Certain efficient techniques to solve the unreported cases of 2019−nCoV epidemic model

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Abstract. One of the furthermost intimidations that the death faced after the second World War is 2019-nCoV epidemic and most crucial large-scale health disaster of this century. We devote the current work to discuss the epidemic prediction for the epidemic model created for 2019-nCoV in Wuhan, China by certain approximate analytical methods such as differential transform method and variational iteration method. Further, we recognize unreported cases in numbers and the parameters of model are due to reported case data. For the considered system demonstrating the model of coronavirus, the series solution is conventional in the structure of the differential transform method. The obtained solutions are discussed in figures which show the performance of considered model. The results show that the used schemes are definite and trouble-free to execution for the system of nonlinear ODEs. The solutions exposed that the

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both schemes are in total agreement, correct and well-organized for solving systems of nonlinear differential equations.

Keywords: mathematical models, epidemic model, corona-virus (2019-nCoV), reported and unreported cases, differential transform method, variational iteration method.

1. Introduction

The entire globe is facing tough time, since December 2019 due to 2019-nCoV, which was first identified in Wuhan, China. The detailed history of this virus, its behavior, its characteristics, how it has from animal to human and how this spreads exponentially person to person and its origin all those and what not, one can refer in the following list of works [2, 10, 12, 13, 16, 17, 18, 20, 30, 31, 32, 33, 34, 35, 36, 37, 40, 41, 42, 44, 46].

From the time when the mathematical epidemiological model was developed, there has been triggering concentration to apply its application to various biological transmittable diseases to understand the disease and its behavior. In view of the fact that these models be capable of replicate the special effects of the infection at countless levels like within-host to metapopulation. Also, these models are constructive in comprising, proposing, planning, implementing, testing theories, prevention, evaluating various detection, therapy and control programs. All the epidemic models are framed by means of ODEs may be linear or nonlinear with initial conditions. To control the disease one requires strong formation of epidemic mathematical models and effective methods to solve. Generally, the solutions will be obtained by numerical methods or exact solution methods or approximate method. But mainly these methods are rigorous in computation, need complex agent to compute. Usually, obtaining the exact solutions for such proposed epidemic models are out-of-the-way and exceptionally complex.

The differential transform method (DTM) is a numerical and analytic method in solving differential equations. The conception of DTM was first initiated by Zhou [45] in the filed of electrical circuit which showed the new path in solving both linear and nonlinear initial value and boundary value problems. This method is an iterative procedure for obtaining Taylor series solutions of differential equations and partial differential equations (see [28, 29]). The main advantage of this method is that it can be applied directly to linear and nonlinear ODEs without requiring linearization, discretization or perturbation. Another important advantage is that this method is capable of greatly reducing the size of computational work while still accurately providing the series solution with fast convergence rate.

In 2009, Arenas et al. [5] employed differential transform approach to solve seasonal diseases of epidemic model associated with non-autonomous ODEs in the sense of nonlinear and found solution in series form and compared with numerical results. In 2012, Do and Jang [14] considered the population models namely simple and Gauss type prey - predator models which involve the complex nonlinear system of ODEs and obtained solutions by the numerical and analytical methods namely variational iteration method (VIM), RK4 and DTM and proved that DTM is efficient method to solve such a nonlinear differential equations. In 2014, Akinboro et al. [3] (see also [27]) investigated the applications of DTM and VIM methods to solve SIR epidemic nonlinear model and showed that DT method has its own advantage to solve a large set of ODEs which occur in diverse area of study. In 2017, Chakraborty et al. [9] used DTM successfully to SAEIQRS epidemic model which consists the six nonlinear ODEs with constraints and compared the solutions with RK4 and proved that the solutions obtained by DTM are efficient than RK4. In 2019, Astuti et al. [6] employed DTM to solve the SEIR model which is modeled for influenza virus with disease resistance and obtained the exact solution. Recently Harir et al. [20] introduced the four compartmental model namely SEIR coronavirus epidemiological mathematical model with the help of four differential equations which are in the sense of non-linear and studied its behavior with the help of VIM and DTM, also they showed that the solutions obtained by both methods are in good agreement.

In this paper, we applied DTM for the considered system of nonlinear differential equations describing the reported and unreported cases which results our solution to be analytical with the minimum number of orders.

