Cubic fuzzy $\beta-$ideals of $\beta-$algebras

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Abstract. This paper extends fuzzy $\beta-$ideal into cubic fuzzy $\beta-$ideal of a $\beta-$algebra. Further, some related results using Cartesian product and homomorphism are also studied.
Keywords: $\beta-$algebra, $\beta-$ideals, fuzzy $\beta-$ideal, cubic ideals, cubic $\beta-$ideal.

1. Introduction

Fuzzy sets have been introduced by Zadeh [12, 13] in 1965 to manipulate data and information possessing non statistical uncertainties. He also discussed about the concept of a linguistic variable which presents a means of approximate characterization of phenomena which are too complicated in conventional quantitative terms. Atanassov [2] initiated Intuitionistic fuzzy sets as an extension of fuzzy set whose components have degrees of membership and non-membership. Neggers and Kim [5] presented the thought of $\beta-$algebras as a generalization of $BCK-$algebras and $BCI-$algebras. Abu Ayub Ansari and

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chandramouleeswaran [1] proposed the notion of fuzzy $\beta-$ideals of $\beta-$algebras. In [3, 4], Hemavathi et al. described the concept of interval valued fuzzy $\beta-$subalgebras also they have extended the thought of $\beta-$ideals in interval valued fuzzy set. In 2012, the thought of cubic sets has been initiated by Jun et al. [10]. The notion of cubic subalgebras and ideals of $BCK/BCI-$ algebras was described by Jun et al. [8, 9]. Also they have applied the cubic structures in to ideals of $BCI-$algebras. Senapathi et al. [7] originated the perception of cubic implicative ideal of $BCK-$algebras. The notion of cubic $BRK-$ideal of $BRK-$algebra was initiated by Osama [6] in 2015. Recently Muralikrishna et al. [11] described some aspects on cubic fuzzy $\beta-$subalgebras of $\beta-$algebra. With all these inspiration, this paper provides the study of cubic fuzzy $\beta-$ideals of $\beta-$algebras and discusses some fascinating results.

2. Preliminaries

This section reveals the necessary definitions required for the work.

Definition 2.1. *A $\beta-$ algebra is a non-empty set X with a constant* 0 *and two binary operations* $+$ *and* $-$ *are satisfying the following axioms:*

(*i*) $x - 0 = x$;

(*ii*) $(0 - x) + x = 0$;

(*iii*) $(x - y) - z = x - (z + y)$, $\forall\ x, y, z \in X$.

Example 2.1. The following Cayley table shows $(X = \{0, 1, 2, 3\}, +, -, 0)$ is a $\beta-$algebra.

+	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	1	0
3	3	2	0	1

−	0	1	2	3
0	0	1	3	2
1	1	0	2	3
2	2	3	0	1
3	3	2	1	0

Example 2.2. Consider Set of all integers Z. $(Z, +, -, 0)$ is an infinite $\beta-$algebra where 0, + and − have usual meanings.

Definition 2.2. *A non-empty subset I of a $\beta-$algebra $(X, +, -, 0)$ is called a $\beta-$ideal of X, if*

(*i*) $0 \in I$;

(*ii*) $x + y \in I$;

(*iii*) $x - y,\ y \in I$ *then* $x \in I, \forall x, y \in X$.

Example 2.3. Consider the $\beta-$algebra $(X, +, -, 0)$ in the following Cayley's table

$+$	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	0	1
3	3	2	1	0

$-$	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	0	1
3	3	2	1	0

The subset $I_1 = \{0, 1\}$ is a $\beta-$ideal of X.

