Continuous mappings in soft lattice topological spaces

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Abstract. The fuzzy set, soft set and their extensions have proven to be a fruitful bridge between precise classical mathematics and the imprecise real world. Soft lattices, which are a generalization of soft sets, are a novel mathematical approach to the study of uncertainty. Soft lattice topological spaces are introduced over an initial universe X with a fixed set of parameters P. In this paper, we introduce the concept of soft lattice continuous mappings(soft L-continuous mappings) in soft lattice topological spaces which are defined with a fixed set of parameters over an initial universe. Further we investigate some properties regarding the continuity of mappings in soft lattice topological spaces. Finally, open mappings, closed mappings and homeomorphism on soft L-topological spaces are defined and some interesting results are obtained.
Keywords: soft L-topology, soft L-continuity, soft L-homeomorphism.

1. Introduction

The concept of soft set theory begins with Molodtsov [17, 18] in the year 1999. It is completely new approach for modelling, vagueness and uncertainties. Few applications in many directions of soft set theory have been shown by Molodtsov in [17, 18]. Also Maji et.al [16, 20] studied soft sets introduced by Molodtsov [17, 18] and gave the definitions based on equality of two soft sets, subset and super set of a soft set, complement of a soft set, null soft set, and absolute soft set with examples and basic properties are also defined. The algebraic structure of set theory dealing with uncertainties has also been studied by some authors [1, 2, 3, 4, 5, 10, 11, 12, 13, 14, 23]. The concept of soft set has been extended to soft lattices and soft fuzzy sets by Li F [15] in the year 2010. Shabir and Naz [22]

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introduced the concept of soft topological spaces in the year 2011 and studied some basic properties. In our work, we use the notion of soft set initiated by Molodtsov [17, 18] and extend this idea to the field of soft lattices and obtain the topological properties of soft lattices. In 2016, Cigdem Gunduz Aras, Ayse Sonmez and Huseyin Cakalli [6] introduced soft continuous mappings. Some of its properties are studied. In 2012, Hazra, Majumdar and Samanta [8] gave the definition of continuity of soft mappings with their properties. Soft mappings have been studied by a variety of other authors [7, 9, 19].

Soft Lattice topological spaces (Soft L-topological spaces or Soft L-space) [21] are introduced with a fixed set of parameters P over an initial universe X. We have defined some basic properties of soft L-topological spaces and also gave the definition of soft L-open and soft L-closed sets. The soft L-closure of a soft lattice is also defined which is a generalization of closure of a set. The concept of parameters plays a major role with the set of parameterized topologies on the initial universe. We define a topological space corresponding to each parameter, and it is more essential. We show that a soft topological space gives a parameterized family of topologies on the initial universe whereas the converse need not be true. It means if we are given some topologies for each parameter, it is not possible to construct a soft topological space. Consequently, we tell that the soft topological spaces are more generalization of classical topological spaces. Similarly, soft lattice topological spaces are generalized than soft topological spaces.

In this paper, our purpose is to study soft L-continuity on soft L-topological spaces. Soft lattice continuous mappings(soft L-continuous mappings) are introduced with a fixed set of parameters P over an initial universe set X. We also discuss some algebraic properties of soft L-mappings and study their continuity properties under soft L-topology. Finally, we give the definitions of soft L-open mappings, soft L-closed mappings and soft L-homeomorphism and some interesting results are proved.

2. Preliminaries and basic definitions

Throughout this paper, we consider L as a complete lattice and we denote its universal bounds as $\bot$ and $\top$. Our assumption is L is consistent i.e. floor is different from top. Therefore, $\bot \leq \alpha \leq \top$ for every $\alpha \in L$. Also $\vee\phi = \bot$ and $\wedge\phi = \top$. The two point lattice $\{\bot, \top\}$ is denoted by 2. A unary operation $\prime \colon L \longrightarrow L$ is quasi complementation. It is an involution (i.e., $\alpha'' = \alpha$ for all $\alpha \in L$) that inverts the ordering. (i.e., $\alpha \leq \beta \Longrightarrow \beta' \leq \alpha'$). De Morgan's laws also hold in $(L, \prime)$. (i.e., $(\vee A)' = \wedge\{\alpha' : \alpha \in A\}$ and $(\wedge A)' = \vee\{\alpha' : \alpha \in A\}$ for every $A \subset L$). In addition, $\bot' = \top$ and $\top' = \bot$. Based on these concepts, we use a completely distributive lattice $(L, \prime)$ as a complete lattice equipped with an order reserving involution in this paper.

Definition 2.1 ([1])**.** Assume X as an initial universe set and P be a set of parameters. The power set of X is denoted as $\wp(X)$ and $B \subset P$. Then, a pair (P, B) is said to be a *soft set* over X, where the mapping P is given by $P : B \to \wp(X)$ i.e., a soft set over X is regarded as a parametrized family of subsets of the universe X. For $b \in B$, the set of approximate elements of the soft set (P, B) denoted by $P(b)$.

Definition 2.2 ([1])**.** Let (P, A) and (Q, B) be soft sets over a common universe X, then (Q, B) is a soft subset of (P, A) if (i) $B \subset A$, and (ii) for all $b \in B$, $Q\,(b)$ and $P\,(b)$ are identical approximations.

We denote it by $(Q, B)\,\tilde{\subset}\,(P, A)$. If (P, A) is a soft subset of (Q, B) then (Q, B) is a soft super set of (P, A) and it is denoted by $(Q, B)\,\tilde{\supset}\,(P, A)$.

