sgp-resolvable and sgp-irresolvable spaces

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Abstract. The fundamental intent of this paper is to developing the idea of SSGP-continuous and SSGP-open functions using sgp-open sets. In addition sgp-resolvable and sgp-irresolvable spaces are introduced. Its various characterisations and properties are established.
Keywords: sgp-open set, SSGP-continuous, SSGP-irresolute , SSGP-open functoin, sgp-Resolvable space and sgp-Irresolvable space.

1. Introduction

The remarkable development in the field of general topology is the introduction of generalized closed sets by Levine [4] which plays a significant role. Many researchers studied extensively this notion. The investigation of generalized closed sets had paved the way for many interesting concepts. In [5] the idea of semigeneralized pre closed (briefly, sgp-closed) set was developed by G.B. Navalagi et al. In [6,7,8,9,10] authors continued the research using the set defined. Gentry and Hoyle [2] developed the concepts of "somewhat continuous and somewhat open functions".

In this paper, we will continue research related functions with sgp-closed and sgp-open sets. We introduce and characterize the concept of "somewhat

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sgp-continuous, Somewhat sgp-irresolute and Somewhat sgp-open functions". In addition to this sgp-resolvable and sgp-irresolvable spaces are introduced. Using these spaces characterization of defined open and continuous functions is done.

2. Preliminaries

In entire paper (X τ), (Y, σ) (or shortly X_τ, Y_σ) symbolize topological space(in short TS)s on which no separation axioms are assumed unless explicitly stated.

Definition 2.1. *A subset R of a TS X_τ is called:*

(1) *semi-open set [3] if $R \subset Cl(Int(R))$.*

(2) *semi-closed set [1] if $Int(Cl(Int(R))) \subset R$.*

Definition 2.2 ([5])**.** *A subset W of X_τ is semi generalized pre-closed (briefly, sgp-closed)set if $pCl(W) \subset \Delta$ whenever $W \subset \Delta$ and Δ is semi-open in X_τ. The complement of sgp-closed set(SGPCS in short) is semi generalized-pre open (briefly,sgp-open). The collection of all sgp-closed set (SGPCS in short) of X_τ is labelled as $SGPC(X_\tau)$ and sgp-open set(SGPOS in short)s by $SGPO(X_\tau)$.*

Definition 2.3. *A function $\mu \colon X_\tau \to Y_\sigma$ is named as:*

(*i*) *semi generalized pre-irresolute (shortly, sgp-irresolute)* [5] *if $\mu^{-1}(J)$ is SGPCS in X_τ for every SGPCS J of Y_σ;*

(*ii*) *semi generalized pre-continuous (shortly, sgp-continuous)* [5] *if $\mu^{-1}(C_1)$ is SGPCS in X_τ for each closed set C_1 of Y_σ;*

(*iii*) *somewhat continuous [2] if for $S_1 \in \sigma$ along with $\mu^{-1}(S_1) \neq \phi$ there prevails an open set K in X_τ thereby $K \neq \phi$ together with $K \subseteq \mu^{-1}(S_1)$.*

Definition 2.4 ([5])**.** *A function $\eta \colon X_\tau \to Y_\sigma$ is termed as sgp-open (resp., sgp-closed) if $\eta(N_0)$ is sgp-open (resp., sgp-closed) in Y_σ for each open set (resp., closed) N_0 in X_τ.*

3. Somewhat sgp-continuous functions

Definition 3.1. A function $\mu : X_\tau \to Y_\sigma$ is named as Somewhat sgp-continuous (in short, SSGP-continuous) if each $K \in Y_\sigma$ and $\mu^{-1}(K) \neq \phi$ there arises a SGPOS W in X_τ such that and $W \neq \phi$ and $V \subseteq \mu^{-1}(K)$.

Theorem 3.1. *Each somewhat continuous function is SSGP-continuous .*

Proof. Consider $\eta : X_\tau \to Y_\sigma$ is SSGP-continuous function. Take K as an open set in Y_σ thereby $\eta^{-1}(K) \neq \phi$. As η is somewhat continuous function, there exists an open set R in X_τ such that $R \neq \phi$ and $R \subseteq \eta^{-1}(K)$. As each open set is SGPOS, there prevails SGPOS, R so that $R \neq \phi$ and $R \subseteq \eta^{-1}(K)$, which gives η is SSGP-continuous function. □

Remark 3.2. The counter part of the above theorem is not possible.

