Decision making application based on parameterization of fuzzy hypersoft set with fuzzy setting

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Abstract. Fuzzy soft set is considered as an apt parameterization tool to deal with vagueness and uncertainties. There are many situations when attributes need to be further classified into attribute-valued disjoint sets. Such situations are not tackled by the existing fuzzy soft-like structures. Hypersoft set, an extension of soft set, employs multi-argument approximate function which addresses the inadequacy of existing structures for attribute-valued disjoint sets. In this study, theory of fuzzy parameterized fuzzy hypersoft set is characterized and some of its essential properties are discussed. In order to cope with the decision making problems, an algorithm is proposed which employs the fuzzy decision set of fuzzy parameterized fuzzy hypersoft set for dealing with decision making problems under uncertain scenarios. The proposed algorithm is validated with the help of a numerical example.

Keywords: fuzzy set, soft set, fuzzy parameterized soft set, hypersoft set, decision making.

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1. Introduction

In 1965, Zadeh [1] initiated the theory of fuzzy set (FST) as an emerging field of mathematics to cope with the uncertainty-based scenarios. There are many real life cases (i.e. decision making setting) where some sorts of parameterization tools are needed to handle problems having base on certain attributes. The existing theories including FST dealing uncertainties, are insufficient for such kind of setting. In 1999, Molodtsov [2] addressed this limitation with the introduction of new theory called soft set theory (SST) which is a parameterized family of subsets of universal set. Maji et al. [3] extended the concept and introduced some fundamental terminologies and operations of soft set. They also defined fuzzy soft set in [4] and successfully applied it in decision making. Some authors [5, 6, 7, 8, 9, 10, 11] discussed some more properties and relational features of soft set theory. Çağman et al. [12, 13] conceptualized fuzzy parameterized soft set which is the ordered pair of membership function of fuzzy set and approximate function of fuzzy soft. They discussed its properties and applied this theory to different fields. Kamacı [15] characterized the theory of interval-valued fuzzy parameterized intuitionistic fuzzy soft sets and discussed its applications.

In 2018, Smarandache [14] developed a new structure hypersoft set (HSS) to adequate the soft set for multi attribute-valued functions. In 2020, Saeed et al. [16, 17] characterized the necessary basic axioms, properties, laws and set theoretic operations of HSS with the help of appropriate examples. In 2020, Rahman et al. [18, 19] enhanced the study of HSS to develop fuzzy-like structures with complex sets and also studied certain properties of convexity under HSS-environment. In 2021, Rahman et al. [20, 21, 22] studied decision making applications based on neutrosophic parameterized hypersoft set, fuzzy parameterized hypersoft set and rough hypersoft set. Saeed et al. [23, 24, 25, 26, 27] discussed decision making techniques for mappings on hypersoft classes, neutrosophic hypersoft mapping and complex multi-fuzzy hypersoft set. They also developed hypersoft graphs with some properties. Ihsan et al. [28] investigated hypersoft expert set with application in decision making for the best selection of product. Yolcu et al. [29, 30] conceptualized the theories of fuzzy and intuitionistic fuzzy hypersoft sets with their employment in decision making. Ozturk et al. [31] introduced neutrosophic hypersoft topological spaces and discussed their important properties. Saqlain et al. [32, 33, 34] developed single and multi-valued neutrosophic hypersoft sets along with calculation of tangent similarity measure of single valued neutrosophic hypersoft sets, characterized aggregate operators of neutrosophic hypersoft set and employed TOPSIS method for neutrosophic hypersoft sets using accuracy function with application. Kamacı [35] made very valuable research on the hybrid structures of hypersoft sets and rough sets. Recently Kamacı et al. [36] investigated some new hybridized structures i.e. n-ary fuzzy hypersoft expert sets which are the extensions of existing structures (i.e. n-ary fuzzy soft expert sets).

1.1 Motivation

In many real life situations, distinct attributes are further partitioned into disjoint attribute-valued sets. Decision makers may suffer some kind of inclination and penchant while ignoring such partitioning of attributes during the judgment. The existing soft set theory is not projected for such sets. Therefore a new structure demands its place in literature for addressing such impediment, so hypersoft set theory is conceptualized to tackle such situations. This novel structure has increased the flexibility and reliability of decision making process. It not only addresses the inadequacy of existing soft-like structures for multi-argument approximate functions but also helps the decision makers to decide the matters with deep observation. Although this structure face some kind of challenge (limitation) regarding the consideration of overlapping attribute-valued sets. Motivating from the work discussed in [12, 14, 17], fuzzy parameterized fuzzy hypersoft set is characterized in order to adequate the literature regarding fuzzy parameterized soft set for multi attribute-valued approximate functions and its some essential elementary properties are discussed. A decision making based algorithm is proposed with successful application for the best choice of product.

1.2 Paper layout

The rest of the paper is organized as:

In section 2, some fundamental definitions and terms are recalled from already published relevant literature, section 3 formulates the theory of fuzzy parameterized fuzzy hypersoft set, section 4 proposes a decision-making algorithm based on decision system of fuzzy parameterized fuzzy hypersoft set with application for the best selection of an appropriate product, section 5 presents the validation for generalization and comparison analysis of proposed model and finally, section 6 concludes the paper with future directions.

2. Preliminary

In this section some basic terms are recalled from existing literature to support the proposed work. Throughout the paper, $\mathcal{U}$, $P(\mathcal{U})$ and $\mathbb{I}^\bullet$ will denote the universe of discourse, power set of $\mathcal{U}$ and closed unit interval respectively.

Definition 2.1 ([1]). *A* fuzzy set $\mathcal{X} \subseteq \mathcal{U}$ *is defined as* $\mathcal{X} = \{(\epsilon, \zeta_{\mathcal{X}}(\epsilon)) | \epsilon \in \mathcal{U}\}$ *such that* $\zeta_{\mathcal{X}} : \mathcal{U} \to \mathbb{I}^\bullet$ *where* $\zeta_{\mathcal{X}}(\epsilon)$ *denotes the belonging value of* $\epsilon \in \mathcal{X}$.

