Decomposition of continuity via bioperation associated with new sets

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Abstract. In this paper, we introduce some new types of sets in bioperation topological space. Also we discuss some properties and characterize these new sets and finally, we find some relationships between $(\gamma,\gamma')-(\beta,\beta')-A_1$ continuous, $(\gamma,\gamma')-(\beta,\beta')-t$ continuous and $(\gamma,\gamma')-(\beta,\beta')-B_1$ continuous functions.

Keywords: (γ,γ')-open set, $(\gamma,\gamma')-A_1$-set, $(\gamma,\gamma')-A_2$-set, $(\gamma,\gamma')-t$-set, $(\gamma,\gamma')-B_1$-set.

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1. Introduction

Generalized open sets actually play an important role in General Topology and represent an interesting research topics of many topologist worldwide. The study, characterization, modified forms and separation axioms of the notions of generalized open sets, represent an important theme in General Topology and Real analysis. It is well known that, Kasahara [2], introduced and studied the concept of an operation γ on topological spaces. Ogata and Maki [5], introduced and characterized the notions of γ-open sets in a topological space (X, τ) and denote by τ_γ as the collection of all γ-open sets in a topological space (X, τ). Carpintero et al. [3], studied and investigate the concept of (γ, γ')-preopen set in a topological space and described some important properties. Again Carpintero et al. [4], using the notions of bioperations, introduced some new types of sets and obtained a new decomposition of bioperation continuity. In this paper, we define and study new types of sets in a bioperation topological space. Also, we discuss some properties and give a characterization and relationships between these new sets.

Finally, we find some relationships between $(\gamma, \gamma') - (\beta, \beta') - A_1$ continuous, $(\gamma, \gamma') - (\beta, \beta') - t$ continuous and $(\gamma, \gamma') - (\beta, \beta') - B_1$ continuous functions.

2. Preliminaries

Definition 2.1 ([2]). *Let (X, τ) be a topological space. An operation γ on the topology τ is a function from τ into the power set $\mathcal{P}(X)$ of X such that $V \subset V^\gamma$ for each $V \in \tau$, where V^γ denotes the value of γ at V. It is denoted by $\gamma : \tau \to \mathcal{P}(X)$.*

The notions of closure and the interior of a subset A of (X, τ) associated with an operation γ are denoted by $\gamma - \mathrm{Cl}(A)$ and $\gamma - \mathrm{Int}(A)$, respectively.

Definition 2.2 ([5]). *A topological space (X, τ) equipped with two operations, say, γ and γ' defined on τ is called a bioperation topological space, it is denoted by $(X, \tau, \gamma, \gamma')$.*

Definition 2.3 ([5]). *A subset A of a bioperation topological space $(X, \tau, \gamma, \gamma')$ is said to be (γ, γ')-open set if for each $x \in A$ there exist open neighborhoods U and V of x such that $U^\gamma \cup V^{\gamma'} \subset A$. The complement of a (γ, γ')-open set is called a (γ, γ')-closed set. $\tau_{(\gamma,\gamma')}$ denotes the set of all (γ, γ')-open sets in $(X, \tau, \gamma, \gamma')$.*

Definition 2.4 ([5]). *A subset A of a bioperation topological space $(X, \tau, \gamma, \gamma')$, $(\gamma, \gamma') - \mathrm{Cl}(A)$ denotes the intersection of all (γ, γ')-closed sets containing A, that is, $(\gamma, \gamma') - \mathrm{Cl}(A) = \cap\{F : A \subset F, X \backslash F \in \tau_{(\gamma,\gamma')}\}$.*

Definition 2.5. *Let A be any subset of X. The $(\gamma, \gamma') - \mathrm{Int}(A)$ is defined as $(\gamma, \gamma') - \mathrm{Int}(A) = \cup\{U : U$ is a (γ, γ')-open set and $U \subset A\}$.*

Definition 2.6. *Let (X, τ) be a topological space, γ and γ' be operations on τ. A subset A of X is said to be:*