If (P, A) is a soft subset of (Q, B) and (Q, B) is a soft subset of (P, A), then these two soft sets (P, A) and (Q, B) over a common universe X are said to be soft equal.

Definition 2.3 ([1])**.** Let $P = \{g_1, g_2, g_3, ..., g_i\}$ be a set of parameters. The NOT set of P denoted by $\neg P$ is defined by $\neg P = \{\neg g_1, \neg g_2, \neg g_3, ..., \neg g_i\}$, where $\neg g_i = not g_n$ for all n.

Definition 2.4 ([1])**.** Let (Q, B) be a soft set, then $(Q, B)^c$ is complement or neg-complement of a soft set (Q, B) and is defined by $Q^c = (Q^c, \neg B)$, where $Q^c\colon \neg B \longrightarrow \wp(X)$ is a mapping given by $Q^c(\neg\beta) = X - Q(\beta)$, for every $\neg\beta \in \neg B$. Also $(Q^c)^c = Q$ and $((Q, B)^c)^c = (Q, B)$.

Definition 2.5 ([1])**.** Let (Q, B) be a soft set over X, then (Q, B) is said to be a NULL soft set if $\forall\, b \in B$, $Q\,(b) = \phi$,(null-set) and is denoted by Q_ϕ.

Definition 2.6 ([1])**.** A soft set (Q, B) over X is said to be absolute soft set denoted by $\overline{B}$, if $\forall\, b \in B$, $Q\,(b) = X$. Clearly $\overline{B}^c = Q_\phi$, $Q_\phi^c = \overline{B}$.

Definition 2.7 ([15])**.** Consider $M = (f, X, L)$, where L is a complete lattice, $f : X \longrightarrow \wp(L)$ is a mapping, X is a universe set, then M is called the soft lattice denoted by f_P^L, i.e., for every $x \in X$, f_P^L is a soft lattice over L, if $f(x)$ is a sub lattice of L.

Definition 2.8 ([21])**.** The relative complement of a soft lattice f_P^L is denoted by $(f_P^L)'$ and is defined as $(f_P^L)' = (f_P'^L)$ where $f' : P \longrightarrow \wp(L)$ is a mapping given by $f'(\alpha) = L - f(\alpha)$ for all $\alpha \in P$.

Definition 2.9 ([21])**.** Consider X as an initial universe set and P as the non-empty set of parameters.

Let τ be the set of complete, uniquely complemented soft lattices over L, then τ is said to be a soft lattice topology on L if:

(i) ϕ, L belongs to τ.

(ii) The arbitrary union of soft Lattices in τ belongs to τ.

(iii) The finite intersection of soft Lattices in τ belongs to τ.

Then, (L, τ, P) is called a soft lattice topological space (soft topological lattice space or soft L -space) over L.

Definition 2.10 ([21])**.** Consider (L, τ, P) as a soft lattice topological space over L, then the members of τ are called as soft L-open sets in L.

Definition 2.11 ([21])**.** Let (L, τ, P) be a soft lattice topological space over L. A soft lattice f_P^L over L is said to be a soft L-closed set in L, if its relative complement $(f_P^L)'$ belongs to τ.

Definition 2.12 ([21])**.** We consider L as a lattice, P be the set of parameters and $\tau = \{\phi, L\}$. Then τ is called the soft indiscrete lattice topology on L and (L, τ, P) is said to be a soft indiscrete lattice topological space over L.

Definition 2.13 ([21])**.** Consider L be a lattice, P be the set of parameters and let τ be the collection of all soft lattices which can be defined over L. Then τ is called the soft discrete lattice topology on L and (L, τ, P) is said to be a soft discrete lattice topological space over L.

Definition 2.14 ([21])**.** We consider (L, τ, P) as a soft lattice topological space over L and f_P^L be a soft lattice over L. Then the soft lattice closure of f_P^L, denoted by $\overline{f}_P^L$, is the intersection of all soft L-closed super sets of f_P^L .

Definition 2.15 ([21])**.** Let (L, τ, P) be a soft lattice topological space over L and f_P^L be a soft lattice over L. Then we associate with f_P^L, a soft lattice L, denoted by $\overline{f}_P^L$ and defined as $\overline{f}(\alpha) = \overline{f(\alpha)}$, where $\overline{f(\alpha)}$ is the soft L-closure of $f(\alpha)$ in τ_α for each $\alpha \in P$.

Definition 2.16 ([21])**.** Consider (L, τ, P) as a soft lattice topological space over L, g_P^L be a soft lattice over L and $x \in L$. Then x is said to be a soft L-interior point of g_P^L if there exists a soft L-open set f_P^L such that $x \in f_P^L \subset g_P^L$. It is denoted by $(f_P^L)^o$.

Definition 2.17 ([21])**.** Let (L, τ, P) be a soft lattice topological space over L, g_P^L be a soft lattice over L and $x \in L$. Then g_P^L is said to be a soft lattice neighbourhood of x if there exists a soft L-open set f_P^L such that $x \in f_P^L \subset g_P^L$.

2.1 Soft lattice mapping and soft lattice continuous mapping

Assume X as an initial universe set and P_1 and P_2 are two non-empty set of parameters.