Definition 2.2 ([2]). *A pair* (ζ_S, Λ) *is called a* soft set *over* $\mathcal{U}$*, where* $\zeta_S : \Lambda \to P(\mathcal{U})$ *and* Λ *be a set of attributes.*

Definition 2.3 ([3]). *A soft set* (ζ_{S_1}, Λ_1) *is a* soft subset *of another soft set* (ζ_{S_2}, Λ_2) *if*

(*i*) $\Lambda_1 \subseteq \Lambda_2$;

(ii) $\forall \omega \in \Lambda_1, \zeta_{S_1}(\omega)$ *and* $\zeta_{S_2}(\omega)$ *are identical approximations.*

For more detail on soft set, see [3]-[11]

Definition 2.4 ([18]). *The pair* (Ψ, G) *is called a hypersoft set over* $\mathcal{U}$, *where* G *is the Cartesian product of* n *disjoint sets* $G_1, G_2, G_3, ..., G_n$ *having attribute values of* n *distinct attributes* $g_1, g_2, g_3, ..., g_n$ *respectively and* $\Psi : G \to P(\mathcal{U})$.

For more definitions and operations of hypersoft set (see, [16]-[18]).

3. Fuzzy parameterized fuzzy hypersoft set ($fpfhs$-set) theory

In this section, fuzzy parameterized fuzzy hypersoft set is conceptualized and some of its fundamentals are discussed.

Definition 3.1. *Let* $\mathcal{A} = \{\mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_3, ..., \mathcal{A}_n\}$ *be a collection of disjoint attribute-valued sets corresponding to* n *distinct attributes* $\alpha_1, \alpha_2, \alpha_3, ..., \alpha_n$ *respectively. An FPF-hypersoft set (*$fpfhs$*-set)* $\Psi_{\mathcal{W}}$ *over* $\mathcal{U}$ *is defined as*

$$\Psi_{\mathcal{W}} = \left\{ \begin{array}{l} (\zeta_{\mathcal{W}}(g)/g, \psi_{\mathcal{W}}(g)) : \\ g \in G, \ \psi_{\mathcal{W}}(g) \in F(\mathcal{U}), \ \zeta_{\mathcal{W}}(g) \in \mathbb{I}^{\bullet} \end{array} \right\}$$

where

(i) $F(\mathcal{U})$ *is a collection of all fuzzy sets over* $\mathcal{U}$

(ii) $G = \mathcal{A}_1 \times \mathcal{A}_2 \times \mathcal{A}_3 \times .... \times \mathcal{A}_n$

(iii) $\mathcal{W}$ *is a fuzzy set over* G *with* $\zeta_{\mathcal{W}} : G \to \mathbb{I}^{\bullet}$ *as membership function of* $fpfhs$*-set.*

(iv) $\psi_{\mathcal{W}}(g)$ *is a fuzzy set for all* $g \in G$ *with* $\psi_{\mathcal{W}} : G \to F(\mathcal{U})$ *and is called approximate function of* $fpfhs$*-set.*

Now, throughout the remaining part of the paper, $\Omega_{FPFHS}(\mathcal{U})$, $\zeta_{\mathcal{W}}$ *and* $\psi_{\mathcal{W}}$ *will represent the collection of all* $fpfhs$*-sets over* $\mathcal{U}$*, membership function and approximate function respectively.*

Example 3.1. Suppose that Mrs. Andrew wants to buy a washing machine from market. There are eight kinds of washing machines (options) which form the set of discourse $\mathbb{X} = \{\hat{W}_1, \hat{W}_2, \hat{W}_3, \hat{W}_4, \hat{W}_5, \hat{W}_6, \hat{W}_7, \hat{W}_8\}$. The best selection may be evaluated by observing the attributes i.e. b_1 = Company, b_2 = Power in Watts, b_3 = Voltage, b_4 = Capacity in kg, and b_5 = Color. The attribute-valued sets corresponding to these attributes are:
$B_1 = \{b_{11} = National, b_{12} = Hier\}$,
$B_2 = \{b_{21} = 400, b_{22} = 500\}$,
$B_3 = \{b_{31} = 220, b_{32} = 240\}$,
$B_4 = \{b_{41} = 7, b_{42} = 10\}$,
$B_5 = \{b_{51} = White\}$,
then $\mathbb{Q} = B_1 \times B_2 \times B_3 \times B_4 \times B_5$,

$\mathbb{Q} = \{q_1, q_2, q_3, q_4, ..., q_{16}\}$ where each $q_i, i = 1, 2, ..., 16$, is a 5-tuples element. we can construct $fpfhs$-set $\Psi_{\mathcal{W}}$ as $\Psi_{\mathcal{W}} =$

$$\left\{\begin{array}{l} (0.1/q_1, \{0.21/W_1, 0.51/W_2\}), (0.2/q_2, \{0.32/W_1, 0.16/W_2, 0.1/W_3\}), \\ (0.3/q_3, \{0.34/W_2, 0.46/W_6, 0.48/W_8\}), (0.4/q_4, \{0.63/W_4, 0.13/W_5, 0.91/W_6\}), \\ (0.5/q_5, \{0.42/W_4, 0.53/W_5, 0.84/W_8\}), (0.6/q_6, \{0.23/W_2, 0.54/W_4, 0.35/W_7\}), \\ (0.7/q_7, \{0.81/W_1, 0.72/W_3, 0.43/W_5\}), (0.8/q_8, \{0.14/W_2, 0.35/W_3, 0.36/W_7\}), \\ (0.9/q_9, \{0.34/W_2, 0.65/W_3, 0.86/W_6\}), (0.16/q_{10}, \{0.27/W_6, 0.68/W_7, 0.29/W_8\}), \\ (0.25/q_{11}, \{0.22/W_2, 0.63/W_4, 0.14/W_7\}), (0.45/q_{12}, \{0.15/W_1, 0.36/W_3, 0.77/W_7\}), \\ (0.35/q_{13}, \{0.11/W_2, 0.12/W_3, 0.93/W_5\}), (0.75/q_{14}, \{0.43/W_1, 0.75/W_5, 0.37/W_8\}), \\ (0.65/q_{15}, \{0.27/W_2, 0.15/W_3, 0.23/W_8\}), (0.85/q_{16}, \{0.46/W_4, 0.64/W_5, 0.27/W_6\}) \end{array}\right\}$$