2. Basics of Differential Transform Method (DTM)

The differential transform of a function is defined as follows:

$$\mathcal{V}(p) = \frac{1}{p!} \left[\nu^{(p)}(t)\right]_{t=t_0}. \tag{1}$$

In equation (1), ν is the original function and $\mathcal{V}$ is the transformed function, which is called $T-$function. Differential inverse transform for $\mathcal{V}$ is defined as:

$$\nu(t) = \sum_{p=0}^{\infty} \mathcal{V}(p)\,(t-t_0)^p. \tag{2}$$

It follows from (1) and (2), we get

$$\nu(t) = \sum_{p=0}^{\infty} \frac{(t-t_0)^p}{p!} \left[\nu^{(p)}(t)\right]_{t=t_0}. \tag{3}$$

Equation (3) shows the concept of differential transform is derived from the Taylor series expansion and relative derivatives are calculated by an iterative way which is described by the transformed equations of the original functions.

Using equations (1) and (2), the following mathematical operations can be obtained. Few of the properties listed below are applied in this work. For more properties one could refer recent work [21]:

(i) If $\nu(t) = \nu_1(t) \pm \nu_2(t)$, then $\mathcal{V}(p) = \mathcal{V}_1(p) \pm \mathcal{V}_2(p)$.

(ii) If $\nu(t) = \nu_1^{(m)}(t)$, then $\mathcal{V}(p) = (p+m)!\,\mathcal{V}_1(p+m)$.

(iii) If $\nu(t) = t^m$, then

$$\mathcal{V}(p) = \delta(p-m) = \begin{cases} 1, & \text{if } p = m \\ 0, & \text{if } p \neq m, \end{cases}$$

where δ is the Kronecker delta.

(iv) If $\nu(t) = \nu_1(t)\,\nu_2(t)$, then

$$\mathcal{V}(p) = \sum_{m=0}^{p} \mathcal{V}_1(m)\,\mathcal{V}_2(p-m).$$

If $\nu(t)$ is finite series then (2) can be written

$$\nu(t) \approx \sum_{p=0}^{N} \mathcal{V}(p)\,(t-t_0)^p \tag{4}$$

with N is a convergence of natural frequency.

2.1 Applications of DTM to 2019-nCoV epidemic model

Recently, Liu et al. [36] considered a mathematical model for 2019-nCoV to identify the unreported cases essentially inspired by the works of [15, 38]. Also, Liu et al. [36] discussed the severity of this epidemic due to unreported cases and other related issues. Actually, this model was developed by Tang et al. [41], and it was modeled without unreported cases. This current model consist of four sections β_i, where $i = 1, 2, 3, 4$ namely susceptible as individuals, asymptomatically infectious, reported symptomatically infectious and unreported symptomatically infectious respectively.

$$\beta_1'(t) = -\alpha\,\beta_1(t)\,[\beta_2(t) + \beta_4(t)], \tag{5}$$

$$\beta_2'(t) = \alpha\,\beta_1(t)\,[\beta_2(t) + \beta_4(t)] - \beta\,\beta_2(t), \tag{6}$$

$$\beta_3'(t) = \lambda\,\beta_2(t) - \gamma\,\beta_3(t), \tag{7}$$

$$\beta_4'(t) = \phi\,\beta_2(t) - \gamma\,\beta_4(t). \tag{8}$$

where $'$ represents the time derivative.

Further, we apply the differential transformation techniques discussed in previous section for the considered mathematical model (5) - (8) with initial conditions $\mathcal{A}(0) = \mathcal{A}_0$, $\mathcal{B}(0) = \mathcal{B}_0$, $\mathcal{C}(0) = \mathcal{C}_0$, $\mathcal{D}(0) = \mathcal{D}_0$, we have

$$\mathcal{A}(p+1) = \frac{1}{(p+1)}\left[-\alpha\sum_{l=0}^{p}\mathcal{A}(l)\mathcal{B}(p-l) - \alpha\sum_{l=0}^{p}\mathcal{A}(l)\mathcal{D}(p-l)\right], \tag{9}$$

$$\mathcal{B}(p+1) = \frac{1}{(p+1)}\left[\alpha\sum_{l=0}^{p}\mathcal{A}(l)\mathcal{B}(p-l) + \alpha\sum_{l=0}^{p}\mathcal{A}(l)\mathcal{D}(p-l) - \beta\,\mathcal{B}(p)\right], \tag{10}$$

$$\mathcal{C}(p+1) = \frac{1}{(p+1)}\left[\lambda\,\mathcal{B}(p) - \gamma\,\mathcal{C}(p)\right], \tag{11}$$

$$\mathcal{D}(p+1) = \frac{1}{(p+1)}\left[\phi\,\mathcal{B}(p) - \gamma\,\mathcal{D}(p)\right]. \tag{12}$$

