Interval valued vague ideals in Γ-nearrings

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Abstract. In the present paper we propose interval valued (I-V)vague ideals in Γ-nearrings, explain some of their properties and provide some applications related to this work. Also, look at the direct product of I-V vague ideals, as well as the normal I-V vague ideals Γ-nearrings.

Keywords: vague ideals, Γ-nearrings, vague ideal in Γ-nearrings, I-V ideals, direct product, normal vague ideals.

1. Introduction

Zadeh introduced fuzzy set in 1965 [5]. In this he talked about the true membership value of a function. Gau and Buehrer introduced the concept of vague theory in [13]. In vague set they talk about the membership value of a function as well as non-membership value. To increase the study of vague sets, many authors have considered several extension works of fuzzy sets. In a vague set, the universal set $\mathfrak{X}$ can be represented by a pair of functions $(\tau_{\mathfrak{X}}, \sigma_{\mathfrak{X}})$. In this $\tau_{\mathfrak{X}}$ and $\sigma_{\mathfrak{X}}$ are the functions from $\mathfrak{X}$ to [0,1] such as $\tau_{\mathfrak{X}}(\mathrm{i}) + \sigma_{\mathfrak{X}}(\mathrm{i}) \leq 1$. Here, $\tau_{\mathfrak{X}}$ gives membership value and $\sigma_{\mathfrak{X}}$ is non-membership value. In the domain of fuzzy sets, a vague set is referred to as an interval membership function, as opposed to point membership. This notion is used in a variety of fields including fuzzy control systems, decision making, fault diagnostics, knowledge discovery and many more. Vague in group theory introduced by R. Biswas [11]. A similar concept of vague set is intuitionistic set is given by many researchers in [14],[8].

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Vague ideals in Γ-nearrings introduced by Y.Bhargav and S.Ragamayi in [12]. L. Bhaskar et. al sum, vague ideal and a nearring [7].

Vague set is interval membership function but it is not I-V function because in vague set first value represent membership value and second value represent non-membership value, but in I-V function whole interval represent either membership value or non-membership values.

Here, we introduced I-V vague ideals in Γ-nearring and show that vague interval and interval valued vague ideal is different and give some examples related to this Also, talk about normal interval valued vague ideal and direct product of vague ideals. For standard definitions and results we refer to Book and article [3], [6], [1]. A vague collection $\mathfrak{A}$ in the discourse universe $\mathfrak{X}$ is a pair ($\tau_{\mathfrak{A}}$, $\sigma_{\mathfrak{A}}$) where $\tau_{\mathfrak{A}}$: $\mathfrak{X} \to [0, 1]$ and $\sigma_{\mathfrak{A}}$: $\mathfrak{X} \to [0, 1]$ are mappings such as $\tau_{\mathfrak{A}}(\mathrm{i}) + \sigma_{\mathfrak{A}}(\mathrm{i}) \leq 1 \ \forall \ i \in \mathfrak{X}$. The membership function and non-membership function in $[0, 1]$ is $\tau_{\mathfrak{A}}$ and $\sigma_{\mathfrak{A}}$ correspondingly. And $[\tau_{\mathfrak{A}}, 1 - \sigma_{\mathfrak{A}}]$ is called interval vague value of i in $\mathfrak{A}$ and it is indicated by $\mathfrak{V}_{\mathfrak{A}}(i)$ that is $\mathfrak{V}_{\mathfrak{A}}(i) = [\tau_{\mathfrak{A}}(i),\, 1 - \sigma_{\mathfrak{A}}(i)]$.

2. Interval valued vague ideals in Γ-nearrings

Definition 2.1. *A vague collection $\mathfrak{A}$ in Γ-nearring N is I-V vague ideals if it is satisfy following properties for all i, j, k $\in N$ and $\rho \in \Gamma$:*

(i) $\overline{\mathfrak{V}_{\mathfrak{A}}}(i - j) \geq \min\{\overline{\mathfrak{V}_{\mathfrak{A}}}(i), \overline{\mathfrak{V}_{\mathfrak{A}}}(j)\}$;

(ii) $\overline{\mathfrak{V}_{\mathfrak{A}}}(j + i - j) \geq \overline{\mathfrak{V}_{\mathfrak{A}}}(i)$;

(iii) $\overline{\mathfrak{V}_{\mathfrak{A}}}(i\rho(j + k) - i\rho k) \geq \overline{\mathfrak{V}_{\mathfrak{A}}}(j)$;

(iv) $\overline{\tau_{\mathfrak{A}}}(i - j) \geq \min\{\overline{\tau_{\mathfrak{A}}}(i), \overline{\tau_{\mathfrak{A}}}(j)\}$;

(v) $\overline{\tau_{\mathfrak{A}}}(j + i - j) \geq \overline{\tau_{\mathfrak{A}}}(i)$;

(vi) $\overline{\tau_{\mathfrak{A}}}(i\rho(j + k) - i\rho k) \geq \overline{\tau_{\mathfrak{A}}}(j)$;

(vii) $1 - \overline{\sigma_{\mathfrak{A}}}(i - j) \geq \min\{1 - \overline{\sigma_{\mathfrak{A}}}(i), 1 - \overline{\sigma_{\mathfrak{A}}}(j)\}$, *or* $\overline{\sigma_{\mathfrak{A}}}(i - j) \leq \max\{\overline{\sigma_{\mathfrak{A}}}(i), \overline{\sigma_{\mathfrak{A}}}(j)\}$;

(viii) $1 - \overline{\sigma_{\mathfrak{A}}}(j + i - j) \geq 1 - \overline{\sigma_{\mathfrak{A}}}(i)$, *or* $\overline{\sigma_{\mathfrak{A}}}(j + i - j) \leq \overline{\sigma_{\mathfrak{A}}}(i)$;

(ix) $1 - \overline{\sigma_{\mathfrak{A}}}(i\rho(j + k) - i\rho k) \geq 1 - \overline{\sigma_{\mathfrak{A}}}(j)$, *or* $\overline{\sigma_{\mathfrak{A}}}(i\rho(j + k) - i\rho k) \leq \overline{\sigma_{\mathfrak{A}}}(j)$.

Example 2.1. Consider integers modulo 4, $N = \{0^*, i^*, j^*, k^*\}$ be a collection with two binary operations + and ρ as described below:

$+$	0^*	i^*	j^*	k^*
0^*	0^*	i^*	j^*	k^*
i^*	i^*	j^*	k^*	0^*
j^*	j^*	k^*	0^*	i^*
k^*	k^*	0^*	i^*	j^*

ρ	0^*	i^*	j^*	k^*
0^*	0^*	0^*	0^*	0^*
i^*	0^*	j^*	i^*	i^*
j^*	0^*	j^*	0^*	k^*
k^*	0^*	0^*	0^*	k^*

Clearly, N is a Γ-nearring. Define as $\overline{\mathfrak{V}_{\mathfrak{A}}}(0^*) = [[0.8, 0.9], [0, 0.1]]$, $\overline{\mathfrak{V}_{\mathfrak{A}}}(i^*) = [[0.5, 0.5], [0.4, 0.5]]$, $\overline{\mathfrak{V}_{\mathfrak{A}}}(j^*) = [[0.1, 0.3], [0.6, 0.7]]$, $\overline{\mathfrak{V}_{\mathfrak{A}}}(k^*) = [[0.6, 0.7], [0.2, 0.3]]$. Now, by routine verification we can show that $\overline{\mathfrak{V}_{\mathfrak{A}}} = [\overline{\tau_{\mathfrak{A}}}, \overline{\sigma_{\mathfrak{A}}}]$ is an I-V vague ideal in Γ-nearring.

