An extended outranking approach for multi attribute group decision making problems with intuitionistic fuzzy data

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Abstract. In group decision making problems, decision matrix depends on different attributes. The weight of each attribute indicates its relative importance comparing to others. The objective of this paper, however, is to introduce a linear model in order to determine the importance of each attribute in multi attribute group decision making problems with intuitionistic fuzzy data; so that, this model prevents the decision makers from making mistake in determining the weight and shape of standardization of their mental measurement units. Next, using the determined weights, a new method of ranking alternatives based on ELECTRE III is introduced and illustrated. In this method, group satisfaction index is used to exclude decision makers' personal opinion for selection the best alternative. Finally, a numerical example is considered to elucidate the details of the proposed method, and then the obtained results are compared with other current methods.

Keywords: intuitionistic fuzzy set, multi attribute group decision making, ranking, weight of attributes.

1. Introduction

The human is always seeking a balance between his/her requierments and goals even in making the tiniest personal decisions. For this necessity, he/she needs to use multi criteria decision making (MCDM) as a solution, which is one of the subsets of decision making. In this subset of decision making, the

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determiner selects the best choice based on some criteria from the options he/she has.

MCDM is the accepted designation for all concepts dealing with multi objective decision making (MODM) and/or multi attribute decisin making (MADM). The main difference between MODM and MADM is related to the definition of alternatives. In MODM, criteria is defined implicitly by a mathematical programming structure that results with continuous alternatives, while in MADM, the set of decision alternatives is defined explicitly by a finite list of alternative actions where discrete alternatives exit.

As lots of attributes that are sometimes antithetical involve in choosing the most appropriate alternative, this issue will be investigated as a multi attribute group decision making (MAGDM) problem. Since the decision matrix has different attributes in group decision making (GDM) models, being aware of the importance coefficient or weight for each attribute is necessary in decision making. Each attribute weight expresses its relative importance toward others. Choosing the weights knowingly and correctly is a great help in order to reach the intended target.

Imprecise and uncertainity data have considerable influence in studying and solving decision making and optimization problems. These data contain statistical or random, fuzzy, interval, rough and even a combination of the aforementioned imprecise data. One of the most important imprecise data is fuzzy set theory proposed by Zadeh [36]. He established the basics of modelling the imprecise data and approximate reasoning with mathematical equations by introducing the fuzzy sets theory which caused a massive change of its kind in mathematics and classical logic. As the distribution and density functions play the fundamental role in random or statistical environments, membership function has the main role in fuzzy data.

One of the other imprecise data is the intuitionistic fuzzy sets (IFSs) proposed by Attanassov [1] which are an extension of ordinary fuzzy sets. He believed that the non-membership degree does not always equal to one minus membership degree of ordinary fuzzy sets and there may be hesitancies in its value, too. Therefore, he introduced the IFSs that have both membership and non-membership degrees for modelling uncertainty concept in real world problems. A lot of researchers in the area of intuitionistic fuzziness, used another concept like intuitionistic fuzzy pairs [3]. Also, Attanassov [2] offered a comprehensive survey of intuitionistic fuzzy logics by reporting on both the author's research and others' findings.

Xu [28] investigated the GDM problems in which all the provided information by the decision makers (DMs) is expressed as intuitionistic fuzzy decision matrices and the information about attribute weights is partially known. A deviation-based approach to intuitionistic fuzzy MAGDM has been introduced by Xu [29] in which attributes importance is determined through an specified method. Wang et al. [25] employed projection method for MAGDM problems in intuitionistic fuzzy environment, where the weights of attributes and DMs are predetermined. Intuitionistic fuzzy MAGDM models based on the information entropy weights for attributes have been presented by Duan et al. [8]. Yue [35] presented a TOPSIS-based fuzzy GDM methodology based on IFSs for fuzzy MAGDM where the weight of attributes is predetermined. Chen and Tan [7] proposed techniques which allow that degrees of satisfiability and non-satisfiability of each alternative with respect to a set of criteria to be presented by vague values. Furthermore, the proposed techniques allow the DM to assign a different degree of importance to each attribute. Xian et al. [27] introduced an operator for linguistic fuzzy data and evaluated its usage in GDM where the weight of each attribute is predetermined. A method for determining weights of DMs with interval numbers under GDM environment has been developed by Yue [34] . Chen et al. [6] studied the MAGDM problems with IFSs and evidential reasoning method in which weight or importance of each attribute and DMs is inductively specified. Wu et al. [26] proposed a model for GDM based on a fuzzy VIKOR approach using linguistic variables in which the attribute importance is obtained by linguistic variables. Sevastjanov and Dymova [18] studied the ranking of alternatives using hesitant fuzzy values within the Dempster-Shafer theory (DST) in which the attribute importance is predetermined.

Elimination and choice translating reality (ELECTRE) method is one of the most well-known ranking methods in decision making problems. This methodology has been proposed by Benayoun [4] and then has been developed by Van Delft and Nijkamp [22]. The dominance concept is used implicitly in this method and the alternatives are compared with each other pairwise, then dominant and weak (nondominant) alternatives are recognized and finally the weak or non-dominant alternatives are deleted [17]. Due to the complexity of new decision making problems and developing usage of MADM methods, different versions of this method have been proposed as its subsets. The primary proposed method by Benayon has been noted as ELCTRE I. Since the principle of all the new methods are the same as ELECTRE I, they have been named ELECTRE II, ELECTRE III, ELECTRE IV and ELECTRE TRI. The difference in various versions is in the mathematical operations and the kind of problems these methods are able to solve. Method I and TRI are specifically used to solve selection and allocation problems, respectively. Methods II, III and IV are used for ranking problems [23]. Shen et al. [19, 20] proposed two ranking models using ELECTRE III in MCGDM with intuitionistic fuzzy data in which the weight of criteria and DMs are previously specified. Hashemi et al. [10] evaluated the GDM problems using ELECTRE III with interval-valued intuitionistic fuzzy data in which each criterion weight is defined as interval-valued intuitionistic fuzzy values by experts however group satisfaction index is not applied.