Definition 2.18. Consider (L_1, τ_1, P_1) and (L_2, τ_2, P_2) as two soft lattice topological spaces over L_1 and L_2 respectively. The mapping f_g is called a soft L-mapping from L_1 to L_2 denoted by $f_g \colon (L_1, \tau_1, P_1) \longrightarrow (L_2, \tau_2, P_2)$, where $f \colon L_1 \longrightarrow L_2$ and $g \colon P_1 \longrightarrow P_2$ are two mappings.

Assume $F_{P_1}^L \in (L_1, \tau_1, P_1)$, then the image of $F_{P_1}^L$ under the soft L-mapping f_g is the soft lattice set over L_2 denoted by $f_g(F_{P_1}^L)$ and defined by

$$f_g(F_{P_1}^L)(p_2) = \begin{cases} \bigcup_{p_1 \in g^{-1}(p_2) \cap P_1} f(F_{P_1}^L(p_1)), & g^{-1}(p_2) \cap P_1 \neq \phi \\ \phi, & \text{otherwise} \end{cases}.$$

Let $G^L_{P_2} \in (L_2, \tau_2, P_2)$, then the pre-image of $G^L_{P_2}$ under the soft L-mapping f_g is the soft lattice set over L_1 denoted by $f_g(G^L_{P_2})$ and defined by

$$f_g^{-1}(G^L_{P_2})(p_1) = \begin{cases} f^{-1}(G^L_{P_2}(f(p_1))), & g(p_1) \in P_2 \\ \phi, & \text{otherwise} \end{cases}.$$

The soft L-mapping f_g is called injective if f and g are injective. Similarly, f_g is also surjective and bijective.

Example 2.1. Suppose $L_1 = \{l_1, l_2, l_3\}, L_2 = \{h_1, h_2, h_3\}, P_1 = \{p_1, p_2\}, P_2 = \{p_3, p_4\}$ and $\tau_1 = \{\phi, L_1, F^{L_1}_{1P_1}, F^{L_1}_{2P_1}, F^{L_1}_{3P_1}, F^{L_1}_{4P_1}\}$ is a soft L-topological space over L_1, $\tau_2 = \{\phi, L_2, G^{L_2}_{1P_2}, G^{L_2}_{2P_2}, G^{L_2}_{3P_2}, G^{L_2}_{4P_2}\}$ is a soft L-topological space over L_2, where $F^{L_1}_{1P_1}, F^{L_1}_{2P_1}, F^{L_1}_{3P_1}, F^{L_1}_{4P_1}$ are soft lattices over L_1 and $G^{L_2}_{1P_2}, G^{L_2}_{2P_2}, G^{L_2}_{3P_2}, G^{L_2}_{4P_2}$ are soft lattices over L_2, defined as follows,

$$\begin{aligned}
&F_1(p_1) = \{l_2\}, F_1(p_2) = \{l_1\},\\
&F_2(p_1) = \{l_2, l_3\}, F_2(p_2) = \{l_1, l_2\},\\
&F_3(p_1) = \{l_1, l_2\}, F_3(p_2) = L_1,\\
&F_4(p_1) = \{l_1, l_2\}, F_4(p_2) = \{l_1, l_3\},
\end{aligned}$$

$$\begin{aligned}
&G_1(p_3) = \{h_1\}, G_1(p_4) = \{h_2\},\\
&G_2(p_3) = \{h_2, h_3\}, G_2(p_4) = \{h_1, h_2\},\\
&G_3(p_3) = \{h_1, h_2\}, G_3(p_4) = \{h_1, h_2\},\\
&G_4(p_3) = \{h_1\}, G_3(p_4) = \{h_1, h_3\}.
\end{aligned}$$

Let f_g be a mapping, where $f\colon L_1 \longrightarrow L_2$ and $g\colon P_1 \longrightarrow P_2$.

Assume $f^L_{P_1} \in (L_1, \tau_1, P_1)$, then

$$f_g(F^L_{P_1})(p_2) = \begin{cases} \bigcup_{p_1 \in g^{-1}(p_2)\cap P_1} f(F^L_{P_1}(p_1)), & g^{-1}(p_2) \cap P_1 \neq \phi \\ \phi, & \text{otherwise} \end{cases}.$$

Then, f_g is a soft L-mapping over L_2.

But $f\colon L_1 \longrightarrow L_1$ is not soft L-mapping.

Let $G^L_{P_2} \in (L_2, \tau_2, P_2)$, then

$$f_g^{-1}(G^L_{P_2})(p_1) = \begin{cases} f^{-1}(G^L_{P_2}(f(p_1))), & g(p_1) \in P_2 \\ \phi, & \text{otherwise} \end{cases}.$$

Then, f_g is a soft L-mapping over L_1.

We consider that X as an initial universe set and P represents the non-empty set of parameters.

Definition 2.19. Consider f_P^L as a soft lattice over L. The soft lattice f_P^L is called a soft L-point, denoted by (l_p, P), for the element $p \in P$, $f(p) = \{l\}$ and $f(p') = \phi$ for all $p' \in P - \{l\}$.

Definition 2.20. Let (L_1, τ_1, P) and (L_2, τ_2, P) be two soft lattice topological spaces. The mapping f_g is called a soft L-mapping from L_1 to L_2 denoted by $f_g\colon (L_1, \tau_1, P) \longrightarrow (L_2, \tau_2, P)$, where $f\colon L_1 \longrightarrow L_2$ and $g\colon P \longrightarrow P$ are two mappings. For each soft L-neighbourhood g_P^L of $(f(l)_p, P)$, if there exist a soft L-neighbourhood f_P^L of (l_p, P) such that $f_g(f_P^L \subset g_P^L)$, then f_g is said to be soft L-continuous mapping at (l_p, P).

