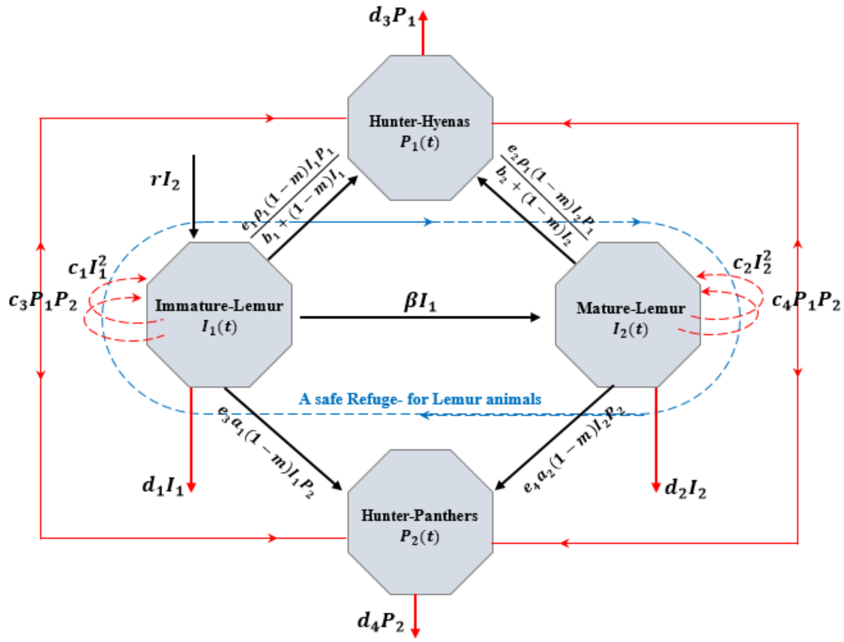

Figure 1: *Sketch, showing the idea of mathematical simulation of an Ecological-model.*

(Hyenas) $P_1(t)$, and a top-level predator (the black panthers) $P_2(t)$. Each of, I_1, I_2, P_1, and P_2 are representing the densities of populations at time (t). Furthermore, all the parameters used are positive and will be described biologically through Table 1.

Table 1: The inputs of the mathematical model(1).

Parameters Code in Model (1)	Biological Description
r	Actual increase average of a mature prey
β	Actual increase average of an immature prey
$c_{1,2}$	Competition average for an immature and mature prey
$d_{1,2}$	Natural death average for an immature and mature prey
$\rho_{1,2}$	Predation average for the prey- by a mid- level predator
$b_{1,2}$	Semi saturation average for a mid- level predator
$a_{1,2}$	Predation average for the prey- by a top- level predator
$c_{3,4}$	Competition average between a predators species
$d_{3,4}$	Death average for a predators after loss prey species
m	Refuge average
$(1-m)$	The number of prey exposed to predation by a predators
$e_{1,\dots,4}$	Conversion average of a sustenance

Using the dimensionless variables technique, we have,

$$t = rT, \quad x = \frac{c_1}{r} I_1, \quad y = \frac{c_2}{r} I_2, \quad z = \frac{\rho_1 c_1}{r^2} P_1 \quad ,and \quad w = \frac{a_1(1-m)}{r} P_2.$$

A coordination to these assumptions, the model becomes as,

$$\begin{aligned}
\frac{dx}{dt} &= \upsilon_1 y - x\left(\frac{z}{\upsilon_2 + x} + w + (\upsilon_3 + \upsilon_4) + x\right) = f_1(x,y,z,w); x(0) \geq 0,\\
\frac{dy}{dt} &= \upsilon_5 x - y\left(\frac{\upsilon_6 z}{\upsilon_7 + y} + \upsilon_8 w + \upsilon_9 + y\right) = f_2(x,y,z,w); y(0) \geq 0,\\
\frac{dz}{dt} &= z\left(\frac{\upsilon_{10} x}{\upsilon_2 + x} + \frac{\upsilon_{11} y}{\upsilon_7 + y} - \upsilon_{12} - \upsilon_{13} w\right) = f_3(x,y,z,w); z(0) \geq 0,\\
\frac{dw}{dt} &= w\left(\upsilon_{14} x + \upsilon_{15} y - \upsilon_{16} - \upsilon_{17} z\right) = f_4(x,y,z,w); w(0) \geq 0.
\end{aligned} \tag{2.2}$$

