Signed product cordial labeling of corona product between paths and second power of fan graphs

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Abstract. In this work, we contribute some new results on signed product cordial labeling and investigate necessary and sufficient conditions of the corona product between paths and second power of fan graphs to be signed product cordial.
Keywords: corona operation, second power, signed product cordial, path graph, fan graph.

1. Introduction

The field of graph theory plays an important role in various areas of pure and applied sciences. One of the main problems in this field is graph labeling which is an assignment of integers to the vertices or edges, or both, subject to certain conditions. It is a very powerful tool that eventually makes things in different fields very ease to be handled in mathematical way. While the labeling of graphs is perceived to be a primarily theoretical subject in the field of graph theory and discrete mathematics, it serves as models in a wide range of application like astronomy, coding theory, x-ray crystallography, circuit design and communication networks addressing [1]. There are many contributions and different types of labeling (see, [2], [3], [4]). An excellent reference for this purpose is the survey written by Gallian [5].

In this paper, all graphs are finite, simple and un directed. The original concept of cordial graphs is due to Cahit [6] who called a graph G cordial if there is a vertex labeling $f : V(G) \to 0, 1$ such that the induced labeling $f^* : E(G) \to 0, 1$, defined by $f^*(xy) = |f(x) - f(y)|$, for all edges $x, y \in E(G)$ and with the following inequalities holding: $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$, where $v_f(i)$ (respectively $e_f(i)$) is the number of vertices (respectively, edges)

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labeled with i. Motivated through the concept of cordial labeling the product cordial labeling was introduced by Sundaram et al [7] where absolute difference of vertex labels is replaced by product of vertex labels.

Definition 1. A graph $G = (V, E)$ is called signed product cordial [8] if it is possible to label the vertex by the function $f : V \to -1, 1$ and label the edges by $f^* : E \to -1, 1$, where $f^*(uv) = f(u).f(v)$, $u, v \in V$ so that $|v_{-1} - v_1| \leq 1$ and $|e_{-1} - e_1| \leq 1$.

Definition 2. The corona $G \bigodot H$ of two graphs G (with n_1 vertices and m_1 edges) and H (with n_2 vertices and m_2 edges) is the graph denoted by $G \bigodot H$ and is obtained by taking one copy of G and n_1 copies of H, and then joining the i^{th} vertex of G with an edge to every vertex in the i^{th} copy of H (see, [9]). It follows from the definition of the corona that $G \bigodot H$ has $n_1 + n_1.n_2$ vertices and $m_1 + n_1.m_2 + n_1.n_2$ edges.

It is easy to see that $G \bigodot H$ is not in general isomorphic to $H \bigodot G$.

Definition 3. A second power of a fan F_m^2 is the graph obtained from the join of the second power of a path P_m^2 and a null graph N_1, i.e.$F_m^2 = P_m^2 + N_1$. So the order of F_m^2 is $m + 1$ and its size is $3m - 3$, in particular $F_1^2 \equiv P_2$,$F_2^2 \equiv C_3$ and $F_3^2 \equiv K_4$.

In this paper we proposed the corona $P_k \bigodot F_m^2$ and show that is signed product cordial for all $K \geq 1$ and $m \geq 4$.

2. Terminology and notation

A path with n vertices and $n - 1$ edges is denoted by P_n, and second power of fan graph has $n + 1$ vertices and $3n - 3$ edges is denoted by F_n^2. We let $M_r$denote the labeling $(-1)1(-1)1...(-1)1$, zero-one repeated r-times if r is even and $(-1)1(-1)1...(-1)1(-1)$ if r is odd; for example, $M_6 = (-1)1(-1)1(-1)1$ and $M_5 = (-1)1(-1)1(-1)$. We let M'_{2r} denote the labeling $1(-1)1(-1)...1(-1)$. Sometimes, we modify the labeling M_r or M'_r by adding symbols at one end or the other (or both). We let L_{4r} denote the labeling $(-1)_2 11$ $(-1)_2 11...(-1)_2 11$ (repeated r-times), Let L'_{4r} denote the labeling $(-1)11(-1)(-1)11(-1)...(-1)11$ (-1) (repeated r-times). The labeling $11(-1)_2 11(-1)_2...11(-1)_2$ (repeated r-times) and labeling $1(-1)_2 11(-1)_2 1...1(-1)_2 1$ (repeated r-times) are written S_{4r} and S'_{4r}. Let M_r denote the labeling $(-1)1$ $(-1)1...(-1)1$, zero-one repeated $r$times if r is even and $(-1)1$ $(-1)1...(-1)1(-1)$ if r is odd; for example, $M_6 = (-1)1(-1)1(-1)1$ and $M_5 = (-1)1(-1)1(-1)$. We let M'_r denote the labeling $1(-1)1(-1)...1(-1)$. Sometimes, we modify the labeling M_r or M'_r by adding symbols at one end or the other (or both). Also, L_{4r} (or L'_{4r}) with extra labeling from right or left (or both sides) [9-13]. For specific labeling L and M of $G \bigodot H$ where G is path and H is a second power of fan graph, we let $[L; M]$ denote the corona labeling. Additional notation that we use is the following. For a given labeling of the corona $G \bigodot H$, we let v_i and e_i (for $i = (-1), 1$) be the numbers of labels that are i as before, we let x_i and a_i be the corresponding quantities for G, and we let $y_i$and b_i be those for H, which are connected to the vertices labeled (-1) of G. Likewise, let y_i' and b_i' be those for H, which are connected to the vertices labeled 1 of G. In case it increases by one more vertex, so $y_i^{"}$ and $b"_i$ will be those for H, which are connected to the vertex labeled 1 or (-1) of G. It is easy to verify that $v_{-1} = x_{-1} + y_{-1}x_{-1} + y'_{-1}x_1, v_1 = x_1 + y_1x_{-1} + y'_1x_1$ and $e_{-1} = a_{-1}+b_{-1}x_{-1}+b'_{-1}x_1+y_1x_{-1}+y'_1x_{-1}, e_1 = a_1+b_1x_{-1}+b'_1x_1+y_1x_1+y'_{-1}x_{-1}$ Thus, $v_{-1} - v_1 = (x_{-1} - x_1) + x_{-1}(y_{-1} - y_1) + x_1(y'_{-1} - y'_1)$ and $e_{-1} - e_1 = (a_{-1} - a_1) + x_{-1}(b_{-1} - b_1) + x_1(b'_{-1} - b'_1) - x_{-1}(y_{-1} - y_1) + x_1(y'_{-1} - y'_1)$. When it comes to the proof, we only need to show that, for each specified combination of labeling, $|v_{-1} - v_1| \leq 1$ and $|e_{-1} - e_1| \leq 1$.