If f_g is soft L-continuous mapping for all (l_p, P), then f_g is called soft L-continuous mapping.

Definition 2.21. Let (L_1, τ_1, P) and (L_2, τ_2, P) be two soft lattice topological spaces, $f_g\colon (L_1, \tau_1, P) \longrightarrow (L_2, \tau_2, P)$ be a mapping. Then:

(a) If the image $f_g(F_P^L)$ of each soft L-open set F_P^L over L_1 is a soft L-open set in L_2, then f_g is said to be a soft L-open mapping.

(b) If the image $f_g(H_P^L)$ of each soft L-closed set H_P^L over L_1 is a soft L-closed set in L_2, then f_g is said to be a soft L-closed mapping.

Theorem 2.1. We know that (L_1, τ_1, P) and (L_2, τ_2, P) are two soft lattice topological spaces, $f_g\colon (L_1, \tau_1, P) \longrightarrow (L_2, \tau_2, P)$ be a mapping. Then the following conditions are equivalent:

(1) $f_g\colon (L_1, \tau_1, P) \longrightarrow (L_2, \tau_2, P)$ is a softL-continuous mapping.

(2) For each soft L-open set G_P^L over L_2, $f_g^{-1}(G_P^L)$ is a soft L-open set over L_1.

(3) For each soft L-closed set H_P^L over L_2, $f_g^{-1}(H_P^L)$ is a soft L-closed set over L_1.

(4) For each soft L-set F_P^L over L_1, $f_g\overline{(F_P^L)} \subset \overline{f_g(F_P^L)}$.

(5) For each soft L-set G_P^L over L_2, $\overline{f_g^{-1}(G_P^L)} \subset f_g\overline{(G_P^L)}$.

(6) For each soft L-set G_P^L over L_2, $f_g^{-1}((G_P^L)^o) \subset (f_g^{-1}(G_P^L))^o$.

Proof. $\underline{(1) \Rightarrow (2)}$:
Let G_P^L be a soft lattice open set over L_2 and $(l_p, P) \in f_g^{-1}(G_P^L)$ be an arbitrary soft L-point. Then $f_g(l_p, P) = (f_g(l)_p, P) \in F_P^L$.

Since f_g is soft L-continuous mapping, there exists $(l_p, P) \in F_P^L \in \tau$ such that $f_g(F_P^L) \subset G_P^L$. This implies $(l_p, P) \in F_P^L \subset f_g^{-1}(G_P^L)$, $f_g^{-1}(G_P^L)$ is a soft L-open set over L_1.

$\underline{(2) \Rightarrow (1)}$:
Let (l_p, P) be a soft L-point and $(f(l)_p, P) \in G_P^L$ be an arbitrary soft L-neighbourhood. Then $(f(l)_p, P) \in f_g^{-1}(G_P^L)$ is a soft L-neighbourhood and $f_g(f_g^{-1}(G_P^L)) \subset G_P^L$.

$\underline{(2) \Rightarrow (3)}$:
If for each soft L-open set G_P^L over L_2, $f_g^{-1}(G_P^L)$ is a soft L-open set L_1, then for each soft L-closed set H_P^L over L_2, $f_g^{-1}(H_P^L)$ is a soft L-closed set over L_1.

$\underline{(3) \Rightarrow (4)}$:
Let F_P^L be a soft L-open set over L_1.

Since $F_P^L \subset f_g^{-1}(f_g(F_P^L))$ and $f_g(F_P^L) \subset \overline{(f_g(F_P^L))}$, we have $F_P^L \subset f_g^{-1}(f_g(F_P^L)) \subset f_g^{-1}\overline{(f_g(F_P^L))}$.

By (3), since $f_g^{-1}\overline{(f_g(F_P^L))}$ is a soft L-closed set over L_1, $\overline{(F_P^L)} \subset f_g^{-1}\overline{(f_g(F_P^L))}$. Thus, $f_g(\overline{(F_P^L)}) \subset f_g(\overline{f_g^{-1}(F_P^L)}) \subset \overline{f_g(F_P^L)}$ is obtained.

$\underline{(4) \Rightarrow (5)}$:
Let G_P^L be a soft lattice open set over L_2 and $f_g^{-1}(G_P^L) = F_P^L$. By (4), $f_g(\overline{(F_P^L)}) = f_g(\overline{f_g^{-1}(G_P^L)}) \subset \overline{f_g(f_g^{-1}(G_P^L))} \subset \overline{(G_P^L)}$. Then, $\overline{f_g^{-1}(G_P^L)} = (\overline{(F_P^L)}) \subset f_g^{-1}\overline{(f_g(F_P^L))} \subset f_g^{-1}\overline{(G_P^L)}$.

$\underline{(5) \Rightarrow (6)}$:
Let G_P^L be a soft lattice open set over L_2.

Substitute $(F_P^L)'$ in (5), then $\overline{f_g^{-1}((G_P^L)')} \subset f_g^{-1}\overline{((G_P^L)')}$.

