Neural dynamic optimization algorithm based on event triggered algorithm and its application

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Abstract. Wireless sensor networks consist of microprocessor controlled sensors that communicate with each other over multi-hop communication networks. In WSN, the energy consumption of sensor networks for communication can be obviously bigger than the energy required to operate computation that would bring us unimaginable benefits if communication and computation between each node can be somehow isolated. In this paper, a neurodynamic optimization approach is proposed based on the event-triggered algorithm for handling standard NUM problem in WSN. We first confirm that the equilibrium point set of the designed neural network model based on event-triggered algorithm corresponds to the optimal solution of the NUM problem. Then, it is proved that the proposed neural network model is stable in the sense of Lyapunov and is convergent to the optimal solution. Finally, a numerical example is provided to illustrate the performance of the proposed neural network.

Keywords: wireless sensor networks, neural network, distributed optimization, event triggered, network utility maximization.

1. Introduction

In recent years, the distributed optimization algorithm, which is more powerful than traditional optimization in large-scale problems, has attracted attention from more and more researchers. Various optimization problems in sensor networks, smart grids, computation, etc. [1, 2, 3, 4], have been studied by using distributed algorithms.

The *wireless sensor network* (WSN) consists of nodes with limited energy and memory and helps to monitor the far located areas out of the human reach. The sensor nodes gather and send information to the public mobile communication base station. The nodes can only communicate with the nearby nodes. The human administrators control the sensor network convey orders in time and get reactions throughout the public mobile communication base station. The sensor nodes and the battery are both small in size only to provide limited energy and power to the nodes.

Generally speaking, it is impractical and sometimes impossible to replace the battery to maintain a longer network lifetime. The sensor nodes utilize high amount of energy in sensing the environmental activities and communicating with other nodes in the network, sensation and communication with affects the network lifetime. The lifetime of the network can be increased by using various protocols that conserve the residual energy of the sensor nodes [5]. For instance, an artificial bee colony algorithm can be applied to extend the network lifetime [6]. The reference [7] proposed distributed algorithms to calculate and compute the best routing scheme that maximizes the time where the initial node in the network runs out of energy.

One way of conserving energy of the sensor nodes is reducing the complexity of messaging by means of applying a novel distributed algorithms called the *event-triggered algorithm*. Under the event-triggered algorithm, each agent sends information from itself to its neighbours when a local “error” signal is bigger than a state dependent threshold. The activation of the event-triggered system is due to the occurrence of a major event. In a time-triggered system, the activities are initiated periodically at a preset point in real-time [8]. In order to reduce the demand of communication in smart grid, Li et al. [9] presented a distributed optimization approach based on an event-triggered communication under the economic dispatch problem. In order to reduce the demand of information and communication in WSN, Pu and Lemmon [10] presented a distributed optimization approach based on the event-triggered communication. Similar approaches of resource allocation were used in [11] and [12]. However, these approaches are traditional optimization algorithms and lower in efficiency in large-scale computing problems. Because of the inborn large-scale parallelism, the neural network method can solve optimization problems in calculating time at the order of magnitude. It is much faster than those optimization algorithms implemented on general-purpose digital computers [13].

Since the mid-1980s, the neurodynamic optimization approach based on continuous-time *recurrent neural networks* (RNNs) has been extensively studied. Hopfield and Tank [14] proposed a neural network approach to solve linear programming problems. In [15], an RNN to handle a class of nonlinear optimization problem was discovered by Kennedy and Chua. Since then, many neural network models were proposed and studied. Yan, Wang and Li [16] presented a neurodynamic approach for bound-constrained global optimization problem. Qin et al. [17] presented a neurodynamic approach for solving a class of convex optimization problems with equality and inequality constraints. In [18], a complex-valued neural network was presented to handle a class of complex-valued nonlinear convex optimization problem. In [19], a one-layer RNN was proposed for solving a class of non-linear non-smooth pseudoconvex optimization problem with linear equality constraints. Recently, a collaborative neurodynamic approach to multiobjective optimization was presented to attain both goals of pareto optimality and solution diversity[20]. The collaborative neurodynamic approach demonstrates higher efficiency in seeking for the global optimal solution [21]. A collaborative neurodynamic approach based on the distributed constrained optimization was proposed in [22]. A collaborative neurodynamic optimization approach for solving global and combinatorial optimization was designed in [23]. [16] presented a new collaborative neurodynamic optimization approach for solving a class of nonconvex optimization problems with bound constraints.