Example 3.3. For $X_\tau = Y_\sigma = \{v_1, v_2, v_3\}$, $\tau = \{X, \phi, \{v_1\}\}$, $\sigma = \{Y, \phi, \{v_1\}, \{v_1, v_2\}\}$. We have SGPO(X) $= \{X, \phi, \{v_1\}, \{v_1, v_2\}, \{v_1, v_3\}\}$. Then, the identity map is SSGP-continuous yet it isn't somewhat continuous .

Theorem 3.4. *If $\lambda : X_\tau \to Y_\sigma$ and $\mu : Y_\sigma \to Z_\eta$ be any two functions. Whenever λ is SSGP-continuous and μ is continuous function, thereupon $\mu \circ \lambda$ is SSGP-continuous function.*

Proof. Take Δ be any open set in Z_η. Assume that $\mu^{-1}(\Delta) \neq \phi$. As $\Delta \in Z_\eta$ and μ is continuous function, $\mu^{-1}(\Delta) \in Z_\eta$. Guess that $\lambda^{-1}(\mu^{-1}(\Delta)) \neq \phi$. As par hypothesis λ is SSGP-continuous, there exists a SGPOS, K in X_τ such that $K \neq \phi$ and $K \subseteq \lambda^{-1}(\mu^{-1}(\Delta)) = (\mu \circ \lambda)^{-1}(\Delta)$, which implies that $K \subseteq (\mu \circ \lambda)^{-1}(\Delta)$. Therefore, $\mu \circ \lambda$ is SSGP-continuous function. □

Definition 3.2. *A subset W of a TS X is named as sgp-dense in X_τ whenever there is no proper SGPCS R in X_τ so as $W \subset R \subset X$.*

Theorem 3.5. *The consecutive are identical for $d : X_\tau \to Y_\sigma$:*

(*i*) *d is SSGP-continuous function;*

(*ii*) *Whenever $\mathcal{F}$ is a closed subset of Y_σ thereby $d^{-1}(\mathcal{F}) \neq X_\tau$, subsequently there is proper sgp-closed member D of X_τ so as $d^{-1}(\mathcal{F}) \subset D$;*

(*iii*) *Whenever $\mathcal{P}$ is a sgp-dense member of X_τ, then $d(\mathcal{P})$ is a dense subset of Y_σ.*

Proof. (i)⇒(ii): Consider $\mathcal{F}$ as a closed subset of Y_σ so, that $d^{-1}(\mathcal{F}) \neq X_\tau$. Thereupon $d^{-1}(Y_\sigma - \mathcal{F}) = X_\tau - d^{-1}(\mathcal{F}) \neq \phi$. From (i) there prevails a SGPOS V in X_τ thereby $V \neq \phi$ and $V \subset d^{-1}(Y - \mathcal{F}) = X_\tau - d^{-1}(\mathcal{F})$. This intend $d^{-1}(\mathcal{F}) \subset X_\tau - V$ and $X_\tau - V = D$ is a SGPCS in X_τ. Therefore (ii) holds.

(ii)⇒ (i): Take U as an open set in Y_σ and $d^{-1}(U) \neq \phi$. Accordingly, $d^{-1}(Y - U) = X - d^{-1}(U) = \phi$. By hypothesis, there arises a proper SGPCS, D such that $d^{-1}(Y - U) \subset D$. This implies that $X - D \subset d^{-1}(U)$ and $X_\tau - D$ is sgp-open and $X_\tau - D \neq \phi$.

(ii)⇒(iii): For a sgp-dense set $\mathcal{P}$ in X_τ. Suppose $d(\mathcal{P})$ is not dense subset of Y_σ, then there exists a proper SGPCS, D such that $\mathcal{P} \subset d^{-1}(\mathcal{F}) \subset D \subset X_\tau$. This disprove the fact that $\mathcal{P}$ is a sgp-dense set in X_τ. Therefore, (iii) holds.