Definition 3.2. *Let $\Psi_{\mathcal{W}} \in \Omega_{FPFHS}(\mathcal{U})$. If $\zeta_{\mathcal{W}}(g) = 0$ and $\psi_{\mathcal{W}}(g) = \phi$, for all $g \in G$, then $\Psi_{\mathcal{W}}$ is called an $\mathcal{W}$-empty $fpfhs$-set, denoted by $\Psi_{\Phi_{\mathcal{W}}}$. If $\mathcal{W} = \phi$, then $\Psi_{\mathcal{W}}$ is called an empty $fpfhs$-set, denoted by Ψ_{Φ}.*

Definition 3.3. *Let $\Psi_{\mathcal{W}} \in \Omega_{FPFHS}(\mathcal{U})$. If $\zeta_{\mathcal{W}}(g) = 1$ and $\psi_{\mathcal{W}}(g) = \mathcal{U}$ for all $g \in \mathcal{W}$, then $\Psi_{\mathcal{W}}$ is called $\mathcal{W}$-universal $fpfhs$-set, denoted by $\Psi_{\tilde{\mathcal{W}}}$. If $\mathcal{W} = G$, then the $\mathcal{W}$-universal $fpfhs$-set is called universal $fpfhs$-set, denoted by $\Psi_{\tilde{G}}$.*

Example 3.2. Consider $\mathcal{U} = \{u_1, u_2, u_3, u_4, u_5\}$ and $\mathcal{A} = \{\mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_3\}$ with $\mathcal{A}_1 = \{a_{11}, a_{12}\}$, $\mathcal{A}_2 = \{a_{21}, a_{22}\}, \mathcal{A}_3 = \{a_{31}\}$, then $G = \mathcal{A}_1 \times \mathcal{A}_2 \times \mathcal{A}_3$

$$G = \left\{\begin{array}{l} (a_{11}, a_{21}, a_{31}), (a_{11}, a_{22}, a_{31}), \\ (a_{12}, a_{21}, a_{31}), (a_{12}, a_{22}, a_{31}) \end{array}\right\}$$
$$G = \{g_1, g_2, g_3, g_4\}.$$

Case 1. If $\mathcal{W}_1 = \{0.2/g_2, 0.5/g_3, 1.0/g_4\}$ and $\psi_{\mathcal{W}_1}(g_2) = \{0.4/u_2, 0.6/u_4\}$, $\psi_{\mathcal{W}_1}(g_3) = \phi$, and $\psi_{\mathcal{W}_1}(g_4){=}\mathcal{U}$, then $\Psi_{\mathcal{W}_1}{=}\{(0.2/g_2, \{0.4/u_2, 0.6/u_4\}), (0.5/g_3, \phi)$, $(1.0/g_4, U)\}$.
Case 2. If $\mathcal{W}_2 = \{0/g_2, 0/g_3\}, \psi_{\mathcal{W}_2}(g_2) = \phi$ and $\psi_{\mathcal{W}_2}(g_3) = \phi$, then $\Psi_{\mathcal{W}_2} = \Psi_{\Phi_{\mathcal{W}_2}}$.
Case 3. If $\mathcal{W}_3 = \phi$, then $\Psi_{\mathcal{W}_3} = \Psi_{\Phi}$.
Case 4. If $\mathcal{W}_4 = \{1.0/g_1, 1.0/g_2, 1.0/g_3, 1.0/g_4\}, \psi_{\mathcal{W}_4}(g_1) = \psi_{\mathcal{W}_4}(g_2) = \psi_{\mathcal{W}_4}(g_3)$ $= \psi_{\mathcal{W}_4}(g_4) = \mathcal{U}$, then $\Psi_{\mathcal{W}_4} = \Psi_{\tilde{\mathcal{W}}_4}$.

Definition 3.4. *Let $\Psi_{\mathcal{W}_1}$, $\Psi_{\mathcal{W}_2} \in \Omega_{FPFHS}(\mathcal{U})$ then $\Psi_{\mathcal{W}_1}$ is an $fpfhs$-subset of $\Psi_{\mathcal{W}_2}$, denoted by $\Psi_{\mathcal{W}_1} \tilde{\subseteq} \Psi_{\mathcal{W}_2}$ if $\zeta_{\mathcal{W}_1}(g) \leq \zeta_{\mathcal{W}_2}(g)$ and $\psi_{\mathcal{W}_1}(g) \subseteq \psi_{\mathcal{W}_2}(g)$ for all $g \in G$.*

Proposition 3.1. *Let $\Psi_{\mathcal{W}_1}, \Psi_{\mathcal{W}_2}, \Psi_{\mathcal{W}_3} \in \Omega_{FPFHS}(\mathcal{U})$ then:*

(i) $\Psi_{\mathcal{W}_1} \tilde{\subseteq} \Psi_{\tilde{G}}$;

(ii) $\Psi_{\phi} \tilde{\subseteq} \Psi_{\mathcal{W}_1}$;

(iii) $\Psi_{\mathcal{W}_1} \tilde{\subseteq} \Psi_{\mathcal{W}_1}$;

(iv) $\Psi_{\mathcal{W}_1} \tilde{\subseteq} \Psi_{\mathcal{W}_2}$; $\Psi_{\mathcal{W}_2} \tilde{\subseteq} \Psi_{\mathcal{W}_3} \Rightarrow \Psi_{\mathcal{W}_1} \tilde{\subseteq} \Psi_{\mathcal{W}_3}$.

Proof. The proofs of (i)- (iv) can be obtained easily by using Definition 3.4. □

Definition 3.5. *Let $\Psi_{\mathcal{W}_1}, \Psi_{\mathcal{W}_2} \in \Omega_{FPFHS}(\mathcal{U})$ then, $\Psi_{\mathcal{W}_1}$ and $\Psi_{\mathcal{W}_2}$ are fpfhs-equal, represented as $\Psi_{\mathcal{W}_1} = \Psi_{\mathcal{W}_2}$, iff $\zeta_{\mathcal{W}_1}(x) = \zeta_{\mathcal{W}_2}(x)$ and $\psi_{\mathcal{W}_1}(g) = \psi_{\mathcal{W}_2}(g)$ for all $g \in G$.*

Proposition 3.2. *Let $\Psi_{\mathcal{W}_1}, \Psi_{\mathcal{W}_2}, \Psi_{\mathcal{W}_3} \in \Omega_{FPFHS}(\mathcal{U})$ then:*

(i) If $\Psi_{\mathcal{W}_1} = \Psi_{\mathcal{W}_2}$ and $\Psi_{\mathcal{W}_2} = \Psi_{\mathcal{W}_3} \Rightarrow \Psi_{\mathcal{W}_1} = \Psi_{\mathcal{W}_3}$.