3. The signed product cordial of corona between paths and second power of Fan graphs

In this section, we show that the corona between paths and second power of Fan graphs $P_k \bigodot F_m^2$ is signed product cordial for all $k \geq 1$, and $m \geq 4$. This target will be achieved after the following series of lemmas.

Lemma 3.1. *$P_k \bigodot F_m^2$ is signed product cordial for all $k \geq 1$ and $m \equiv 0(mod4)$.*

Proof of Lemma 3.1. We need to examine the following cases:
Case (1). $k \equiv 0(mod4)$.

Let $k = 4r, r \geq 1$. Then, one can choose the labeling $[L_{4r} : (-1)M'_{4s}, (-1) M'_{4s}, 1M_{4s}, 1M_{4s}, \ldots (r - times)]$ for $P_{4r} \bigodot F_{4s}^2$. Therefore $x_{-1} = x_1 = 2r, a_{-1} = 2r - 1, a_1 = 2r, y_{-1} = 2s + 1, y_1 = 2s, b_{-1} = 6s - 1, b_1 = 6s - 2, y'_{-1} = 2s, y'_1 = 2s+1, b'_{-1} = 6s-1$ *and* $b'_1 = 6s-2$. Hence, one can easily show that $v_{-1}-v1 = 0$ and $e_{-1} - e_1 = -1$. Thus $P_{4r} \bigodot F_{4s}^2$, $s \geq 1$ is signed product cordial. As an example, Figure 1 illustrates this case $P_4 \bigodot F_4^2$.

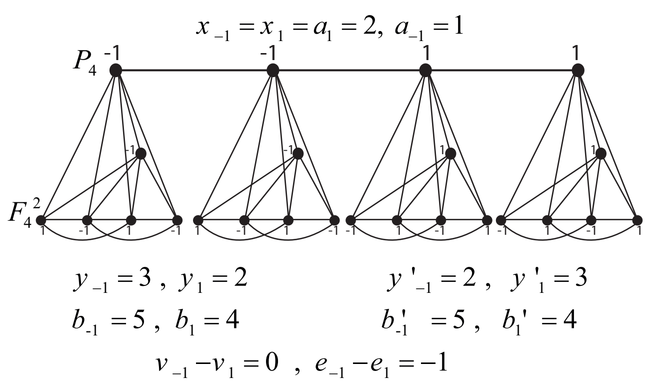

Figure 1: $P_4 \bigodot F_4^2$ is signed product cordial

Case (2). $k \equiv$ *1(mod 4).*

Let $k = 4r+1$, $r > 0$. Then, one can choose the labelling $[L_{4r}1 : (-1)M_{4s}, (-1)M_{4s}, 1M_{4s}, 1M_{4s}, \dots (r-times), (-1)M_{4s}]$ for $P_{4r+1} \odot F^2_{4s}$. Therefore $x_{(-1)} = 2r, x_1 = a_{(-1)} = 2r-1, a_1 = 2r+1, y_{-1} = 2s+1, y_1 = 2s, b_{-1} = 6s-2, b_1 = 6s-1, y'_{-1} = 2s, y'_1 = 2s+1, b'_{-1} = 6s-2, b'_1 = 6s-1, y''_{-1} = 2s+1, y''_1 = 2s, b''_{-1} = 6s-2$ *and* $b''_1 = 6s-1$. Hence, one can easily show that $v_{-1} - v_1 = 0$ and $e_{-1} - e_1 = 0$. Thus $P_{4r+1} \odot F^2_{4s}$, $s \geq 1$ is signed product cordial.