Example 2.2. Write $N = \mathbb{Z}$ (set of integers) under operations "+" and $\Gamma = \mathbb{Z}$. Thus $(N, +, \Gamma)$ is a Γ-nearring and with the mapping, $\overline{\mathfrak{V}_{\mathfrak{A}}}$: $N \to [0, 1]$ define by

$$\overline{\tau_{\mathfrak{A}}}(i) = \begin{cases} [0.5, 0.7], & \text{if } i = 2\text{n for some n} \in \mathbb{Z}, \\ 0, & \text{if } i = 2\text{n} + 1, \text{n} \in \mathbb{Z}\ , \\ 1, & \text{if } i = 0 \end{cases}$$

$$\overline{\sigma_{\mathfrak{A}}}(i) = \begin{cases} [0.1, 0.3], & \text{if } i = 2\text{n for some n} \in \mathbb{Z}, \\ 1, & \text{if } i = 2\text{n} + 1, \text{n} \in \mathbb{Z}\ , \\ 0, & \text{if } i = 0 \end{cases} .$$

By routine verification, we can show that $\overline{\mathfrak{V}_{\mathfrak{A}}}$ is an I-V vague ideal.

Example 2.3. Write $N = \mathbb{Z}_7$ (set of integers modulo 7) and $\Gamma = \mathbb{Z}$, thus $(N, +, \Gamma)$ is a Γ-nearring. Then, the vague collection $\overline{\mathfrak{V}_{\mathfrak{A}}} = (\overline{\tau_{\mathfrak{A}}}, \overline{\sigma_{\mathfrak{A}}})$ becomes I-V vague ideal in N with the following mappings:

$$\overline{\tau_{\mathfrak{A}}}(i) = \begin{cases} [0.6, 0.9], & \text{if } i \in \mathbb{Z}_7, \\ [0, 0.1], & \text{if } i \notin \mathbb{Z}_7 \end{cases}$$

$$\overline{\sigma_{\mathfrak{A}}}(i) = \begin{cases} [0.1, 0.3], & \text{if } i \in \mathbb{Z}_7, \\ [0.5, 0.6], & \text{if } i \notin \mathbb{Z}_7 \end{cases}$$

Now, by routine calculation we can show that $\overline{\mathfrak{V}_{\mathfrak{A}}} = [\overline{\tau_{\mathfrak{A}}}, \overline{\sigma_{\mathfrak{A}}}]$ is an I-V vague ideals in Γ-nearring.

Proposition 2.1. *If $\overline{\mathfrak{V}_{\mathfrak{A}}}$ is an I-V vague ideals Γ-nearring N and if satisfies the condition $\overline{\mathfrak{V}_{\mathfrak{A}}}(j + i - j) \geq \overline{\mathfrak{V}_{\mathfrak{A}}}(i)$, then $\overline{\mathfrak{V}_{\mathfrak{A}}}(i + j) = \overline{\mathfrak{V}_{\mathfrak{A}}}(j + i)$.*

Proposition 2.2. *Let $\mathfrak{A}$ be a I-V vague ideal of Γ-nearring N and if $\overline{\mathfrak{V}_{\mathfrak{A}}}(i-j) = \overline{\mathfrak{V}_{\mathfrak{A}}}(0)$, then $\overline{\mathfrak{V}_{\mathfrak{A}}}(i) = \overline{\mathfrak{V}_{\mathfrak{A}}}(j)$.*

Lemma 2.1. *Let $\overline{\mathfrak{V}_{\mathfrak{A}}}$ be an I-V vague ideals in N and if $\overline{\mathfrak{V}_{\mathfrak{A}}}(i) < \overline{\mathfrak{V}_{\mathfrak{A}}}(j)$, then $\overline{\mathfrak{V}_{\mathfrak{A}}}(i - j) = \overline{\mathfrak{V}_{\mathfrak{A}}}(i) = \overline{\mathfrak{V}_{\mathfrak{A}}}(j - i)$.*

Lemma 2.2. *Let $\Re$ is an I-V vague ideal in N than for any $\overline{e} \leq \overline{f} \subset \overline{(0, 1]}$, where $\overline{e} + \overline{f} \leq 1$, then there exists an I-V vague ideal $\overline{\mathfrak{V}_{\mathfrak{A}}}$ in N, such that $\overline{\mathfrak{V}_{\mathfrak{A}_{\overline{e}, \overline{f}}}} = \Re$. Here, $\overline{e}$ and $\overline{f}$ also an intervals.*

Proof. Consider an I-V vague ideal $\Re$ of N. Define a function $\overline{\mathfrak{V}_{\mathfrak{A}}}$: $N \to [0, 1]$ by

$$\overline{\mathfrak{V}_{\mathfrak{A}}}(i) = \begin{cases} [\overline{e}, \overline{f}] & \text{if } i \in \Re \\ [0, 0] & \text{if } i \notin \Re \end{cases}$$

where $\overline{e}$, $\overline{f} \subset (0, 1]$. Clearly $\overline{\mathfrak{V}_{\mathfrak{A}_{\overline{e},\overline{f}}}} = \Re$.

(i) Let i, $j \in \Re$ now $\overline{\mathfrak{V}_{\mathfrak{A}}}(i - j) = [\overline{e}, \overline{f}] \geq \min\{\overline{\mathfrak{V}_{\mathfrak{A}}}(i), \overline{\mathfrak{V}_{\mathfrak{A}}}(j)\}$. If there exists at least one of i and j is not in $\Re$, then $i - j \notin \Re$, thus $\overline{\mathfrak{V}_{\mathfrak{A}}}(i - j) = [\overline{0}, \overline{0}] \geq \min\{\overline{\mathfrak{V}_{\mathfrak{A}_1}}(i), \overline{\mathfrak{V}_{\mathfrak{A}_1}}(j)\}$.

(ii) Let $i \in \Re$ now $\overline{\mathfrak{V}_{\mathfrak{A}}}(j + i - j) = [\overline{e}, \overline{f}] \geq \overline{\mathfrak{V}_{\mathfrak{A}}}(i)$. If there $i \notin \Re$, then $j + i - j \notin \Re$ thus $\overline{\mathfrak{V}_{\mathfrak{A}}}(j + i - j) = [\overline{0}, \overline{0}] \geq \overline{\mathfrak{V}_{\mathfrak{A}}}(i)$.

(iii) Let $j \in \Re$ now $\overline{\mathfrak{V}_{\mathfrak{A}}}(i\rho(j + k) - i\rho k) = [\overline{e}, \overline{f}] \geq \overline{\mathfrak{V}_{\mathfrak{A}}}(j)$. If there $j \notin \Re$, then $i\rho(j + k) - i\rho k \notin \Re$ thus $\overline{\mathfrak{V}_{\mathfrak{A}}}(i\rho(j + k) - i\rho k) = [\overline{0}, \overline{0}] \geq \overline{\mathfrak{V}_{\mathfrak{A}_1}}(j)$. Thus, $\overline{\mathfrak{V}_{\mathfrak{A}}}$ is an I-V vague ideal of N. □

Corollary 2.1. *If $\overline{\mathfrak{V}_{\mathfrak{A}}}$ is an I-V vague ideal of N, then the collection $N_{\overline{\mathfrak{V}_{\mathfrak{A}}}}(i) = \{i \in N \mid \overline{\mathfrak{V}_{\mathfrak{A}}}(i) = \overline{\mathfrak{V}_{\mathfrak{A}}}(0)\}$ is a ideal of N.*

Proposition 2.3. *If $\overline{\mathcal{P}} \subseteq \overline{Q}$ are I-V vague ideals in Γ-nearring N, then $\overline{\tau_{\mathcal{P}}} \subseteq \overline{\tau_{Q}}$ and $\overline{\sigma_{Q}} \subseteq \overline{\sigma_{\mathcal{P}}}$.*

Theorem 2.1. *Let $\overline{\mathfrak{V}_{\mathfrak{A}}}$ is an I-V vague ideal of a Γ-nearring N iff each level sub collection $\overline{\mathfrak{V}_{\mathfrak{A}_{\overline{\mathfrak{b}}}}}$, $\overline{\mathfrak{b}} \in Im(\overline{\mathfrak{V}_{\mathfrak{A}}})$ is an ideal of N.*

Proof. Suppose $\overline{\mathfrak{V}_{\mathfrak{A}}}$ is an I-V vague ideal of N. Let $\overline{\mathfrak{b}} \in \text{Im}(\overline{\mathfrak{V}_{\mathfrak{A}}})$ and i, j, $k \in N$.