1. $(\gamma, \gamma') - \alpha$*-open if* $A \subset (\gamma, \gamma') - \mathrm{Int}((\gamma, \gamma') - \mathrm{Cl}((\gamma, \gamma') - \mathrm{Int}(A)))$
2. (γ, γ')*-semi-preopen if* $A \subset (\gamma, \gamma') - \mathrm{Cl}((\gamma, \gamma') - \mathrm{Int}((\gamma, \gamma') - \mathrm{Cl}(A)))$
3. (γ, γ')*-regular open if* $A = (\gamma, \gamma') - \mathrm{Int}((\gamma, \gamma') - \mathrm{Cl}(A))$.
4. $(\gamma, \gamma') - t$ *set if* $(\gamma, \gamma') - \mathrm{Int}((\gamma, \gamma') - \mathrm{Cl}(S)) = (\gamma, \gamma') - \mathrm{Int}(S)$,

Definition 2.7. *A function* $f : (X, \tau, \gamma, \gamma') \longrightarrow (Y, \sigma, \beta, \beta')$ *is said to be:*

1. $(\gamma, \gamma') - (\beta, \beta')$ *continuous if* $f^{-1}(V)$ *is* (γ, γ')*-open in* X *for eve-ry* $V \in \sigma_{(\beta,\beta)}$,
2. $(\gamma, \gamma') - (\beta, \beta') - \alpha$ *continuous if* $f^{-1}(V)$ *is* $(\gamma, \gamma') - \alpha$*-open in* X *for every* $V \in \sigma_{(\beta,\beta)}$,
3. $(\gamma, \gamma') - (\beta, \beta')$ *semi precontinuous if* $f^{-1}(V)$ *is* (γ, γ')*-semi-preopen in* X *for every* $V \in \sigma_{(\beta,\beta)}$.

3. Some subsets in bioperation topological space

Given a bioperation topological space $(X, \tau, \gamma, \gamma')$, we define new sets and find its relations between them.

Definition 3.1. *A subset A of a bioperation topological space $(X, \tau, \gamma, \gamma')$ is called:*

1. $(\gamma, \gamma') - A_1$ *set if* $(\gamma, \gamma') - \mathrm{Int}(A) = (\gamma, \gamma') - \mathrm{Int}((\gamma, \gamma') - \mathrm{Cl}((\gamma, \gamma') - \mathrm{Int}(A)))$
2. $(\gamma, \gamma') - A_2$ *set if* $(\gamma, \gamma') - \mathrm{Cl}(A) = (\gamma, \gamma') - \mathrm{Cl}((\gamma, \gamma') - \mathrm{Int}((\gamma, \gamma') - \mathrm{Cl}(A)))$.

Definition 3.2. *A subset A of a bioperation topological space $(X, \tau, \gamma, \gamma')$ is called:*

1. $(\gamma, \gamma') - B_1$ *set if* $A = U \cap V$*, where* $U \in \tau_{(\gamma,\gamma')}$ *and* V *is a* $(\gamma, \gamma') - A_1$ *set.*
2. $(\gamma, \gamma') - B_2$ *set if* $A = U \cap V$*, where* $U \in \tau_{(\gamma,\gamma')}$ *and* V *is a* $(\gamma, \gamma') - A_2$ *set.*

Proposition 3.3. *Let $(X, \tau, \gamma, \gamma')$ be a bioperation topological space. The following properties hold:*

1. *Every* (γ, γ')*-regular open set is a* $(\gamma, \gamma') - A_2$ *set.*
2. *Every* (γ, γ')*-regular closed set is a* $(\gamma, \gamma') - A_1$ *set.*

Observe that in the following example, the converse of the Proposition 3.3 is not true.

Example 3.4. Let $X = \{a, b, c\}$ and $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\}, X\}$. Define the operations $\gamma, \gamma' : \tau \longrightarrow \mathcal{P}(X)$ as follows

$$A^{\gamma} = \begin{cases} A & \text{if b} \notin \text{A}, \\ \text{Cl}(A) & \text{if b} \in \text{A}, \end{cases} \quad \text{and} \quad A^{\gamma'} = \begin{cases} \text{Cl}(A) & \text{if b} \notin \text{A}, \\ A & \text{if b} \in \text{A}. \end{cases}$$

The $\tau_{(\gamma,\gamma')} = \{\emptyset, \{b\}, \{a, b\}, \{a, c\}, X\}$. Observe that, $\{a, b\}$ is a $(\gamma, \gamma') - A_2$ set but is not a (γ, γ')-regular open and $\{c\}$ is a $(\gamma, \gamma') - A_1$ set but is not (γ, γ')-regular closed.