In most of the performed studies for solving decision making problems, the importance of each attribute is either specified with ascribing a weight by the DM or with one of the defined methods, for example entropy, LINMAP, weighted least squares, eigenvector, etc. In this paper, it is tried to determine and evaluate the importance of each attribute by solving a linear programming model. Then, an introduction to a ranking model based on ELECTRE III is carried out. Also, an MAGDM approach based on intuitionistic information is proposed. Preliminaries and the mathematical formulation is presented in Sect. 2. In Sect. 3, the ELECTRE III method and linear programming are briefly described. The proposed algorithmic method is illustrated in Sect. 4. Finally, short conclusions are given in Sect. 5.

2. Preliminaries

In the following, the definition and operations of IFSs are briefly reviewed.

Definition 2.1. *Let X be a reference set. A fuzzy set $\tilde{A}$ in X can be defined as $\tilde{A} = \{\langle x, \mu_{\tilde{A}}(x)\rangle | x \in X\}$, where the function $\mu_{\tilde{A}} : X \to [0,1]$ is the membership function of $\tilde{A}$. The function value $\mu_{\tilde{A}}(x)$ is called the grade of membership of x in $\tilde{A}$.*

Definition 2.2. *Let X be fixed. An IFS P in X is defined as $P = \{\langle x, \mu_P(x), \nu_P(x)\rangle | x \in X\}$, where the functions $\mu_P : X \to [0,1]$ and $\nu_P : X \to [0,1]$ are membership function and non-membership function of IFS P, respectively. The hesitancy degree of x in P is $\pi_P(x) = 1 - \mu_P(x) - \nu_P(x)$.*

Remark 2.1. For every $x \in X, 0 \leq \mu_P(x) \leq 1, 0 \leq \nu_P(x) \leq 1, 0 \leq \mu_P(x) + \nu_P(x) \leq 1$ and it can be seen that $\pi_P(X) \in [0,1]$. Xu [30] named $(\mu_\alpha, \nu_\alpha, \pi_\alpha)$ intuitionistic fuzzy number (IFN), denoted as α where $0 \leq \mu_\alpha \leq 1$, $0 \leq \nu_\alpha \leq 1$, $0 \leq \mu_\alpha + \nu_\alpha \leq 1$ and $\pi_\alpha = 1 - \mu_\alpha - \nu_\alpha$. If $\mu_\alpha + \nu_\alpha = 1$ then the $(\mu_\alpha,\ \nu_\alpha,\ \pi_\alpha)$ reduced to $(\mu_\alpha,\ 1 - \nu_\alpha)$.

Definition 2.3 ([7, 11])**.** *Let $\alpha = (\mu_\alpha, \nu_\alpha, \pi_\alpha)$ be an IFN. Score function $s(\alpha)$ which is used to measure the degree of suitability and the accuracy function $h(\alpha)$ which is used to evaluate the accuracy degree of α, as follows:*

$$s(\alpha) = \mu_\alpha - \nu_\alpha,$$
$$h(\alpha) = \mu_\alpha + \nu_\alpha.$$

In many theoretical and practical decision making problems, DMs often need to compare two IFSs or IFNs, in the same universe. Xu and Yager [31] presented some relations between any two IFNs α and β.

Definition 2.4. *Let $\alpha = (\mu_\alpha, \nu_\alpha, \pi_\alpha)$ and $\beta = (\mu_\beta, \nu_\beta, \pi_\beta)$ be two IFNs, the score and accuracy degree of α and β are defined as $s(\alpha)$ and $h(\alpha)$, respectively. Then:*

- $\alpha \prec \beta$ *if* $s(\alpha) < s(\beta)$
- $\alpha \succ \beta$ *if* $s(\alpha) > s(\beta)$

- *if* $s(\alpha) = s(\beta)$ *then*

$$\begin{cases} \alpha \prec \beta, & \text{if } h(\alpha) < h(\beta); \\ \alpha \succ \beta, & \text{if } h(\alpha) > h(\beta); \\ \alpha \approx \beta, & \text{if } h(\alpha) = h(\beta). \end{cases}$$

Definition 2.5 ([21]). *Let* $\alpha = (\mu_\alpha, \nu_\alpha, \pi_\alpha)$ *and* $\beta = (\mu_\beta, \nu_\beta, \pi_\beta)$ *be two IFNs, an intuitionistic fuzzy distance between* α *and* β *is defined as follows:*

(1) $$d_{IFN}(\alpha, \beta) = \sqrt{\frac{1}{2}[(\mu_\alpha - \mu_\beta)^2 + (\nu_\alpha - \nu_\beta)^2 + (\pi_\alpha - \pi_\beta)^2]}$$

Lemma 2.1 ([30]). *Let* $\alpha = (\mu_\alpha, \nu_\alpha, \pi_\alpha)$ *and* $\beta = (\mu_\beta, \nu_\beta, \pi_\beta)$ *be two* $IFNs$, *then* $0 \leq d_{IFN}(\alpha, \beta) \leq 1$, $d_{IFN}(\alpha, \alpha) = 0$ *and* $d_{IFN}(\alpha, \beta) = d_{IFN}(\beta, \alpha)$.

3. The proposed MAGDM approach based on intuitionistic fuzzy information

In the MAGDM ranking and selection problems, the final decision is result from the preferences of a group of DMs. Uncertainty data have considerable influence in studying and solving decision making problems. For a given MAGDM problem under intuitionistic environment the decision matrix can be constructed based on IFNs for each DM. These problems consist of m alternatives, denoted by $A = \{a_1, a_2, \ldots, a_m\}$, and n attributes denoted by $C = \{c_1, c_2, \ldots, c_n\}$. Also, let $D = \{d_1, d_2, \ldots, d_q\}$ be the set of DMs. The decision matrix of DM $d_k; (k = 1, 2, .., q)$ is defined as $R_k = (r_{ijk})_{m\times n}$ in which $r_{ijk}; (i = 1, 2, \ldots, m, j = 1, 2, \ldots, n, k = 1, 2, .., q)$ is an IFN and is provided by DM d_k for the alternative a_i with respect to the attribute c_j.

3.1 The weight of attributes

In MADM techniques for evaluation of alternatives based on the attributes, the optimal alternative is selected. In almost of techniques, weight of attributes are not determined through specific methods. In most of these techniques, determining weight of attributes is based on DMs' ideas. Also in a series of the other techniques, DMs have calculated the weight by proposing a special model. In these models, there are different alternatives that are evaluated based on several attributes independently. Finally, alternatives based on the obtained values, are ranked. The importance coefficient or weight of each attribute is essential in decision making. The weight of each attribute indicates its relative importance comparing to others. A knowing and accurate selection of weights will be a great help in the way of achieving the intended goal. Therefore a linear model in order to determine the importance of each attribute in MAGDM problems with intuitionistic fuzzy data is introduced.