(i) For any $i, j \in \overline{\mathfrak{V}_{\mathfrak{A}_{\overline{\mathfrak{b}}}}}$ and $\rho \in \Gamma$, we have $\overline{\mathfrak{V}_{\mathfrak{A}}}(i-j) \geq \min\{\overline{\mathfrak{V}_{\mathfrak{A}}}(i), \overline{\mathfrak{V}_{\mathfrak{A}}}(j)\} = \overline{\mathfrak{b}}$ thus, $i - j \in \overline{\mathfrak{V}_{\mathfrak{A}_{\overline{\mathfrak{b}}}}}$.

(ii) For any $i \in \overline{\mathfrak{V}_{\mathfrak{A}_{\overline{\mathfrak{b}}}}}$ we have $\overline{\mathfrak{V}_{\mathfrak{A}}}(j+i-j) \geq \overline{\mathfrak{V}_{\mathfrak{A}}}(i) = \overline{\mathfrak{b}}$ thus, $j+i-j \in \overline{\mathfrak{V}_{\mathfrak{A}_{\overline{\mathfrak{b}}}}}$.

(iii) For any $j \in \overline{\mathfrak{V}_{\mathfrak{A}_{\overline{\mathfrak{b}}}}}$, we have $\overline{\mathfrak{V}_{\mathfrak{A}}}(i\rho(j+k)-i\rho k) \geq \overline{\mathfrak{V}_{\mathfrak{A}}}(j) = \overline{\mathfrak{b}}$ thus $i\rho(j+k)-i\rho k \in \overline{\mathfrak{V}_{\mathfrak{A}_{\overline{\mathfrak{b}}}}}$. Hence, $\overline{\mathfrak{V}_{\mathfrak{A}_{\overline{\mathfrak{b}}}}}$ is an ideal of N. Conversely, assume that $\overline{\mathfrak{V}_{\mathfrak{A}_{\overline{\mathfrak{b}}}}}$ is an ideal of N for every $\overline{\mathfrak{b}} \in \text{Im}(\overline{\mathfrak{V}_{\mathfrak{A}}})$ suppose that $\overline{\mathfrak{V}_{\mathfrak{A}}}(i_\circ - j_\circ) < \min\{\overline{\mathfrak{V}_{\mathfrak{A}}}(i_\circ), \overline{\mathfrak{V}_{\mathfrak{A}}}(j_\circ)\}$, for some $i_\circ$, $j_\circ \in N$ then, by taking $\overline{\mathfrak{b}}_\circ = \frac{1}{2}[\overline{\mathfrak{V}_{\mathfrak{A}}}(i_\circ - j_\circ) + \min\{\overline{\mathfrak{V}_{\mathfrak{A}}}(i_\circ), \overline{\mathfrak{V}_{\mathfrak{A}}}(j_\circ)\}]$, we have $\overline{\mathfrak{V}_{\mathfrak{A}}}(i_\circ - j_\circ) < \overline{\mathfrak{b}}_\circ, \overline{\mathfrak{V}_{\mathfrak{A}}}(i_\circ) > \overline{\mathfrak{b}}_\circ$ and $\overline{\mathfrak{V}_{\mathfrak{A}}}(j_\circ) > \overline{\mathfrak{b}}_\circ$. Hence, $(i_\circ - j_\circ) \notin \overline{\mathfrak{V}_{\mathfrak{A}_{\overline{\mathfrak{b}}_\circ}}}, i_\circ \in \overline{\mathfrak{V}_{\mathfrak{A}_{\overline{\mathfrak{b}}_\circ}}}$ and $j_\circ \in \overline{\mathfrak{V}_{\mathfrak{A}_{\overline{\mathfrak{b}}_\circ}}}$ this is a contradiction.

(ii) Suppose that $\overline{\mathfrak{V}_{\mathfrak{A}}}(j_\circ+i_\circ-j_\circ) < \overline{\mathfrak{V}_{\mathfrak{A}}}(i_\circ)$ for some $i_\circ, j_\circ \in N$, then by taking $\overline{\mathfrak{b}}_\circ = \frac{1}{2}[\overline{\mathfrak{V}_{\mathfrak{A}}}(j_\circ + i_\circ - j_\circ) + \overline{\mathfrak{V}_{\mathfrak{A}}}(i_\circ)]$, we have $\overline{\mathfrak{V}_{\mathfrak{A}}}(j_\circ + i_\circ - j_\circ) < \overline{\mathfrak{b}}_\circ and \overline{\mathfrak{V}_{\mathfrak{A}}}(i_\circ) > \overline{\mathfrak{b}}_\circ$. Hence, $(j_\circ + i_\circ - j_\circ) \notin \overline{\mathfrak{V}_{\mathfrak{A}_{\overline{\mathfrak{b}}_\circ}}}$ and $i_\circ \in \overline{\mathfrak{V}_{\mathfrak{A}_{\overline{\mathfrak{b}}_\circ}}}$ this is an contradiction.

(iii) Suppose that $\overline{\mathfrak{V}_{\mathfrak{A}}}(i_\circ\rho_0(j_\circ+k_\circ)-i_\circ\rho_0 k_\circ) < \overline{\mathfrak{V}_{\mathfrak{A}}}(v_\circ)$ for some $i_\circ, j_\circ, k_\circ \in N$ and $\rho_0 \in \Gamma$ then, by taking $\overline{\mathfrak{b}}_\circ = \frac{1}{2}[\overline{\mathfrak{V}_{\mathfrak{A}}}(i_\circ\rho_0(j_\circ + k_\circ) - i_\circ\rho_0 k_\circ) + \overline{\mathfrak{V}_{\mathfrak{A}}}(j_\circ)]$, we have $\overline{\mathfrak{V}_{\mathfrak{A}}}(i_\circ\rho_0(j_\circ + k_\circ) - i_\circ\rho_0 k_\circ) < \overline{\mathfrak{b}}_\circ$ and $\overline{\mathfrak{V}_{\mathfrak{A}}}(j_\circ) > \mathfrak{b}_\circ$. Hence $i_\circ\rho_0(j_\circ + k_\circ) - i_\circ\rho_0 k_\circ \notin \overline{\mathfrak{V}_{\mathfrak{A}_{\overline{\mathfrak{b}}_\circ}}}$ and $j_\circ \in \overline{\mathfrak{V}_{\mathfrak{V}_{\overline{\mathfrak{b}}_\circ}}}$ a contradiction. Thus, $\overline{\mathfrak{V}_{\mathfrak{A}}}$ is a I-V vague ideal of N. □

Theorem 2.2. *Let* $\overline{\mathfrak{V}_{\mathfrak{A}}}$ *be an I-V vague sub-collection of* N *then* $\overline{\mathfrak{V}_{\mathfrak{A}}} = [\overline{\mathfrak{V}_{\mathfrak{A}}}^-, \overline{\mathfrak{V}_{\mathfrak{A}}}^+]$ *is an I-V vague ideal of* N *if and only if* $\overline{\mathfrak{V}_{\mathfrak{A}}}^-$, $\overline{\mathfrak{V}_{\mathfrak{A}}}^+$ *are I-V vague ideals of* N.