Since $(G_P^L)^o = (\overline{(G_P^L)'})'$, then we have

$$
\begin{aligned}
&f_g^{-1}((G_P^L)^o) = f_g^{-1}((\overline{(G_P^L)'})') \\
&= (f_g^{-1}((\overline{(G_P^L)'})') \subset \overline{(f_g^{-1}(((G_P^L)')')} \\
&= (\overline{(f_g^{-1}(((G_P^L)')}))' = (f_g^{-1}(G_P^L))^o.
\end{aligned}
$$

$\underline{(6) \Rightarrow (2)}$:
Let G_P^L be a soft lattice open set over L_2. Then, since $(f_g^{-1}((G_P^L))^o \subset f_g^{-1}(G_P^L) = f_g^{-1}(G_P^L)^o) \subset (f_g^{-1}((G_P^L))^o$, $(f_g^{-1}((G_P^L))^o = f_g^{-1}(G_P^L)$ is obtained. This implies $f_g^{-1}(G_P^L)$ is a soft lattice open set over L_1. □

Example 2.2. Suppose $L = \{l_1, l_2, l_3\}, P = \{p_1, p_2\}$ and $\tau_1 = \{\phi, L, F_{1P}^L, F_{2P}^L\}$, $\tau_2 = \{\phi, L, G_{1P}^L, G_{2P}^L\}$ be two soft L-topologies over L, where $F_{1P}^L, F_{2P}^L, G_{1P}^L, G_{2P}^L$ are soft lattices over L, defined as follows,

$$
\begin{aligned}
&F_1(p_1) = \{l_1, l_2\}, F_1(p_2) = L, \\
&F_2(p_1) = \{l_3\}, F_2(p_2) = \{l_3\},
\end{aligned}
$$

and

$$
\begin{aligned}
&G_1(p_1) = \{l_1\}, G_1(p_2) = \{l_3\}, \\
&G_2(p_1) = \{l_1, l_3\}, G_2(p_2) = \{l_2, l_3\}.
\end{aligned}
$$

If $f_g : L \longrightarrow L$ defined as $f_g(h_1) = f_g(h_2) = h_1, f_g(h_3) = h_3$, then since $f_g^{-1}(G_{1P}^L) = F_{1P}^L$ and $f_g^{-1}(G_{2P}^L) = F_{2P}^L$, f_g is a soft L-continuous mapping.

Example 2.3. Suppose $L = \{l_1, l_2, l_3\}, P = \{p_1, p_2\}$ and $\tau_1 = \{\phi, L, F_{1P}^L, F_{2P}^L, F_{3P}^L, F_{4P}^L\}$, $\tau_2 = \{\phi, L, G_{1P}^L, G_{2P}^L, G_{3P}^L, G_{4P}^L\}$ be two soft L-topologies over

L, where $F^L_{1P}, F^L_{2P}, F^L_{3P}, F^L_{4P}, G^L_{1P}, G^L_{2P}, G^L_{3P}, G^L_{4P}$ are soft lattices over L, defined as follows,

$$F_1(p_1) = \{l_2\}, F_1(p_2) = \{l_1\},$$
$$F_2(p_1) = \{l_2, l_3\}, F_2(p_2) = \{l_1, l_2\},$$
$$F_3(p_1) = \{l_3\}, F_3(p_2) = \{l_1, l_2\},$$
$$F_4(p_1) = \{\phi\}, F_4(p_2) = L$$

and

$$G_1(p_1) = \{l_2\}, G_1(p_2) = \{l_1\},$$
$$G_2(p_1) = \{l_2, l_3\}, G_2(p_2) = \{l_1, l_2\},$$
$$G_3(p_1) = \{l_1, l_2\}, G_3(p_2) = L,$$
$$G_4(p_1) = \{l_2\}, G_4(p_2) = \{l_1, l_2\}.$$

Thus, (L, τ_1, P) and (L, τ_2, P) are two soft L-topological spaces and $f_g : 1_L : L \longrightarrow L$ is not soft L-continuous mapping.

Theorem 2.2. If $f_g \colon (L_1, \tau_1, P) \longrightarrow (L_2, \tau_2, P)$ is a soft L-continuous mapping, then for each $\alpha \in P$, $f_{g\alpha} \colon (L_1, \tau_{1\alpha}) \longrightarrow (L_2, \tau_{2\alpha})$ is a soft continuous mapping.

Proof. Let $V \in \tau_{2\alpha}$. Then there exist a soft L-open set G^L_P over L_2 such that $V = G(\alpha)$.

Since $f_g \colon (L_1, \tau_1, P) \longrightarrow (L_2, \tau_2, P)$ is a soft L-continuous mapping, $f_g^{-1}(G^L_P)$ is a soft L-open set over L_1 and $f_g^{-1}(G^L_P)(\alpha) = f_g^{-1}(G(\alpha)) = f^{-1}(V)$ is a soft open set.

This implies $f_{g\alpha}$ is a soft continuous mapping.

Now, this is an example given to show that the converse of above theorem does not hold. □