Let d_{ijk} be the distance of alternative $a_i; (i = 1, 2, \ldots, m)$ with respect to the attribute $c_j; (j = 1, 2, \ldots, n)$ from the ideal alternative $a^*_{jk}; (j = 1, 2, \ldots, n, k = 1, 2, \ldots, q)$, by DM $d_k; (k = 1, 2, \ldots, q)$, where d_{ijk} is defined as follows:

$$d_{ijk} = \sqrt{\tfrac{1}{2}[(\mu_{r_{ijk}} - \mu_{a^*_{jk}})^2 + (\nu_{r_{ijk}} - \nu_{a^*_{jk}})^2 + (\pi_{r_{ijk}} - \pi_{a^*_{jk}})^2]}$$

such that a^*_{jk} is calculated with Algorithms 3.1 and 3.2 in the following.

Algorithm 3.1.

Step 1. *For each attribute c_j with positive aspect in decision matrix R_k, find:*

$$\Gamma_1 := \{t \ \ |\mu_{r_{tjk}} = \max_{1\le i\le m} (\mu_{r_{ijk}})\}$$

Step 2. *If $|\Gamma_1| = 1$, define $a^*_{jk} = (\mu_{r_{tjk}}, \nu_{r_{tjk}}, \pi_{r_{tjk}})$, $t \in \Gamma_1$. Else:*

$$a^*_{jk} = (\mu_{r_{tjk}}, \min_{t\in\Gamma_1}(\nu_{r_{tjk}}), \pi_{r_{tjk}}).$$

Algorithm 3.2.

Step 1. *For each attribute c_j with negative aspect in decision matrix R_k, find:*

$$\Gamma_2 := \{t \ \ |\nu_{r_{tjk}} = \max_{1\le i\le m} (\nu_{r_{ijk}})\}$$

Step 2. *If $|\Gamma_2| = 1$, define $a^*_{jk} = (\mu_{r_{tjk}}, \nu_{r_{tjk}}, \pi_{r_{tjk}})$, $t \in \Gamma_2$. Else:*

$$a^*_{jk} = (\min_{t\in\Gamma_2}(\mu_{r_{tjk}}), \nu_{r_{tjk}}, \pi_{r_{tjk}}).$$

Now, consider the linear programming model (P), as follows:

$$(2) \qquad \begin{array}{lll} (P) & \gamma := & \max \frac{1}{q} \sum_{k=1}^{q} \varepsilon_k \\ s.t. & & \sum_{j=1}^{n} x_j = 1 \\ & & x_j \bar{a}_{jk} - \varepsilon_k \ge 0 \quad ; j = 1, 2, \ldots, n; k = 1, 2, \ldots, q. \end{array}$$

where x_j's are decision variables and $\bar{a}_{jk}$'s are obtained from the following relations:

$$(3) \qquad \begin{cases} \bar{a}_{jk} := \exp(\mu_{a^*_{jk}}), & \text{For attributes with positive aspect} \\ \bar{a}_{jk} := \exp(\nu_{a^*_{jk}}), & \text{For attributes with negative aspect} \end{cases}$$

it is clear that $\bar{a}_{jk} > 0$.

Lemma 3.1. *Let γ is the optimal solution of the linear programming model (2). Then $\gamma > 0$.*

Proof of Lemma 3.1. At first, consider the dual of (2), i.e. the problem (4), as follows:

$$
(4) \qquad
\begin{array}{lll}
(D) & \min & \alpha \\
s.t. & & \alpha + \sum_{k=1}^{q} \beta_{jk}\bar{a}_{jk} = 0; \quad j = 1,2,\ldots,n \\
 & & -\sum_{j=1}^{n} \beta_{jk} = \frac{1}{q}; \quad k = 1,2,\ldots,q \\
 & & \beta_{jk} \leqslant 0; \quad k = 1,2,\ldots,q, j = 1,2,\ldots,n.
\end{array}
$$

Now, by contradiction assume that γ not greater than 0, i.e. $\gamma \leq 0$. If $\gamma = 0$ then according to the dual problem $\alpha = \gamma = 0$. Therefore $\sum_{k=1}^{q} \beta_{jk}\bar{a}_{jk} = 0, \forall j$. It is contradict with constraints in (4). If $\gamma < 0$ then $\alpha = \gamma < 0$, but it is also contradict with constraints in (4) and relations (3).

According to the previous explenation, let d_{ijk} is the distance of alternative a_i respect to the attribute c_j from the ideal alternative $a^*_{jk}; (j = 1,2,\ldots,n, k = 1,2,\ldots,q)$, in decision matrix R_k which can be obtained from the Algorithm 3.1 or Algorithm 3.2, γ and $\bar{a}_{jk}; (j = 1,2,\ldots,n, k = 1,2,\ldots,q)$ calculated from (2) and (3), respectively.

Now, let $W_k = \{w_{1k}, w_{2k}, \ldots, w_{nk}\}; (k = 1,2,\ldots,q)$ be a vector of weights for n attributes corresponsive with each decision matrix $R_k; (k = 1,2,\ldots,q)$, then this vector can be obtained by the following linear programming model:

$$
(5) \qquad
\begin{array}{ll}
\min & \sum_{i=1}^{m}\sum_{j=1}^{n} w_{jk} d_{ijk} \\
s.t. & \sum_{j=1}^{n} w_{jk} = 1 \\
 & w_{jk}\bar{a}_{jk} \geq \gamma\ ; \quad j = 1,2,\ldots,n.
\end{array}
$$

3.2 Ranking the alternatives

In this section ,by the weight vectors $W_k = \{w_{1k}, w_{2k}, \ldots, w_{nk}\}$;$(k = 1,2,\ldots,q)$, which are obtained by the stated Algorithms and models (specially (5)), a new ranking method is introduced based on ELECTRE III method with intuitionistic fuzzy information. In this method, after constructing the intuitionistic fuzzy decision matrix via DMs and the definition of the thresholds p, q and v, concordance and discordance matrixes are presented based on score function. At last, through the proposed group satisfaction degree by Goshtasby [9], the last ranking of alternatives are achieved.