Definition 2.3. *An interval valued fuzzy set (briefly i-v fuzzy set) A defined on a non empty set X is given by* A=$\{(x, [\zeta_A^L(x), \zeta_A^U(x)])\}, \forall\, x \in X$ *(briefly denoted by* $A = [\zeta_A^L, \zeta_A^U]$*), where* ζ_A^L *and* ζ_A^U *are two fuzzy sets in X such that* $\zeta_A^L(x) \leq \zeta_A^U(x), \forall\, x \in X$*. Let* $\overline{\zeta}_A(x) = [\zeta_A^L(x), \zeta_A^U(x)], \forall\, x \in X$ *and let* $D[0,1]$ *denotes the family of all closed sub intervals of* $[0,1]$*. If* $\zeta_A^L(x) = \zeta_A^U(x) = c$*, say, where* $0 \leq c \leq 1$*, then we have which we also assume, for the sake of convenience, to belong to* $D[0,1]$*. Thus,* $\overline{\zeta}_A(x) \in D[0,1], \forall x \in X$*, and* $\overline{\zeta}_A(x) \in D[0,1], \forall\, x \in X$*, and therefore the i-v fuzzy set A is given by* $A = \{(x, \overline{\zeta}_A(x))\}, \forall\, x \in X$*, where* $\overline{\zeta}_A : X \to D[0,1]$*. Now, let us define what is known as* $refined$ *mimimum (briefly* $r\min$*) of two elements in* $D[0,1]$*. We also define the symbols "* $\geq$ *" , "* $\leq$ *", and "* $=$ *" in case of two elements in* $D[0,1]$*. Consider two elements* $D_1 := [a_1, b_1]$ *and* $D_2 := [a_2, b_2] \in D[0,1]$*. Then, we have* $r\min(D_1, D_2) = [\min\{a_1, a_2\}, \min\{b_1, b_2\}]$*;* $D_1 \geq D_2$ *if and only if* $a_1 \geq a_2,\ b_1 \geq b_2$*; Similarly we may have* $D_1 \leq D_2$ *and* $D_1 = D_2$*.*

Definition 2.4. *Let* ζ *be a fuzzy set in a* $\beta-$ *algebra. Then* ζ *is called a fuzzy* $\beta-$*ideal of X, if:*

(i) $\zeta(0) \geq \zeta(x)$;

(ii) $\zeta(x + y) \geq \min\{\zeta(x), \zeta(y)\}$;

(iii) $\zeta(x) \geq \min\{\zeta(x - y), \zeta(y)\}, \forall x, y \in X$.

Definition 2.5. *Let* $A = \{\langle x, \overline{\zeta}_A(x)\rangle : x \in X\}$ *be an i-v fuzzy set in a* $\beta-$*algebra of X. Then A is called an interval valued fuzzy* $\beta-$ *ideal of X if:*

(i) $\overline{\zeta}_A(0) \geq \overline{\zeta}_A(x), \forall x \in X$;

(ii) $\overline{\zeta}_A(x + y) \geq r\min\{\overline{\zeta}_A(x), \overline{\zeta}_A(y)\}, \forall x, y \in X$;

(iii) $\overline{\zeta}_A(x) \geq r\min\{\overline{\zeta}_A(x - y), \overline{\zeta}_A(y)\}, \forall x, y \in X$.

Definition 2.6. *Let X be a non empty set. By a cubic set in X we mean a structure* $C = \{\langle x, \overline{\zeta}_C(x), \eta_C(x)\rangle :\ x \in\ X\}$ *in which* $\overline{\zeta}_C$ *is an interval valued fuzzy set in X and* η_C *is a fuzzy set in X.*

Definition 2.7. *Let* $C = \{\langle x, \overline{\zeta}_C(x), \eta_C(x)\rangle : x \in X\}$ *be a cubic fuzzy (CF) set in a non empty set X. Then Then, the set C is a cubic fuzzy* $\beta-$ *subalgebra if it satisfies the following conditions*

(i) $\overline{\zeta}_C(x+y) \geq r\min\{\overline{\zeta}_C(x), \overline{\zeta}_C(y)\}$ & $\overline{\zeta}_C(x-y) \geq r\min\{\overline{\zeta}_C(x), \overline{\zeta}_C(y)\}$;

(ii) $\eta_C(x+y) \leq \max\{\eta_C(x), \eta_C(y)\}$ & $\eta_C(x-y) \leq \max\{\eta_C(x), \eta_C(y)\}$, $\forall\ x, y \in X$.

3. Cubic fuzzy $\beta-$ ideals

This section starts with the definitions of cubic fuzzy $\beta-$ideals of $\beta-$algebras. Also throughout the paper, X is a $\beta-$algebra unless and otherwise specified.

Definition 3.1. *Let $C = \{x, \overline{\zeta}_C(x), \eta_C(x) : x \in X\}$ be a cubic fuzzy set in a $\beta-$algebra of X. C is called a cubic fuzzy $\beta-$ideal of X, if $\forall x, y \in X$:*

(i) $\overline{\zeta}_C(0) \geq \overline{\zeta}_C(x)$ & $\eta_C(0) \leq \eta_C(x)$;

(ii) $\overline{\zeta}_C(x+y) \geq r\min\{\overline{\zeta}_C(x), \overline{\zeta}_C(y)\}$&$\eta_C(x+y) \leq \max\{\eta_C(x), \eta_C(y)\}$;

(iii) $\overline{\zeta}_C(x) \geq r\min\{\overline{\zeta}_C(x-y), \overline{\zeta}_C(y)\}$ & $\eta_C(x) \leq \max\{\eta_C(x-y), \eta_C(y)\}$.

