Solving the fuzzy assignment problems via utilizing branch and bound algorithm-practically treated

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Abstract. In this paper described of assignment cost for the numbers that are fuzzy. Here, the assignment problem, which is fuzzy or invalid, is turned into crisp values throughout robust ranking technique, and by using branch and bound algorithm we gated the optimal solution and at a less cost.

An applied example is taken from Source No. (9) to solving the assignment problem for the Cotton Industries Company in Iraq and find the optimal assignment and total cost.

Keywords: fuzzy assignment problem, branch and bound algorithm, assignment problem, robust ranking technique.

Introduction

In 1965, Zadeh presented the fuzzy set for the sake of tackling information-uncertainty. The Problem of Assignment is a specific kind of Programming Problem that is Linear. Here the crucial aim is to identify a group of jobs to identical number of individuals with cost or profit that is minimum or maximum respectively ([3]).

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Made use of in manufacturing and service systems, the Assignment Problem (AP) is to obtain the assignment schedule of the optimal jobs-total costs in which n jobs are specialized for n workers (or machines), and everyone gets a job only. The AP is a specific kind of 0–1 programming which is linear. To add more, the jobs and workers of m and n respectively can also be found, m 6 n. For every worker, there must be only one jib assigned. To add more,, the related cost C_{ij} is provided in accordance with the allocation that for the j worker the I job is specialized. Finding out such an assignment is the main problem. There will be many extensions because, in the literature, the assignment problem has been proposed. To start with, in some circumstances, the costs that are in literature are not crisp. In such a thing, some system-parameters are distinguished by information which might be uncertain such as the variables that are fuzzy and random. Thus, the AP is tackled in uncertain environments ([7]).

Literature review

Recently, a variety of papers have used the set, in other words, the technique of branch and bound , to characterize optimal solution with data-uncertainty. Liu, L., & Li, Y. (2006) presented the fuzzy quadratic assignment problem with penalty is formulated as expected value model, chance-constrained programming and dependent-opportunity programming relying on the numerous characteristics of decision, and the crisp counterparts are provided.

Strouvalis, A. M., Heckl, I., Friedler, F., & Kokossis, A. C. (2002) they proposed a methodology for implementing logic and engineering knowledge within a Branch-and-Bound algorithm and with an aim for accelerating convergence. The utility networks- assignment problem is addressed by the development emphasizing on the most favorable allocation of units for the problems of maintenance. Hahn, P., Hightower, W., Johnson, T., Guignard-Spielberg, M., & Roucairol, C. (2001), presented a new strategy for selecting nodes in a branch and bound algorithm for solving exactly the Quadratic Problem of Assignment (QAP). If it is learned, then it will be developed. This means that there is a failure on the larged-sized problems on the part of the order strategies.

The ranking method of Yager is utilized for numbers-ranking that are fuzzy, after the transformation of the problem of assignment into a crisp one via the use of linguistic variables, the (AP) has been worked out by the technique of branch and bound, presented by Gotmare, D., & Khot, P. G. (2016).