(iii)⇒ (ii): Guess (iii) is doesn't holds. Then there arises a closed set $\mathcal{F}$ in Y_σ so long as $d^{-1}(\mathcal{F}) \neq X$. Though there is no proper SGPCS that $d^{-1}(\mathcal{F}) \subset D$. This means that $d^{-1}(\mathcal{F})$ is sgp-dense in X_τ. But from hypothesis $d(d^{-1}(\mathcal{F})) = \mathcal{F}$ must be dense in Y_σ, which negate the selection of $\mathcal{F}$. Hence (ii) hold. □

Theorem 3.6. *If $\mu : X_\tau \to Y_\sigma$ be a function and $X_\tau = N_1 \cup N_2$, N_1 and N_2 are open subsets of X_τ so as (μ/N_1) and (μ/N_2) are SSGP-continuous functions then μ is SSGP-continuous.*

Proof. Consider an open set $\mathcal{K}$ in Y_σ so as $\mu^{-1}(\mathcal{K}) \neq \phi$. Thereupon $(\mu/N_1)^{-1}(U) \neq \phi$ of $(\mu/N_2)^{-1}(\mathcal{K}) \neq \phi$ or both $(\mu/N_1)^{-1}(\mathcal{K}) \neq \phi$ and $(\mu/N_2)^{-1}(\mathcal{K}) \neq \phi$.
Case (i). Assume $(\mu/N_1)^{-1}(\mathcal{K}) \neq \phi$. Since (μ/N_1) is SSGP-continuous, then there exists a SGPOS C_1 in N_1 so that $C_1 \neq \phi$ and $C_1 \subset (\mu/N_1)^{-1}(\mathcal{K}) \subseteq \mu^{-1}(\mathcal{K})$. Since C_1 is SGPOS in $N_1$and N_1 is open in X_τ, C_1 is SGPOS in X_τ. Hence μ is SSGP-continuous function.
Case(ii). Presume that $(\mu/N_2)^{-1}(\mathcal{K}) \neq \phi$. As (μ/N_2) is SSGP-continuous function, then there exists a SGPOS C_1 in N_2 thereby $C_1 \neq \phi$ and $C_1 \subset (\mu/N_2)^{-1}(\mathcal{K}) \subset \mu^{-1}(\mathcal{K})$. As C_1 is SGPOS in N_2 as well as N_2 is open in X_τ, C_1 is SGPOS in X_τ. Hence μ is SSGP-continuous function.
Case(iii). Assume that $(\mu/N_1)^{-1}(\mathcal{K}) \neq \phi$ and $(\mu/N_2)^{-1}(\mathcal{K}) \neq \phi$. Follows from case(i) as well as case(ii). □

Definition 3.3. *A TS X_τ is termed as sgp-separable if there exists a countable subset R of X_τ whichever is sgp-dense in X_τ.*

Theorem 3.7. *Whenever η is SSGP-continuous from X_τ onto Y_σ along with X_τ as sgp-separable, thereupon Y_σ also separable.*

Proof. Consider η be SSGP-continuous function in such a way that X_τ is sgp-separable. Due to definition there prevails a countable member Δ of X_τ which is sgp-dense in X_τ. With reference to Theorem 3.5, $\eta(\Delta)$ is dense in Y_σ. As Δ is countable $\eta(\Delta)$ is also countable which is dense in Y_σ, so Y_σ is also separable. □

4. Somewhat sgp-irresolute function

Definition 4.1. A function $\mu : X_\tau \to Y_\sigma$ is termed as Somewhat sgp-irresolute (in short SSGP-irresolute) if for $J_0 \in SGPO(\sigma)$ and $\mu^{-1}(J_0) \neq \phi$, there exists a non-empty SGPOS set R in X_τ so that $R \subset \mu^{-1}(J_0)$.

Theorem 4.1. *If λ is SSGP-irresolute along with μ is sgp-irresolute function, then $(\mu \circ \lambda)$ is SSGP-irresolute.*

Proof. Consider $M \in SGPO(\sigma)$. Presume that $\mu^{-1}(M) \neq \phi$. As $M \in SGPO(\sigma)$ along with μ is SSGP-irresolute function, there exists a SGPOS W in X_τ so $W \neq \phi$ and $W \subseteq \lambda^{-1}(\mu^{-1}(M))$. However, $\lambda^{-1}(\mu^{-1}(M)) = (\mu \circ \lambda)^{-1}(M)$ $\Rightarrow W \subseteq (\mu \circ \lambda)^{-1}(M)$. Thus, $(\mu \circ \lambda)$ is SSGP-irresolute function. □

Theorem 4.2. *For $\alpha : X_\tau \to Y_\sigma$ subsequents are equal:*

- *(i) α is SSGP-irresolute function.*
- *(ii) Whenever K is a closed subset of Y_σ so as $\alpha^{-1}(K) \neq X$, then there is proper sgp-closed member D_1 of X thereby $\alpha^{-1}(K) \subset D_1$.*
- *(iii) Whenever K is a sgp-dense member of X_τ, thereupon $\alpha(K)$ is a dense subset of Y_σ.*