Step 1 (create decision matrix). The first step is determination of the decision matrix. The characteristic of alternatives in decision matrix respect to the attributes are considered as IFN. The matrix elements related to alternative

$a_i; (i = 1, 2, \ldots, m)$, and attribute $c_j; (j = 1, 2, \ldots, n)$, by the DM $d_k; (k = 1, 2, \ldots, q)$, which is displayed as r_{ijk}.

Step 2 (define thresholds and assign weights to the attributes). In this step, in order to strengthen of the ability of identifing superior alternatives, three new thresholds are introduced as follows:

- The preference threshold p_j, which the DM shows a strongly preference of one alternative over all the others in therms of attribute c_j.
- The indifference threshold q_j, which the DM is indifferent between two alternatives outranking relationship with respect to attribute c_j.
- The veto threshold v_j, blocks the outranking relationship between alternatives for the attribute c_j.

One of the other important parameters in choosing the appropriate alternative in this step, is weight or degree of importance of each attribute. For this purpose, the weight vector W_k is obtained using the model (5) after solving (2) problem.

Step 3 (create concordance matrix for each attribute). At this level using the constructed decision matrices in the first step, the thresholds in the second step and the alternatives' relation for each attribute by any DM, a concordance matrix $\widetilde{C}_k$ is formed by any DM $d_k; (k = 1, 2, \ldots, q)$. Each of the concordance matrix's elements using the following formula are obtained:

$$\widetilde{C}_k(a_i, a_l) = \prod_{j=1}^{n} (\widetilde{c}_{jk}(a_i, a_l))^{w_{jk}}; \quad k = 1, 2, \ldots, q, \forall i \neq l \tag{6}$$

such that

$$\widetilde{c}_{jk}(a_i, a_l) = \begin{cases} 1, & \text{if } s_{ij} + q_j \geq s_{lj} \\ 0, & \text{if } s_{ij} + p_j \leq s_{lj} \\ \frac{p_j - (s_{lj} - s_{ij})}{p_j - q_j} & \text{otherwise} \end{cases} \tag{7}$$

where s_{ij} is the score function for the alternative a_i with respect to the attribute c_j i.e. $s_{ij} = \mu_{a_{ij}} - \nu_{a_{ij}}$.

Step 4 (create the discordance matrix for each attribute). As the third step, using the decision matrix and the thresholds set, a discordance matrix $\widetilde{D}$ is formed by any DM $d_k; (k = 1, 2, \ldots, q)$. Each of the discordance matrix's elements using the following formula are comparable:

$$\widetilde{D}_k(a_i, a_l) = \prod_{j=1}^{n} (\widetilde{d}_{jk}(a_i, a_l))^{w_{jk}}; \quad k = 1, 2, \ldots, q, \forall i \neq l \tag{8}$$

such that $\widetilde{d}_{jk}$ with respect to each attribute c_j by each DM using the following relations is formed:

$$\widetilde{d}_{jk}(a_i, a_l) = \begin{cases} 0, & \text{if } s_{ij} + p_j \geq s_{lj} \\ 1, & \text{if } s_{ij} + v_j \leq s_{lj} \\ \frac{s_{ij}+p_j-s_{lj}}{p_j-v_j}, & \text{otherwise} \end{cases}. \tag{9}$$

Note that, $\widetilde{d}_{jk}(a_i, a_l)$ shows the credibility of the proposition which alternative a_i is not at least as good as alternative a_l.

Remark 3.1. Note that if $s_j(a_i)$ be the score function between alternative $a_i; (i = 1, 2, \ldots, m)$ and ideal alternative with respect to the benefit or cost attribute $c_j; (j = 1, 2, \ldots, n)$, then the concept of the thresholds of preference (p_j), indifference (q_j) and veto (v_j) aims at defining the statement $a_iSa_l, (i, l = 1, 2, \ldots, m)$ for every couple a_i and a_l of the alternatives set. The statement a_iSa_l means that alternative a_i outranks alternative a_l, when a_i is at least as good as a_l in the most of the attributes and never significantly worse in the rest of them. Respectively, the statement $a_iS_ja_l$ is defined for every attribute c_j. Then the attribute c_j is in agreement with a_iSa_l if only $a_iS_ja_l$ and even if $s_j(a_i) > s_j(a_l) - q_j$. The ELECTRE III method is based on the definition of two matrices, the concordance and the discordance matrices, which determine if the statement a_iSa_l is acceptable. The rule of concordance requests that the majority of the attributes, have to be in favour of the statement a_iSa_l, while the rule of discordance requests that no attribute from the minority that does not support the statement a_iSa_l be strongly against it. The veto threshold (v_j) allows the complete rejection of the a_iSa_l statement when the relation $s_j(a_l) > s_j(a_i) + v_j$ is valid for every attribute c_j in concordance matrices.

Step 5 (create the credibility matrix). After creating the concordance matrix $\widetilde{C}_k$ and discordance matrix $\widetilde{D}_k$, the credibility matrix $\widetilde{S}_k(a_i, a_l), \forall i \neq l$ for each DM $d_k; (k = 1, 2, \ldots, q)$ among the different alternatives and based on these two matrices is formed. Outranking matrix elements' using the following mathematical relationship can be calculated:

$$\widetilde{S}_k(a_i, a_l) = \begin{cases} \widetilde{C}_k(a_i, a_l), & \text{if } \widetilde{D}_k(a_i, a_l) \leq \widetilde{C}_k(a_i, a_l), \\ \widetilde{D}_k(a_i, a_l), & \text{if } \widetilde{D}_k(a_i, a_l) > \widetilde{C}_k(a_i, a_l). \end{cases} \tag{10}$$

Step 6 (create the net outranking flow matrix). According to the credibility matrix $\widetilde{S}_k$, the net outranking flow matrix $F_k(a_i, a_l); (k = 1, 2, \ldots, q)$ in order to represnet the net outranking alternative a_i over a_l is stated as follows:

$$F_k(a_i, a_l) = \widetilde{S}_k(a_i, a_l) - \widetilde{S}_k(a_l, a_i); \quad \forall i \neq l. \tag{11}$$

Step 7 (the net outranking flow index). In this step, the net outranking flow index $\Phi_k(a_i); (i = 1, 2, \ldots, m)$, i.e. the net outranking alternative a_i is overall to the remaining alternatives, is calculated with the following formula:

$$\Phi_k(a_i) = \sum_{l=1, l\neq i}^{m} F_k(a_i, a_l), \quad \forall k = 1, 2, \ldots, q. \tag{12}$$

Therefore, according to the net outranking flow index $\Phi_k(a_i)$, the alternatives can be ranked.