Proof. Assume that $\overline{\mathfrak{V}_{\mathfrak{A}}}$ is an I-V vague ideal of N. For any $i, j \in N$, we have

$$\begin{aligned}\overline{\mathfrak{V}_{\mathfrak{A}}}(i–j) &\geq \min\{\overline{\mathfrak{V}_{\mathfrak{A}}}(i), \overline{\mathfrak{V}_{\mathfrak{A}}}(j)\} = \min\{[\overline{\mathfrak{V}_{\mathfrak{A}}}^-(i), \overline{\mathfrak{V}_{\mathfrak{A}}}^+(i)], [\overline{\mathfrak{V}_{\mathfrak{A}}}^-(j), \overline{\mathfrak{V}_{\mathfrak{A}}}^+(j)]\} \\ &= \min\{[\overline{\mathfrak{V}_{\mathfrak{A}}}^-(i), \overline{\mathfrak{V}_{\mathfrak{A}}}^-(j)], [\overline{\mathfrak{V}_{\mathfrak{A}}}^+(i), \overline{\mathfrak{V}_{\mathfrak{A}}}^+(j)]\} \\ &= [\min\{\overline{\mathfrak{V}_{\mathfrak{A}}}^-(i), \overline{\mathfrak{V}_{\mathfrak{A}}}^-(j)\}], [\min\{\overline{\mathfrak{V}_{\mathfrak{A}}}^+(i), \overline{\mathfrak{V}_{\mathfrak{A}}}^+(j)\}].\end{aligned}$$

It follow that $\overline{\mathfrak{V}_{\mathfrak{A}}}^-(i-j) \geq \min\{\overline{\mathfrak{V}_{\mathfrak{A}}}^-(i), \overline{\mathfrak{V}_{\mathfrak{A}}}^-(j)\}$ and

$$\overline{\mathfrak{V}_{\mathfrak{A}}}^+(i-j) \geq \min\{\overline{\mathfrak{V}_{\mathfrak{A}}}^+(i), \overline{\mathfrak{V}_{\mathfrak{A}}}^+(j)\}.$$

Similarly, we can prove all the conditions.

Now, conversely assume that $\overline{\mathfrak{V}_{\mathfrak{A}}}^-$ and $\overline{\mathfrak{V}_{\mathfrak{A}}}^+$ is a vague ideal of N.

Let $i, j \in N$, then $\overline{\mathfrak{V}_{\mathfrak{A}}}(i-j) = [\overline{\mathfrak{V}_{\mathfrak{A}}}^-(i-j), \overline{\mathfrak{V}_{\mathfrak{A}}}^+(i–j)] \geq [\min\{\overline{\mathfrak{V}_{\mathfrak{A}}}^-(i), \overline{\mathfrak{V}_{\mathfrak{A}}}^-(j)\}, \min\{\overline{\mathfrak{V}_{\mathfrak{A}}}^+(i), \overline{\mathfrak{V}_{\mathfrak{A}}}^+(j)\}] = \min\{[\overline{\mathfrak{V}_{\mathfrak{A}}}^-(i), \overline{\mathfrak{V}_{\mathfrak{A}}}^+(i)], [\overline{\mathfrak{V}_{\mathfrak{A}}}^-(j), \overline{\mathfrak{V}_{\mathfrak{A}}}^+(j)]\} = \min\{\overline{\mathfrak{V}_{\mathfrak{A}}}(i), \overline{\mathfrak{V}_{\mathfrak{A}}}(j)\}$. Similarly, we can prove all other conditions of I-V vague ideals of N. □

Theorem 2.3. *If* $\{\overline{\mathfrak{V}_{\mathfrak{A}}} \mid \dot{a} \in \Omega\}$ *is a family of I-V vague ideals of* N, *then* $\bigcap_{\dot{a}\in\Omega}\overline{\mathfrak{V}_{\mathfrak{A}}}$ *is also an I-V vague ideals of* N, *where* Ω *is any index set.*

Proof. Let $i, j \in N$ and $\rho \in \Gamma$. Then, $\bigcap_{\dot{a}\in\Omega}\overline{\mathfrak{V}_{\mathfrak{A}}}(i-j) = \inf_{\dot{a}\in\Omega}\overline{\mathfrak{V}_{\mathfrak{A}}}(i-j) : \dot{a} \in \Omega\} \geq \inf_{\dot{a}\in\Omega}\min\{\overline{\mathfrak{V}_{\mathfrak{A}}}(i), \overline{\mathfrak{V}_{\mathfrak{A}}}(j)\} = \min\{\inf_{\dot{a}\in\Omega}\overline{\mathfrak{V}_{\mathfrak{A}}}(i), \inf_{\dot{a}\in\Omega}\overline{\mathfrak{V}_{\mathfrak{A}}}(i)\} = \min\{\bigcap_{\dot{a}\in\Omega}\overline{\mathfrak{V}_{\mathfrak{A}}}(i), \bigcap_{\dot{a}\in\Omega}\overline{\mathfrak{V}_{\mathfrak{A}}}(j)\}$.

$\bigcap_{\dot{a}\in\Omega}\overline{\mathfrak{V}_{\mathfrak{A}}}(j+i-j) = \inf_{\dot{a}\in\Omega}\overline{\mathfrak{V}_{\mathfrak{A}}}(j+i-j) \geq \inf_{\dot{a}\in\Omega}\overline{\mathfrak{V}_{\mathfrak{A}}}(i) = \bigcap_{\dot{a}\in\Omega}\overline{\mathfrak{V}_{\mathfrak{A}}}(i)$. Now, $\bigcap_{\dot{a}\in\Omega}\overline{\mathfrak{V}_{\mathfrak{A}}}(i\rho(j+k) - i\rho k) = \inf_{\dot{a}\in\Omega}\overline{\mathfrak{V}_{\mathfrak{A}}}(i\rho(j+k) - i\rho k) \geq \inf_{\dot{a}\in\Omega}\overline{\mathfrak{V}_{\mathfrak{A}}}(j) = \bigcap_{\dot{a}\in\Omega}\overline{\mathfrak{V}_{\mathfrak{A}}}(j)$. Therefore, $\bigcap_{\dot{a}\in\Omega}\overline{\mathfrak{V}_{\mathfrak{A}}}$ is an I-V vague ideal of N. □

Theorem 2.4. *If* $\{\overline{\mathfrak{V}_{\mathfrak{A}}} \mid \dot{a} \in \Omega\}$ *be a family of an is an I-V vague ideal of* N, *then* $\bigvee_{\dot{a}\in\Omega}\overline{\mathfrak{V}_{\mathfrak{A}\dot{a}}}$ *is also I-V vague ideal of* N.

Proof. Proof is similar. □

Definition 2.2. *Consider two* Γ*-nearrings* N *and* M. *A mapping* ß: $N \to M$ *is called homomorphism it satisfies the following conditions* $ß(i+j) = ß(i)+ß(j)$ *and* $ß(i\rho j) = ß(i)\rho ß(j)$, *for all* $i, j \in N$ *and* $\rho \in \Gamma$.

Theorem 2.5. *Let* N *and* M *be* Γ*-nearrings and* ß: $N \to M$ *is an onto homomorphism. If* $\overline{\mathfrak{V}_{\mathfrak{A}}}$ *is a* ß *invariant I-V vague ideal of* N, *then* ß$(\overline{\mathfrak{V}_{\mathfrak{A}}})$ *is an I-V vague ideal of* M.

Proof. Suppose that N and M be Γ-nearrings, ß: $N \to M$ is an onto homomorphism, $\overline{\mathfrak{V}_{\mathfrak{A}}}$ be an ß invariant I-V vague ideal of N and $i \in M$. Suppose $p \in M, t \in ß^{-1}(i), p = ß(t)$. Then $p \in ß^{-1}(i)$ that implies $ß(t) = i = ß(p)$, since $\overline{\mathfrak{V}_{\mathfrak{A}}}$ is a ß invariant, $\overline{\mathfrak{V}_{\mathfrak{A}}}(t) = \overline{\mathfrak{V}_{\mathfrak{A}}}(p) \Rightarrow ß(\overline{\mathfrak{V}_{\mathfrak{A}}})(i) = \inf_{t \in ß^{-1}(i)} \overline{\mathfrak{V}_{\mathfrak{A}}}(t) = \overline{\mathfrak{V}_{\mathfrak{A}}}(p)$. Hence, $ß(\overline{\mathfrak{V}_{\mathfrak{A}}})(i) = \overline{\mathfrak{V}_{\mathfrak{A}}}(p)$.

(i) Let $i, j \in M$. Then, there exists $p, q \in N$such that $ß(p) = i, ß(q) = j$ implies $ß(p - q) = i - j \Rightarrow ß(\overline{\mathfrak{V}_{\mathfrak{A}}})(i - j) = \overline{\mathfrak{V}_{\mathfrak{A}}}(p - q) \geq \min\{\overline{\mathfrak{V}_{\mathfrak{A}}}(p), \overline{\mathfrak{V}_{\mathfrak{A}}}(q)\} = \min\{ß(\overline{\mathfrak{V}_{\mathfrak{A}}})(i), ß(\overline{\mathfrak{V}_{\mathfrak{A}}})(j)\}$.