Theorem 3.5. *Let $(X, \tau, \gamma, \gamma')$ be a bioperation topological space and A a subset of X. The following properties hold:*

1. *A is a $(\gamma, \gamma') - A_1$ set if, and only if it is (γ, γ')-semi-preclosed.*
2. *A is a $(\gamma, \gamma') - A_2$ set if, and only if it is (γ, γ')-semi-preopen.*

Proof. (1) Let A be a $(\gamma, \gamma') - A_1$ set. Then we have the $(\gamma, \gamma') - \text{Int}(A) = (\gamma, \gamma') - \text{Int}((\gamma, \gamma') - \text{Cl}((\gamma, \gamma') - \text{Int}(A)))$. Then the $(\gamma, \gamma') - \text{Int}((\gamma, \gamma') - \text{Cl}((\gamma, \gamma') - \text{Int}(A))) \subset A$. Therefore A is (γ, γ')-semi-preclosed. Conversely, assume that A is a (γ, γ')-semi-preclosed. Then the $(\gamma, \gamma') - \text{Int}((\gamma, \gamma') - \text{Cl}((\gamma, \gamma') - \text{Int}(A))) \subset A$. Then the $(\gamma, \gamma') - \text{Int}((\gamma, \gamma') - \text{Cl}((\gamma, \gamma') - \text{Int}(A))) \subset (\gamma, \gamma') - \text{Int}(A)$. Since we also have the $(\gamma, \gamma') - \text{Int}(A) \subset (\gamma, \gamma') - \text{Int}((\gamma, \gamma') - \text{Cl}((\gamma, \gamma') - \text{Int}(A)))$. Then the $(\gamma, \gamma') - \text{Int}(A) = (\gamma, \gamma') - \text{Int}((\gamma, \gamma') - \text{Cl}((\gamma, \gamma') - \text{Int}(A)))$. Hence A is a $(\gamma, \gamma') - A_1$ set.

(2) Let A be a $(\gamma, \gamma') - A_2$ set. Then we have the $(\gamma, \gamma') - \text{Cl}(A) = (\gamma, \gamma') - \text{Cl}((\gamma, \gamma') - \text{Int}((\gamma, \gamma') - \text{Cl}(A)))$. Then $A \subset (\gamma, \gamma') - \text{Cl}((\gamma, \gamma') - \text{Int}((\gamma, \gamma') - \text{Cl}(A)))$. Therefore, A is (γ, γ')-semi-preopen.

Conversely, assume that A is (γ, γ')-semi-preopen. We have $A \subset (\gamma, \gamma') - \text{Cl}((\gamma, \gamma') - \text{Int}((\gamma, \gamma') - \text{Cl}(A)))$. Hence $(\gamma, \gamma') - \text{Cl}(A) \subset (\gamma, \gamma') - \text{Cl}((\gamma, \gamma') - \text{Int}((\gamma, \gamma') - \text{Cl}(A)))$. Since the $(\gamma, \gamma') - \text{Cl}((\gamma, \gamma') - \text{Int}((\gamma, \gamma') - \text{Cl}(A))) \subset A$, $(\gamma, \gamma') - \text{Cl}(A) = (\gamma, \gamma') - \text{Cl}((\gamma, \gamma') - \text{Int}((\gamma, \gamma') - \text{Cl}(A)))$. Hence A is $(\gamma, \gamma') - A_2$ set. $\square$

Theorem 3.6. *Let $(X, \tau, \gamma, \gamma')$ be a bioperation topological space and A and B subsets of X. The following properties hold:*

1. *If A and B are $(\gamma, \gamma') - A_1$ sets, then $A \cap B$ is a $(\gamma, \gamma') - A_1$ set.*
2. *If A and B are $(\gamma, \gamma') - A_2$ sets, then $A \cup B$ is a $(\gamma, \gamma') - A_2$ set.*