(ii) $\Psi_{\mathcal{W}_1} \tilde{\subseteq} \Psi_{\mathcal{W}_2}$ and $\Psi_{\mathcal{W}_2} \tilde{\subseteq} \Psi_{\mathcal{W}_1} \Leftrightarrow \Psi_{\mathcal{W}_1} = \Psi_{\mathcal{W}_2}$.

Proof. The properties (i)- (ii) can be proved easily by using Definition 3.5. □

Definition 3.6. *Let $\Psi_{\mathcal{W}} \in \Omega_{FPFHS}(\mathcal{U})$ then, complement of $\Psi_{\mathcal{W}}$ (i.e. $\Psi^{\tilde{c}}_{\mathcal{W}}$) is an fpfhs-set given as $\zeta^{\tilde{c}}_{\mathcal{W}}(g) = 1 - \zeta_{\mathcal{W}}(g)$ and $\psi^{\tilde{c}}_{\mathcal{W}}(g) = \mathcal{U} \setminus \psi_{\mathcal{W}}(g)$*

Proposition 3.3. *Let $\Psi_{\mathcal{W}} \in \Omega_{FPFHS}(\mathcal{U})$ then,*

- $(\Psi^{\tilde{c}}_{\mathcal{W}})^{\tilde{c}} = \Psi_{\mathcal{W}}$.
- $\Psi^{\tilde{c}}_{\Phi} = \Psi_{\tilde{G}}$.

Definition 3.7. *Let $\Psi_{\mathcal{W}_1}, \Psi_{\mathcal{W}_2} \in \Omega_{FPFHS}(\mathcal{U})$ then, union of $\Psi_{\mathcal{W}_1}$ and $\Psi_{\mathcal{W}_2}$, denoted by $\Psi_{\mathcal{W}_1} \tilde{\cup} \Psi_{\mathcal{W}_2}$, is defined by*

- $\zeta_{\mathcal{W}_1 \tilde{\cup} \mathcal{W}_2}(g) = \max\{\zeta_{\mathcal{W}_1}(x), \zeta_{\mathcal{W}_2}(g)\}$ *and*
- $\psi_{\mathcal{W}_1 \tilde{\cup} \mathcal{W}_2}(g) = \psi_{\mathcal{W}_1}(g) \cup \psi_{\mathcal{W}_2}(g)$, *for all $g \in G$.*

Proposition 3.4. *Let $\Psi_{\mathcal{W}_1}, \Psi_{\mathcal{W}_2}, \Psi_{\mathcal{W}_3} \in \Omega_{FPFHS}(\mathcal{U})$ then,*

(i) $\Psi_{\mathcal{W}_1} \tilde{\cup} \Psi_{\mathcal{W}_1} = \Psi_{\mathcal{W}_1}$.

(ii) $\Psi_{\mathcal{W}_1} \tilde{\cup} \Psi_{\Phi} = \Psi_{\mathcal{W}_1}$.

(iii) $\Psi_{\mathcal{W}_1} \tilde{\cup} \Psi_{\tilde{G}} = \Psi_{\tilde{G}}$.

(iv) $\Psi_{\mathcal{W}_1} \tilde{\cup} \Psi_{\mathcal{W}_2} = \Psi_{\mathcal{W}_2} \tilde{\cup} \Psi_{\mathcal{W}_1}$.

(v) $(\Psi_{\mathcal{W}_1} \tilde{\cup} \Psi_{\mathcal{W}_2}) \tilde{\cup} \Psi_{\mathcal{W}_3} = \Psi_{\mathcal{W}_1} \tilde{\cup} (\Psi_{\mathcal{W}_2} \tilde{\cup} \Psi_{\mathcal{W}_3})$.

Proof. The proofs of (i)- (v) are obvious in accordance with Definition 3.7. □

Definition 3.8. *Let $\Psi_{\mathcal{W}_1}, \Psi_{\mathcal{W}_2} \in \Omega_{FPFHS}(\mathcal{U})$ then intersection of $\Psi_{\mathcal{W}_1}$ and $\Psi_{\mathcal{W}_2}$, denoted by $\Psi_{\mathcal{W}_1} \tilde{\cap} \Psi_{\mathcal{W}_2}$, is an fpfhs-set defined by $\zeta_{\mathcal{W}_1 \tilde{\cap} \mathcal{W}_2}(g) = \min\{\zeta_{\mathcal{W}_1}(g), \zeta_{\mathcal{W}_2}(g)\}$ and $\psi_{\mathcal{W}_1 \tilde{\cap} \mathcal{W}_2}(g) = \psi_{\mathcal{W}_1}(g) \cap \psi_{\mathcal{W}_2}(g)$*

Proposition 3.5. *Let $\Psi_{\mathcal{W}_1}, \Psi_{\mathcal{W}_2}, \Psi_{\mathcal{W}_3} \in \Omega_{FPFHS}(\mathcal{U})$ then*

(i) $\Psi_{\mathcal{W}_1} \tilde{\cap} \Psi_{\mathcal{W}_1} = \Psi_{\mathcal{W}_1}$;

(ii) $\Psi_{\mathcal{W}_1} \tilde{\cap} \Psi_\Phi = \Psi_\Phi$;

(iii) $\Psi_{\mathcal{W}_1} \tilde{\cap} \Psi_{\tilde{G}} = \Psi_{\tilde{\mathcal{W}}_1}$;

(iv) $\Psi_{\mathcal{W}_1} \tilde{\cap} \Psi_{\mathcal{W}_2} = \Psi_{\mathcal{W}_2} \tilde{\cap} \Psi_{\mathcal{W}_1}$;

(v) $(\Psi_{\mathcal{W}_1} \tilde{\cap} \Psi_{\mathcal{W}_2}) \tilde{\cap} \Psi_{\Psi_{\mathcal{W}_3}} = \Psi_X \tilde{\cap} (\Psi_{\mathcal{W}_2} \tilde{\cap} \Psi_{\Psi_{\mathcal{W}_3}})$.