Here:

$\upsilon_1 = \frac{c_1}{c_2}$; $\upsilon_2 = \frac{b_1 c_1}{r(1-m)}$; $\upsilon_3 = \frac{\beta}{r}$; $\upsilon_4 = \frac{d_1}{r}$; $\upsilon_5 = \frac{\beta c_2}{r c_1}$; $\upsilon_6 = \frac{\rho_2 c_2}{\rho_1 c_1}$; $\upsilon_7 = \frac{b_2 c_2}{r(1-m)}$; $\upsilon_8 = \frac{a_2}{a_1}$; $\upsilon_9 = \dfrac{d_2}{r}$; $\upsilon_{10} = \frac{e_1 \rho_1}{r}$; $\upsilon_{11} = \frac{e_2 \rho_2}{r}$; $\upsilon_{12} = \frac{d_3}{r}$; $\upsilon_{13} = \frac{c_2}{a_1(1-m)}$; $\upsilon_{14} = \frac{e_3 a_1 (1-m)}{c_1}$; $\upsilon_{15} = \frac{e_4 a_2 (1-m)}{c_2}$; $\upsilon_{16} = \frac{d_4}{r}$; $\upsilon_{17} = \frac{r c_4}{\rho_1 c_1}$.

Since these functions are Lipschitzian on $\mathbb{R}^4_+ = \{(x,y,z,w) \in \mathbb{R}^4_+ : x(0) \geq 0, y(0) \geq 0, z(0) \geq 0 \text{ and } w(0) \geq 0\}$, then the solution of the model (2) exists and unique.

3. Boundedness

Theorem 1. *All the trajectories of model (2), with the initial points in $\mathbb{R}^4_+$ are uniformly bounded. For the proof, we refer the reader to see [12].*

4. Existence and stability analysis

The model (2) have at most five- biologically reasonable equilibrium points $H_i = (x,y,z,w),\ i = 0,...,4$, which are exist under the conditions established in [12].

In the following, the stability of model (2) near proper equilibrium points $H_i, i = 0,...,4$ is discussed in [12].

1. The trivial point $H_0 = (0,0,0,0)$, if the following condition hold

$$u_5 < \frac{u_9(u_3 + u_4)}{u_1}. \tag{4.1}$$

Then, the trajectories of model (2) tending to the asymptotically stable point H_0.

2. The predators-free point $H_1 = (\bar{x}, \bar{y}, 0, 0)$, if the following conditions hold

$$u_{12} + u_{16} > n_3 + n_4, \tag{4.2}$$

$$n_3 n_4 + u_{12} u_{16} > u_{16} n_3 + u_{12} n_4. \tag{4.3}$$

Then, the trajectories of model (2) tending to the asymptotically stable point H_1.

3. The mid-level predator-free point $H_2 = (\bar{\bar{x}}, \bar{\bar{y}}, 0, \bar{\bar{w}})$, if the following conditions hold

$$\bar{\bar{w}} > \max\{\Gamma_1, \Gamma_2\}, \tag{4.4}$$

$$\Gamma_3 > \Gamma_4, \tag{4.5}$$

$$\Gamma_5 > \Gamma_6. \tag{4.6}$$

Then, the trajectories of model(2)tending to the asymptotically stable point H_2. For more details see [12].

4. The top-level predator-free point $H_3 = (\bar{\bar{\bar{x}}}, \bar{\bar{\bar{y}}}, \bar{\bar{\bar{z}}}, 0)$, if the following conditions hold

$$\bar{\bar{\bar{z}}} > \frac{u_{14}\bar{\bar{\bar{x}}} + u_{15}\bar{\bar{\bar{y}}}}{u_{17}}, \tag{4.7}$$

$$u_{12} < c_3 < u_{12} + c_1 + c_2, \tag{4.8}$$

$$((u_{12} + c_1 + c_2) - c_3)\psi_1 > (c_3 - (u_{12} + c_1 + c_2))\psi_2 + Q_3. \tag{4.9}$$

Then, the trajectories of model(2) tending to the asymptotically stable point H_2. For more details see [12].