Proof. Obvious. □

Theorem 4.3. *If $f : X_\tau \to Y_\sigma$ be a function and $X_\tau = K_1 \cup K_2$, K_1 and K_2 are open members of X_τ such that (f/K_1) and (f/K_2) are SSGP-irresolute function then f is also SSGP-irresolute .*

Proof. Obvious. □

Definition 4.2. *If X_τ is a set and τ as well as σ are topologies for X_τ, so τ is named as sgp-equivalent to σ provided whenever $J \in \tau$ along with $J \neq \phi$, then there is a SGPOS R in Y_σ thereby $R \neq \phi$ and $R \subset J$ and if $J \in \sigma$ and $J \neq \phi$ accordingly there is SGPOS R in X_τ so as $R \neq \phi$ and $R \subset J$.*

Theorem 4.4. *Let $\lambda : X_\tau \to Y_\sigma$ be a somewhat continuous function and τ^* be a topology for X_τ, which is sgp-equivalent to τ then $\lambda : X^*_\tau \to Y_\sigma$ is SSGP-continuous function.*

Proof. Consider an open set D_1 in Y_σ so as $\lambda^{-1}(D_1) \neq \phi$. In view of hypothesis λ is somewhat continuous function then there arises an open set in Δ in X_τ thereby $\Delta \neq \phi$ and $\Delta \subseteq \lambda^{-1}(D_1)$. As Δ is an open set in X_τ so that $\Delta \neq \phi$ and due to postulate τ is sgp-equivalent to τ^* so, there exists a SGPOS C_1 in X^*_τ so as $C_1 \neq \phi$ and $C_1 \subset \Delta \subset \lambda^{-1}(D_1)$. Henceforth $\Delta \subset \lambda^{-1}(D_1)$. Accordingly, for any open set D_1 in Y_σ so as $\lambda^{-1}(D_1) \neq \phi$ there prevails a SGPOS, C_1 in X^*_τ such that $C_1 \subset \lambda^{-1}(D_1)$. So $\lambda : X^*_\tau \to Y_\sigma$ is SSGP-continuous function. □

Theorem 4.5. *For SSGP-continuous function $\mu : X_\tau \to Y_\sigma$ along with σ^* be a topology for Y_σ, which is equivalent to σ. Then $\mu : X_\tau \to Y^*_\sigma$ is SSGP-continuous.*

Proof. Consider $\mathcal{G}$ be any open set in Y^*_σ thereby $\mu^{-1}(\mathcal{G}) \neq \phi \Rightarrow \mathcal{G} \neq \phi$. Since σ as well as σ^* are equivalent, then there arises an open set W in Y_σ thereby $W \neq \phi$ and $W \subset U$. Now, W is open set hereupon $W \neq \phi \Rightarrow \mu^{-1}(W) \neq \phi$. By the postulate $\mu : X_\tau \to Y_\sigma$ is SSGP-continuous. Therefore there prevails a SGPOS, V_1 in (X, τ) so that $V_1 \subseteq \mu^{-1}(W)$. Now, $W \subset \mathcal{G} \Rightarrow \mu^{-1}(W) \subset \mu^{-1}(\mathcal{G})$. This impart $V_1 \subset \mu^{-1}(W) \subset \mu^{-1}(\mathcal{G})$. So, we have $V_1 \subset \mu^{-1}(\mathcal{G}) \Rightarrow \mu : X_\tau \to Y^*_\sigma$ is SSGP-continuous. □

Theorem 4.6. *Let $\mu : X_\tau \to Y_\sigma$ be a SSGP-irresolute surjection function and let σ^* be a topology for Y_σ, which is equivalent to σ. Then, the function $\eta : X_\tau \to Y^*_\sigma$ is SSGP-irresolute function.*