Lin, C. J., & Wen, U. P. (2004) they proposed an effective algorithm dependent upon the labeling technique for working out the programming case that is linear fractional. The algorithm starts with primary feasibility and progresses to get double feasibility while keeping complementary slackness till the primary ultimate answer is obtained. And branch-and-bound-with-underestimates, has been applied and there is the definition of the two functions which are underestimate. By Chen, G. H., & Yur, J. S. (1990, January) Their effectiveness has been shown via experiment. This is done by making a comparison between the proposed algorithm and the Wang and Tsai's algorithm and an algorithm with $h(x) = O$. And also in our algorithm for calculating the lower bounds on solutions to the QAP. Resulting from some recently developed improvements, By Hahn, P., Grant, T., & Hall, N. (1998) the DP provides bounds that are lower and as tight as any that could be important in a branch-and-bound algorithm. These are of cheap production relatively, particularly on greater problems. The elements of the profit matrix of the problem of assignment are regarded as the triangular fuzzy numbers by Muruganandam, S., & Hema, K. (2017), and ultimate solution can be obtained through the use of branch and bound method without changing the fuzzy numbers into crisp ones. To add more, the Lone, M. A., Mir, S. A., Ismail, Y., & Majid, R. (2017) tackle the assignment problem which is Intuitionistic and fuzzy whose cost has been considered as an Intuitionistic triangular fuzzy numbers. The assignment costs are turned into crisp values via defuzzifying, and the optimum solution is got through the use of Branch and Bound method. And the Vinoliah, E. M., & Ganesan, K. (2018) have suggested a new method for getting an ultimate solution, that is fuzzy, for a fuzzy problem of assignment with the numbers that are octagonal and fuzzy. By checking its optimality via the best method of computations that is minimum, the problem is solved and considered regarded. Too reviews the Srinivas, B., & Ganesan, G. (2015) a new procedure for ranking that could be found in and is utilized for comparing the intuitionistic fuzzy numbers so that an Intuitionistic Fuzzy Branch Bound method could be used for solving the issue of assignment that is intuitionistic and fuzzy. A branch and bound algorithm is developed by Ross, G. T., & Soland, R. M. (1975) that works out the problem of assignment generalized throughout working out a binary-group of the problems of knapsack for the sake of identifying the bounds.

Definition of the problem of assignment

The Assignment problem is an NP one of hard optimization, in which n workers are assigned to n jobs, with minimum cost. The quadratic model can directly be solved as a regular model of transportation. The model of assignment is actually a certain issue of the transportation-model where the sources are represented by the workers, and the destinations are by the jobs, the amount of supply at each source and the amount of demand at each destination actually equal 1 as in table (1). The "transportation-cost" of worker i to j is C_{ij} . In effect, nevertheless, the fact that all the supply and demand amounts equal 1. The problem of assignment where n workers are assigned to n jobs, or the problem contains assignment of n facilities (or warehouses) to n locations (or sites) with the objective of reducing the costs of transportation which are associated with the material-flow between facilities and distances between locations. It did not assign many facilities to the same location or assign many locations to the same

Table 1: The general form of assignment problems

C_{11}	C_{12}	...	C_{1n}	1
C_{21}	C_{22}	...	C_{2n}	1
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
C_{m1}	C_{m2}	...	C_{mn}	1
1	1	...	1	

facility. The evaluation function of an assignment problem is given as ([1])

$$Minimize(z) = \sum_{i=1}^{n} C_{ij}X_{ij}, \tag{1}$$

$$\sum_{i=1}^{n} X_{ij} = 1,\ i = 1, 2, \ldots, n \tag{2}$$

$$\sum_{j=1}^{n} X_{ij} = 1,\ j = 1, 2, \ldots, n\ X_{ij} = 0or1 \tag{3}$$

$$X_{ij} = \begin{cases} 1, & \text{if worker } i \text{ is assigned to job } j, \\ 0, & \text{otherwise.} \end{cases}$$

Preliminaries

It has been stated earlier that the Fuzzy set was firstly presented by Zadeh in 1965. It was presented mathematically as a way for reflecting ambiguity or inaccuracy in daily life.

Definition. The $\tilde{A}$ set that is fuzzy and may be defined depending on the R, which represents real numbers- universal set, is defined to be a number that is fuzzy in case that its job of membership is having the characteristics stated below:

1. $\mu_{\tilde{A}} : R[0, 1]$ is permanent.
2. $\mu_{\tilde{A}}(x) = 0$, for all $x \in (-\infty, a] \cup [d, \infty)$.
3. $\mu_{\tilde{A}}(x)$ is increasing strictly on [a, b] and strictly decreasing on [c, d] is decreasing strictly.
4. $\mu_{\tilde{A}}(x) = 1$, for all $x \in [b, c]$, where $a < b < c < d$.