(ii) Let $i, j \in M$. Then, there exists $p, q \in N$ such that $ß(p) = i, ß(q) = j \Rightarrow ß(q+p-q) = j+i-j$ implies $ß(\overline{\mathfrak{V}_{\mathfrak{A}}})(j+i-j) = \overline{\mathfrak{V}_{\mathfrak{A}}}(q+p-q) \geq \overline{\mathfrak{V}_{\mathfrak{A}}}(p) = ß(\overline{\mathfrak{V}_{\mathfrak{A}}})(i)$.

(iii) Let $i, j, k \in M$ and $\rho \in \Gamma$. Then, there exists $p, q, r \in N$ such that $ß(p) = i, ß(q) = j, ß(r) = k$ implies $ß(p\rho(q + r) - q\rho r) = i\rho(j + k) - i\rho k \Rightarrow ß(\overline{\mathfrak{V}_{\mathfrak{A}}})(i\rho(j + k) - i\rho k) = \overline{\mathfrak{V}_{\mathfrak{A}}}(p\rho(q + r) - q\rho r) \geq \overline{\mathfrak{V}_{\mathfrak{A}}}(q) = ß(\overline{\mathfrak{V}_{\mathfrak{A}}})(j)$. Hence, $ß(\overline{\mathfrak{V}_{\mathfrak{A}}})$ is an I-V vague ideal of M. □

Theorem 2.6. *Let N and M be two Γ-nearrings and ß: $N \to M$ a homomorphism mapping. If $\overline{\mathfrak{V}_{\mathfrak{A}}}$ is a I-V vague ideal of M then $ß^{-1}(\overline{\mathfrak{V}_{\mathfrak{A}}})$ is an I-V vague ideal of N.*

Proof. Proof is straight forward. □

3. Normal interval valued vague ideal

Definition 3.1. *An vague set $\overline{\mathfrak{V}_{\mathfrak{A}}}$ is said to be a normal I-V vague ideal if $\overline{\mathfrak{V}_{\mathfrak{A}}}(0) = [1, 1]$ it means that $\overline{\tau_{\mathfrak{A}}}(0) = 1$ and $1 - \overline{\sigma_{\mathfrak{A}}}(0) = 1$.*

Note. Example(3) is an example of normal interval valued vague ideal.

Theorem 3.1. *Let $t \in (0, 1]$ be a real number. If $\overline{\mathfrak{V}_{\mathfrak{A}}}$ is a normal I-V vague ideal of N, then $\overline{\mathfrak{V}^t_{\mathfrak{A}}}$ is also a normal vague ideal of N and $N_{\overline{\mathfrak{V}^t_{\mathfrak{A}}}} = N_{\overline{\mathfrak{V}_{\mathfrak{A}}}}$. Here, $\overline{\mathfrak{V}^t_{\mathfrak{A}}}(i) = [\overline{\mathfrak{V}_{\mathfrak{A}}}(i)]^t$.*

Proof. Let i, j, $k \in N$ and $\rho \in \Gamma$ then, (i) $\overline{\mathfrak{V}^t_{\mathfrak{A}}}(i - j) = [\overline{\mathfrak{V}_{\mathfrak{A}}}(i - j)]^t \geq [\min\{\overline{\mathfrak{V}_{\mathfrak{A}}}(i), \overline{\mathfrak{V}_{\mathfrak{A}}}(j)\}]^t = \min\{(\overline{\mathfrak{V}_{\mathfrak{A}}}(i))^t, (\overline{\mathfrak{V}_{\mathfrak{A}}}(j))^t\} = \min\{\overline{\mathfrak{V}^t_{\mathfrak{A}}}(i), \overline{\mathfrak{V}^t_{\mathfrak{A}}}(j)\}$.

(ii) $\overline{\mathfrak{V}^t_{\mathfrak{A}}}(j + i - j) = [\overline{\mathfrak{V}_{\mathfrak{A}}}(j + i - j)]^t \geq [\overline{\mathfrak{V}_{\mathfrak{A}}}(i)]^t = \overline{\mathfrak{V}^t_{\mathfrak{A}}}(i)$.

(iii) $\overline{\mathfrak{V}^t_{\mathfrak{A}}}(i\rho(j + k) - i\rho k) = [\overline{\mathfrak{V}_{\mathfrak{A}}}(i\rho(j + k) - i\rho k)]^t \geq [\overline{\mathfrak{V}_{\mathfrak{A}}}(j)]^t = \overline{\mathfrak{V}^t_{\mathfrak{A}}}(j)$

Hence, $\overline{\mathfrak{V}^t_{\mathfrak{A}}}$ is I-V vague ideal of the N.

Now, $\overline{\mathfrak{V}^t_{\mathfrak{A}}}(0) = [\overline{\mathfrak{V}_{\mathfrak{A}}}(0)]^t = [1, 0]^t\} = [1, 0]$. Therefore, $\overline{\mathfrak{V}^t_{\mathfrak{A}}}$ is a normal I-V vague ideal of the N. Now, $N_{\overline{\mathfrak{V}^t_{\mathfrak{A}}}} = \{i \in N \mid \overline{\mathfrak{V}^t_{\mathfrak{A}}}(i) = \overline{\mathfrak{V}^t_{\mathfrak{A}}}(0)\} = \{i \in N \mid \{\overline{\mathfrak{V}_{\mathfrak{A}}}(i)\}^t = [\overline{\mathfrak{V}_{\mathfrak{A}}}(0)]^t\} = \{i \in N \mid \{\overline{\mathfrak{V}_{\mathfrak{A}}}(i)\}^t = [1, 0]\} = \{i \in N \mid \overline{\mathfrak{V}_{\mathfrak{A}}}(i) = [1, 0]\} = \{i \in N \mid \overline{\mathfrak{V}_{\mathfrak{A}}}(i) = \overline{\mathfrak{V}_{\mathfrak{A}}}(0)\} = N_{\overline{\mathfrak{V}_{\mathfrak{A}}}}$. □

Theorem 3.2. *Let $\overline{\mathfrak{V}_{\mathfrak{A}}} = [\overline{\tau_{\mathfrak{A}}}, \overline{\sigma_{\mathfrak{A}}}]$ be an I-V vague ideal of N such that if $\overline{\tau_{\mathfrak{A}}}(i) + \overline{\sigma_{\mathfrak{A}}}(i) \leq \overline{\tau_{\mathfrak{A}}}(0) + \overline{\sigma_{\mathfrak{A}}}(0) \forall i \in N$, where $\overline{\mathfrak{V}_{\mathfrak{A}^+}} = [\overline{\tau_{\mathfrak{A}^+}}, \overline{\sigma_{\mathfrak{A}^+}}]$ defined as*

$\overline{\tau_{\mathfrak{A}^+}}(i) = \overline{\tau_{\mathfrak{A}}}(i) + \overline{\tau_{\mathfrak{A}}}(0) - 1$ and $\overline{\sigma_{\mathfrak{A}^+}}(i) = \overline{\sigma_{\mathfrak{A}}}(i) - \overline{\sigma_{\mathfrak{A}}}(0)$, then $\overline{\mathfrak{V}_{\mathfrak{A}^+}}$ is normal I-V vague ideal of N.