Proof. Consider an open set $\mathcal{K}$ in Y^*_σ ther by $\eta^{-1}(\mathcal{K}) \neq \phi \Rightarrow \mathcal{K} \neq \phi$. As σ and σ^* are identical, thereupon there exists an open set W in Y_σ so as $W \neq \phi$ and $W \subset \mathcal{K}$. Now, W is open set such that $W \neq \phi$, which leads to $\mu^{-1}(W) \neq \phi$. Now, by postulation μ is SSGP-irresolute function. Therefore, there exists a SGPOS, S_1 in X_τ such that $S_1 \subseteq \mu^{-1}(W)$. Now, $W \subset \mathcal{K} \Rightarrow \mu^{-1}(W) \subset \eta^{-1}(\mathcal{K})$. This gives $S_1 \subset \mu^{-1}(W) \subset \eta^{-1}(\mathcal{K})$. So, we have $S_1 \subset \eta^{-1}(\mathcal{K})$, this leads to η is SSGP-irresolute function. □

5. Somewhat sgp-open functions

Definition 5.1. A function $\lambda : X_\tau \to Y_\sigma$ is termed as Somewhat sgp-open (SSGP-open, for short) provided that for each $K \in \tau$ with $K \neq \phi$, there exists a SGPOS, W in Y_σ such that and $W \neq \phi$ and $W \subseteq \lambda(Q)$.

Theorem 5.1. Each somewhat open function is SSGP-open.

Proof. Consider somewhat open function $\eta : X_\tau \to Y_\sigma$. Take $K \in \tau$ with $K \neq \phi$. As η is somewhat open function, there exists an open set J_1 in Y_σ such that $J_1 \neq \phi$ and $J_1 \subseteq \eta(K)$. However, each open is sgp-open. So, there exists a SGPOS, J_1 in Y_σ so that $J_1 \neq \phi$. Thus, η is SSGP-open function. □

Remark 5.2. The example below makes clear that reverse of the above theorem impossible.

Example 5.3. Let $X_\tau = Y_\sigma = \{k_1, k_2, k_3\}$, $\tau = \{X, \phi, \{k_1\}, \{k_2\}, \{k_1, k_2\}\}$, $\sigma = \{Y, \phi, \{k_1, k_2\}\}$. We have SGPO(Y) $= \{Y, \phi, \{k_1\}, \{k_2\}, \{k_1, k_2\}, \{k_1, k_3\}, \{k_2, k_3\}\}$. Then, the identity function is SSGP-open yet it isn't not somewhat open.

Theorem 5.4. *If $k : X_\tau \to Y_\sigma$ is an open function and $j : Y_\sigma \to Z_\eta$ is SSGP-open then $(j \circ k)$ is SSGP-open function.*

Proof. Consider $\mathcal{M} \in \tau$. Suppose $\mathcal{M} \neq \phi$. Since k is an open function, $k(\mathcal{M})$ is open and $k(\mathcal{M}) \neq \phi$. Consequently $k(\mathcal{M}) \in \sigma$ and $k(\mathcal{M}) \neq \phi$. As j is SSGP-open function and $k(\mathcal{M}) \in \sigma$ such that $k(\mathcal{M}) \neq \phi$, there exists a SGPOS $R_0 \in \eta$, $R_0 \subset j(k(\mathcal{M}))$, which indirect $(j \circ k)$ is SSGP-open function. □

Theorem 5.5. *For a bijective function $h : X_\tau \to Y_\sigma$ succeeding are equal;*
(i) h is SSGP-open function.
(ii) If $\mathcal{Q}$ is a closed subset of Y_σ such that $h(\mathcal{Q}) \neq Y$, then there exists a sgp-closed subset J of Y_σ such that $J \neq \phi$ and $h(\mathcal{Q}) \subset J$.

Proof. (i)⇒(ii): Consider closed subset $\mathcal{Q}$ of Y_σ such that $h(\mathcal{Q}) \neq Y_\sigma$. From (i), there exists a SGPOS W in X_τ such that $W \neq \phi$ such that $W \subset h(X_\tau - \mathcal{Q})$. Put $J = Y_\sigma - W$. Clearly J is a SGPCS in Y_σ and we claim that $J \neq \phi$. If $J = Y_\sigma$, then $W = \phi$ which is a negation. Since $W \subset h(X_\tau - \mathcal{Q})$, $J = Y_\sigma - W \subset Y_\sigma - [h(X_\tau - \mathcal{Q})] = h(\mathcal{Q})$.