Proof. The proofs of (i)- (v) are vivid by using Definition 3.8. □

Remark 3.3. Let $\Psi_\mathcal{W} \in \Omega_{FPFHS}(\mathcal{U})$. If $\Psi_\mathcal{W} \neq \Psi_{\tilde{G}}$, then $\Psi_\mathcal{W} \tilde{\cup} \Psi^{\tilde{c}}_\mathcal{W} \neq \Psi_{\tilde{G}}$ and $\Psi_\mathcal{W} \tilde{\cap} \Psi^{\tilde{c}}_\mathcal{W} \neq \Psi_\Phi$

Proposition 3.6. *Let $\Psi_{\mathcal{W}_1}, \Psi_{\mathcal{W}_2} \in \Omega_{FPFHS}(\mathcal{U})$ D Morgan's laws are valid*

(i) $(\Psi_{\mathcal{W}_1} \tilde{\cup} \Psi_{\mathcal{W}_2})^{\tilde{c}} = \Psi^{\tilde{c}}_{\mathcal{W}_1} \tilde{\cap} \Psi^{\tilde{c}}_{\mathcal{W}_2}$.

(ii) $(\Psi_{\mathcal{W}_1} \tilde{\cap} \Psi_{\mathcal{W}_2})^{\tilde{c}} = \Psi^{\tilde{c}}_{\mathcal{W}_1} \tilde{\cup} \Psi^{\tilde{c}}_{\mathcal{W}_2}$.

Proof. For all $g \in G$, (i) Since $(\zeta_{\mathcal{W}_1 \tilde{\cup} \mathcal{W}_2})^{\tilde{c}}(g) = 1 - \zeta_{\mathcal{W}_1 \tilde{\cup} \mathcal{W}_2}(g) = 1 - \max\{\zeta_{\mathcal{W}_1}(g), \zeta_{\mathcal{W}_2}(g)\} = \min\{1 - \zeta_{\mathcal{W}_1}(g), 1 - \zeta_{\mathcal{W}_2}(g)\} = \min\{\zeta_{\mathcal{W}_1}\tilde{c}(g), \zeta_{\mathcal{W}_2}\tilde{c}(g)\} = \zeta_{\mathcal{W}^{\tilde{c}}_1 \tilde{\cap} \mathcal{W}^{\tilde{c}}_2}(g)$ and $(\psi_{\mathcal{W}_1 \tilde{\cup} \mathcal{W}_2})^{\tilde{c}}(g) = \mathcal{U} \setminus \psi_{\mathcal{W}_1 \tilde{\cup} \mathcal{W}_2}(g) = \mathcal{U} \setminus (\psi_{\mathcal{W}_1}(g) \cup \psi_{\mathcal{W}_2}(g)) = (\mathcal{U} \setminus \psi_{\mathcal{W}_1}(g)) \cap (\mathcal{U} \setminus \psi_{\mathcal{W}_2}(g)) = \psi_{\mathcal{W}^{\tilde{c}}_1}(g) \tilde{\cap} \psi_{\mathcal{W}^{\tilde{c}}_2}(g) = \psi_{\mathcal{W}^{\tilde{c}}_1 \tilde{\cap} \mathcal{W}^{\tilde{c}}_2}(g)$. similarly (ii) can be proved easily. □

Proposition 3.7. *Let $\Psi_{\mathcal{W}_1}, \Psi_{\mathcal{W}_2}, \Psi_{\mathcal{W}_3} \in \Omega_{FPFHS}(\mathcal{U})$ then*

(i) $\Psi_{\mathcal{W}_1} \tilde{\cup} (\Psi_{\mathcal{W}_2} \tilde{\cap} \Psi_{\mathcal{W}_3}) = (\Psi_{\mathcal{W}_1} \tilde{\cup} \Psi_{\mathcal{W}_2}) \tilde{\cap} (\Psi_{\mathcal{W}_1} \tilde{\cup} \Psi_{\mathcal{W}_3})$.

(ii) $\Psi_{\mathcal{W}_1} \tilde{\cap} (\Psi_{\mathcal{W}_2} \tilde{\cup} \Psi_{\mathcal{W}_3}) = (\Psi_{\mathcal{W}_1} \tilde{\cap} \Psi_{\mathcal{W}_2}) \tilde{\cup} (\Psi_{\mathcal{W}_1} \tilde{\cap} \Psi_{\mathcal{W}_3})$.

Proof. For all $g \in G$, (i).

Since $\zeta_{\mathcal{W}_1 \tilde{\cup} (\mathcal{W}_2 \tilde{\cap} \mathcal{W}_3)}(g) = \max\{\zeta_{\mathcal{W}_1}(g), \zeta_{\mathcal{W}_2 \tilde{\cap} \mathcal{W}_3}(g)\} = \max\{\zeta_{\mathcal{W}_1}(g), \min\{\zeta_{\mathcal{W}_2}(g), \zeta_{\mathcal{W}_3}(g)\}\} = \min\{\max\{\zeta_{\mathcal{W}_1}(g), \zeta_{\mathcal{W}_2}(g)\}, \max\{\zeta_{\mathcal{W}_1}(g), \zeta_{\mathcal{W}_3}(g)\}\} = \min\{\zeta_{\mathcal{W}_1 \tilde{\cup} \mathcal{W}_2}(g), \zeta_{\mathcal{W}_1 \tilde{\cup} \mathcal{W}_3}(g)\} = \zeta_{(\mathcal{W}_1 \tilde{\cup} \mathcal{W}_2) \tilde{\cap} (\mathcal{W}_1 \tilde{\cup} \mathcal{W}_3)}(g)$ and $\psi_{\mathcal{W}_1 \tilde{\cup} (\mathcal{W}_2 \tilde{\cap} \mathcal{W}_3)}(g) = \psi_{\mathcal{W}_1}(g) \cup \psi_{\mathcal{W}_2 \tilde{\cap} \mathcal{W}_3}(g) = \psi_{\mathcal{W}_1}(g) \cup (\psi_{\mathcal{W}_2}(g) \cap \psi_{\mathcal{W}_3}(g)) = (\psi_{\mathcal{W}_1}(g) \cup \psi_{\mathcal{W}_2}(g)) \cap (\psi_{\mathcal{W}_1}(g) \cup \psi_{\mathcal{W}_3}(g)) = \psi_{\mathcal{W}_1 \tilde{\cup} \mathcal{W}_2}(g) \cap \psi_{\mathcal{W}_1 \tilde{U} \mathcal{W}_3}(g) = \psi_{(\mathcal{W}_1 \tilde{\cup} \mathcal{W}_2) \tilde{\cap} (\mathcal{W}_1 \tilde{\cup} \mathcal{W}_3)}(g)$. In the same way, (ii) can be proved. □

4. Fuzzy decision set of $fpfhs$-set

In this section novel algorithm is proposed with the help of characterization of fuzzy decision set on $fpfhs$-set which based on decision making technique and is explained with example.