Definition. The $\tilde{A}$, which represents a fuzzy number, equals (a, b, c) is a triangular number that is fuzzy in case that its function of membership is provided by ([3])

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1}, & \text{if } a_1 \leqslant x \leqslant a_2 \\ \frac{a_3-x}{a_3-a_2}, & \text{if } a_2 \leqslant x \leqslant a \\ 0, & \text{otherwise.} \end{cases} \tag{4}$$

Robust ranking technique

Finding the measures of performance in terms of crisp values, it is important to clarify them within crisp ones by a ranking method of a fuzzy number. The technique of Robust ranking that satisfies and ensures compensation, linearity, and additional properties. What is more is providing the results that are consistent with the intuition of human. Give a convex fuzzy number ã, the Robust Ranking Index is defined by

$$R(\tilde{a}) = \int_0^1 (0.5)(a_\alpha^L a_\alpha^U) \tag{5}$$

$(a_\alpha^L a_\alpha^U) = \{(b-a)\alpha + a + c - (c-b)\alpha\}$, here $(a_\alpha^L a_\alpha^U)$ is the α–*level* cut of the fuzzy number a. This technique is utilized here for the fuzzy numbers- ranking. The Robust ranking index R(ã) Robust ranking index indicates the fuzzy number typical value ã. It is satisfiable for linearity and additional property ([6]).

Branch and bound algorithm

There is an m level number in the branching tree in Branch and bound technique applied to assignment problem, but is has been found zero for root node.

$$K_\gamma = [C_ij]_{i,j\in T} + \sum_{i\in x}^{n} C_{ij}(\sum_{j\in y}^{n} lessC_{ij}) \tag{6}$$

such that:
K_γ; be the assignment lower bound which is partial, A up to Q_γ^m .
γ; be the assignment made in the present branching tree-node.
T; be a cells-set that is assigned up to the node from Q_γ^m the root node.
C_{ij}; is the cell-entry of the cost matrix with respect to the i row and j column.
X; be the rows-set that is not omitted up to the node Q_γ^m from the root node which is found in branching tree.
γ; be the columns-set which is not omitted up to the node Q_γ^m from the node of the root in the branching tree.
Q_γ^m; be an m level- assignment of a branching tree.

Rules of branching

- The row that is assigned as m of the problem of assignment at m level, would be designated with the most suitable column of that issue.

- If a tie exists on the lower bound, and the final node at the lower level is to be apt for additional branching.
- Stopping rule: The optimality is reached in case that the minimal lower bound is to be at a terminal node at the (n-1) level. Then the optimum solution will be shaped or formed by the assignments existed in the passage from the node of the root to that one in accordance with the row-column combination-pair which is missing ([3]).

Application

General of Company for Cotton Industries Iraq.

It is considered one of the large companies with diversified production, established in 1945, and includes four factories. A sample of production machines was selected, as each machine produces a type of cotton products, which are as follows;

- Cowboy pants production machine.
- Business suits production machine.
- Gauze and medical bandage production machine.
- Dress for men and women production machine.

The goal is to allocate four workers on the machines, and the table below shows the fuzzy times for each worker on each production machine. ([11]).

Table 2: The fuzzy number for each worker machines

	Cowboy Pants	Business Suits	Gauze and Medical Bandage	Dress for Men and Women
Worker (1)	146, 230, 424	146, 240, 434	166, 260, 454	140, 195, 489
Worker (2)	163, 263, 457	160, 200, 394	166, 260, 454	160, 210, 404
Worker (3)	146, 300, 494	146, 195, 380	140, 195, 380	158, 252, 446
Worker (4)	160, 200, 394	130, 197, 391	160, 210, 404	140, 196, 310