5. Finally, the coexistence equilibrium point $H_4 = (x^*, y^*, z^*, w^*)$, if the following conditions hold

$$p_{12}^2 < \frac{4}{9}\, p_{11}p_{22}, \tag{4.10}$$

$$p_{13}^2 < \frac{4}{9}\, p_{11}p_{33}, \tag{4.11}$$

$$p_{14}^2 < \frac{4}{9}\, p_{11}p_{44}, \tag{4.12}$$

$$p_{23}^2 < \frac{4}{9}\, p_{22}p_{33}, \tag{4.13}$$

$$p_{24}^2 < \frac{4}{9}\, p_{22}p_{44}, \tag{4.14}$$

$$p_{34}^2 > \frac{4}{9}\, p_{33}p_{44}. \tag{4.15}$$

Then, the trajectories of model(2) tending to the asymptotically stable point H_2. For more details see [12].

5. Numerical simulations

In this section, the quantitative behavior of model (2) is determined based on Runge-Kutta predictor-corrector method using MATLAB. These simulations demonstrate the previously obtained theoretical results of stability and equilibrium of the proposed model given in [12]. Also, the global dynamics and persistence have been proven and materialized numerically. As in the Figures 2-9. Furthermore, the effects of changing parameter values of model (2) were investigated. The proposed system was simulated numerically for the following parameter values:

$$\begin{aligned}
&\upsilon_1 = 0.9, \upsilon_2 = 0.4, \upsilon_3 = 0.3, \upsilon_4 = 0.4, \upsilon_5 = 2.5, \upsilon_6 = 0.4,\\
&\upsilon_7 = 0.6, \upsilon_8 = 0.2, \upsilon_9 = 0.7, \upsilon_{10} = 0.9, \upsilon_{11} = 0.9,\\
&\upsilon_{12} = 0.25, \upsilon_{13} = 0.09, \upsilon_{14} = 0.9, \upsilon_{15} = 1.1, \upsilon_{16} = 0.04, \upsilon_{17} = 0.33.
\end{aligned} \tag{5.1}$$

Taking the above data into consideration, the time series of the trajectories of model (2) are shown in Figure 2.