Definition 4.1. *Let $\Psi_\mathcal{Z} \in \Omega_{FPFHS}(\mathcal{U})$ then a fuzzy decision set of $\Psi_\mathcal{Z}$ (i.e. $\Psi^D_\mathcal{Z}$) is represented as $\Psi^D_\mathcal{Z} = \left\{\zeta^D_\mathcal{Z}(u)/u : u \in \mathcal{U}\right\}$, where $\zeta_{\Psi^D_\mathcal{Z}} : \mathcal{U} \to \mathbb{I}^\bullet$ and*

$$\zeta_{\Psi^D_\mathcal{Z}}(u) = \frac{1}{|S(\mathcal{Z})|} \sum_{v \in S(\mathcal{Z})} \zeta_\Psi(v) \Gamma_{\psi_\mathcal{Z}(v)}(u),$$

where $S(\mathcal{Z})$ is the support set of $\mathcal{Z}$ with

$$\Gamma_{\psi_{\mathcal{Z}}(v)}(u) = \begin{cases} \psi_{\mathcal{Z}}(v), & u \in \Gamma_{\psi_{\mathcal{Z}}}(v), \\ 0, & u \notin \Gamma_{\psi_{\mathcal{Z}}}(v) \end{cases}.$$

4.1 Proposed algorithm

Once $\Psi^{D}_{\mathcal{Z}}$ has been established, it may be indispensable to select the best single substitute from the options. Therefore, decision can be set up with the help of following algorithm (see, Figure 1):

Step 1 Determine $\mathcal{Z} = \{\zeta_{\mathcal{Z}}(g)/g : \zeta_{\mathcal{Z}}(g) \in \mathbb{I}^{\bullet}, g \in G\}$,

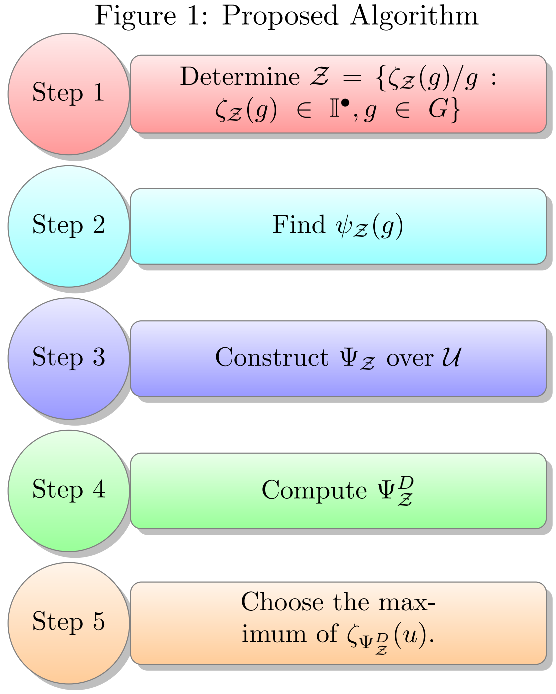
Step 2 Find $\psi_{\mathcal{Z}}(g)$,

Step 3 Construct $\Psi_{\mathcal{Z}}$ over $\mathcal{U}$,

Step 4 Compute $\Psi^{D}_{\mathcal{Z}}$,

Step 5 Choose the maximum of $\zeta_{\Psi^{D}_{\mathcal{Z}}}(u)$.

Figure 1: Proposed Algorithm



Example 4.1. Suppose that Mr. Smith wants to buy a mobile tablet from a mobile market. There are eight kinds of mobile tablets (options) which form the set of discourse $\mathcal{U} = \{T_1, T_2, T_3, T_4, T_5, T_6, T_7, T_8\}$. The best selection may be evaluated by observing the attributes i.e. a_1 = Company, a_2 = Camera Resolution (pixels), a_3 = Size (inches), a_4 = Storage (GB), and a_5 = Battery power (mAh). The attribute-valued sets corresponding to these attributes are: $B_1 = \{b_{11} = Samsung, b_{12} = Oppo\}$, $B_2 = \{b_{21} = 8, b_{22} = 16\}$, $B_3 = \{b_{31} = 9, b_{32} = 9.5\}$, $B_4 = \{b_{41} = 32, b_{42} = 64\}$, $B_5 = \{b_{51} = 4000\}$, then $G = B_1 \times B_2 \times B_3 \times B_4 \times B_5$, $G = \{g_1, g_2, g_3, g_4, ..., g_{16}\}$, where each $g_i, i = 1, 2, ..., 16$, is a 5-tuples element.
Step 1: From Table 1, we can construct $\mathcal{Z}$ as

Table 1: Degrees of Membership $\zeta_{\mathcal{Z}}(g_i)$

$\zeta_{\mathcal{Z}}(g_i)$	Degree	$\zeta_{\mathcal{Z}}(g_i)$	Degree
$\zeta_{\mathcal{Z}}(g_1)$	0.1	$\zeta_{\mathcal{Z}}(g_9)$	0.9
$\zeta_{\mathcal{Z}}(g_2)$	0.2	$\zeta_{\mathcal{Z}}(g_{10})$	0.16
$\zeta_{\mathcal{Z}}(g_3)$	0.3	$\zeta_{\mathcal{Z}}(g_{11})$	0.25
$\zeta_{\mathcal{Z}}(g_4)$	0.4	$\zeta_{\mathcal{Z}}(g_{12})$	0.45
$\zeta_{\mathcal{Z}}(g_5)$	0.5	$\zeta_{\mathcal{Z}}(g_{13})$	0.35
$\zeta_{\mathcal{Z}}(g_6)$	0.6	$\zeta_{\mathcal{Z}}(g_{14})$	0.75
$\zeta_{\mathcal{Z}}(g_7)$	0.7	$\zeta_{\mathcal{Z}}(g_{15})$	0.65
$\zeta_{\mathcal{Z}}(g_8)$	0.8	$\zeta_{\mathcal{Z}}(g_{16})$	0.85

$$\mathcal{Z} = \left\{ \begin{array}{l} 0.1/g_1, 0.2/g_2, 0.3/g_3, 0.4/g_4, \\ 0.5/g_5, 0.6/g_6, 0.7/g_7, 0.8/g_8, \\ 0.9/g_9, 0.16/g_{10}, 0.25/g_{11}, 0.45/g_{12}, \\ 0.35/g_{13}, 0.75/g_{14}, 0.65/g_{15}, 0.85/g_{16} \end{array} \right\}.$$

Step 2: Table 2 presents $\psi_{\mathcal{Z}}(g_i)$ corresponding to each element of G.