(ii)⇒ (i): Take L_1 as a non empty open set in X_τ. Substitute $\mathcal{Q} = X_\tau - L_1$. Thereupon $\mathcal{Q}$ is a closed member of X_τ and $h(X - L_1) = h(\mathcal{Q}) = Y_\sigma - h(L_1)$ which imparts $h(\mathcal{Q}) \neq \phi$. With reference to (ii), there is a sgp-closed subset J of Y_σ so that $h(L_1) \subset J$. Put $V = X_\tau - J$, clearly V is SGPOS and $V \neq \phi$. Further $V = X_\tau - J \subset Y_\sigma - h(\mathcal{Q}) = Y_\sigma - [Y_\sigma - h(\mathcal{Q})] = h(L_1)$. □

Theorem 5.6. *If $k : X_\tau \to Y_\sigma$ be SSGP-open function and R be any open subset of X. Then $k/R : (R, \tau/R) \to (Y, \sigma)$ is also SSGP-open funtion.*

Proof. Consider $\mathcal{B} \in \tau/\mathcal{R}$ such that $\mathcal{B} \neq \phi$. Since $\mathcal{B}$ is open in $\mathcal{R}$ and $\mathcal{R}$ is open in X_τ, $\mathcal{B}$ is open in X_τ and due to the postulation k is SSGP-open function, then there exists a SGPOS, $\mathcal{L}$ in Y_σ, such that $\mathcal{L} \subset k(\mathcal{B})$. Thus, for any open set $\mathcal{B}$ in $(\mathcal{R}, \tau/\mathcal{R})$ with $\mathcal{B} \neq \phi$, there prevails a SGPOS, $\mathcal{L}$ in Y_σ thereby $\mathcal{L} \subset k(\mathcal{B})$ which imparts $k/\mathcal{R}$ is SSGP-open funtion. □

Theorem 5.7. *Let $\mu : X_\tau \to Y_\sigma$ be a function such that μ/K_1 and μ/K_2 are SSGP-open, then μ is SSGP-open function, where $X = K_1 \cup K_2$, K_1 and K_2 are open subsets of X.*

Proof. Obvious. □

Theorem 5.8. *The statements are equivalent:*

(1) *k be SSGP-open function.*

(2) *Whenever S_1 is a sgp-dense subset of Y_σ, then $k^{-1}(S_1)$ is a dense subset of X_τ.*

Proof. (i)⇒(ii): Presume S_1 is a sgp-dense set in Y_σ. If $k^{-1}(S_1)$ is not dense in X_τ, then there arises a closed set N_1 in X_τ so as $k^{-1} \subset N_1 \subset X_\tau$. As k is SSGP-open and $X_\tau - N_1$ is open, there exists a nonempty SGPOS H_1 in Y_σ so that $H_1 \subset k(X_\tau - N_1)$. Thereupon, $H_1 \subset k(k^{-1}(Y_\sigma - S_1)) \subset Y_\sigma - S_1$ implies $S_1 \subset Y_\sigma - H_1 \subset Y_\sigma$. Now, $Y_\sigma - H_1$ is a SGPCS along with $S_1 \subset Y_\sigma - H_1 \subset Y_\sigma$. This indicates that S_1 is not a sg-dense set in Y_σ, which is negation. So, $k^{-1}(S_1)$ is a dense in X_τ.

(ii)⇒(i): Suppose S_1 is a nonempty open subset of X_τ. We need to prove that $sgp(k(S_1))^0 \neq \phi$. Presume $sgp(k(S_1))^0 = \phi$. Thereupon, $sgp\overline{(k(S_1))} = Y_\sigma$. So by (ii), $k^{-1}Y_\sigma - k(S_1)$ is dense in X_τ. But $k^{-1}(Y_\sigma - k(S_1)) \subset X_\tau - S_1$. Now, $X_\tau - S_1$ is closed. Accordingly, $k^{-1}(Y_\sigma - k(S_1)) \subset X_\tau - S_1$ gives $X_\tau = \overline{k^{-1}(Y_\sigma - k(S_1)))} \subset X_\tau - S_1$. This implies that $S_1 = \phi$ which contrary to $S_1 \neq \phi$. Therefore, $sgp(k(S_1))^0 \neq \phi$. Thereupon k is SSGP-open. □

6. sgp-resolvable spaces

Definition 6.1. *A TS X_τ is named as sgp-resolvable if there arises a sgp-dense set K in X_τ so that $X_\tau - K$ is also sgp-dense. Otherwise, X_τ is termed as sgp-irresolvable.*

Theorem 6.1. *The succeeding statements are similar*

(*i*) *X_τ is sgp-resolvable;*

(*ii*) *X_τ has a pair of sgp-dense sets Δ_1 as well as Δ_2 such that $\Delta_1 \subset \Delta_2$.*

Proof. (i)⇒(ii): Assume that X_τ is sgp-resolvable. There exists a sgp-dense set Δ_1 such that $X_\tau - \Delta_1$ is sgp-dense. Set $\Delta_1 = X_\tau - \Delta_2$.