Example 3.1. Let $X = \{0, a, b, c\}$ be a $\beta-$algebra with constant 0 and binary operations + and − are defined on X as in the following Cayley's table

$+$	0	a	b	c
0	0	a	b	c
a	a	b	c	0
b	b	c	0	a
c	c	0	a	b

$-$	0	a	b	c
0	0	c	b	a
a	a	0	c	b
b	b	a	0	c
c	c	b	a	0

The Cubic fuzzy set C in X defined as

$$\overline{\zeta}_C = \begin{cases} [0.4, 0.5] : & x = 0 \\ [0.3, 0.4] : & x = b \\ [0.2, 0.3] : & x = a, c \end{cases} \qquad \eta_C = \begin{cases} 0.5 : & x = a, b \\ 0.4 : & x = c \\ 0.2 : & x = 0 \end{cases}$$

Then C is a Cubic fuzzy $\beta-$ideal of X .

Theorem 3.1. *Let C be a Cubic fuzzy $\beta-$ ideal of a $\beta-$algebra X. If $x \leq y$ then $\overline{\zeta}_C(x) \geq \overline{\zeta}_C(y)$ and $\eta_C(x) \leq \eta_C(y)$.*

Proof. For $x, y \in X, x \leq y \Rightarrow x - y = 0$ then

$$\overline{\zeta}_C(x) \geq r\min\{\overline{\zeta}_C(x-y), \overline{\zeta}_C(y)\} = r\min\{\overline{\zeta}_C(0), \overline{\zeta}_C(y)\} = \overline{\zeta}_C(y),$$
$$\eta_C(x) \leq \max\{\eta_C(x-y), \eta_C(y)\} = \max\{\eta_C(0), \eta_C(y)\} = \eta_C(y). \qquad \square$$

Theorem 3.2. *Let $C = \{x, \overline{\zeta}_C(x), \eta_C(x) : x \in X\}$ be a cubic fuzzy $\beta-$ ideals of a $\beta-$algebra X. If $x \leq z + y$ then $\overline{\zeta}(x) \geq r\min\{\overline{\zeta}(z), \overline{\zeta}(y)\}$ & $\eta(x) \leq \max\{\eta(z), \eta(y)\}, \forall\ x, y, z \in X$.*

Proof. For $x, y, z \in X$

$$\begin{aligned}
\overline{\zeta}(x) &\geq r\min\{\overline{\zeta}(x-y), \overline{\zeta}(y)\} \\
&= r\min\{r\min\{\overline{\zeta}((x-y)-z), \overline{\zeta}(z)\}, \overline{\zeta}(y)\} \\
&= r\min\{r\min\{\overline{\zeta}((x-(z+y)), \overline{\zeta}(z)\}, \overline{\zeta}(y)\} \\
&= r\min\{r\min\{\overline{\zeta}(0), \overline{\zeta}(z)\}, \overline{\zeta}(y)\} \\
&= r\min\{\overline{\zeta}(z), \overline{\zeta}(y)\}, \\
\eta(x) &\leq \max\{\eta(x-y), \eta(y)\} \\
&= \max\{\max\{\eta((x-y)-z), \eta(z)\}, \eta(y)\} \\
&= \max\{\max\{\eta((x-(z+y)), \eta(z)\}, \eta(y)\} \\
&= \max\{\max\{\eta(0), \eta(z)\}, \eta(y)\} \\
&= \max\{\eta(z), \eta(y)\}.
\end{aligned}$$

□

The definitions of product, homomorphic image and inverse image of cubic fuzzy sets are defined in the following and some related results are discussed.

Definition 3.2. *Consider the cubic fuzzy sets $A = \{\langle x, \overline{\zeta}_A(x), \eta_A(x)\rangle : x \in X\}$ of X and $B = \{\langle y, \overline{\zeta}_B(y), \eta_B(y)\rangle : y \in Y\}$ of Y. Then, cubic Cartesian product of A and B is defined as $A \times B = \{\overline{\zeta}_{A\times B}(x,y)$ & $\eta_{A\times B}(x,y) : x, y \in X \times Y\}$, where $\overline{\zeta}_{A\times B} : X \times Y \to D[0,1]$ is given by $\overline{\zeta}_{A\times B}(x,y) = r\min\{\overline{\zeta}_A(x), \overline{\zeta}_B(y)\}$ and $\eta_{A\times B} : X \times Y \to [0,1]$ is given by $\eta_{A\times B}(x,y) = \max\{\eta_A(x), \eta_B(y)\}$.*