Table 2: Approximate functions $\psi_{\mathcal{Z}}(g_i)$

g_i	$\psi_{\mathcal{Z}}(g_i)$	g_i	$\psi_{\mathcal{Z}}(g_i)$
g_1	$\{0.2/T_1, 0.5/T_2\}$	g_9	$\{0.3/T_2, 0.6/T_3, 0.8/T_6\}$
g_2	$\{0.3/T_1, 0.1/T_2, 0.1/T_3\}$	g_{10}	$\{0.2/T_6, 0.6/T_7, 0.2/T_8\}$
g_3	$\{0.3/T_2, 0.4/T_6, 0.4/T_8\}$	g_{11}	$\{0.2/T_2, 0.6/T_4, 0.1/T_7\}$
g_4	$\{0.6/T_4, 0.1/T_5, 0.9/T_6\}$	g_{12}	$\{0.1/T_1, 0.3/T_3, 0.7/T_7\}$
g_5	$\{0.4/T_4, 0.5/T_5, 0.8/T_8\}$	g_{13}	$\{0.1/T_2, 0.1/T_3, 0.9/T_5\}$
g_6	$\{0.2/T_2, 0.5/T_4, 0.3/T_7\}$	g_{14}	$\{0.4/T_1, 0.7/T_5, 0.3/T_8\}$
g_7	$\{0.8/T_1, 0.7/T_3, 0.4/T_5\}$	g_{15}	$\{0.2/T_2, 0.1/T_3, 0.2/T_8\}$
g_8	$\{0.1/T_2, 0.3/T_3, 0.3/T_7\}$	g_{16}	$\{0.4/T_4, 0.6/T_5, 0.7/T_6\}$

Step 3: With the help of Tables 1 and 2, we can construct $\Psi_{\mathcal{Z}}$ as

$$\Psi_{\mathcal{Z}} = \left\{ \begin{array}{l} (0.1/g_1, \{0.2/T_1, 0.5/T_2\}), (0.2/g_2, \{0.3/T_1, 0.1/T_2, 0.1/T_3\}), \\ (0.3/g_3, \{0.3/T_2, 0.4/T_6, 0.4/T_8\}), (0.4/g_4, \{0.6/T_4, 0.1/T_5, 0.9/T_6\}), \\ (0.5/g_5, \{0.4/T_4, 0.5/T_5, 0.8/T_8\}), (0.6/g_6, \{0.2/T_2, 0.5/T_4, 0.3/T_7\}), \\ (0.7/g_7, \{0.8/T_1, 0.7/T_3, 0.4/T_5\}), (0.8/g_8, \{0.1/T_2, 0.3/T_3, 0.3/T_7\}), \\ (0.9/g_9, \{0.3/T_2, 0.6/T_3, 0.8/T_6\}), (0.16/g_{10}, \{0.2/T_6, 0.6/T_7, 0.2/T_8\}), \\ (0.25/g_{11}, \{0.2/T_2, 0.6/T_4, 0.1/T_7\}), (0.45/g_{12}, \{0.1/T_1, 0.3/T_3, 0.7/T_7\}), \\ (0.35/g_{13}, \{0.1/T_2, 0.1/T_3, 0.9/T_5\}), (0.75/g_{14}, \{0.4/T_1, 0.7/T_5, 0.3/T_8\}), \\ (0.65/g_{15}, \{0.2/T_2, 0.1/T_3, 0.2/T_8\}), (0.85/g_{16}, \{0.4/T_4, 0.6/T_5, 0.7/T_6\}) \end{array} \right\}.$$

Step 4:

Table 3: Membership values $\zeta_{\Psi^D_{\mathcal{Z}}}(T_i)$

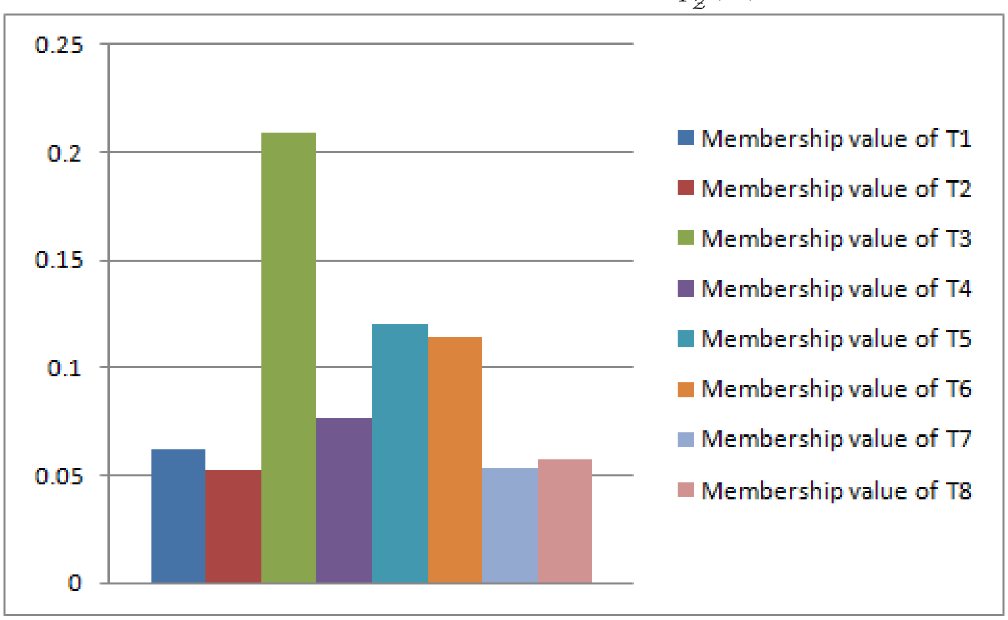
T_i	$\zeta_{\Psi^D_{\mathcal{Z}}}(T_i)$	T_i	$\zeta_{\Psi^D_{\mathcal{Z}}}(T_i)$
T_1	0.0616	T_5	0.1200
T_2	0.0528	T_6	0.1142
T_3	0.2091	T_7	0.0535
T_4	0.0769	T_8	0.0567