Proof. (1) Let A and B be $(\gamma, \gamma') - A_1$ sets. Then, the $(\gamma, \gamma') - \text{Int}(A \cap B) \subset (\gamma, \gamma') - \text{Int}((\gamma, \gamma') - \text{Cl}((\gamma, \gamma') - \text{Int}(A \cap B))) \subset (\gamma, \gamma') - \text{Int}((\gamma, \gamma') - \text{Cl}((\gamma, \gamma') - \text{Int}(A)) \cap (\gamma, \gamma') - \text{Cl}((\gamma, \gamma') - \text{Int}(B))) = (\gamma, \gamma') - \text{Int}((\gamma, \gamma') - \text{Cl}((\gamma, \gamma') - \text{Int}(A))) \cap (\gamma, \gamma') - \text{Int}((\gamma, \gamma') - \text{Cl}((\gamma, \gamma') - \text{Int}(B))) = (\gamma, \gamma') - \text{Int}(A \cap B)(\gamma, \gamma') - \text{Int}(A \cap B) \subset (\gamma, \gamma') - \text{Int}((\gamma, \gamma') - \text{Cl}((\gamma, \gamma') - \text{Int}(A \cap B))) \subset (\gamma, \gamma') - \text{Int}(A \cap B) = (\gamma, \gamma') - \text{Int}((\gamma, \gamma') - \text{Cl}((\gamma, \gamma') - \text{Int}(A \cap B)))) = A \cap B$ is a $(\gamma, \gamma') - A_1$ set.

(2) Let A and B be $(\gamma,\gamma') - A_2$ sets. Then the $(\gamma,\gamma') - \mathrm{Cl}(A \cup B) = (\gamma,\gamma') - \mathrm{Cl}((\gamma,\gamma') - \mathrm{Int}((\gamma,\gamma') - \mathrm{Cl}(A) \cup (\gamma,\gamma') - \mathrm{Cl}(B))) \subset (\gamma,\gamma') - \mathrm{Cl}((\gamma,\gamma') - \mathrm{Int}((\gamma,\gamma') - \mathrm{Cl}(A)) \cup (\gamma,\gamma') - \mathrm{Int}((\gamma,\gamma') - \mathrm{Cl}(B)))(\gamma,\gamma') - \mathrm{Cl}(A \cup B) \subset (\gamma,\gamma') - \mathrm{Cl}((\gamma,\gamma') - \mathrm{Int}((\gamma,\gamma') - \mathrm{Cl}(A \cup B)))$. Also the $(\gamma,\gamma') - \mathrm{Int}((\gamma,\gamma') - \mathrm{Cl}(A \cup B)) \subset (A \cup B))(\gamma,\gamma') - \mathrm{Cl}((\gamma,\gamma') - \mathrm{Int}((\gamma,\gamma') - \mathrm{Cl}(A \cup B))) \subset (\gamma,\gamma') - \mathrm{Cl}(A \cup B)(\gamma,\gamma') - \mathrm{Cl}(A \cup B) = (\gamma,\gamma') - \mathrm{Cl}((\gamma,\gamma') - \mathrm{Int}((\gamma,\gamma') - \mathrm{Cl}(A \cup B)))) = A \cup B$ is a $(\gamma,\gamma') - A_2$ set. □

Theorem 3.7. *A set A is a $(\gamma,\gamma') - A_1$ set if, and only if its complement is a $(\gamma,\gamma') - A_2$ set.*

Proof. The proof follows from Definition 3.1. □

Theorem 3.8. *Let (X,τ,γ,γ') be a bioperation topological space. A subset A of X is a (γ,γ')-open set if, and only if it is a $(\gamma,\gamma') - \alpha$-open set and $(\gamma,\gamma') - B_1$ set.*