Next, in view of (2), we find the following iterative series from (9) - (12):

$$\beta_1(t) = \sum_{p=0}^{\infty}\mathcal{A}(p)\,t^p$$
$$= 11081000 - 3.847323200\,t - 0.7058240125\,t^2 - 0.09105854710\,t^3 + \cdots \tag{13}$$
$$\beta_2(t) = \sum_{p=0}^{\infty}\mathcal{B}(p)\,t^p$$
$$= 3.62 + 3.330180343\,t + 0.4679539880\,t^2 + 0.06877502387\,t^3 + \cdots \tag{14}$$
$$\beta_3(t) = \sum_{p=0}^{\infty}\mathcal{C}(p)\,t^p$$
$$= 0.4137142858\,t + 0.1607449992\,t^2 + 0.01017229482\,t^3 + \cdots \tag{15}$$
$$\beta_4(t) = \sum_{p=0}^{\infty}\mathcal{D}(p)\,t^p$$
$$= 4.13 - 0.4865714286\,t + 0.08232910695\,t^2 + 0.0005362709833\,t^3 + \cdots. \tag{16}$$

3. Basics of Variational Iteration Method (VIM)

In 1997, He [22] introduced an another semi-analytical method namely variational iteration method (VIM) (also see [23, 24, 25, 26]) to solve large class of non-linear differential equations effectively and it is observed that this method helps to get fast convergence. Following He [22] there are numerous works among those [1, 3, 8, 19, 20, 39, 43] have been considered and applied the VIM to solve the system of linear or non-linear ODEs and obtained solutions .

Now, consider the non-linear equation:

$$L\beta(t) + N\beta(t) = \gamma(t),$$

where L is a linear operator, N is nonlinear operator and $\gamma(t)$ is analytic function. Due to VIM, we take the correction functional as

$$\beta^{(n+1)}(t) = \beta^{(n)}(t) + \int_0^t \lambda(\tau)(L\beta(\tau) + N\beta(\tau) - \gamma(\tau))d\tau,$$

where λ is known as a general Lagrange multiplier and evaluated by variational theory, $\beta^{(0)}(t)$ is an initial approximation with possible unknowns and $\tilde{\beta}^{(n)}(t)$ is a restricted variation, that is $\delta\tilde{\beta}^{(n)}(t) = 0$.

3.1 Applications of VIM to 2019-nCOV epidemic model

Now, we apply the techniques of VIM to solve the equations (5) - (8) and write the correction functional as:

$$\begin{aligned} \beta_1^{(n+1)}(t) &= \beta_1^{(n)}(t) \\ &+ \int_0^t \lambda_1(\tau) \left[\frac{d\beta_1^{(n)}(\tau)}{d\tau} + \alpha\beta_1^{(n)}(\tau)\beta_2^{(n)}(\tau) + \alpha\beta_1^{(n)}(\tau)\beta_4^{(n)}(\tau)\right] d\tau, \end{aligned} \tag{17}$$

$$\begin{aligned} \beta_2^{(n+1)}(t) &= \beta_2^{(n)}(t) + \int_0^t \lambda_2(\tau) \left[\frac{d\beta_2^{(n)}(\tau)}{d\tau} - \alpha\beta_1^{(n)}(\tau)\beta_2^{(n)}(\tau)\right. \\ &\left. -\alpha\beta_1^{(n)}(\tau)\beta_4^{(n)}(\tau) + \beta\beta_2^{(n)}(\tau)\right] d\tau, \end{aligned} \tag{18}$$

$$\beta_3^{(n+1)}(t) = \beta_3^{(n)}(t) + \int_0^t \lambda_3(\tau) \left[\frac{d\beta_3^{(n)}(\tau)}{d\tau} - \lambda\beta_2^{(n)}(\tau) + \gamma\beta_3^{(n)}(\tau)\right] d\tau, \tag{19}$$