(ii)⇒(i): Presume that (ii) is true. Take X_τ as sgp-irresolvable. A that time $X_\tau - \Delta_2$ is not sgp-dense and $sgp\overline{(\Delta_1)} \subset sgp\overline{(X_\tau - \Delta_2)} \neq X$. Hence, Δ_1 is not sgp-dense. This negates the presumption. □

Theorem 6.2. *For X_τ the succeeding statements are identical:*

(*i*) *X_τ is sgp-irresolvable;*

(*ii*) *For any sgp-dense set Δ in X_τ $sgp(\Delta)^o \neq \phi$.*

Proof. (i)⇒(ii): Consider Δ be any sgp-dense set of X_τ. Then, we have $sgp\overline{(X_\tau - \Delta)} \neq X_\tau$; then $sgp(\Delta)^o \neq \phi$.

(ii)⇒(i): Presume that X_τ is sgp-resolvable space. Then, there arises a sgp-dense set Δ in X_τ so that Δ^c is also sgp-dense in X_τ. It follows that $sgp(\Delta)^o = \phi$, which is a negation; henceforth X_τ is sgp-irresolvable. □

Definition 6.2. *A TS is termed as strongly sgp-irresolvable if for a nonempty set K in X_τ $sgp(K)^o = \phi$ implies $sgp(sgp\overline{K})^o = \phi$.*

Theorem 6.3. *Whenever X_τ is a strongly sgp-irresolvable space along with $sgp\overline{K} = X_\tau$ for a nonempty subset K of X, then $sgp\overline{(sgp(K)^o)} = X_\tau$.*

Theorem 6.4. *If X_τ is a strongly sgp-irresolvable space as well as $sgp\overline{K} = X_\tau$ along with $sgp(sgp\overline{K})^o \neq \phi$ for any nonempty subset K in X_τ, thence $sgp(K)^o \neq X_\tau$.*

Theorem 6.5. *Every strongly-irresolvable space is sgp-irresolvable. However converse is not true.*

Example 6.6. Consider $X_\tau = Y_\sigma$=$\{r_1, r_2, r_3\}$ with $\tau = \{X, \phi, \{r_1\}\}$. Then (X_τ, τ) is sgp-irresolvable space however it isn't strongly sgp-irresolvable.

Theorem 6.7. *If η is SSGP-open and $sgp(R)^o = \phi$ for a nonempty set R in Y_σ, then $(\eta^{-1}(R))^o = \phi$.*

Proof. Consider a nonempty set R in Y_σ so that $sgp(R)^o = \phi$. Thereupon $sgp\overline{(Y_\sigma - R)} = Y_\sigma$. As η is SSGP-open and $Y_\sigma - R$ is sgp-dense in Y_σ, by Theorem 5.9, $\eta^{-1}(Y_\sigma - R)$ is dense in X_τ. Accordingly $\overline{(X_\tau - \eta^{-1}(R))} = X_\tau$; so $(\eta^{-1}(R))^o = \phi$. □

Theorem 6.8. *For a function $\eta : X_\tau \to Y_\sigma$, whenever X is irresolvable, then Y_σ is sgp-irresolvable.*

Proof. Consider a nonempty set R in Y so that $sgp\overline{(R)} = Y$. We need to prove that $sgp(R)^o \neq \phi$. Suppose not, then $sgp\overline{(Y - R)} = Y$. As η is SSGP-open and $Y - R$ is sgp-dense in Y, with reference to Theorem 5.8, $\eta^{-1}(Y - R)$ is dense in X. Accordingly, $(\eta^{-1}(R))^o = \phi$. Now, since R is sgp-dense in Y, $\eta^{-1}(R)$ is dense in X. Therefore, for the dense set $\eta^{-1}(R)$, we have $(\eta^{-1}(R))^o = \phi$, which is a contradiction to theorem 6.3. So, we should have $sgp(R)^o \neq \phi$, for all sgp-dense sets R in Y. Hence, by Theorem 6.3, Y is sgp-irresolvable. □

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