Because of the inherent massive parallelism, the neurodynamic optimization approach can solve optimization problems in running time much faster than those of the most popular optimization algorithms executed on general-purpose digital computers [25]. However, in WSN, there is a dearth of literature on the neurodynamic optimization algorithms. In this paper, we use a neurodynamic optimization approach to solve the *network utility maximization* (NUM) problems in WSN.

The rest of this paper is organized as follows: Some basic concepts of the problem are introduced in Section 2. Descriptions of the neurodynamic approach are provided in Section 3. In Section 4, the simulation examples are given to show the performance and effectiveness of the proposed neural network model. In Section 5, we give the conclusion of this paper.

2. Problem formulation

In this section, we introduce some background information. The data gathering problem in WSN studied in [10] is formulated as

$$\begin{array}{ll} \max & U(x) = \sum_{i \in V} U_i(x_i) \\ \text{s.t.} & Ax \le \bar{c} \\ & (e_{tx} + e_{rx})Ax - e_{rx}x \le b(t), \end{array}$$

where $x = [x_1, x_2, \cdots, x_N]^T$, $x_i \in \mathbb{R}$ and $x_i \ge 0$ stands for the data rate of the node i. $\bar{c} \in \mathbb{R}^M$ is the vector of node capacitie. $A \in \mathbb{R}^{N \times N}$ is the routing matrix of the relaying relationship between nodes. A_{ji}, the ji-th component, is 1 if node i communicates with j and is 0 if node i does not communicate with j. The j-th row of Ax means the total data rates node j requires to send, which is not higher than its capacity $\bar{c}_j$. e_{tx} represents the energy used in transmitting and e_{rx} represents the energy used in receiving one unit of data. In the last inequality constraint, $b(t) = [b_1(t), b_2(t), \cdots, b_N(t)]^T$ represents the expected energy rate of reduction on node i at instant t. The cost function U is the sum of the node utility functions $U_i(x_i)$. By simplifying the notations and descriptions, we obtain that

$$\begin{array}{ll} \max & U(x) = \sum_{i \in V} U_i\left(x_i\right) \\ \text{s.t.} & Ax \leqslant c, \quad x \geqslant 0, \end{array} \tag{1}$$

where

$$c = \min\left\{\bar{c}, \frac{b(t)}{e_{tx} + e_{rt}}\right\}.$$

Equation (1) is a NUM problem and c is a constant.

In NUM problem, we have the following equation by applying the augmented Lagrangian method,

$$\begin{aligned} \bar{L}(x, s; \lambda, w) = & -\sum_{i \in V} U_i\left(x_i\right) + \sum_{j \in V} \lambda_j \left(a_j^T x - c_j + s_j\right) \\ & + \tfrac{1}{2} \sum_{j \in V} \tfrac{1}{w_j} \left(a_j^T x - c_j + s_j\right), \end{aligned} \tag{2}$$

where $s \in \mathbb{R}^n$ represents the slack variable and $s_j \ge 0, j \in \mathcal{V}$. The vector $a_j^T = [A_{j1}, A_{j2}, \cdots, A_{jN}]$ is the j-th row of the routing matrix A. The penalty parameter w_j is related to all constraints, and $w = diag\{w_1, w_2 \cdots, w_N\}$ is the diagonal matrix and the elements in the matrix are made up of penalty parameters. Under the Karush-Kuhn-Tucker condition, λ_j is the Lagrange multiplier as related to node constraint $c_j - a_j^T x \ge 0$.