Step 8 (the group net outranking flow index). The group net outranking flow index $\Phi_G(a_i); (i = 1, 2, \ldots, m)$, is represented as follows:

$$\Phi_G(a_i) = \sum_{k=1}^{q} \lambda_k \Phi_k(a_i). \tag{13}$$

where $\lambda_k; (k = 1, 2, \ldots, q)$, indicate the DMs weights.

Note that, $\Phi_G(a_i)$ shows the net outranking alternative a_i is overall to the other alternatives.

Step 9 (the group's satisfaction index). To gain the group's satisfaction index, the greatest deviation [9] is used between individual and group preorders. Assume $u_k(a_i)$ is the ranking order of alternative a_i and $U(a_i)$ is the group ranking order based on the obtained values using (12) and (13), respectively. Also, let:

$$d_i^k = \sum_{j=1}^{i} I\left[u_k(a_i) \leq i < U(a_j)\right]; \quad \forall k \tag{14}$$

where $I[E] = 1$ if E is true and $I[E] = 0$ if E is false. Finally, let

$$D_i^k = \sum\nolimits_{j=1}^{i} I\left[n + 1 - u_k(a_i) > U(a_j)\right]; \quad \forall k. \tag{15}$$

Then the greatest deviation between $u_k(a_i)$ and $U(a_i)$ is calculated as follows:

$$R_g^k = \frac{\max\limits_i D_i^k - \max\limits_i d_i^k}{\frac{m}{2}}; \quad \forall k. \tag{16}$$

Note that for each k, $-1 \leq R_g^k \leq 1$, R_g^k close to 1 means that there is little difference between the personal ranking order and group sorting result. Also, R_g^k close to -1 means that the group sorting result is almost the reverse of the personal order, therefore the personal disagreement is maximized.

Then the group's satisfaction index is:

$$R_G = \sum_{k=1}^{q} \lambda_k R_g^k. \tag{17}$$

Remark 3.2. If $R_G \geq \theta$, where $\theta(-1 \leq \theta \leq 1)$ is the threshold of the acceptable group satisfaction level, then R_G is an acceptable group satisfaction degree and if $R_G < \theta$, then R_G is an unacceptable group satisfaction degree, such that θ is usually determined by the DMs.

This satisfaction degree which is very useful in the interactive procedures is required to reach a consensus assignments. The DMs rank the alternatives according to the net outranking flow indices. Therefore, if the majority of preferences of the net outranking flow matrix $F_k(a_i, a_l)$ is close and a few of DMs' preferences are in disagreement, it is reasonable to change only these particular preferences.

Step 10 (the weighted deviation degree). Let $F_k(a_i, a_l) = f_{il(k)}$ and $\prod_{il} = \{f_{il(1)}, f_{il(2)}, \ldots, f_{il(q)}\}$ be the set of DMs' net outranking flow values, where $f_{il(k)}$ is the net outranking flow value of DM d_k with respect to the pair of alternatives $(a_i, a_l); (a_i, a_l \in A, i \neq l)$. Then, the weighted deviation degree ξ_{il} is calculated as follows:

$$\xi_{il} = \sqrt{\sum_{k=1}^{q} \lambda_k (f_{il(k)} - \overline{f}_{il})^2} \tag{18}$$

where $\overline{f}_{il} = \sum_{k=1}^{q} \lambda_k f_{il(k)}$ is the weighted mean of the set $\prod_{il}$.

The automatic adjustment of the group satisfaction consists of two processes (see, [19]):

- Choose the maximum element $\xi_{i^*l^*}$ from the matrix $\xi = [\xi_{il}]_{m \times m}$, then find the element $f_{i^*l^*(k^*)}$ using $\lambda_{k^*}(f_{i^*l^*(k^*)} - \overline{f}_{i^*l^*})^2 = \max\limits_{1 \leq k \leq q} \lambda_k (f_{i^*l^*(k)} - \overline{f}_{i^*l^*})^2$.

- Replace $f_{i^*l^*(k^*)}$ with a numerical value $\acute{f}_{i^*l^*(k^*)} = \overline{f}_{il}$, and other elements do not change i.e., let

$$f_{il(k)}^{h+1} = \begin{cases} \overline{f}_{il}^{h}, & \text{if } i = i^*, l = l^*, and \;\; k = k^* \\ f_{il(k)}^{h}, & \text{otherwise.} \end{cases} \tag{19}$$

Let $\prod_{il}^0 = \{f_{il(1)}^0, f_{il(2)}^0, \ldots, f_{il(q)}^0\}$ be an original set of the DMs' net outranking flow values and ξ_{il}^0 be its weighted deviation degree, and $\prod_{il}^1 = \{f_{il(1)}^1, f_{il(2)}^1, \ldots, f_{il(q)}^1\}$ be the corresponding first adjusted set and ξ_{il}^1 be its corresponding weighted deviation degree, obviously then, ξ_{il}^1 is smaller than ξ_{il}^0. Moreover, if one continue to change some elements of the set in this method, the weighted deviation degree approaches zero. Obviously, the recent automatic adjustment strategy is convergent. In fact, the following theorem is stated toward convergence.

Theorem 3.1. *Let* $\prod_{il}^{h} = \{f_{il(1)}^{h}, f_{il(2)}^{h}, \dots, f_{il(q)}^{h}\}; (i, l = 1, 2, ...m, i \neq l)$ *be a set of the DMs' net outranking flow values after* h *times, and the above adjustment* ξ_{il}^{h} *be the corresponding weighted deviation degree, then*

$$\lim_{h \longrightarrow \infty} \xi_{il}^{h} = 0 \quad (20)$$

Proof of Theorem 3.1. See [19].

4. Numerical example

Due to globalization, supply chains are getting more and more risky than before. Supplier selection is one of the main concern in the context of supply chain network. In essence, supplier evaluation and selection can be taken into account as an MAGDM problem. In order to elucidate the details of the proposed method in this paper and demonstrate the method in practice, in the following a supplier evaluation problem in a high-tech company is considered.