Using equation No. 5, convert the fuzzy numbers in Table (2) to Crisp number:
$R(\tilde{C}_{11}) = R(146, 230, 424) = \int_0^1 (0.5)\{(230-146)\alpha+146+424-(424-230)\alpha\}d\alpha = 257.5$
$R(\tilde{C}_{12}) = R(146, 240, 434) = \int_0^1 (0.5)\{(240-146)\alpha+146+434-(434-240)\alpha\}d\alpha = 265$
$R(\tilde{C}_{13}) = R(166, 260, 454) = \int_0^1 (0.5)\{(260-166)\alpha+166+454-(454-260)\alpha\}d\alpha = 285$
$R(\tilde{C}_{14}) = R(140, 195, 489) = \int_0^1 (0.5)\{(195-140)\alpha+140+489-(489-195)\alpha\}d\alpha = 254.8$
$R(\tilde{C}_{21}) = 286.5$, $R(\tilde{C}_{22}) = 238.5$, $R(\tilde{C}_{23}) = 285$, $R(\tilde{C}_{24}) = 246$
$R(\tilde{C}_{31}) = 310$, $R(\tilde{C}_{32}) = 229$, $R(\tilde{C}_{33}) = 227.5$, $R(\tilde{C}_{34}) = 277$
$R(\tilde{C}_{41}) = 238.5$, $R(\tilde{C}_{42}) = 228.8$, $R(\tilde{C}_{43}) = 246$, $R(\tilde{C}_{44}) = 210.5$
These values are replaced with their similar $R(\tilde{C}_{ij})$ and use the branch and bound method in to solve the problem of assignment.

Table 3: The crisp number for each worker machines

	Cowboy Pants	Business Suits	Gauze and Medical Bandage	Dress for Men and Women
Worker (1)	257.5	265	285	254.8
Worker (2)	286.5	238.5	285	246
Worker (3)	310	229	227.5	277
Worker (4)	238.5	228.8	246	210.5

In Fig 1, the sub-problems, which are four, under the nodes of the root are shown, lower bound the problems of solution.

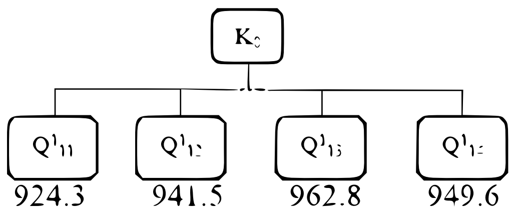

Figure 1: Further branch and bound the root nodes

Q^1_{11} is the lower bound and by applying equation (6) we get;
$\gamma = \{(1,1)\}$, $T = \{(1,1)\}$, $x = \{2,3,4\}$, $\gamma = \{2,3,4\}$
$Q^1_{11} = 257.5 + (228.8 + 227.5 + 210.5) = \mathbf{924.3}$
$Q^1_{12} = 264 + (238.5 + 227.5 + 210.5) = 941.5$
$Q^1_{13} = 285 + (238.5 + 228.8 + 210.5) = 962.8$
$Q^1_{14} = 254.8 + (238.5 + 228.8 + 227.5) = 949.6$
Further branching is performed out of the node that is terminal and possesses the least lower bound at this stage, the terminal nodes are Q^1_{11}, Q^1_{12}, Q^1_{13}, Q^1_{14}. The node Q^1_{11} has the least lower bound. Thus, in Fig(2), further branching from this node is shown:

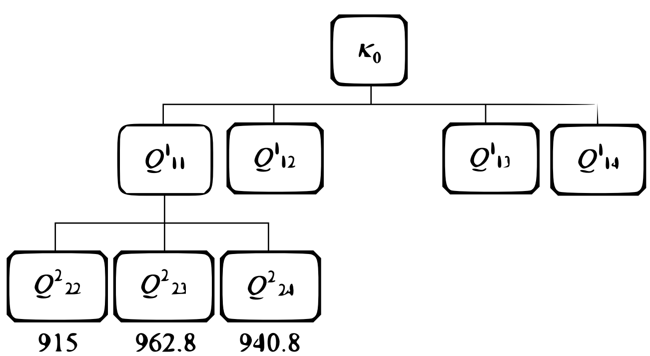

Figure 2: Further branch and bound this node

Q^2_{22} is the lower bound and by applying equation (6) we get:

$T = \{(1,1),(2,2)\}$, $x = \{3,4\}$, $\gamma = \{3,4\}$
$Q^2_{22} = 238.5 + (238.5 + 227.5 + 210.5) = \mathbf{915}$
$Q^2_{23} = 285 + (238.5 + 228.8 + 210.5) = 962.8$
$Q^2_{24} = 246 + (238.5 + 228.8 + 227.5) = 940.8$

An extra branching is performed by the final node that possesses the least lower bound at this part, the terminal nodes are Q^2_{22}, Q^2_{23}, Q^2_{24}. The node that has the minimum lower bound is Q^2_{22}. Therefore, in Fig (3) shows a further branching that is from this node:

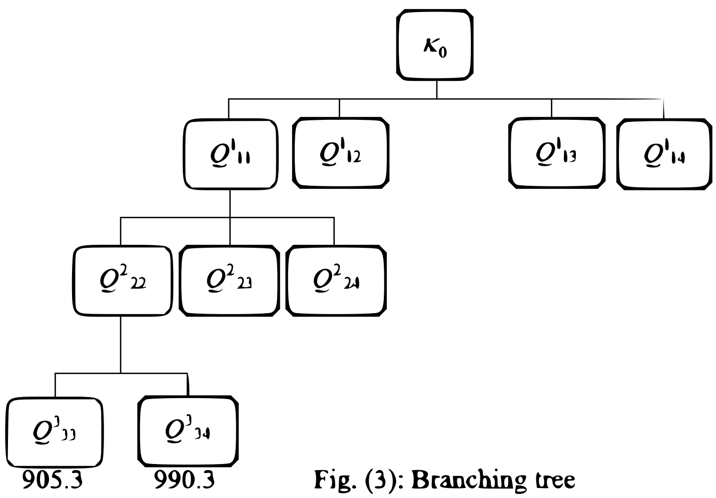

Figure 3: Branching tree

Q^3_{33} is the lower bound and by applying equation (6) we get;
$T = \{(1,1),(2,2),(3,3)\}$, $x = \{4\}$, $\gamma = \{4\}$
$Q^3_{33} = 227.5 + (238.5 + 228.8 + 210.5) = \mathbf{905.3}$
$Q^3_{34} = 277 + (238.5 + 228.8 + 246) = 990.3$
final assignment table is (1,1), (2,2), (3,3), (4,4)
Fuzzy final cost is (146, 230, 424) + (160, 200, 394) + (140, 195, 380) + (140, 196, 310). And at a cost (257.5 + 238.5 + 227.5 + 210.5 = **934**).

Conclusions

Decision making greatly relies on Assignment problem since it is of great use, e.g. problems of resource allocation, such as jobs personnel assignment, machines-tasks, etc. Not only do the managers, in real world, hope to promote each job-quality in case it is assigned to a certain individual, but also to decrease the total cost that is used. Regardless of this, each job-cost, if quality is dependent upon here, is not a deterministic number, nor the planned total cost.

Here, the cost of assignment is regarded as Intuitionistic fuzzy numbers. By demulsifying with the preciseness function, the assignment costs are converted into crisp values on the way robust ranking technique and the optimum solution is obtained by using the Branch and Bound method.

And therefore; Q_{11}^{1} be the lower bound of the first partial assignment, and the value is. 924.3 and be fuzzy assignment (146, 230, 424), And also Q_{22}^{2} be the lower bound of the second assignment that is partial, and the value is. 915 and be fuzzy assignment (160, 200, 394), Q_{33}^{3} be the lower bound of the third assignment that is partial, and the value is. 905.3 and be fuzzy assignment (140, 195, 380), and at a total cost (934).

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