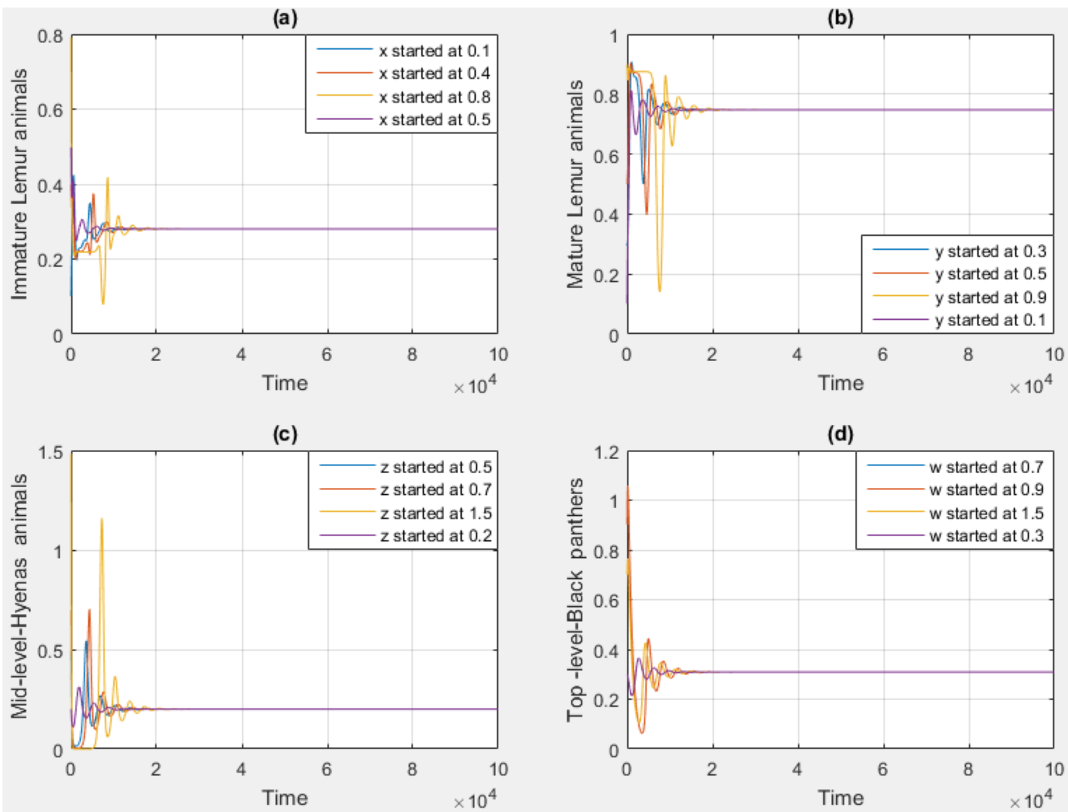

Figure 2: The time series of system (2), starting with four different initial points $(0.1, 0.3, 0.5, 0.7)$, $(0.4, 0.5, 0.7, 0.9)$, $(0.8, 0.9, 1.5, 1.5)$ and $(0.5, 0.7, 0.2, 0.4)$.

It illustrates that model (2) has globally asymptotically stable as the solution of model (2) approaches asymptotically to the positive equilibrium point $H_4 = (0.254, 0.714, 0.2, 0.309)$, which confirmed the obtained analytical results.

Next, we need to analyze the results of the asymptotic stability of points $H_i, i = 0, 1, 2, 3$. Some parameter values affect the dynamical behavior of model (2). At each time, the effect of varying of one parameter while the others are fixed is discussed. The results are summarized in Table 1.

It can be seen that varying the value of parameters $\upsilon_i, i = 3, 4, 5$ does not affect the dynamic of model(2). Therefore, the solution of the model (2) still converges to the coexisting (positive) equilibrium point $H_4 = (x^*, y^*, z^*, w^*)$. Figures (3-6) show the time series of model (2) according to different parameters, which converge to the equilibrium points H_i ; $for\ i = 0, 1, 2, 3$.

Table 2: The numerical behaviors and persistence of model (2) by changing of a specific parameter and fixing the other.