Proof. For any $i \in N$, $\overline{\tau_{\mathfrak{A}^+}}(i) + \overline{\sigma_{\mathfrak{A}^+}}(i) = \overline{\tau_{\mathfrak{A}}}(i) + \overline{\tau_{\mathfrak{A}}}(0) - 1 + \overline{\sigma_{\mathfrak{A}}}(i) - \overline{\sigma_{\mathfrak{A}}}(0) \leq 1$, thus $\overline{\tau_{\mathfrak{A}^+}}, \overline{\sigma_{\mathfrak{A}^+}}$ is I-V vague ideal with $\overline{\tau_{\mathfrak{A}^+}}(0) = 1, \overline{\sigma_{\mathfrak{A}^+}}(0) = 0$, therefore $\overline{\mathfrak{V}_{\mathfrak{A}^+}}$ is normal I-V vague ideal of N. □

Corollary 3.1. *If $\overline{\mathfrak{V}_{\mathfrak{A}}}$ is a I-V vague ideal of N and if $\overline{\mathfrak{V}_{\mathfrak{A}^+}}(i) = 0$, for some $i \in N$ then $\overline{\mathfrak{V}_{\mathfrak{A}}}(i) = 0$, for all $i \in N$.*

Theorem 3.3. *An I-V vague ideal of $\overline{\mathfrak{V}_{\mathfrak{A}}}$ is normal if and only if $\overline{\mathfrak{V}_{\mathfrak{A}}} = \overline{\mathfrak{V}_{\mathfrak{A}^+}}$.*

Proof. Suppose that $\overline{\mathfrak{V}_{\mathfrak{A}}}$ is normal I-V vague ideal of N and let $i \in N$. Then, $\overline{\tau_{\mathfrak{A}^+}}(i) = \overline{\tau_{\mathfrak{A}}}(i) + \overline{\tau_{\mathfrak{A}}}(0) - 1 = \overline{\tau_{\mathfrak{A}}}(i)$ and $\overline{\sigma_{\mathfrak{A}^+}}(i) = \overline{\sigma_{\mathfrak{A}}}(i) + \overline{\sigma_{\mathfrak{A}}}(0) = \overline{\sigma_{\mathfrak{A}}}(i)$. Converse is obvious. □

4. Direct product of interval valued vague ideal of Γ-nearrings

Definition 4.1. *Let $\overline{\mathfrak{V}_{\mathfrak{A}_l}}$ be an I-V vague ideal of N_l for l = 1, 2... n, then direct product of $\overline{\mathfrak{V}_{\mathfrak{A}_l}}$ is a function $\overline{\mathfrak{V}_{\mathfrak{A}_1}} \times \overline{\mathfrak{V}_{\mathfrak{A}_2}} \times \ldots \times \overline{\mathfrak{V}_{\mathfrak{A}_n}}$: $N_1 \times N_2 \times \ldots \times N_n \to \overline{[0,1]}$ defined by $(\overline{\mathfrak{V}_{\mathfrak{A}_1}} \times \overline{\mathfrak{V}_{\mathfrak{A}_2}} \times \ldots \times \overline{\mathfrak{V}_{\mathfrak{A}_n}})(i_1, i_2 \ldots, i_n) = \min\{\overline{\mathfrak{V}_{\mathfrak{A}_1}}(i_1), \overline{\mathfrak{V}_{\mathfrak{A}_2}}(i_2) \ldots \overline{\mathfrak{V}_{\mathfrak{A}_n}}(i_n)\}$.*

Theorem 4.1. *If $\overline{\mathfrak{V}_{\mathfrak{A}_1}}$ and $\overline{\mathfrak{V}_{\mathfrak{A}_2}}$ be two I-V vague ideals in Γ-nearrings M and N respectively then $\overline{\mathfrak{V}_{\mathfrak{A}_1}} \times \overline{\mathfrak{V}_{\mathfrak{A}_2}}$ is an I-V vague ideal of $M \times N$.*

Corollary 4.1. *The direct product of I-V vague ideals of a Γ-nearrings N is also an I-V vague ideal of N.*

Proof. Let $\overline{\mathfrak{V}_{\mathfrak{A}_i}}$ be I-V vague ideal of Γ-nearrings N_i, for $i = 1, 2 \ldots n$. Now, let $N = N_1 \times N_2 \times \ldots \times N_n$ be the direct product of Γ-nearrings and $\rho_1, \rho_2 \ldots \rho_n \in \Gamma$ be an direct product of Γ-nearrings.

Let $i = (i_1, i_2 \ldots i_n), j = (j_1, j_2 \ldots j_n), k = (k_1, k_2 \ldots k_n) \in N$. Take $\overline{\mathfrak{V}_{\mathfrak{A}_1}} \times \overline{\mathfrak{V}_{\mathfrak{A}_2}} \times \ldots \times \overline{\mathfrak{V}_{\mathfrak{A}_n}} = \overline{\mathfrak{V}_{\mathfrak{A}}}$. Now, (i) $\overline{\mathfrak{V}_{\mathfrak{A}}}(i - j) = \overline{\mathfrak{V}_{\mathfrak{A}}}((i_1, i_2 \ldots i_n) - (j_1, j_2 \ldots j_n)) = \overline{\mathfrak{V}_{\mathfrak{A}}}(i_1 - j_1, i_2 - j_2 \ldots i_n - j_n) = \min\{\overline{\mathfrak{V}_{\mathfrak{A}_1}}(i_1 - j_1), \overline{\mathfrak{V}_{\mathfrak{A}_2}}(i_2 - j_2) \ldots \overline{\mathfrak{V}_{\mathfrak{A}_n}}(i_n - j_n)\} \geq \min\{\min\{\overline{\mathfrak{V}_{\mathfrak{A}_1}}(i_1), \overline{\mathfrak{V}_{\mathfrak{A}_1}}(j_1)\}, \min\{\overline{\mathfrak{V}_{\mathfrak{A}_2}}(i_2), \overline{\mathfrak{V}_{\mathfrak{A}_2}}(j_2)\} \ldots \min\{\overline{\mathfrak{V}_{\mathfrak{A}_n}}(i_n), \overline{\mathfrak{V}_{\mathfrak{A}_n}}(j_n)\} = \min\{(\overline{\mathfrak{V}_{\mathfrak{A}_1}} \times \overline{\mathfrak{V}_{\mathfrak{A}_2}} \times \ldots \times \overline{\mathfrak{V}_{\mathfrak{A}_n}})(i_1, i_2 \ldots i_n), (\overline{\mathfrak{V}_{\mathfrak{A}_1}} \times \overline{\mathfrak{V}_{\mathfrak{A}_2}} \times \ldots \times \overline{\mathfrak{V}_{\mathfrak{A}_n}})(j_1, j_2 \ldots j_n)\} = \min\{\overline{\mathfrak{V}_{\mathfrak{A}}}(i), \overline{\mathfrak{V}_{\mathfrak{A}}}(j)\}$.

(ii) $\overline{\mathfrak{V}_{\mathfrak{A}}}(j + i - j) = \overline{\mathfrak{V}_{\mathfrak{A}}}((j_1, j_2 \ldots j_n) + (i_1, i_2 \ldots i_n) - (j_1, j_2 \ldots j_n)) = \overline{\mathfrak{V}_{\mathfrak{A}}}\{j_1 + i_1 - j_1, j_2 + i_2 - j_2 \ldots j_n + i_n - j_n\} = \min\{\overline{\mathfrak{V}_{\mathfrak{A}_1}}(j_1 + i_1 - j_1), \overline{\mathfrak{V}_{\mathfrak{A}_2}}(j_2 + i_2 - j_2) \ldots \overline{\mathfrak{V}_{\mathfrak{A}_n}}(j_n + i_n - j_n)\} \geq \min\{\overline{\mathfrak{V}_{\mathfrak{A}_1}}(i_1), \overline{\mathfrak{V}_{\mathfrak{A}_2}}(i_2) \ldots \overline{\mathfrak{V}_{\mathfrak{A}_n}}(i_n)\} = (\overline{\mathfrak{V}_{\mathfrak{A}_1}} \times \overline{\mathfrak{V}_{\mathfrak{A}_2}} \times \ldots \times \overline{\mathfrak{V}_{\mathfrak{A}_n}})(i_1, i_2 \ldots i_n) = \overline{\mathfrak{V}_{\mathfrak{A}}}(i)$.