Theorem 3.3. *Let A and B be two cubic fuzzy sets of X and Y such that $A\times B$ is also a cubic fuzzy $\beta-$ ideal of $X \times Y$. Then either A is a cubic fuzzy $\beta-$ideal of X or B is a cubic fuzzy $\beta-$ideal of Y.*

Proof. We know that

$$(1) \qquad \overline{\zeta}_A(0) \geq \overline{\zeta}_A(x), \overline{\zeta}_B(0) \geq \overline{\zeta}_B(y) \text{ \& } \eta_A(0) \leq \eta_A(x), \eta_B(0) \leq \eta_B(y),$$

then $\overline{\zeta}_{A\times B}(0,y) = r\min\{\overline{\zeta}_A(0), \overline{\zeta}_B(y)\}$, $\eta_{A\times B}(0,y) = \max\{\eta_A(0), \eta_B(y)\}$. Since $A \times B$ is a cubic fuzzy $\beta-$ideals of $X \times Y$,

$$\overline{\zeta}_{A\times B}((x_1,y_1),(x_2,y_2)) \geq r\min\{\overline{\zeta}_{A\times B}((x_1,y_1)-(x_2,y_2)), \overline{\zeta}_{A\times B}(x_2,y_2)\}$$

and since

$$\begin{aligned}
&\overline{\zeta}_{A\times B}((x_1,y_1),(x_2,y_2)) \geq r\min\{\overline{\zeta}_{A\times B}((x_1,y_1)-(x_2,y_2)), \overline{\zeta}_{A\times B}((x_2,y_2)\}, \\
(2)\ &\overline{\zeta}_{A\times B}((x_1,y_1),(x_2,y_2)) \geq r\min\{\overline{\zeta}_{A\times B}((x_1-x_2),(y_1-y_2)), \overline{\zeta}_{A\times B}(x_2,y_2)\}, \\
&\overline{\zeta}_{A\times B}((x_1-x_2),(y_1-y_2)) \geq r\min\{\overline{\zeta}_{A\times B}((x_1,y_1), \overline{\zeta}_{A\times B}(x_2,y_2)\}.
\end{aligned}$$

Putting $x_1 = x_2 = 0$ in (6). Then

$$\overline{\zeta}_{A\times B}((0,y_1) \geq r\min\{\overline{\zeta}_{A\times B}(0,(y_1-y_2)), \overline{\zeta}_{A\times B}(0,y_2)\}$$

and

$$\overline{\zeta}_{A\times B}(0,(y_1-y_2)) \geq r\min\{\overline{\zeta}_{A\times B}(0,y_1),\overline{\zeta}_{A\times B}(0,y_2)\}. \tag{3}$$

Using equations (1) in (3) which yields $\overline{\zeta}_B(y_1) \geq r\min\{\{\overline{\zeta}_B(y_1-y_2),\overline{\zeta}_B(y_2)\}$ and $\overline{\zeta}_B(y_1-y_2) \geq r\min\{\overline{\zeta}_B(y_1),\overline{\zeta}_B(y_2)\}$. Similarly, $\eta_B(y_1) \leq \max\{\{\eta_B(y_1-y_2),\eta_B(y_2)\}$ and $\eta_B(y_1-y_2) \leq \max\{\eta_B(y_1),\eta_B(y_2)\}$. Hence, B is a cubic fuzzy $\beta-$ideal of Y. □

Using the definition of product of cubic fuzzy sets and the above theorem, one can prove the following theorem.

Theorem 3.4. *If A and B are two cubic fuzzy $\beta-$ideals of X and Y respectively then $A\times B$ is also a cubic fuzzy $\beta-$ ideal of $X\times Y$.*

Definition 3.3. *Let $C=\{\langle x,\overline{\zeta}_C(x),\eta_C(x)\rangle : x\in X\}$ be a cubic fuzzy set with the degree membership $\overline{\zeta}_C(x) : X\to D[0,1]$ and the degree of non-membership function $\eta_C(x) : X\to[0,1]$ is said to have rsup-inf property if for any subset T of X, there exists $x_0\in T$, such that $\overline{\zeta}_C(x_0)=r\sup_{x\in T}\overline{\zeta}_C(x)$ and $\eta_C(x_0)=\inf_{x\in T}\ \eta_C(x)$*