Variable parameters in model(2)	Numerical behavior of model(2)	Persistence of model(2)
$0.01 \leqslant \upsilon_1 < 0.9$	Converge to stable point $H_0 = (0,0,0,0)$	Not Persist
$0.9 \leqslant \upsilon_1 \leqslant 1$	Converge to stable point in Int.$\mathbb{R}_+^4$	Persist
$\upsilon_1 > 1$	Converge to stable point in $xy - plane$	Persist
$0.3 < \upsilon_2 \leqslant 1.1$	Converge to stable point in Int.$\mathbb{R}_+^4$	Persist
$0.01 \leqslant \upsilon_5 < 0.6$	Converge to stable point $H_0 = (0,0,0,0)$	Not Persist
$0.6 \leqslant \upsilon_5 < 2.5$	Converge to stable point in $xyw - space$	Persist
$2.5 \leqslant \upsilon_5 < 3$	Converge to stable point in Int.$\mathbb{R}_+^4$	Persist
$\upsilon_5 \geqslant 3$	Converge to periodic dynamics in Int.$\mathbb{R}_+^4$	Persist
$0.13 < \upsilon_7 < 0.6$	Converge to periodic dynamics in Int.$\mathbb{R}_+^4$	Persist
$0.6 \leqslant \upsilon_7 < 1$	Converge to stable point in Int.$\mathbb{R}_+^4$	Persist
$0.01 \leqslant \upsilon_8 \leqslant 0.35$	Converge to stable point in Int.$\mathbb{R}_+^4$	Persist
$0.3 < \upsilon_9 < 0.7$	Converge to stable point in $xyz - space$	Persist
$0.7 \leqslant \upsilon_9 \leqslant 0.95$	Converge to stable point in Int.$\mathbb{R}_+^4$	Persist
$0.8 \leqslant \upsilon_{10} < 1$	Converge to stable point in Int.$\mathbb{R}_+^4$	Persist
$1 \leqslant \upsilon_{10} < 2$	Converge to stable point in $xyz - space$	Persist
$0.9 \leqslant \upsilon_{11} < 2$	Converge to stable point in Int.$\mathbb{R}_+^4$	Persist
$\upsilon_{11} \geqslant 2$	Converge to periodic dynamics in Int.$\mathbb{R}_+^4$	Persist
$0.2 < \upsilon_{12} \leqslant 0.25$	Converge to stable point in Int.$\mathbb{R}_+^4$	Persist
$0.01 \leqslant \upsilon_{13} \leqslant 0.1$	Converge to stable point in Int.$\mathbb{R}_+^4$	Persist
$0.1 < \upsilon_{13} \leqslant 0.7$	Converge to stable point in $xyw - space$	Persist
$0.01 \leqslant \upsilon_{14} \leqslant 0.9$	Converge to stable point in Int.$\mathbb{R}_+^4$	Persist
$0.3 < \upsilon_{15} \leqslant 1.1$	Converge to stable point in Int.$\mathbb{R}_+^4$	Persist
$0.035 < \upsilon_{16} \leqslant 0.5$	Converge to stable point in Int.$\mathbb{R}_+^4$	Persist
$0.5 < \upsilon_{16} \leqslant 2$	Converge to stable point in $xyz - space$	Persist
$0.33 \leqslant \upsilon_{17} < 0.8$	Converge to stable point in Int.$\mathbb{R}_+^4$	Persist
$0.8 \leqslant \upsilon_{17} \leqslant 2$	Converge to stable point in $xy - plane$	Persist

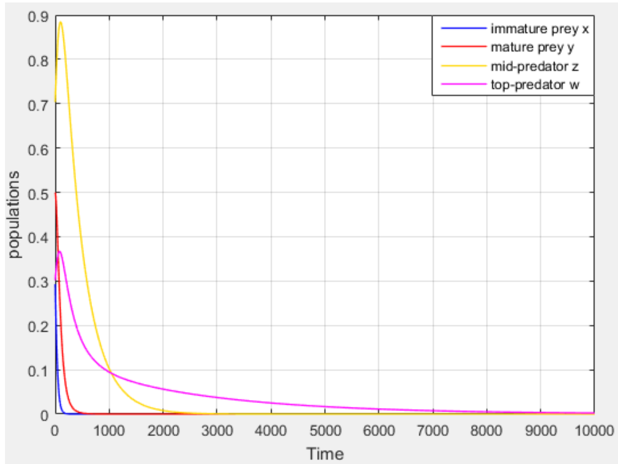

Figure 3: Time series of the trajectories for the data given in equation (18), with $\upsilon_1 = 0.01$, which shows that the trajectories converge asymptotically to the vanishing equilibrium point $H_0 = (0, 0, 0, 0)$.

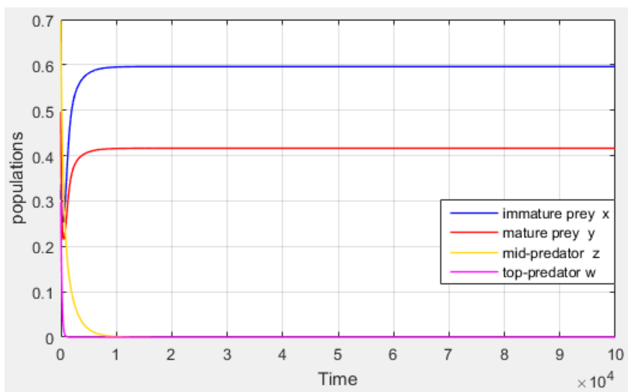

Figure 4: Time series of the trajectories for the data given in equation (18), with $\upsilon_{17} = 0.8$, which shows that the trajectories converge asymptotically to the predators-free equilibrium point $H_1 = (0.596, 0.416, 0, 0)$.