$$\beta_4^{(n+1)}(t) = \beta_4^{(n)}(t) + \int_0^t \lambda_4(\tau) \left[\frac{d\beta_4^{(n)}(\tau)}{d\tau} - \phi\beta_2^{(n)}(\tau) + \gamma\beta_4^{(n)}(\tau)\right] d\tau, \tag{20}$$

where λ_i, $i = 1, 2, 3, 4$. Making the above functions stationary with respect to $\beta_i^{(n)}(t)$, $i = 1, 2, 3, 4$ it can be identified as $\lambda_i(\tau) = -1$, $i = 1, 2, 3, 4$. Hence, the iteration are:

$$\begin{aligned} \beta_1^{(n+1)}(t) &= \beta_1^{(n)}(t) \\ &- \int_0^t \left[\frac{d\beta_1^{(n)}(\tau)}{d\tau} + \alpha\beta_1^{(n)}(\tau)\beta_2^{(n)}(\tau) + \alpha\beta_1^{(n)}(\tau)\beta_4^{(n)}(\tau)\right] d\tau, \end{aligned} \tag{21}$$

$$\begin{aligned} \beta_2^{(n+1)}(t) &= \beta_2^{(n)}(t) - \int_0^t \left[\frac{d\beta_2^{(n)}(\tau)}{d\tau} - \alpha\beta_1^{(n)}(\tau)\beta_2^{(n)}(\tau)\right. \\ &\left. -\alpha\beta_1^{(n)}(\tau)\beta_4^{(n)}(\tau) + \beta\beta_2^{(n)}(\tau)\right] d\tau, \end{aligned} \tag{22}$$

$$\beta_3^{(n+1)}(t) = \beta_3^{(n)}(t) - \int_0^t \left[\frac{d\beta_3^{(n)}(\tau)}{d\tau} - \lambda\beta_2^{(n)}(\tau) + \gamma\beta_3^{(n)}(\tau)\right] d\tau, \tag{23}$$

$$\beta_4^{(n+1)}(t) = \beta_4^{(n)}(t) - \int_0^t \left[\frac{d\beta_4^{(n)}(\tau)}{d\tau} - \phi\beta_2^{(n)}(\tau) + \gamma\beta_4^{(n)}(\tau)\right] d\tau. \tag{24}$$

Applying the initial approximations $\beta_i(0) = \beta_i^{(0)}$, $i = 1,\ 2,\ 3,\ 4$, we have

$$
\begin{cases}
\beta_1^{(1)}(t) = 11081000.0 - 3.847323200\,t, \\
\beta_2^{(1)}(t) = 3.62 + 3.330180343\,t, \\
\beta_3^{(1)}(t) = 0.4137142858\,t, \\
\beta_4^{(1)}(t) = 4.13 - 0.4865714286\,t.
\end{cases} \tag{25}
$$

For the solution after two terms:

$$
\begin{cases}
\beta_1^{(2)}(t) = 11081000.0 - 3.847323200\,t - 0.7058240126\,t^2, \\
\beta_2^{(2)}(t) = 3.62 + 3.330180343\,t + 0.4679539881\,t^2, \\
\beta_3^{(2)}(t) = 0.4137142858\,t + 0.1607449992\,t^2, \\
\beta_4^{(2)}(t) = 4.13 - 0.4865714286\,t + 0.08232910695\,t^2.
\end{cases} \tag{26}
$$

Similarly, we get the following iterative series as:

$$
\begin{cases}
\beta_1^{(3)}(t) = 11081000.0 - 3.847323200\,t - 0.7058240127\,t^2 \\
\qquad\qquad - 0.09105854712\,t^3, \\
\beta_2^{(3)}(t) = 3.62 + 3.330180343\,t + 0.4679539882\,t^2 + 0.06877502388\,t^3, \\
\beta_3^{(3)}(t) = 0.4137142858\,t + 0.1607449992\,t^2 + 0.01017229483\,t^3, \\
\beta_4^{(3)}(t) = 4.13 - 0.4865714286\,t + 0.08232910694\,t^2 \\
\qquad\qquad + 0.0005362709867\,t^3.
\end{cases} \tag{27}
$$