Definition 3.4. *Let f be a mapping from a set X into a set Y. Let $A=\{\langle x,\overline{\zeta}_A(x),\eta_A(x)\rangle : x\in X\}$ be a cubic fuzzy set in X, then the cubic set $B=\{\langle x,\overline{\zeta}_B(x),\eta_B(x)\rangle : x\in X\}$ of Y defined by*

$$\overline{\zeta}_A(f^-1(y))=\overline{\zeta}_B(y)=\begin{cases} r\sup_{x\in\ f^{-1}(y)}\overline{\zeta}_A(x), & \text{if } f^{-1}=\{x\in X,f(x)=y\}\neq\emptyset\\ [0,0], & \text{if otherwise}\end{cases}$$

$$\eta_A(f^{-1}(y))=\eta_B(y)=\begin{cases} \inf_{x\in\ f^{-1}(y)}\eta_A(x), & \text{if } f^{-1}=\{x\in X,f(x)=y\}\neq\emptyset\\ 1 & \text{if otherwise}\end{cases}$$

$\forall y\in Y$ is called the image of A under f.

Similarly, if B is a cubic set of Y, then the cubic set $A=B\circ f$ in X defined by $\overline{\zeta}_B(f(x))=\overline{\zeta}_A(x),\eta_B(f(x))=\eta_A(x),\forall\ x\in X$, also to be the inverse image of B under under f.

Theorem 3.5. *Let $f : X\to X$ be an endomorphism on X and $C=\{x,\overline{\zeta}_C(x),\eta_C(x) : x\in X\}$ be a cubic fuzzy $\beta-$ideal of X. Then, $C_f=\{f(x),\overline{\zeta}_f(x),\eta_f(x) : x\in X\}$, where $\overline{\zeta}_f : X\to D[0,1]$ and $\eta_f : X\to[0,1]$ are defined by $\overline{\zeta}_f(x)=\overline{\zeta}(f(x))$ and $\eta_f(x)=\eta(f(x)),\forall x\in X$, is a cubic fuzzy $\beta-$ideal of X.*

Proof. Let C be a cubic fuzzy $\beta-$ideal of X. For $x\in X$, $\overline{\zeta}_f(0)=\overline{\zeta}(f(0))=\overline{\zeta}(0)\leq\overline{\zeta}(x)$, $\eta_f(0)=\eta(f(0))=\eta(0)\geq\eta(x),\forall x\in X$. Then

$$\begin{aligned}\overline{\zeta}_f(x+y)&=\overline{\zeta}(f(x+y))\\ &\geq\overline{\zeta}(f(x)+f(y))\\ &=r\min\{\overline{\zeta}(f(x)),\overline{\zeta}(f(y))\}\\ &=r\min\{\overline{\zeta}_f(x),\overline{\zeta}_f(y)\}.\end{aligned}$$

and

$$\begin{aligned}\eta_f(x+y) &= \eta(f(x+y)) \\ &\leq \eta(f(x)+f(y)) \\ &= \max\{\eta(f(x)),\eta(f(y))\} \\ &= \max\{\eta_f(x),\eta_f(y)\}.\end{aligned}$$

Also,

$$\begin{aligned}\overline{\zeta}_f(x) &= \overline{\zeta}(f(x)) \\ &\geq r\min\{\overline{\zeta}(f(x)-f(y)),\overline{\zeta}(f(y))\} \\ &= r\min\{\overline{\zeta}(f(x-y)),\overline{\zeta}(f(y))\} \\ &= r\min\{\overline{\zeta}_f(x-y),\overline{\zeta}_f(y)\}.\end{aligned}$$

Similarly,

$$\begin{aligned}\eta_f(x) &= \eta(f(x)) \\ &\leq \max\{\eta(f(x)-f(y)),\eta(f(y))\} \\ &= \max\{\eta(f(x-y)),\eta(f(y))\} \\ &= \max\{\eta_f(x-y),\eta_f(y)\}.\end{aligned}$$

Hence, C_f is a cubic fuzzy $\beta-$ideal of X. □

Using the definition of homomorphic image and the above theorem, one can prove the following theorem.

Theorem 3.6. *Let $f : X \to Y$ be an onto homomorphism of $\beta-$algebras. If C is a cubic fuzzy $\beta-$ideal of Y, then the inverse image $f^{-1}(C)$ is a cubic fuzzy $\beta-$ideal of X.*

Conclusion

This paper introduces cubic fuzzy $\beta-$ideal of a $\beta-$algebra. In depth, the study analysed the cubic fuzzy $\beta-$ideal using homomorphic image, cartesion product. This can be extended to the other substructures like $H-$ideals and filters of a $\beta-$algebra.

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