Figure 3 confirms the obtained analytic results regarding the existence of a locally asymptotically stable trivial equilibrium point $H_0 = (0, 0, 0, 0)$, when decreasing the intra-specific competition rate between the prey species (immature and mature prey) relative to the food and a refuge within limits ($0.01 \leqslant \upsilon_1 < 0.9$). Increasing υ_1 in the range ($0.9 \leqslant \upsilon_1 \leqslant 1$) and keeping other parameters constant as equation (18) shows that the solution of model(2) converges asymptotically to the positive equilibrium point $H_4 = (x^*, y^*, z^*, w^*)$ in Int.$\mathbb{R}^4_+$, see Figure 2. In addition, model (2) converges asymptotically to the predators-free

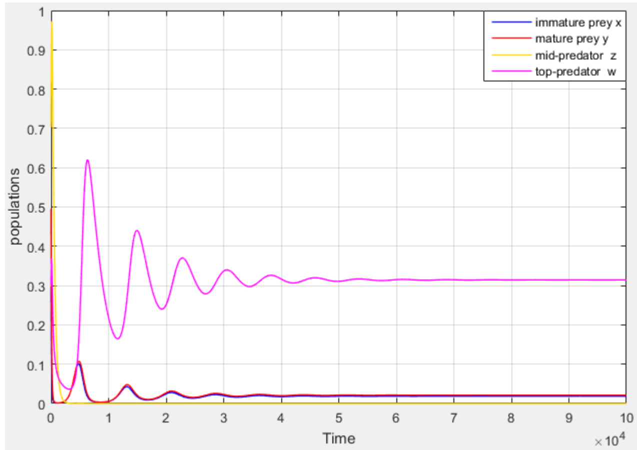

Figure 5: Time series of the trajectories for the data given in equation (18), with $\upsilon_{13} = 0.7$, which shows that the trajectories converge asymptotically to the mid-level predator-free equilibrium point $H_2 = (0.018, 0.212, 0, 0.314)$.

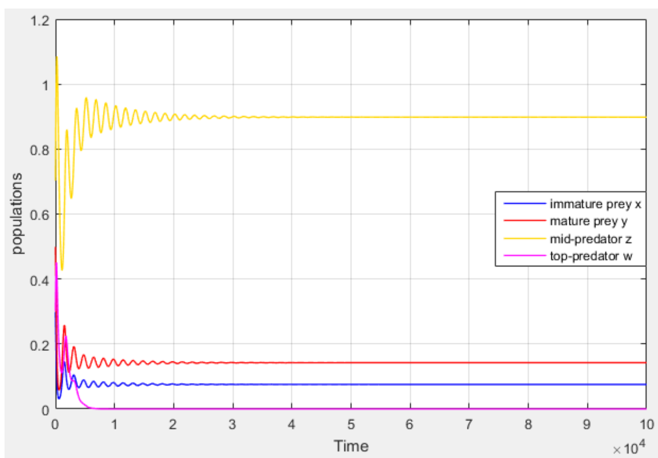

Figure 6: Time series of the trajectories for the data given in equation (18), with $\upsilon_{16} = 0.95$, which shows that the trajectories converge asymptotically to the top-level predator-free equilibrium point $H_3 = (0.075, 0.142, 0.898, 0)$.

point $H_1 = (\bar{x}, \bar{y}, 0, 0)$ when $\upsilon_1 > 1$. In Figure 4, $(0.33 \leqslant \upsilon_{17} < 0.8)$ represents the growth rate of the prey populations (immature and mature prey). To compete the predators for the predation of prey, it will expand, so the solution of model (2) converges asymptotically to the predators-free equilibrium point