(iii) $\overline{\mathfrak{V}_{\mathfrak{A}}}(i\rho(j+k) - i\rho k) = \overline{\mathfrak{V}_{\mathfrak{A}}}((i_1, i_2 \ldots i_n)(\rho_1, \rho_2 \ldots \rho_n)(j_1+k_1, j_2+k_2 \ldots j_n+k_n) - (i_1, i_2 \ldots i_n)(\rho_1, \rho_2 \ldots \rho_n)(k_1, k_2 \ldots k_n)) = \overline{\mathfrak{V}_{\mathfrak{A}}}(i_1\rho_1(j_1+k_1) - i_1\rho_1 k_1, i_2\rho_2(j_2 + j_2) - i_2\rho_2 j_2 \ldots i_n\rho_n(j_n+k_n) - i_n\rho_n k_n) = \min\{\overline{\mathfrak{V}_{\mathfrak{A}_1}}(i_1\rho_1(j_1+k_1) - i_1\rho_1 k_1), \overline{\mathfrak{V}_{\mathfrak{A}_2}}\rho_2(i_2(j_2+k_2) - i_2\rho_2 k_2) \ldots \overline{\mathfrak{V}_{\mathfrak{A}_n}}(i_n\rho_n(j_n+k_n) - i_n\rho_n k_n)\} \geq \min\{\overline{\mathfrak{V}_{\mathfrak{A}_1}}(j_1), \overline{\mathfrak{V}_{\mathfrak{A}_2}}(j_2) \ldots \overline{\mathfrak{V}_{\mathfrak{A}_n}}(j_n)\} = (\overline{\mathfrak{V}_{\mathfrak{A}_1}} \times \overline{\mathfrak{V}_{\mathfrak{A}_2}} \times \ldots \times \overline{\mathfrak{V}_{\mathfrak{A}_n}})(j_1, j_2 \ldots, j_n) = \overline{\mathfrak{V}_{\mathfrak{A}}}(j)$. Hence, $\overline{\mathfrak{V}_{\mathfrak{A}}}$ is I-V vague ideal of N. □

Lemma 4.1. *Let* $\overline{\mathfrak{V}_{\mathfrak{A}_1}}$ *and* $\overline{\mathfrak{V}_{\mathfrak{A}_2}}$ *be a two I-V vague subcollections of* Γ*-nearrings* M *and* N *respectively, if* $\overline{\mathfrak{V}_{\mathfrak{A}_1}} \times \overline{\mathfrak{V}_{\mathfrak{A}_2}}$ *is a I-V vague ideal of* $M \times N$*, then:*

(*i*) $(\overline{\mathfrak{V}_{\mathfrak{A}_1}} \times \overline{\mathfrak{V}_{\mathfrak{A}_2}})(0, 0^{'}) \geq (\overline{\mathfrak{V}_{\mathfrak{A}_1}} \times \overline{\mathfrak{V}_{\mathfrak{A}_2}})(i, j)$;

(*ii*) $(\overline{\mathfrak{V}_{\mathfrak{A}_1}} \times \overline{\mathfrak{V}_{\mathfrak{A}_2}} \ldots \times \overline{\mathfrak{V}_{\mathfrak{A}_n}})(0_1, 0_2 \ldots 0_n) \geq (\overline{\mathfrak{V}_{\mathfrak{A}_1}} \times \overline{\mathfrak{V}_{\mathfrak{A}_2}} \ldots \times \overline{\mathfrak{V}_{\mathfrak{A}_n}})(i_1, i_2 \ldots i_n)$.

Theorem 4.2. *Let* $\overline{\mathfrak{V}_{\mathfrak{A}_1}}$ *and* $\overline{\mathfrak{V}_{\mathfrak{A}_2}}$ *be a two vague subsets of* Γ*-nearrings* M *and* N *respectively. If* $\overline{\mathfrak{V}_{\mathfrak{A}_1}} \times \overline{\mathfrak{V}_{\mathfrak{A}_2}}$ *is a I-V vague ideal of* Γ*-nearring* $M \times N$*, then atleast one of the following must be true*

(*i*) $\overline{\mathfrak{V}_{\mathfrak{A}_1}}(0) \geq \overline{\mathfrak{V}_{\mathfrak{A}_2}}(j) \forall j \in N$;

(*ii*) $\overline{\mathfrak{V}_{\mathfrak{A}_2}}(0^{'}) \geq \overline{\mathfrak{V}_{\mathfrak{A}_1}}(i) \forall i \in M$.

Proof. Let $\overline{\mathfrak{V}_{\mathfrak{A}_1}} \times \overline{\mathfrak{V}_{\mathfrak{A}_2}}$ is the I-V vague ideal of $M \times N$. By contradiction suppose that: there exists $i \in M$ and $j \in N$ such that $\overline{\mathfrak{V}_{\mathfrak{A}_2}}(0') < \overline{\mathfrak{V}_{\mathfrak{A}_1}}(i)$ and $\overline{\mathfrak{V}_{\mathfrak{A}_1}}(0) < \overline{\mathfrak{V}_{\mathfrak{A}_2}}(j)$.

Now, $(\overline{\mathfrak{V}_{\mathfrak{A}_1}} \times \overline{\mathfrak{V}_{\mathfrak{A}_2}})(i, j) = \overline{\mathfrak{V}_{\mathfrak{A}_1}}(i) \wedge \overline{\mathfrak{V}_{\mathfrak{A}_2}}(j) > \overline{\mathfrak{V}_{\mathfrak{A}_1}}(0) \wedge \overline{\mathfrak{V}_{\mathfrak{A}_2}}(0') = (\overline{\mathfrak{V}_{\mathfrak{A}_1}} \times \overline{\mathfrak{V}_{\mathfrak{A}_2}})(0, 0')$. Thus, $\overline{\mathfrak{V}_{\mathfrak{A}_1}} \times \overline{\mathfrak{V}_{\mathfrak{A}_2}}$ is a I-V vague ideals of the $M \times N$ satisfying $(\overline{\mathfrak{V}_{\mathfrak{A}_1}} \times \overline{\mathfrak{V}_{\mathfrak{A}_2}})(i, j) > (\overline{\mathfrak{V}_{\mathfrak{A}_1}} \times \overline{\mathfrak{V}_{\mathfrak{A}_2}})(0, 0')$. This is a contradict to Lemma (26). □

Theorem 4.3. *Let* $\overline{\mathfrak{V}_{\mathfrak{A}_1}}$ *and* $\overline{\mathfrak{V}_{\mathfrak{A}_2}}$ *be an two vague subset of* Γ*-nearrings* M *and* N *respectively, if* $\overline{\mathfrak{V}_{\mathfrak{A}_1}} \times \overline{\mathfrak{V}_{\mathfrak{A}_2}}$ *is a I-V vague ideal of* Γ*-nearring* $M \times N$*, then either* $\overline{\mathfrak{V}_{\mathfrak{A}_1}}$ *is a I-V vague ideal of* M *or* $\overline{\mathfrak{V}_{\mathfrak{A}_2}}$ *is a I-V vague ideal of* N*.*

Proof. Suppose $\overline{\mathfrak{V}_{\mathfrak{A}_1}} \times \overline{\mathfrak{V}_{\mathfrak{A}_2}}$ is a I-V vague ideal $M \times N$. Then, by Theorem 27 $\overline{\mathfrak{V}_{\mathfrak{A}_2}}(0^{'}) \geq \overline{\mathfrak{V}_{\mathfrak{A}_1}}(i)$.

Now, (i) for $i, j \in M$, $\overline{\mathfrak{V}_{\mathfrak{A}_1}}(i-j) \geq \overline{\mathfrak{V}_{\mathfrak{A}_1}}(i-j) \wedge \overline{\mathfrak{V}_{\mathfrak{A}_2}}(0^{'}) = \overline{\mathfrak{V}_{\mathfrak{A}_1}}(i-j) \wedge \overline{\mathfrak{V}_{\mathfrak{A}_2}}(0^{'} - 0^{'}) = (\overline{\mathfrak{V}_{\mathfrak{A}_1}} \times \overline{\mathfrak{V}_{\mathfrak{A}_2}})((i, 0^{'}) - (j, 0^{'})) \geq \overline{\mathfrak{V}_{\mathfrak{A}_1}}(i) \wedge \overline{\mathfrak{V}_{\mathfrak{A}_2}}(0^{'}) \wedge \overline{\mathfrak{V}_{\mathfrak{A}_1}}(j) = \overline{\mathfrak{V}_{\mathfrak{A}_1}}(i) \wedge \overline{\mathfrak{V}_{\mathfrak{A}_1}}(j)$.