Equation (2) can be rewritten as

$$L(x;\lambda,w)=\sum_{j\in V}\psi_j(x;\lambda,w)\ -\sum_{i\in V}U_i\left(x_i\right), \tag{3}$$

where

$$\psi_j(x;\lambda,w)=\begin{cases}-\frac{1}{2}w_j\lambda_j^2, & \text{if } c_j-a_j^Tx-w_j\lambda_j\geq 0,\\ \lambda_j\left(a^Tx-c_j\right)+\frac{1}{2w_j}\left(a^Tx-c_j\right)^2, & \text{otherwise.}\end{cases}$$

The minimizer of $L(x;\lambda,w)$ is sufficiently approximated to the solution of problem (1) when $\lambda_j=0$ and w_j is sufficiently small. The algorithm is as follows [24]:
Step 1. Select initial data rate $x^0>0$, let $\lambda_j=0$ and sufficiently small $w_j>0,\ j\in V$.
Step 2. Minimize $L(x;\lambda,w)$, where γ is a sufficiently small step size,

$$x=\max\left\{0,x^0-\gamma\nabla_xL\left(x^0;\lambda,w\right)\right\},\quad x^0=x.$$

In paper [10], we know that ρ is a constant, $\bar{L}$ is the maximum number of relay nodes and $\bar{S}$ is the maximum number of nodes. For all $i\in\mathcal{V}$, $\ell\in\mathbb{R}^+$ and any $t\in\left[T_j^L[\ell],T_j^L[\ell+1]\right)$,

$$\begin{aligned}z_i(t)&=\dot{x}_i(t)=\left(\nabla U_i\left(x_i(t)\right)-\sum_{j\in\mathcal{L}_i}\hat{\mu}_j(t)\right)^+_{x_i(t)},\\ \mu_j(t)&=\frac{1}{w_j}\left(a_j^Tx(t)-c_j\right)^+,\\ \hat{z}_i(t)&=z_i\left(T_i^S[\ell]\right).\end{aligned}$$

$T_i^S[\ell]$ is the ℓ- th time instant when node i sends information of its user state to all nodes $j\in\mathcal{L}_i$.

$$\hat{\mu}_j(t)=\mu_j\left(T_j^L[\ell]\right).$$

The sequence $T_j^L[\ell]$ represents time instants when node j transmits its link state to the relay nodes. Then, we have the following lemma.

Lemma 1 ([10])**.** *The data rate $x(t)$ asymptotically converges to the unique minimizer of $L(x;\lambda,w)$, while*

$$z_i^2(t)-\rho\hat{z}_i^2(t)\geq 0,$$

for $t\in\left[T_i^S[\ell],T_i^S[\ell+1]\right)$,

$$\rho\sum_{i\in\mathcal{S}_j}\frac{1}{\bar{L}}\hat{z}_i^2(t)-\bar{L}\bar{S}\left(\mu_j(t)-\hat{\mu}_j(t)\right)^2\geq 0,$$

for $t\in\left[T_i^L[\ell],T_i^L[\ell+1]\right)$.

3. Neural network model

Lemma 1 supplies the infrastructure for constructing a neurodynamic optimization approach based on the event-triggered message-passing protocol. Accordingly, we propose the following neural network based on the event-triggered algorithm adopted from [10]

$$\begin{cases} \dot{x}_i = x_i - (x_i + \gamma (z_i - (\mu_j - \hat{\mu}_j) A_{ij}))^+ \\ \hat{z}_i(t) = \begin{cases} z_i(T^+), \ z_i \geq \sqrt{\rho}\hat{z}_i \\ z_i(t), \ z_i < \sqrt{\rho}\hat{z}_i \end{cases} \\ \hat{\mu}_j(t) = \begin{cases} \mu_j(T^+), \ \sqrt{\rho}\frac{1}{\sqrt{\bar{L}}}\hat{z}_i \geq \sqrt{\bar{L}\bar{S}}(\mu_j - \hat{\mu}_j) \\ \mu_j(t), \ \sqrt{\rho}\frac{1}{\sqrt{\bar{L}}}\hat{z}_i < \sqrt{\bar{L}\bar{S}}(\mu_j - \hat{\mu}_j), \end{cases} \end{cases} \tag{4}$$

where, $\bar{L}\sqrt{\bar{S}} \leq A_{ji}$, T^+ is the time instant when $z_i \leqslant \sqrt{\rho}\hat{z}_i$ and $\sqrt{\rho}\frac{1}{\sqrt{\bar{L}}}\hat{z}_i \leqslant \sqrt{\bar{L}\bar{S}}(\mu_j - \hat{\mu}_j)$.