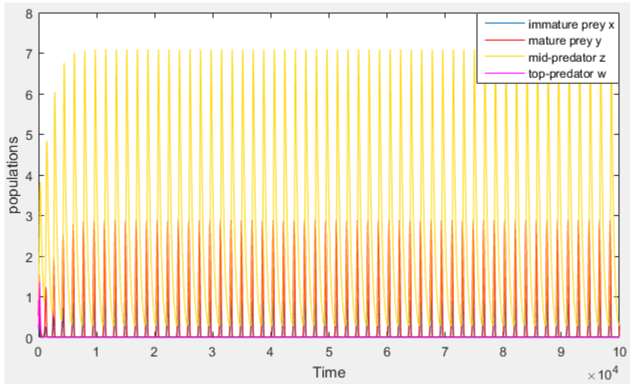

Figure 7: Time series of the trajectories for the data given in equation (18), with $\upsilon_5 = 3$, which shows that the trajectories approach the period dynamics in Int.$\mathbb{R}^4_+$.

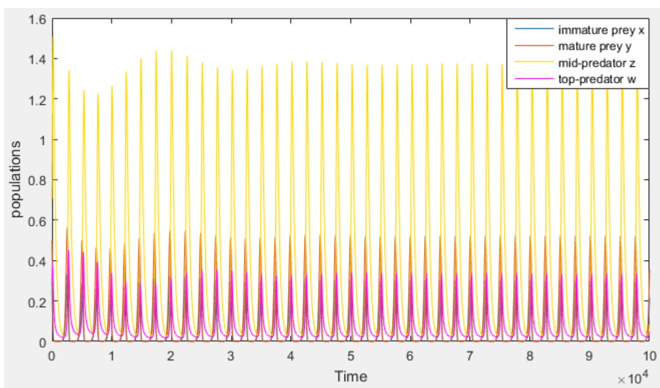

Figure 8: Time series of the trajectories for the data given in equation (18), with $\upsilon_7 = 0.1$, which shows that the trajectories approach the period dynamics in Int.$\mathbb{R}^4_+$.

$H_1 = (0.596, 0.416, 0, 0)$. Decreasing the range $(0.33 \leqslant \upsilon_{17} < 0.8)$, lead to approaching the positive equilibrium point $H_4 = (x^*, y^*, z^*, w^*)$ in Int.$\mathbb{R}^4_+$. For the constant parameters of equation (18), with $(0.1 < \upsilon_{13} \leqslant 0.7)$, which represents the inter-specific competition rate between the predators species (Top and mid-level predators) relative to the food and existence, the solution of model (2) converges asymptotically to the mid-level predator-free equilibrium point $H_2 = (0.018, 0.212, 0, 0.314)$ as shown in Figure 5, while it approaches asymptotically to the positive equilibrium point $H_4 = (x^*, y^*, z^*, w^*)$ when $(0.01 \leqslant \upsilon_{13} \leqslant 0.1)$ as shown in Figure 2. When the natural death rate for the top-level predator population relative to their growth rate is in the range

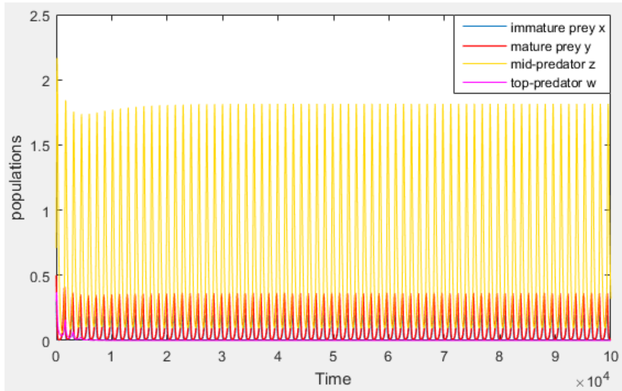

Figure 9: Time series of the trajectories for the data given in equation (18), with $\upsilon_{11} = 2$, which shows that the trajectories approach the period dynamics in Int.$\mathbb{R}^4_+$.

$(0.5 < \upsilon_{16} \leqslant 2)$, the solution of model (2) converges asymptotically to the top-level hunter-free equilibrium point $H_3 = (0.075, 0.142, 0.898, 0)$ as shown in Figure 6. But, it still approaches to positive equilibrium point $H_4 = (x^*, y^*, z^*, w^*)$ in $(0.035 < u_{16} \leqslant 0.5)$.