(ii) $\overline{\mathfrak{V}_{\mathfrak{A}_1}}(j + i - j) \geq \overline{\mathfrak{V}_{\mathfrak{A}_1}}(j + i - j) \wedge \overline{\mathfrak{V}_{\mathfrak{A}_2}}(0^{'}) = (\overline{\mathfrak{V}_{\mathfrak{A}_1}} \times \overline{\mathfrak{V}_{\mathfrak{A}_2}})((j, 0^{'}) + (i, 0^{'}) - (j, 0^{'})) \geq \overline{\mathfrak{V}_{\mathfrak{A}_1}}(i) \wedge \overline{\mathfrak{V}_{\mathfrak{A}_2}}(0^{'}) = \overline{\mathfrak{V}_{\mathfrak{A}_1}}(i)$.

(iii) For i, j, $k \in M$ and $\rho \in \Gamma$ so, $\overline{\mathfrak{V}_{\mathfrak{A}_1}}(j\rho(i + k) - j\rho k) \geq \overline{\mathfrak{V}_{\mathfrak{A}_1}}(k\rho(i + k) - j\rho k) \wedge \overline{\mathfrak{V}_{\mathfrak{A}_2}}(0^{'}) = (\overline{\mathfrak{V}_{\mathfrak{A}_1}} \times \overline{\mathfrak{V}_{\mathfrak{A}_2}})((j, 0^{'})(\rho, 0^{'})((i, 0^{'}) + (k, 0^{'})) - (j, 0^{'})(\rho, 0^{'})(k, 0^{'})) \geq \overline{\mathfrak{V}_{\mathfrak{A}_1}}(i) \wedge \overline{\mathfrak{V}_{\mathfrak{A}_2}}(0^{'}) = \overline{\mathfrak{V}_{\mathfrak{A}_1}}(i)$.

Hence, in this case $\overline{\mathfrak{V}_{\mathfrak{A}_1}}$ is a I-V vague ideal of M. Similarly, we can show $\overline{\mathfrak{V}_{\mathfrak{A}_2}}$ is a I-V vague ideal of N. □

Theorem 4.4. *Let* $\overline{\mathfrak{V}_{\mathfrak{A}_1}}$ *and* $\overline{\mathfrak{V}_{\mathfrak{A}_2}}$ *be vague subsets of* Γ*-nearrings* M *and* N *respectively, such that* $\overline{\mathfrak{V}_{\mathfrak{A}_1}}(0) \geq \overline{\mathfrak{V}_{\mathfrak{A}_1}}(j)$*, for all* $j \in N$ *and* $\overline{\mathfrak{V}_{\mathfrak{A}_1}} \times \overline{\mathfrak{V}_{\mathfrak{A}_2}}$ *is a I-V vague ideal of* $M \times N$*, then* $\overline{\mathfrak{V}_{\mathfrak{A}_2}}$ *is a I-V vague ideal of* N *.*

Corollary 4.2. *Let* $\overline{\mathfrak{V}_{\mathfrak{A}_1}}$, $\overline{\mathfrak{V}_{\mathfrak{A}_2}} \ldots \overline{\mathfrak{V}_{\mathfrak{A}_n}}$ *be a vague subset of* Γ*-nearrings* N_1, $N_2 \ldots N_n$ *respectively and if* $\overline{\mathfrak{V}_{\mathfrak{A}_1}} \times \overline{\mathfrak{V}_{\mathfrak{A}_2}} \times \ldots \times \overline{\mathfrak{V}_{\mathfrak{A}_n}}$ *is a I-V vague ideal of* Γ*-nearring* $N_1 \times N_2 \times \ldots \times N_n$*, then at least for one* l, $\overline{\mathfrak{V}_{\mathfrak{A}_l}}(0_l) \geq \overline{\mathfrak{V}_{\mathfrak{A}_m}}(i_m)$ *for all* $i \in N_m$*, for* m = 1, 2… n *where* 0_l *is the identity in* N_l*, must be true.*

Proof. We know that if N_1, $N_2 \ldots N_n$ are Γ-nearrings, then their product $N_1 \times N_2 \times \ldots \times N_n$ is also a Γ-nearring. Let $\overline{\mathfrak{V}_{\mathfrak{A}_1}} \times \overline{\mathfrak{V}_{\mathfrak{A}_2}} \times \ldots \times \overline{\mathfrak{V}_{\mathfrak{A}_n}}$ is a I-V vague ideal of$N_1 \times N_2 \times \ldots \times N_n$. By contradiction, suppose there is $i_m \in N_m$ such that $\overline{\mathfrak{V}_{\mathfrak{A}_l}}(i_l) > \overline{\mathfrak{V}_{\mathfrak{A}_m}}(0_m)$, for all l. Now, $[(\overline{\mathfrak{V}_{\mathfrak{A}_1}} \times \overline{\mathfrak{V}_{\mathfrak{A}_2}} \times \ldots \times \overline{\mathfrak{V}_{\mathfrak{A}_n}})(i_1, i_2, \ldots, i_n)] = [\overline{\mathfrak{V}_{\mathfrak{A}_1}}(i_1) \wedge \overline{\mathfrak{V}_{\mathfrak{A}_2}}(i_2) \wedge \ldots \overline{\mathfrak{V}_{\mathfrak{A}_n}}(i_n)] > \overline{\mathfrak{V}_{\mathfrak{A}_n}}(0_n) \wedge \overline{\mathfrak{V}_{\mathfrak{A}_{n-1}}}(0_{n-1}) \wedge \ldots \wedge \overline{\mathfrak{V}_{\mathfrak{A}_1}}(0_1) = (\overline{\mathfrak{V}_{\mathfrak{A}_n}} \times \overline{\mathfrak{V}_{\mathfrak{A}_{n-1}}} \times \ldots \times \overline{\mathfrak{V}_{\mathfrak{A}_1}})(0_n, 0_{n-1}, \ldots, 0_1)$. Thus, $\overline{\mathfrak{V}_{\mathfrak{A}_1}} \times \overline{\mathfrak{V}_{\mathfrak{A}_2}} \times \ldots \times \overline{\mathfrak{V}_{\mathfrak{A}_n}}$ is vague with identity $(0_1, 0_2, \ldots, 0_n) of N_1 \times N_2 \times \ldots \times N_n$ satisfying $(\overline{\mathfrak{V}_{\mathfrak{A}_1}} \times \overline{\mathfrak{V}_{\mathfrak{A}_2}} \times \ldots \times \overline{\mathfrak{V}_{\mathfrak{A}_n}})(i_1, i_2 \ldots i_n) > (\overline{\mathfrak{V}_{\mathfrak{A}_1}} \times \overline{\mathfrak{V}_{\mathfrak{A}_2}} \times \ldots \times \overline{\mathfrak{V}_{\mathfrak{A}_n}})(0_n, 0_{n-1} \ldots 0_1)$, this is a contradiction. □

Corollary 4.3. *Let* $\overline{\mathfrak{V}_{\mathfrak{A}_1}}, \overline{\mathfrak{V}_{\mathfrak{A}_2}} \ldots \overline{\mathfrak{V}_{\mathfrak{A}_n}}$ *are I-V vague ideals of* Γ*-nearrings* $N_1, N_2, \ldots, N_n$*, respectively and if* $\overline{\mathfrak{V}_{\mathfrak{A}_1}} \times \overline{\mathfrak{V}_{\mathfrak{A}_2}} \times \ldots \times \overline{\mathfrak{V}_{\mathfrak{A}_n}}$ *is a I-V vague ideal of* $N_1 \times N_2 \times \ldots \times N_n$*, then at least for one of* l*,* $\overline{\mathfrak{V}_{\mathfrak{A}_l}}$ *is a I-V vague ideal of* N_l*.*

Conclusion

In this paper we give about interval vague ideals in Γ-nearrings. After that, we give definition, properties, theorems, examples of vague ideals in Γ-nearrings. We also study normal interval valued vague ideals and the direct product of interval value vague ideals in Γ-nearrings.

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Accepted: May 19, 2021