Lemma 2. *System* (4) *is convergent to the unique optimal solution of problem* (1).

Proof. Let
$$M(x) = -[z - (\mu - \hat{\mu})A] = M,$$
then, we have
$$\dot{x} = x - (x - \gamma M)^+.$$

The dynamic equation of the proposed continuous-time projection neural network model is

$$\begin{aligned} &\frac{dy}{dt} = g(y) - \gamma M(g(y)) - y \\ \textit{output equation} \quad &x = g(y). \end{aligned} \tag{5}$$

Assume that x^* is the solution of (4). According to

$$x^* = [-\gamma M(x^*) + x^*]^+,$$

we obtain that
$$x^* = g(-\gamma M(x^*) + x^*),$$
where $g(x)$ is the projection operator.

Let $y^* = -\gamma M(x^*) + x^*$, then $x^* = g(y^*)$. It follows that

$$\begin{aligned} y^* &= -\gamma M(x^*) + g(y^*), \\ y^* &= -\gamma M(g(y^*)) + g(y^*). \end{aligned}$$

Then $0 = g(y^*) - \gamma M(x^*) - y^*$, thus y^* is an equilibrium point of the system

$$\frac{dy}{dt} = g(y) - \gamma M(g(y)) - y.$$

Assume $\bar{y}$ is an equilibrium point of (5), they satisfies

$$\bar{x} - \gamma M(\bar{x}) - \bar{y} = 0,$$

where $\bar{x} = g(\bar{y})$.

Let

$$\bar{M} = M(\bar{x}) = M(g(\bar{y})) = \left[\bar{z}_i - (\bar{\mu}_j - \bar{\hat{\mu}}_j)A_{ji}\right],$$

where $\bar{z}_i = \bar{z}_i(x) = z_i(\bar{x})$ and $\bar{\hat{\mu}}_j = \bar{\hat{\mu}}_j(x) = \hat{\mu}_j(\bar{x})$.

According to

$$-\bar{y} + \bar{x} - \gamma M(\bar{x}) = 0$$

and

$$\frac{dy}{dt} = -y + g(y) - \gamma M(g(y)),$$

we put the equilibrium point to the origin

$$\frac{dy}{dt} = -\gamma(M(x) - M(\bar{x})) - (y - \bar{y}) + (x - \bar{x}).$$

Consider the following Lyapunov function $V(y) = \|y - \bar{y}\|^2$. According to the chain rule, we have

$$\begin{aligned}
\dot{V}(t) &= \dot{y}(t)(\nabla V(y))^T \\
&= 2[-(y-\bar{y}) + (x-\bar{x}) - \gamma(M - \bar{M}))](y-\bar{y})^T \\
&= -2(y-\bar{y})^T(y-\bar{y}) + 2(y-\bar{y})^T(x-\bar{x}) \\
&\quad - 2\gamma(y-\bar{y})^T(M-\bar{M})) \\
&= 2\gamma\left\{(x-\bar{x})^T\left(M-\bar{M}\right) - \left(M-\bar{M}\right)\right\}.
\end{aligned}$$

According to Lemma of variational inequality [26], we have

$$\begin{aligned}
(x-\bar{x})^T\left(M-\bar{M}\right) &= (x-\bar{x})^T\left(M-\bar{M}+\bar{x}-\bar{x}\right) \\
&= -(x-\bar{x})^T\left(\bar{x}-(M-\bar{M}+\bar{x})\right) \le 0,
\end{aligned}$$

when $\left(M-\bar{M}\right) \ge 0$, we have $\dot{V} \le 0$.