Example 2.4. Suppose $L_1 = \{l_1, l_2, l_3\}, L_2 = \{h_1, h_2, h_3\}, P = \{p_1, p_2\}$ and $\tau_1 = \{\phi, L_1, F^L_{1P}, F^L_{2P}, F^L_{3P}, F^L_{4P}, F^L_{5P}\}$ is a soft L-topological space over L_1, $\tau_2 = \{\phi, L_2, G^L_{1P}, G^L_{2P}, G^L_{3P}\}$ is a soft L-topological space over L_2, where $F^L_{1P}, F^L_{2P}, F^L_{3P}, F^L_{4P}, F^L_{5P}$ are soft lattices over $L_1, G^L_{1P}, G^L_{2P}, G^L_{3P}$ are soft lattices over L_2, defined as follows,

$$F_1(p_1) = \{l_1\}, F_1(p_2) = \{l_1, l_3\},$$
$$F_2(p_1) = \{l_2\}, F_2(p_2) = \{l_1\},$$
$$F_3(p_1) = \{l_1, l_2\}, F_3(p_2) = \{l_1, l_3\},$$
$$F_4(p_1) = \{\phi\}, F_4(p_2) = \{l_1\},$$
$$F_5(p_1) = \{l_1, l_2\}, F_5(p_2) = L_1$$

and

$$G_1(p_1) = \{L_2\}, G_1(p_2) = \{h_2\},$$
$$G_2(p_1) = \{h_1\}, G_2(p_2) = \{h_2\},$$
$$G_3(p_1) = \{h_1, h_2\}, G_3(p_2) = \{h_2\}.$$

If we define a mapping $f_g : L_1 \longrightarrow L_2$ as $f_g(l_1) = h_2, f_g(l_2) = h_1, f_g(l_3) = h_3$, then f_g is not a soft L-continuous mappings, since $f_g^{-1}(G_1) \in \tau_1$, where $f_g^{-1}(G_1)(p_1) = L_1$ and $f_g^{-1}(G_2)(p_2) = \{l_1\}$.

Also, $f_{gp_1}\colon (L_1, \tau_{1p_1}) \longrightarrow (L_2, \tau_{2p_1})$ and $f_{gp_2}\colon (L_1, \tau_{1p_2}) \longrightarrow (L_2, \tau_{2p_2})$ are soft L-continuous mappings.

Here,

$$\begin{aligned}
\tau_{1p_1} &= \{\phi, L_1, \{l_1\}, \{l_2\}, \{l_1, l_2\}\},\\
\tau_{1p_2} &= \{\phi, L_1, \{l_1\}, \{l_1, l_3\}\},\\
\tau_{2p_1} &= \{\phi, L_2, \{h_1\}, \{h_1, h_2\}\},\\
\tau_{2p_2} &= \{\phi, L_2, \{h_2\}\}.
\end{aligned}$$

Theorem 2.3. If $(\bar{F}_P^L)'$ is a soft L-open set over L_1, for each soft L-open set F_P^L, then $f_g\colon (L_1, \tau_1, P) \longrightarrow (L_2, \tau_2, P)$ is a soft L-continuous mapping if and only if $f_{g\alpha}\colon (L_1, \tau_{1\alpha}) \longrightarrow (L_2, \tau_{2\alpha})$ is a soft continuous mapping for each $\alpha \in P$.

Proof. Let $f_{g\alpha}\colon (L_1, \tau_{1\alpha}) \longrightarrow (L_2, \tau_{2\alpha})$ be a soft continuous mapping for each $\alpha \in P$ and let F_P^L be an arbitrary soft L-open set over L_1. Then $f_{g\alpha}(\bar{F}_P^L(\alpha)) \subset \overline{f_{g\alpha}(\overline{F_P}^L)}(\alpha)$ is satisfied for each $\alpha \in P$.

Since $(\bar{F}_P^L)' \in \tau_1$, $(\bar{F}_P^L)' = \overline{(F_P^{L'})}$. Thus, $f_g\overline{((F_P^L))} = \overline{f_g(F_P^L)}$ is obtained.

Let $f_g\colon (L_1, \tau_1, P) \longrightarrow (L_2, \tau_2, P)$ be a soft L-continuous mapping. Then for each $\alpha \in P$ and if F_P^L be an arbitrary soft L-open set over L_1. Then $f_g(\bar{F}_P^L(\alpha)) \subset \overline{f_g(\overline{F_P}^L)}(\alpha)$ is satisfied for each $\alpha \in P$. Thus, $f_{g\alpha}\colon (L_1, \tau_{1\alpha}) \longrightarrow (L_2, \tau_{2\alpha})$ is obtained.

This implies $f_{g\alpha}$ is a soft continuous mapping for each $\alpha \in P$. □

Proposition 2.1. If $f_{g\alpha}\colon (L_1, \tau_{1\alpha}) \longrightarrow (L_2, \tau_{2\alpha})$ is soft L-open(closed) mapping, then for each $\alpha \in P$, $f_{g\alpha}\colon (L_1, \tau_{1\alpha}) \longrightarrow (L_2, \tau_{2\alpha})$ is an soft open(closed) mapping.

Proof. If $f_{g\alpha}\colon (L_1, \tau_{1\alpha}) \longrightarrow (L_2, \tau_{2\alpha})$ is soft L-open(closed) mapping, then for each $\alpha \in P$, $f_{g\alpha}(F_P^L(\alpha)) \subset f_{g\alpha}(F_P{}^L)(\alpha)$.

This implies $f_{g\alpha}\colon (L_1, \tau_{1\alpha}) \longrightarrow (L_2, \tau_{2\alpha})$ is an soft open(closed) mapping. □

Note 2.1. The concepts of soft L-continuous, soft L-open and soft L-closed mappings are all independent of each other.

Example 2.5. Suppose (L, τ_1, P) be a soft discrete L-topological space and (L, τ_2, P) be soft indiscrete L-topological space. Then $f_g : 1_L : L \longrightarrow L$ is a soft L-open mapping and soft L-closed mapping. But it is not soft L-continuous mapping.