4. Results and discussion

The current epidemic model based on COVID-19 pandemic discussed in this work with the initial conditions $\mathcal{A}(0) = \mathcal{A}_0 = \beta_1^{(0)} = 11.081 \times 10^6$, $\mathcal{B}(0) = \mathcal{B}_0 = \beta_2^{(0)} = 3.62$, $\mathcal{C}(0) = \mathcal{C}_0 = \beta_3^{(0)} = 0$, $\mathcal{D}(0) = \mathcal{D}_0 = \beta_4^{(0)} = 4.13$ [36]. Also, we derived the series up to order three to record the procedure of the considered model. Further, we obtained the solutions $\beta_i(t)$, $i = 1,\ 2,\ 3,\ 4$ with respect to time (t) for the parameters $\alpha = 4.48 \times 10^{-8}$, $\beta = \frac{1}{7}$, $\gamma = \frac{1}{7}$, $\lambda = 0.8 \times \gamma$, $\phi = 0.2 \times \gamma$ which were taken from [36]. It is viewed that the projected system purely depends on the time and order. DTM plays a vital role in solving the different types of epidemic models especially nonlinear models and effectively initialized with the given conditions for the considered system which gives the interesting results that helps in better understanding of the current pandemic.

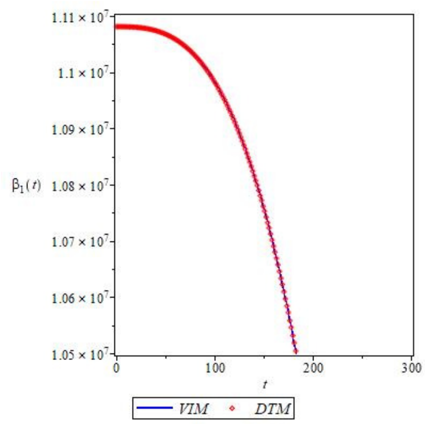

Figure 1: $\beta_1(t)$ in a time t.

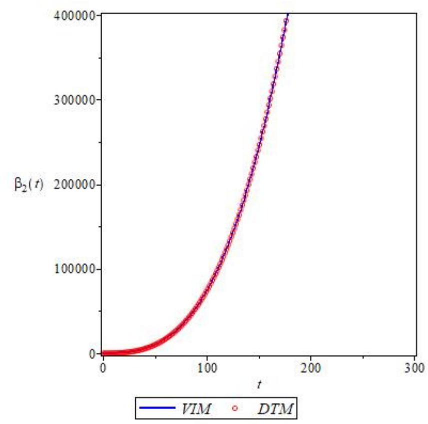

Figure 2: $\beta_2(t)$ in a time t.

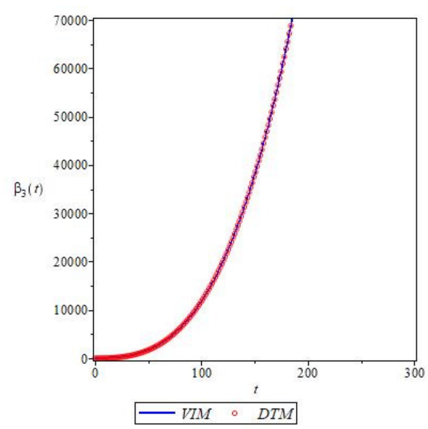

Figure 3: $\beta_3(t)$ in a time t.

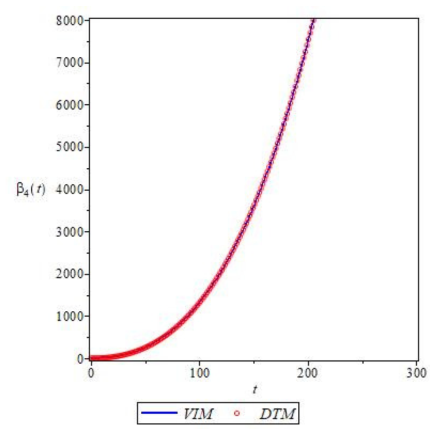

Figure 4: $\beta_4(t)$ in a time t.

5. Conclusion

The present study gives the better understanding of the proposed method and its applications to nonlinear ordinary differential equations. Here, DTM and VIM are effectively applied to the nonlinear differential equations and analyzed that the proposed methods can be implemented to various real life problems for predicting the causes and prevention. The numerical results are presented graphically. The results emphases that one can obtain the analytic form of solutions easily, efficiently by DTM and VIM. The comparison of the results obtained by the proposed methods are in good agreement.

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