According to [10], we have $\bar{\mu}_j = \bar{\hat{\mu}}_j, \bar{z}_i = 0$ then

$$\begin{aligned}
\left(M-\bar{M}\right) &= -[(z-\bar{z}) - A_{ji}\left((\mu-\hat{\mu}_j) - (\bar{\mu}-\bar{\hat{\mu}}_j)\right)] \\
&= -[z - A_{ji}(\mu-\hat{\mu}_j)] \\
&= -[z - \sqrt{\rho}\hat{z} + \sqrt{\rho}\hat{z} - A_{ji}(\mu-\hat{\mu}_j)].
\end{aligned}$$

In the conditions of the proposed model, $z_i \leqslant \sqrt{\rho}\hat{z}_i,\ \sqrt{\rho}\frac{1}{\sqrt{\bar{L}}}\hat{z}_i \leqslant \sqrt{\bar{L}\bar{S}}(\mu_j - \hat{\mu}_j),\ \bar{L}\sqrt{\bar{S}} \le A_{ji}$, we have $z_i \leqslant \sqrt{\rho}\hat{z}_i,\ \sqrt{\rho}\hat{z}_i \leqslant A_{ji}(\mu_j - \hat{\mu}_j)$, then $\left(M-\bar{M}\right) \ge 0$, thus, we have $\dot{V} \le 0$ then, system (4) is convergent to the unique optimal solution of problem (1). □

4. Illustrative example

In this part, the effectiveness of the neural dynamic optimization approach based on event-triggered algorithm is demonstrated by a simulation example.

Consider the following NUM problem

$$\begin{array}{ll} \min & U(x) \\ \text{s.t.} & Ax \leq C \end{array} \tag{6}$$

As a special case, the number of communication nodes we consider is 2, and the node utility function $U(x) = x_1^2 + x_2^2$.

Let $C = [2, 3]^T$, $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, $\rho = 0.25$, $\lambda = 0$, $\gamma = 0.09$, $\omega = 0.05$, and $x \in [0, 10]$. We apply the neurodynamic optimization approach based on event-triggered algorithm (4) to solve this example. By the above assumptions, as shown in Figure 1 and Figure 2, it can be seen that all trajectories converge to the solutions 2.223 and 3.337.

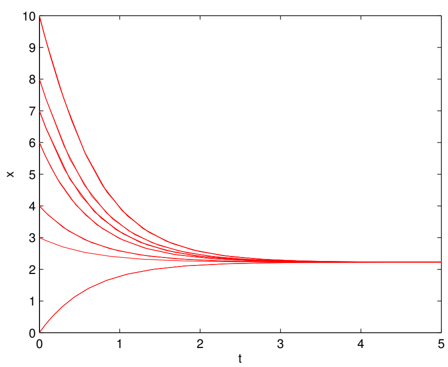

Figure 1: Transient behaviors of x_1

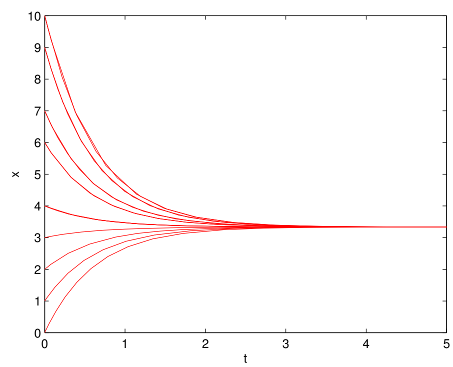

Figure 2: Transient behaviors of x_2

5. Conclusions

In this paper, we propose a neurodynamic optimization approach based on event-triggered algorithm for solving the NUM problems in WSN. The paper shows that the proposed neural network model based on event-triggered algorithm is stable in the sense of Lyapunov and converges to the optimal solution under event-triggered mechanism. Moreover, in traditional optimization algorithms, their efficiency is lower than the neurodynamic optimization approach. Finally, the effectiveness of the neural dynamic optimization method based on event-triggered algorithm is demonstrated by a simulation example. In the future study, we will look more into the neurodynamic optimization approach based on time-triggered algorithm.

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