Example 2.6. Suppose $L = \{l_1, l_2, l_3\}, P = \{p_1, p_2\}$ and $\tau_1 = \{\phi, L, F_{1P}^L, F_{2P}^L, F_{3P}^L, F_{4P}^L, F_{5P}^L, F_{6P}^L, F_{7P}^L\}$, $\tau_2 = \{\phi, L, G_{1P}^L, G_{2P}^L, G_{3P}^L, G_{4P}^L\}$ be two soft L-topologies over L, where $F_{1P}^L, F_{2P}^L, F_{3P}^L, F_{4P}^L, F_{5P}^L, F_{6P}^L, F_{7P}^L, G_{1P}^L, G_{2P}^L, G_{3P}^L, G_{4P}^L$ are soft lattices over L, defined as follows

$$\begin{aligned}
&F_1(p_1) = \{l_2\}, F_1(p_2) = \{l_1\},\\
&F_2(p_1) = \{l_1, l_3\}, F_2(p_2) = \{l_2, l_3\},\\
&F_3(p_1) = \{l_2\}, F_3(p_2) = L,\\
&F_4(p_1) = \{\phi\}, F_4(p_2) = \{l_1\},\\
&F_5(p_1) = \{l_1, l_3\}, F_5(p_2) = L,\\
&F_6(p_1) = \{\phi\}, F_6(p_2) = \{l_2, l_3\},\\
&F_7(p_1) = \{\phi\}, F_7(p_2) = L
\end{aligned}$$

and

$$\begin{aligned}
&G_1(p_1) = \{l_2\}, G_1(p_2) = \{l_1\},\\
&G_2(p_1) = \{l_2, l_3\}, G_2(p_2) = \{l_1, l_2\},\\
&G_3(p_1) = \{l_1, l_2\}, G_3(p_2) = L,\\
&G_4(p_1) = \{l_2\}, G_4(p_2) = \{l_1, l_2\}.
\end{aligned}$$

If we define a mapping $f_g : 1_L : L \longrightarrow L$ as $f_g(l_i) = l_1$, for $1 \leq i \leq 3$, then $f_g^{-1}(G_1)(p_1) = f_g^{-1}(G_4)(p_1) = \phi$, $f_g^{-1}(G_1)(p_2) = f_g^{-1}(G_4)(p_2) = L$, $f_g^{-1}(G_3)(p_1) = f_g^{-1}(G_3)(p_2) = L$. Then, f_g is a soft L-continuous mapping, but $f_g(F_1)(p_1) = \{l_1\}$, $f_g(F_1)(p_2) = \{l_1\}$, $f_g(F_1')(p_1) = \{l_1\}$, $f_g(F_1')(p_2) = \{l_1\}$. Hence it is not both soft L-open and soft L-closed mapping.

Example 2.7. Suppose $L_1 = \{l_1, l_2, l_3\}, L_2 = \{h_1, h_2\}, P = \{p_1, p_2\}$ and $\tau_1 = \{\phi, L_1, F^L_{1P}, F^L_{2P}\}$ is a soft L-topological space over L_1, $\tau_2 = \{\phi, L_2, G^L_{1P}, G^L_{2P}\}$ is a soft L-topological space over L_2, where F^L_{1P}, F^L_{2P} are soft lattices over L_1, G^L_{1P}, G^L_{2P} are soft lattices over L_2, defined as follows $F_1(p_1) = \{l_1, l_2\}, F_1(p_2) = \{l_3\}$, $F_2(p_1) = L_1, F_2(p_2) = \{l_3\}$, $G_1(p_1) = \{L_2\}, G_1(p_2) = \{h_2\}$, $G_2(p_1) = \{h_1\}, G_2(p_2) = \{h_2\}$.

If we define a mapping $f_g : L_1 \longrightarrow L_2$ as $f_g(l_1) = h_1, f_g(l_2) = f_g(l_3) = h_2$, then it is clear that $f_g(F_1)(p_1) = L_2$, $f_g(F_1)(p_2) = \{h_2\}$, $f_g(F_2)(p_1) = L_2$, $f_g(F_2)(p_2) = \{h_2\}$. Then, the mapping $f_g : L_1 \longrightarrow L_2$ is a soft L-open mapping.

Also, since $f_g(F_1')(p_1) = \{h_1\}, f_g(F_1')(p_2) = L_2$, it is not soft L-closed mapping and $f_g(F_2')(p_1) = L_1, f_g(F_2')(p_2) = \{l_2, l_3\}$. Hence, it is not soft L-continuous mapping.

Example 2.8. Suppose $L_1 = \{l_1, l_2, l_3\}, L_2 = \{h_1, h_2\}, P = \{p_1, p_2\}$ and $\tau_1 = \{\phi, L_1, F^L_{1P}, F^L_{2P}, F^L_{3P}\}$ is a soft L-topological space over L_1, $\tau_2 = \{\phi, L_2, G^L_{1P}, G^L_{2P}\}$ is a soft L-topological space over L_2, where $F^L_{1P}, F^L_{2P}, F^L_{3P}$ are soft lattices over L_1, G^L_{1P}, G^L_{2P} are soft lattices over L_2, defined as follows, $F_1(p_1) = \{l_1, l_3\}, F_1(p_2) = \{l_2\}$, $F_2(p_1) = L_1, F_2(p_2) = \{l_2, l_3\}$, $F_3(p_1) = \{l_3\}, F_2(p_2) = \{l_2\}$, and $G_1(p_1) = \phi, G_1(p_2) = \{h_1\}$, $G_2(p_1) = \{h_1\}, G_2(p_2) = L_2$, Now, we define a mapping $f_g : L_1 \longrightarrow L_2$ as $f_g(l_1) = f_g(l_2) = h_1, f_g(l_3) = h_2$. Then, it is clear that $f_g(F_1')(p_1) = f_g(l_2) = \{h_1\}$, $f_g(F_1')(p_2) = f_g(\{l_1, l_3\}) = L_2$,