Proof. Let A be a (γ,γ')-open, then the $(\gamma,\gamma') - \mathrm{Cl}(A) = (\gamma,\gamma') - \mathrm{Cl}((\gamma,\gamma') - \mathrm{Int}(A))$. It is clear that $A \subset (\gamma,\gamma') - \mathrm{Int}((\gamma,\gamma') - \mathrm{Cl}((\gamma,\gamma') - \mathrm{Int}(A)))$. Hence A is a $(\gamma,\gamma') - \alpha$-open set. Let $U = A \in \tau_{(\gamma,\gamma')}$ and $V = X$ be a $(\gamma,\gamma') - A_1$ set containing A. Hence $A = U \cap V$; A is a $(\gamma,\gamma') - B_1$ set. Conversely, let A be a $(\gamma,\gamma') - \alpha$-open set and a $(\gamma,\gamma') - B_1$ set. Then $A = U \cap V$, where $U \in \tau_{(\gamma,\gamma')}$ and V is a $(\gamma,\gamma') - A_1$ set. Since V is a $(\gamma,\gamma') - A_1$ set, $(\gamma,\gamma') - \mathrm{Int}(V) = (\gamma,\gamma') - \mathrm{Int}((\gamma,\gamma') - \mathrm{Cl}((\gamma,\gamma') - \mathrm{Int}(V)))$. Since A is $(\gamma,\gamma') - \alpha$-open, $A \subset (\gamma,\gamma') - \mathrm{Int}((\gamma,\gamma') - \mathrm{Cl}((\gamma,\gamma') - \mathrm{Int}(A))) = (\gamma,\gamma') - \mathrm{Int}((\gamma,\gamma') - \mathrm{Cl}((\gamma,\gamma') - \mathrm{Int}(U \cap V))) = (\gamma,\gamma') - \mathrm{Int}((\gamma,\gamma') - \mathrm{Cl}((\gamma,\gamma') - \mathrm{Int}(U))) \cap (\gamma,\gamma') - \mathrm{Int}((\gamma,\gamma') - \mathrm{Cl}((\gamma,\gamma') - \mathrm{Int}(V)))$. Hence $U \cap V = (\gamma,\gamma') - \mathrm{Int}((\gamma,\gamma') - \mathrm{Cl}(U)) \cap (\gamma,\gamma') - \mathrm{Int}(V)$. Consider $U \cap V = (U \cap V) \cap U \subset U \cap (\gamma,\gamma') - \mathrm{Int}(V)$; hence $V \subset (\gamma,\gamma') - \mathrm{Int}(V)$; A is (γ,γ')-open. □

Theorem 3.9. *Let (X,τ,γ,γ') be a bioperation topological space and A a subset of X. If A is a $(\gamma,\gamma') - t$-set, then $(\gamma,\gamma') - \mathrm{Int}(A)$ is a $(\gamma,\gamma') - t$-set.*

Proof. If A is a $(\gamma,\gamma') - t$-set, then $(\gamma,\gamma') - \mathrm{Int}((\gamma,\gamma') - \mathrm{Cl}((\gamma,\gamma') - \mathrm{Int}(A))) \subset (\gamma,\gamma') - \mathrm{Int}((\gamma,\gamma') - \mathrm{Cl}(A)) = A$. Follows that $(\gamma,\gamma') - \mathrm{Int}((\gamma,\gamma') - \mathrm{Cl}((\gamma,\gamma') - \mathrm{Int}(A))) \subset (\gamma,\gamma') - \mathrm{Int}(A)$. This implies that $(\gamma,\gamma') - \mathrm{Int}(A)$ is a (γ,γ')-semi closed. Follows from [1], that $(\gamma,\gamma') - \mathrm{Int}(A)$ is a $(\gamma,\gamma') - t$-set. □

Theorem 3.10. *Let (X,τ,γ,γ') be a bioperation topological space and A a subset of X. If A is a $(\gamma,\gamma') - t$-set, then A is $(\gamma,\gamma') - A_1$ set.*

Proof. If A is a $(\gamma,\gamma') - t$-set, by Theorem 3.9 and then $(\gamma,\gamma') - \mathrm{Int}((\gamma,\gamma') - \mathrm{Cl}((\gamma,\gamma') - \mathrm{Int}(A))) \subset A$. This implies that A is a (γ,γ')-semi preclosed and then by Theorem 3.5, A is a $(\gamma,\gamma') - A_1$ set. □

The following example, shows that the converse of Theorem 3.10, is not necessarily true.

Example 3.11. Observe that in Example 3.4:

1. $(\gamma, \gamma') - A_1$ sets $= \{\emptyset, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, X\}$
2. $(\gamma, \gamma') - t$ sets $= \{\emptyset, \{b\}, \{c\}, \{a,c\}, \{b,c\}, X\}$

We can see that $\{a, b\}$ is a $(\gamma, \gamma') - A_1$ set but is not a $(\gamma, \gamma') - t$ set.