From Table 3, we can construct $\Psi^D_{\mathcal{Z}}$ as

$$\Psi^D_{\mathcal{Z}} = \left\{ \begin{array}{l} 0.0616/T_1, 0.0528/T_2, 0.2091/T_3, 0.0769/T_4, \\ 0.1200/T_5, 0.1142/T_6, 0.0535/T_7, 0.0567/T_8 \end{array} \right\}.$$

Step 5: Since maximum of $\zeta_{\Psi^D_{\mathcal{Z}}}(T_i)$ is 0.2091 (see, Figure 2) so, the mobile tablet T_3 is selected.

Figure 2: Membership values of $\zeta_{\Psi^D_{\mathcal{Z}}}(T_i)$



5. Discussion

Our proposed structure $fpfhs$-set is very useful in dealing with many decisive systems and it is the generalization of:

(i) Fuzzy Parameterized Hypersoft Set (fphs-set) if the range $F(\mathcal{U})$ of approximate function $\psi_{\mathcal{W}}$ is replaced with $P(\mathcal{U})$.

(ii) Hypersoft Set (hs-set) if domain and range of approximate function $\psi_{\mathcal{W}}$ are replaced with G and $P(\mathcal{U})$ respectively.

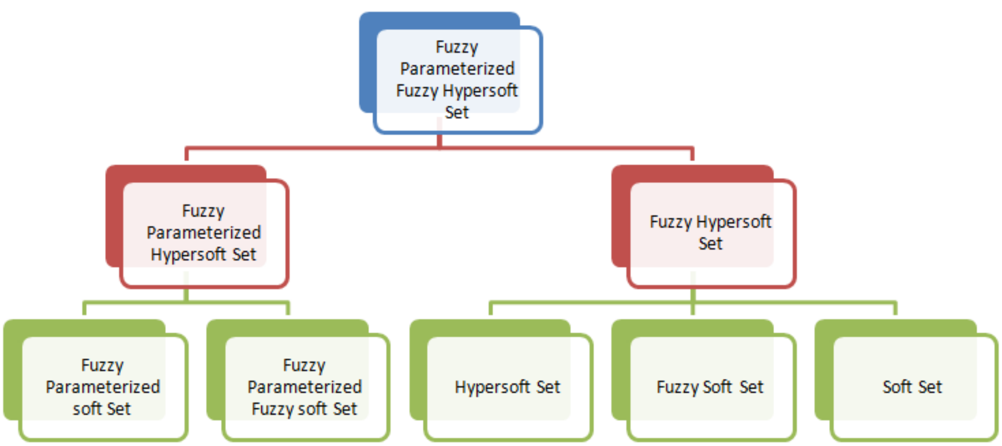
(iii) Fuzzy Parameterized Fuzzy Soft Set (fpfs-set) if disjoint attribute-valued sets are replaced with single set of distinct attributes.

(iv) Fuzzy Parameterized Soft Set (fpfs-set) if disjoint attribute-valued sets are replaced with single set of distinct attributes and the range $F(\mathcal{U})$ of approximate function $\psi_{\mathcal{W}}$ is replaced with $P(\mathcal{U})$.

(v) Fuzzy Soft Set (fs-set) if disjoint attribute-valued sets are replaced with single set E of distinct attributes and the domain G of approximate function $\psi_{\mathcal{W}}$ is replaced with E.

(vi) Soft Set (s-set) if disjoint attribute-valued sets are replaced with single set E of distinct attributes, the domain G and range $F(\mathcal{U})$ of approximate function $\psi_{\mathcal{W}}$ are replaced with E and $P(\mathcal{U})$ respectively.

Figure 3: Generalization of Proposed Model



5.1 Comparison

In this subsection, we compare our propose structure with the existing studies.

Authors	Structures	Remarks
Çağman et al. [13]	Fuzzy parameterized fuzzy soft set theory	Single set of attribute is employed to develop decision system via fuzzy parameterized fuzzy soft set theory,
Enginoğlu et al. [14]	Fuzzy parameterized fuzzy soft matrices	Single set of attribute is employed to develop decision system via fuzzy parameterized fuzzy soft matrix theory,
Kamacı et al. [15]	Interval-valued fuzzy parameterized intuitionistic fuzzy soft sets	Single set of attribute is employed to develop decision system via interval-valued fuzzy parameterized intuitionistic fuzzy soft sets theory,
Proposed structure	fuzzy parameterized fuzzy hypersoft sets theory	• Attributes are further classified into disjoint attribute-valued sets • Decision system is developed via employment of multi-argument approximate functions.

6. Conclusion

In this study, firstly. elementary notions of fuzzy parameterized fuzzy hypersoft set (i.e. an extension of fuzzy parameterized fuzzy soft set) are characterized with examples and generalized results. Secondly, theoretic operations i.e. union, intersection, of proposed structure are investigated with improved results and axiomatic properties. Thirdly, a novel algorithm is proposed for decision making and is validated with the help of an illustrative example for best purchasing of mobile tablet from mobile market. Lastly, comparison of proposed structure with the existing ones, is presented along with the generalization validation of proposed structure. Future work may include the extension of this work for other fuzzy-like environments such as

(i) interval valued fuzzy set;

(ii) intuitionistic fuzzy set;

(iii) pythagorean fuzzy set;

(iv) spherical fuzzy set;

(v) q-rung ortho pair fuzzy set;

(vi) neutrosophic set etc.

and for other soft set-like algebraic structures like

(i) soft topological space;

(ii) soft metric space;

(iii) soft group;

(iv) soft ring;

(v) soft vector space;

(vi) soft functional analysis etc.

with the implementation for solving more real life problems via decision making techniques i.e. TOPSIS etc.

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