$f_g(F_2')(p_1) = \phi$, $f_g(F_2')(p_2) = f_g(\{l_1\}) = \{h_1\}$, $f_g(F_3')(p_1) = f_g(l_2) = \{h_1\}$, $f_g(F_3')(p_2) = f_g(\{l_1, l_3\}) = L_2$, This implies that f_g is not soft L-closed mapping. $f_g(F_2')(p_1) = L_1, f_g(F_2')(p_2) = \{l_2, l_3\}$,

Hence, it is not soft L-continuous mapping. Also, $f_g(F_1)(p_1) = L_2$, $f_g(F_1)(p_2) = \{h_1\}$, $f_g^{-1}(G_1)(p_1) = \phi$, $f_g^{-1}(G_1)(p_2) = \{l_1, l_2\}$. Hence it is not soft L-open and soft L-continuous mappings respectively.

Theorem 2.4. Let (L_1, τ_1, P) and (L_2, τ_2, P) be two soft lattice topological spaces, $f_g\colon (L_1, \tau_1, P) \longrightarrow (L_2, \tau_2, P)$ be a mapping. Then:

(a) f_g is a soft L-open mapping if for each soft L-set F_P^L over L_1, $f_g((F_P^L)^o) \subset (f_g(F_P^L))^o$ is satisfied.

(b) f_g is a soft L-closed mapping if for each soft L-set F_P^L over L_1, $\overline{f_g((F_P^L))} \subset \overline{f_g(F_P^L)}$ is satisfied.

Proof. (a) Let f_g be a soft L-open mapping and F_P^L be a soft L-set over L_1.

$(F_P^L)^o$ is a soft L-open set and $(F_P^L)^o \subset F_P^L$.

Since f_g is a soft L-open mapping, $f_g((F_P^L)^o)$ is a soft L-open set in L_2 and $f_g((F_P^L)^o) \subset (f_g(F_P^L))$.

Thus, $f_g((F_P^L)^o) \subset (f_g(F_P^L))^o$ is obtained. Conversely, let F_P^L be a soft L-open set over L_1. Then, $F_P^L = (F_P^L)^o$. From the given condition(theorem), $f_g((F_P^L)^o) \subset (f_g(F_P^L))^o$. Then, $f_g(F_P^L) = f_g((F_P^L)^o) \subset (f_g(F_P^L))^o \subset f_g(F_P^L)$. This implies $f_g(F_P^L) = (f_g(F_P^L))^o$.

(b) Let f_g be a soft L-closed mapping and F_P^L be a soft L-set over L_1. $\overline{(F_P^L)}$ is a soft L-closed set and $(F_P^L) \subset \overline{F_P^L}$.

Since f_g is a soft L-closed mapping, $f_g\overline{(F_P^L)}$ is a soft L-open set in L_2 and $f_g(F_P^L) \subset f_g\overline{(F_P^L)}$. Thus, $\overline{f_g((F_P^L))} \subset f_g\overline{(F_P^L)}$ is obtained.

Conversely, let F_P^L be a soft L-closed set over L_1. Then $F_P^L = \overline{(F_P^L)}$. From the given condition(theorem), $\overline{(f_g(F_P^L))} \subset f_g\overline{((F_P^L))} \subset f_g(F_P^L) = \overline{(f_g(F_P^L))}$. This implies $\overline{f_g((F_P^L))} = f_g(F_P^L)$. □

Definition 2.22. Consider (L_1, τ_1, P) and (L_2, τ_2, P) as two soft lattice topological spaces, $f_g\colon (L_1, \tau_1, P) \longrightarrow (L_2, \tau_2, P)$ be a mapping. If f_g is a bijection, soft L-continuous and f_g^{-1} is a soft L-continuous mapping, then f_g is said to be soft L-homeomorphism from L_1 to L_2.

When a soft homeomorphism f_g exists between L_1 and L_2, we say that L_1 is soft L-homeomorphic to L_2.

Theorem 2.5. Let (L_1, τ_1, P) and (L_2, τ_2, P) be two soft lattice topological spaces, $f_g\colon (L_1, \tau_1, P) \longrightarrow (L_2, \tau_2, P)$ be a bijection mapping. Then, the following conditions are equivalent:

(1) f_g is a homeomorphism on soft L-topological space;

(2) f_g is a continuous and closed mapping on soft L-topological space;

(3) f_g is a continuous and open mapping on soft L-topological space.

Proof. Proof of the statements are followed from Definition 2.22 and Theorem 2.4. □

3. Conclusion

Topological structures on soft sets are more generalized methods that can be used to assess the similarities and dissimilarities between the objects in a universe which are soft sets. The concept of soft L-topological spaces are defined over a soft lattice with a fixed set of parameter. This paper deals with the concept of soft L-continuous mappings in Soft Lattice topological spaces which are defined with a fixed set of parameters over an initial universe. Also we have given some properties regarding the continuity of mappings in Soft Lattice topological spaces. At the end, open mappings, closed mappings and homeomorphism on soft L-topological spaces are defined and some results are proved.

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