Numerical simulations of model(2) shows that the model has periodic dynamics, as presented in Figures (7-9). For constant parameters of equation (18), Figure 7 shows that the solution curves of model (2) approach to periodic dynamics in Int.$\mathbb{R}^4_+$, when $(\upsilon_5 \geqslant 3)$, which represents the growing rate for immature prey relative to compete the prey population for existence. Reducing the half-saturation constant for mid-level predator relative to compete the mature prey population to refuge within the limits $(0.13 < \upsilon_7 < 0.6)$ which causes an approaching to periodic dynamics in Int.$\mathbb{R}^4_+$, see Figure 8. Expansion the predation rate, the mid-level predator for the mature prey population relative to their growth rate within limits $(\upsilon_{11} \geqslant 2)$ leads to approaching the periodic dynamics in Int.$\mathbb{R}^4_+$, as shown in Figure 9. Otherwise, model (2) still has a globally asymptotically stable positive equilibrium point.

6. Discussion and conclusions

This study aims to analyze a mathematical model (2) describing a food web-model with ecological reactions that occur between different species. We used computational algorithms, through which, the nature of the relationship of these organisms to the external environment and its direct impact on maintaining the balance of nature has been known to them through computer simulation. Initially, the effects of the variation of each parameter on the proposed model are studied and analyzed numerically. This can be summarize as follows:

1. Consider that the parameters' values in equation (18) are fixed, then the time series of the trajectories of model (2) converges to a globally asymptotically stable positive equilibrium point $H_4 = (0.254, 0.714, 0.2, 0.309)$, this can be seen obviously in Figure 2.

2. The trajectories of model (2) again converges to the positive equilibrium point $H_4 = (x^*, y^*, z^*, w^*)$, when changing the parameters values $\upsilon_i, i = 3, 4, 5$, because it does not affect the nature of the dynamic behavior of model(2).

3. The trajectories of model (2) converges to the trivial equilibrium point $H_0 = (0, 0, 0, 0)$, when decreasing the intra-specific competition rate between the prey species (immature and mature prey) are relative to the food and refuge within the limits $(0.01 \leqslant \upsilon_1 < 0.9)$, as shown in Figure 3. Otherwise, it converges to the positive equilibrium point $H_4 = (x^*, y^*, z^*, w^*)$.

4. The trajectories of model (2) converges to the predators-free equilibrium point $H_1 = (0.596, 0.416, 0, 0)$, when expanding the growth rate of the prey species (immature and mature prey) within the limits $(0.33 \leqslant \upsilon_{17} < 0.8)$, as shown in Figure 4. Otherwise, it converges to the positive equilibrium point $H_4 = (x^*, y^*, z^*, w^*)$.

5. The trajectories of model (2) converges to the mid-level predator-free equilibrium point $H_2 = (0.018, 0.212, 0, 0.314)$, when the inter-specific competition rate between the predators species (Top and mid-level predators) are relative to the food and existence within the limits $(0.1 < \upsilon_{13} \leqslant 0.7)$, as shown in Figure 5. Otherwise, it converges to the positive equilibrium point $H_4 = (x^*, y^*, z^*, w^*)$.

6. The trajectories of model (2) converges to the top-level predator-free equilibrium point $H_3 = (0.075, 0.142, 0.898, 0)$, when the natural death rate for the top-level predator population is relative to their growth rate in the range $(0.5 < \upsilon_{16} \leqslant 2)$, as shown in Figure 6. Otherwise, it converges to the positive equilibrium point $H_4 = (x^*, y^*, z^*, w^*)$.

   Moreover, the computer simulations of food web-model (2) showed us that model (2) possesses periodic dynamic behavior, this can be seen obviously in Figures (7-9). For more details see Table 2.

However, from these numerical analyses and results discussion, we conclude that the global stability of such complex ecological model that includes interactions and occur in different organisms is demonstrated. Moreover, such environmental, which has many conflicting can coexist within a common environment. Besides, the numerical experiments give a guarantee that a balance can be reached and the organisms can overcomes the danger of extinction.

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