4. Decomposition of bioperation-continuity

In this section, we define and study the notions of: $(\gamma, \gamma') - (\beta, \beta') - A_1$ continuous, $(\gamma, \gamma') - (\beta, \beta') - A_2$ continuous, $(\gamma, \gamma') - (\beta, \beta') - t$ continuous and $(\gamma, \gamma') - (\beta, \beta') - B_1$ continuous and find some relation between them and with others forms of $(\gamma, \gamma') - (\beta, \beta')$ continuous functions

Definition 4.1. *A function $f : (X, \tau, \gamma, \gamma') \longrightarrow (Y, \sigma, \beta, \beta')$ is said to be:*

1. *$(\gamma, \gamma') - (\beta, \beta') - A_1$ continuous if $f^{-1}(V)$ is a $(\gamma, \gamma') - A_1$ set in X for each $V \in \sigma_{(\beta,\beta')}$,*
2. *$(\gamma, \gamma') - (\beta, \beta') - A_2$ continuous if $f^{-1}(V)$ is a $(\gamma, \gamma') - A_2$ set in X for each $V \in \sigma_{(\beta,\beta')}$,*
3. *$(\gamma, \gamma') - (\beta, \beta') - t$ continuous if $f^{-1}(V)$ is a $(\gamma, \gamma') - t$ set in X for each $V \in \sigma_{(\beta,\beta')}$,*
4. *$(\gamma, \gamma') - (\beta, \beta') - B_1$ continuous if $f^{-1}(V)$ is a $(\gamma, \gamma') - B_1$ set in X for each $V \in \sigma_{(\beta,\beta')}$,*

Theorem 4.2. *Let $(X, \tau, \gamma, \gamma')$ and $(Y, \sigma, \beta, \beta')$ be two bioperation topological spaces. A function $f : (X, \tau, \gamma, \gamma') \longrightarrow (Y, \sigma, \beta, \beta')$ is $(\gamma, \gamma') - (\beta, \beta') - A_2$ continuous if, and only if it is $(\gamma, \gamma') - (\beta, \beta')$-semi-pre continuous.*

Proof. The proof follows from Definition 4.1(2) and Theorem 3.5(2). □

Theorem 4.3. *Let $(X, \tau, \gamma, \gamma')$ and $(Y, \sigma, \beta, \beta')$ be two bioperation topological spaces.A function $f : (X, \tau, \gamma, \gamma') \longrightarrow (Y, \sigma, \beta, \beta')$ is $(\gamma, \gamma') - (\beta, \beta')$ continuous if, and only if it is $(\gamma, \gamma') - (\beta, \beta') - \alpha$ continuous and $(\gamma, \gamma') - (\beta, \beta') - B_1$ continuous.*

Proof. The proof follows from Definition 2.7 and Theorem 3.8. □

Theorem 4.4. *Let $(X, \tau, \gamma, \gamma')$ be a (γ, γ')-extremally disconnected space. If $f : (X, \tau, \gamma, \gamma') \longrightarrow (Y, \sigma, \beta, \beta')$ is $(\gamma, \gamma') - (\beta, \beta') - t$ continuous, then f is $(\gamma, \gamma') - (\beta, \beta') - A_1$ continuous.*

Proof. The proof follows from Definitions 4.1 and Theorem 3.10. □

The following theorem shows that if $f : (X, \tau, \gamma, \gamma') \longrightarrow (Y, \sigma, \beta, \beta')$ is $(\gamma, \gamma') - (\beta, \beta') - A_1$ continuous and $(\gamma, \gamma') - (\beta, \beta')$ continuous, then it is $(\gamma, \gamma') - (\beta, \beta') - B_1$ continuous.

Theorem 4.5. *Let $(X, \tau, \gamma, \gamma')$ and $(Y, \sigma, \beta, \beta')$ be two bioperation topological spaces. A function $f : (X, \tau, \gamma, \gamma') \longrightarrow (Y, \sigma, \beta, \beta')$ is $(\gamma, \gamma') - (\beta, \beta') - A_1$ continuous and $(\gamma, \gamma') - (\beta, \beta')$ continuous, then it is $(\gamma, \gamma') - (\beta, \beta') - B_1$ continuous.*

Proof. The proof follows from Definitions 2.7, 3.2 and 4.1. □

Conclusions

1. We define the new sets: $(\gamma, \gamma') - A_1$ set, $(\gamma, \gamma') - A_2$ set, $(\gamma, \gamma') - B_1$ set, $(\gamma, \gamma') - B_2$ set, $(\gamma, \gamma') - t$ set and find the relationship between them.

2. We find the relationships between $(\gamma, \gamma') - (\beta, \beta') - A_1$ continuous, $(\gamma, \gamma') - (\beta, \beta') - t$ continuous and $(\gamma, \gamma') - (\beta, \beta') - B_1$ continuous functions and with others forms of $(\gamma, \gamma') - (\beta, \beta')$ continuous functions.

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