According to the numerical example that is mentioned by Shen et al. [19], consider a high-technology company which manufactures electronic products intends to rank its five material suppliers $(a_1, a_2, a_3, a_4, a_5)$. For this purpose, the company manager decided to use the GDM to evaluate these five suppliers. Three DMs d_1, d_2 and d_3 are established; an engineering expert, a financial expert, and a quality control expert. The weight vector of experts is considered as $\lambda = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})^T$. To evaluate the five suppliers, company manager decided on four attributes; cost control (c_1), performance (c_2), technique (c_3) and service (c_4).

In order to determine the decision evaluation, the experts may not have a precise or sufficient level of knowledge of problem, and so they maybe unable to be confident about their judgments. To deal with this uncertainty, the experts' preference values are expressed using IFNs.

Three experts (DMs) give their own opinions about the performance of $a_i; (i = 1, \dots, 5)$ with respect to each attribute $c_j; (j = 1, \dots, 4)$, respectively. Their decision matrices with the intuitionistic fuzzy evaluation information are R_1, R_2 and R_3, as follows:

$$R_1 = \begin{pmatrix} (0.45, 0.55, 0.00) & (0.40, 0.55, 0.05) & (0.70, 0.30, 0.00) & (0.70, 0.20, 0.10) \\ (0.50, 0.40, 0.10) & (0.70, 0.30, 0.00) & (0.60, 0.40, 0.00) & (0.60, 0.35, 0.05) \\ (0.70, 0.25, 0.05) & (0.40, 0.55, 0.05) & (0.40, 0.55, 0.05) & (0.90, 0.10, 0.00) \\ (0.40, 0.55, 0.05) & (0.50, 0.50, 0.00) & (0.70, 0.25, 0.05) & (0.60, 0.40, 0.00) \\ (0.40, 0.60, 0.00) & (0.40, 0.50, 0.10) & (0.50, 0.50, 0.00) & (0.50, 0.45, 0.05) \end{pmatrix}$$

$$R_2 = \begin{pmatrix} (0.50, 0.40, 0.10) & (0.40, 0.50, 0.10) & (0.80, 0.20, 0.00) & (0.60, 0.35, 0.05) \\ (0.50, 0.50, 0.00) & (0.60, 0.40, 0.00) & (0.70, 0.30, 0.00) & (0.40, 0.25, 0.05) \\ (0.70, 0.30, 0.00) & (0.40, 0.50, 0.10) & (0.40, 0.60, 0.00) & (0.70, 0.30, 0.00) \\ (0.30, 0.60, 0.10) & (0.60, 0.35, 0.05) & (0.70, 0.25, 0.05) & (0.80, 0.10, 0.10) \\ (0.60, 0.35, 0.05) & (0.50, 0.50, 0.00) & (0.60, 0.30, 0.10) & (0.70, 0.20, 0.10) \end{pmatrix}$$

$$R_3 = \begin{pmatrix} (0.60, 0.30, 0.10) & (0.40, 0.45, 0.15) & (0.90, 0.10, 0.00) & (0.70, 0.25, 0.05) \\ (0.40, 0.55, 0.05) & (0.60, 0.35, 0.05) & (0.65, 0.25, 0.10) & (0.80, 0.15, 0.05) \\ (0.65, 0.25, 0.10) & (0.30, 0.65, 0.05) & (0.50, 0.45, 0.05) & (0.50, 0.20, 0.30) \\ (0.40, 0.50, 0.10) & (0.60, 0.40, 0.00) & (0.40, 0.60, 0.00) & (0.80, 0.15, 0.05) \\ (0.40, 0.60, 0.00) & (0.40, 0.55, 0.05) & (0.60, 0.40, 0.00) & (0.40, 0.60, 0.00) \end{pmatrix}$$

The proposed thresholds by three experts are stated in Table 1. Also, the company manager determine the acceptable group satisfaction degree $\theta = 0.6$.

Table 1: Thresholds given by three experts for attributes

DMs	q_k	p_k	ν_k
d_1	0.05	0.15	0.30
d_2	0.10	0.20	0.35
d_3	0.03	0.13	0.28

Firstly, according to the models (5) and (2), the attributes weight vector $W_k; (k = 1, 2, 3)$ for four atributes is obtained, this weights are stated in Table 2.

Table 2: Weights of attributes

k	w_{1k}	w_{2k}	w_{3k}	w_{4k}
1	0.31	0.24	0.24	0.2
2	0.27	0.29	0.22	0.22
3	0.27	0.31	0.2	0.22

Then, concordance matrices and discordance matrices are calculated using equations (6) until (9), as follows:

$$\widetilde{C}_1 = \begin{pmatrix} - & 22.38698 & 10.84675 & 22.39305 & 30.23373 \\ 23.54024 & - & 15.84675 & 34.24289 & 46.08047 \\ 27.54024 & 39.23373 & - & 39.24444 & 53.08047 \\ 15.69349 & 22.38698 & 10.84675 & - & 30.23373 \\ 14.92006 & 21.42019 & 10.26667 & 21.42596 & - \end{pmatrix}$$

$$\widetilde{C}_2 = \begin{pmatrix} - & 55.46885 & 43.02572 & 11.11707 & 38.26373 \\ 47.29283 & - & 36.62566 & 9.575696 & 32.5757 \\ 67.01011 & 67.01011 & - & 13.29286 & 46.29294 \\ 18.71713 & 18.71713 & 14.45427 & - & 12.85857 \\ 65.32744 & 65.32744 & 50.74287 & 12.97564 & - \end{pmatrix}$$

$$\widetilde{C}_3 = \begin{pmatrix} - & 21.96581 & 39.99846 & 21.96581 & 49.62013 \\ 19.4719 & - & 29.8636 & 16.84187 & 37.31377 \\ 25.93184 & 22.24334 & - & 22.24334 & 50.17518 \\ 19.93184 & 17.24334 & 30.55351 & - & 38.17518 \\ 17.28098 & 14.92847 & 26.57722 & 14.92847 & - \end{pmatrix}$$

$$\widetilde{D}_1 = \begin{pmatrix} - & 65.40884 & 78.14474 & 50.96463 & 45.81636 \\ 44.05049 & - & 48.39999 & 31.69493 & 28.50063 \\ 17.31465 & 15.97031 & - & 12.43004 & 11.16959 \\ 72.05098 & 66.47623 & 79.42794 & - & 46.56289 \\ 72.05098 & 66.47623 & 79.42794 & 51.79541 & - \end{pmatrix}$$

$$\widetilde{D}_2 = \begin{pmatrix} - & 40.81855 & 72.61192 & 78.26022 & 76.24667 \\ 60.90851 & - & 79.21295 & 85.39545 & 83.19147 \\ 37.93031 & 27.73641 & - & 53.07555 & 51.71267 \\ 63.64196 & 46.35283 & 82.80566 & - & 86.97128 \\ 52.41364 & 38.28975 & 68.09317 & 73.38807 & - \end{pmatrix}$$

$$\widetilde{D}_3 = \begin{pmatrix} - & 29.62621 & 18.87567 & 29.62621 & 6.495606 \\ 56.64008 & - & 38.23261 & 61.19735 & 13.48327 \\ 26.53669 & 28.58238 & - & 28.58238 & 6.266492 \\ 55.55732 & 60.02837 & 37.49784 & - & 13.22478 \\ 56.96951 & 61.55227 & 38.45858 & 61.55227 & - \end{pmatrix}$$

In the following, the net outranking flow matrices are constructed using equation (10) and (11).

$$F_1^0 = \begin{pmatrix} - & 21.35835 & 50.6045 & -21.0863 & -26.2346 \\ -21.3583 & - & 9.16626 & -32.2333 & -20.3958 \\ -50.6045 & -9.16626 & - & -40.1835 & -26.3475 \\ 21.08634 & 32.23334 & 40.1835 & - & -5.23252 \\ 26.23462 & 20.39576 & 26.34747 & 5.23252 & - \end{pmatrix}$$

$$F_2^0 = \begin{pmatrix} - & -5.43966 & 5.601816 & 14.61826 & 10.91923 \\ 5.439662 & - & 12.20285 & 39.04262 & 17.86403 \\ -5.60182 & -12.2028 & - & -29.7301 & -16.3805 \\ -14.6183 & -39.0426 & 29.73012 & - & 13.58321 \\ -10.9192 & -17.864 & 16.38049 & -13.5832 & - \end{pmatrix}$$

$$F_3^0 = \begin{pmatrix} - & -27.0139 & 13.46178 & -25.9311 & -7.34938 \\ 27.01387 & - & 9.650228 & 1.168981 & -24.2385 \\ -13.4618 & -9.65023 & - & -8.91546 & 11.7166 \\ 25.93111 & -1.16898 & 8.915461 & - & -23.3771 \\ 7.349381 & 24.2385 & -11.7166 & 23.37708 & - \end{pmatrix}$$

Next, the net outranking flow index and the group net outranking flow index values are calculated using equation (12) and (13) which are abstracted in Table 3.

Table 3: Net outranking flow index values

	Φ_1^0	Φ_2^0	Φ_3^0	Φ_G^0
a_1	24.64188	25.69964	- 46.8326	1.1696
a_2	- 64.8212	74.54915	13.59458	7.7742
a_3	- 126.302	- 63.9153	- 20.3109	- 70.176
a_4	88.27067	- 10.3475	10.30051	29.4079
a_5	78.21038	- 25.986	43.24836	31.8243

Table 4 represents the result of the first individual and gruops' original preorders on the alternatives based on their individual net outranking flow index and the group net outranking flow index values.

Using equations (14) until (17), personal and group's original satisfaction index values are computed and abstracted in Table 5.

According to Remark 3.2, due to the relation between acceptable group satisfaction level θ and the value of R_G^0 in Table 5, i.e. $R_G^0 = 0.53 < 0.6 = \theta$, the

Table 4: The first individual and group's adjusted preorders

u_1^0	u_2^0	u_3^0	u_G^0
3	2	5	4
4	1	2	3
5	5	4	5
1	3	3	2
2	4	1	1

Table 5: The first personal and group's satisfaction index values

R_1^0	R_2^0	R_3^0	R_G^0
0.8	0	0.8	0.53

next stage is going to perform. In the second stage after calculating the value of the individual and group's adjusted preorders, the new result is $R_G^1 = 0.8$ (see Table 6 and Table 7).

Table 6: The second individual and group's adjusted preorders

u_1^1	u_2^1	u_3^1	u_G^1
3	2	5	4
4	3	2	3
5	5	4	5
1	1	3	2
2	4	1	1

Since $R_G^1 = 0.8 > 0.6 = \theta$, then the desired level of group consensus is achieved, and the final group ranking result of the alternatives are given as follows:
$a_5 \succ a_4 \succ a_2 \succ a_1 \succ a_3$.

The comparison between proposed method and ranking alternatives based on Shen et al. [19] is shown in Table 8.

The ranking order obtained by Shen et al. [19] methods are not the same as results obtained from proposed method. The main reason is that in Shen et al. [19] methods, weight of attributes is based on DMs' ideas and not through specific methods, which may lead to unreasonable decision results. By contrast, the proposed method determines weight of attributes by a linear model, which can avoid the untrue randomness and improve convincingness of decision outcomes.

Table 7: The first personal and group's satisfaction index values

R_1^1	R_2^1	R_3^1	R_G^1
0.8	0.8	0.8	0.8

Table 8: Compare the rank of proposed method with Shen et al. method

Alternatives	Proposed method	Shen et al. method
a_1	4	1
a_2	3	2
a_3	5	3
a_4	2	4
a_5	1	5

5. Conclusion

In the present paper, it has been tried to determine and evaluate importance of each attribute by a linear programming model in the MAGDM problems. Also, the application of the ELECTRE III with intuitionistic fuzzy information has been proposed to solve the MAGDM models. An automatic adjustment of group satisfaction has been utilized to overcome the convergence problems that detected in the other consensus approaches. Therefore, because the weight of attribute has been achieved via the proposed model, and it is not based on DMs' opinions, the accuracy of alternatives' ranking should be increase and the decision making errors decrease. Whiles in traditional outranking sorting methods, mainly the focuse is on DMs' opinions for determining the weight of attributes. For practical application, the proposed outranking method has been applied to solve a company supplier evaluation problem which is an important component in the supply chain management. This research could be surveyed by the other types of uncertain environments for future use. Another orientation could be the determination of the DMs' importance in MAGDM problems.

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References

[1] K. T. Atanassov, *Intuitionistic fuzzy sets*, Fuzzy Sets and Systems, 20 (1986), 87-96.

[2] K. T. Atanassov, *Intuitionistic fuzzy logics*, Springer, Cham, 2017.

[3] K. Atanassov, E. Szmidt, J. Kacprzyk, *On intuitionistic fuzzy pairs*, Notes on Intuitionistic Fuzzy Sets, 19 (2013), 1-13.

[4] R. Benayoun, B. Roy, N. Sussman, *Manual de reference du programme electre, note de synthese et formation*, No. 25. Direction Scientifique SEMA, Paris, France, 1966.

[5] T. Y. Chen, *An ELECTRE-based outranking method for multiple criteria group decision making using interval type-2 fuzzy sets,* Information Sciences, 263 (2014), 1-21.

[6] S. M. Chen, S. H. Cheng, C. H. Chiou, *Fuzzy multi attribute group decision making based on intuitionistic fuzzy sets and evidential reasoning methodology,* Inform. Fusion., 27 (2016), 215-227.

[7] S. M. Chen, J. M. Tan, *Handling multicriteria fuzzy decision making problems based on vague set theory*, Fuzzy Sets and Systems, 67 (1994), 163-172.

[8] R. Duan, Q. Han, Z. Wang, *Multi attribute group decision making models under intuitionistic fuzzy environment*, Applied Mechanics and Materials, 263-266 (2012), 3225-3229.

[9] A. A. Goshtasby, *Similarity and dissimilarity measures,* In: Image Registration, Advances in Computer Vision and Pattern Recognition, Springer, London, 2012.

[10] S. S. Hashemi, S. H. Razavi Hajiagha, E. K. Zavadskas, H. Amoozad Mahdiraji, *Multi criteria group decision making with ELECTRE III method based on interval-valued intuitionistic fuzzy information*, Applied Mathematical Modelling, 40 (2016), 1554-1564.

[11] D. H. Hong, C. H. Choi, *Multicriteria fuzzy decision-making problems based on vague set theory*, Fuzzy Sets and Systems, 114 (2000), 103-113.

[12] Y. Ju, A. Wang, *Extension of VIKOR method for multi-criteria group decision making problem with linguistic information*, Appl. Math. Model., 37 (2013), 3112-3125.

[13] Y. Ju, A. Wang, *Projection method for multiple criteria group decision making with incomplete weight information in linguistic setting*, Applied Mathematical Modelling, 37 (2013), 9031-9040.

[14] X. Li, X. Chen, *Multi-criteria group decision making based on trapezoidal intuitionistic fuzzy information*, Applied Soft Computing, 30 (2015), 454-461.

[15] J. J. Peng, J. Q. Wang, J. Wang, L. J. Yang, X. H. Chen, *An extension of ELECTRE to multi- criteria decision-making problems with multi-hesitant fuzzy sets*, Information Sciences, 307 (2015), 113-126.

[16] M. Rogers and M. Bruen, *Choosing realistic values of indifference, preference and veto thresholds for use with environmental criteria within ELECTRE*, European Journal of Operation Research, 107 (1998), 542-551.

[17] B. Roy, *The outranking approach and the foundations of ELECTRE methods*, Theory and Decision, 31 (1991), 49-73.

[18] P. Sevastjanov, L. Dymova, *Generalised operations on hesitant fuzzy values in the framework of Dempster-Shafer theory*, Information Sciences, 311 (2015), 39-58.

[19] F. Shen, J. Xu, Z. Xu, *An automatic ranking approach for multi-criteria group decision making under intuitionistic fuzzy environment*, Fuzzy Optimization and Decision Making, 14 (2015), 311-334.

[20] F. Shen, J. Xu, Z. Xu, *An outranking sorting method for multi-criteria group decision making using intuitionistic fuzzy sets*, Information Sciences, 334-335 (2016), 338-353.

[21] E. Szmidt, J. Kacprzyk, *Distances between intuitionistic fuzzy sets*, Fuzzy Sets and Systems, 114 (2000), 505-518.

[22] A. Van Delft , P. Nijkamp, *A Multi-Objective Decision Making Model For Regional Development, Environ- mental Quality Control and Industrial Lead Use*, Papers in Regional Association, 36 (1976), 35-57.

[23] X. Wang, E.Triantaphylloy, *Ranking irregularities when evaluating alternatives by using some ELECTRE methods*, Omega, 36 (2006), 45-63.

[24] J. Q. Wang, J. Wang, Q. H. Chen, H. Y. Zhang, X. H. Chen, *An outranking approach for multi-criteria decision-making with hesitant fuzzy linguistic term sets*, Information Sciences, 280 (2014), 338-351.

[25] Z. Wang, J. Xu, W. Wang, *Intuitionistic fuzzy multiple attribute group decision making based on projection method*, Control and Decision Conference, Chineese, 2009.

[26] Z. Wu, J. Ahmad, J. Xu, *A group decision making framework based on fuzzy VIKOR approach for machine tool selection with linguistic information*, Applied Soft Computing, 42 (2016), 314-324.

[27] S. Xian, W. Sun, S. Xu, Y. Gao, *Fuzzy linguistic induced OWA Minkowski distance operator and its application in group decision making*, Pattern Anal Applic, 19 (2016), 325-335.

[28] Z. S. Xu, *Multi-person multi-attribute decision making models under intuitionistic fuzzy environment,* Fuzzy. Optim. Decis. Ma., 6 (2007), 221-236.

[29] Z. S. Xu, *A deviation-based approach to intuitionistic fuzzy multiple attribute group decision making*, Group Decision and Negotiation, 19 (2010), 57-76.

[30] Z. S. Xu, *Intuitionistic fuzzy aggregation and clustering*, Heidelberg: Springer, Berlin, 2013.

[31] Z. S. Xu, R. R. Yager, *Some geometric aggregation operators based on intuitionistic fuzzy sets*, International Journal of General Systems, 35 (2006), 417-433.

[32] W. E. Yang , J. Q. Wang, X. F. Wang, *An outranking method for multicriteria decision making with duplex linguistic information*, Fuzzy Sets and Systems, 198 (2012), 20-33.

[33] Z. L. Yue, *Approach to group decision making based on determining the weights of experts by using projection method*, Applied Mathematical Modelling, 36 (2012), 2900-2910.

[34] Z. L. Yue, *Developing a straightforward approach for group decision making based on determining weights of decision makers*, Applied Mathematical Modelling, 36 (2012), 4106-4117.

[35] Z. Yue, *TOPSIS-based group decision-making methodology in intuitionistic fuzzy setting*, Information Sciences, 227 (2014), 141-153.

[36] L.A. Zadeh, *Fuzzy sets*, Information and Control, 8 (1965), 338-353.

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