__Case (3).__ $k \equiv$ *2(mod 4).*

Let $k = 4r+2$, $r > 0$. Then, one can choose the labelling $[L_{4r}1(-1) : (-1)M'_{4s}, (-1)M'_{4s}, 1M_{4s}, 1M_{4s}, \dots (r-times), 1M_{4s}, (-1)M'_{4s}]$ for $P_{4r+2} \odot F^2_{4s}$. Therefore $x_{-1} = x_1 = 2r+1, a_{-1} = 2r, a_1 = 2r+1, y_{-1} = 2s+1, y_1 = 2s, b_{-1} = 6s-1, b_1 = 6s-2, y'_{-1} = 2s, y'_1 = 2s+1, b'_{-1} = 6s-1$ and $b'_1 = 6s-2$. Hence, one can easily show that $v_{-1} - v_1 = 0$ and $e_{-1} - e_1 = -1$. Thus $P_{4r+2} \odot F^2_{4s}$, $s \geq 1$ is signed product cordial.

__Case (4).__ $k \equiv$ *3(mod 4).*

Let $k = 4r+3, r > 0$. Then, one can choose the labelling $[L_{4r}11(-1) : (-1)M'_{4s}, (-1)M'_{4s}, 1M'_{4s}, 1M'_{4s} \dots (r - times), 1M'_{4s}, 1M_{4s}, (-1)M'_{4s}]$ for $P_{4r+3} \odot F^2_{4s}$. Therefore $x_{-1} = 2r+1, x_1 = a_{-1} = 2r, a_1 = 2r+2, y_{-1} = 2s+1, y_1 = 2s, b_{-1} = 6s-1, b_1 = 6s-2, y'_{-1} = 2s, y'_1 = 2s+1, b'_{-1} = 6s-1, b'_1 = 6s-2, y''_{-1} = 2s+1, y''_1 = 2s, b''_{-1} = 6s-1$ and $b''_1 = 6s-2$. Hence, one can easily show that $v_{-1} - v_1 = 0$ and $e_{-1} - e_1 = 0$. Thus $P_{4r+3} \odot F^2_{4s}, s \geq 1$ is signed product cordial.

Lemma 3.2. *$P_k \odot F^2_m$ is signed product cordial for all $k \geq 1$ and $m \equiv 1(mod4)$.*

Proof of Lemma 3.2. We need to examine the following cases:

__Case (1).__ $\boldsymbol{k \equiv 0(mod4)}$.

Let $k = 4r$, $r \geq 1$. Then, one can choose the labeling $[L_{4r} : 11_3(-1)_2M_{4s-4}, 11_3\ (-1)_2M_{4s-4}, (-1)_41_2M'_{4s-4}, (-1)_41_2M'_{4s-4}, \dots (r-times)]$ for $P_{4r} \odot F^2_{4s+1}$. Therefore $x_{-1} = x_1 = a_{-1} = 2r-1, a_1 = 2r, y_{-1} = 2s, y_1 = 2s+2, b_{-1} = 6s-1, b_1 = 6s+1, y'_{-1} = 2s+2, y'_1 = 2s$ and $b'_{-1} = 6s-1, b'_1 = 6s+1$. Hence, one can easily show that $v_{-1} - v_1 = 0$ and $e_{-1} - e_1 = -1$.Thus $P_{4r} \odot F^2_{4s+1}, s \geq 1$ is signed product cordial. As an example, Figure 2 illustrates this case $P_4 \odot F^2_5$.

Case (2). $\boldsymbol{k \equiv 1(mod4)}$.

Let $k = 4r+1, r > 0$. Then, one can choose the labeling $[L4r(-1) : 11_3(-1)_2M_{4s-4}, 11_3(-1)_2M_{4s-4}, (-1)_41_2M'_{4s-4}, (-1)_41_2\ M'_{4s-4}, \dots (r-times), 1(-1)_31_2M_{4s-4}]$ for $P_{4r+1} \odot F^2_{4s+1}$. Therefore $x_{-1} = 2r+1, x_1 = a_{-1} = a_1 = 2r, y_{-1} = 2s+2, y_1 = 2s, b_{-1} = 6s+1, b_1 = 6s-1, y'_{-1} = 2s, y'_1 = 2s+2, b'_{-1} = 6s+1, b'_1 = 6s-1, y''_{-1} = y''_1 = 2s+1$, and $''_{-1} = b''_1 = 6s$. Hence, one can easily show that $v_{-1} - v_1 = 1$ and $e_{-1} - e_1 = 0$. Thus $P_{4r+1} \odot F^2_{4s+1}, s \geq 1$, is signed product cordial.

__Case (3).__ $\boldsymbol{k \equiv 2(mod4)}$.

Let $k = 4r+2, r \geq 0$. Then, one can choose the labeling $[L_{4r}1(-1)1_4(-1)_2 M_{4s-4}, 11_3(-1)_2M_{4s-4}, (-1)_41_2M'_{4s-4}, (-1)_41_2M'_{4s-4}, \dots (r-times), (-1)_41_2$

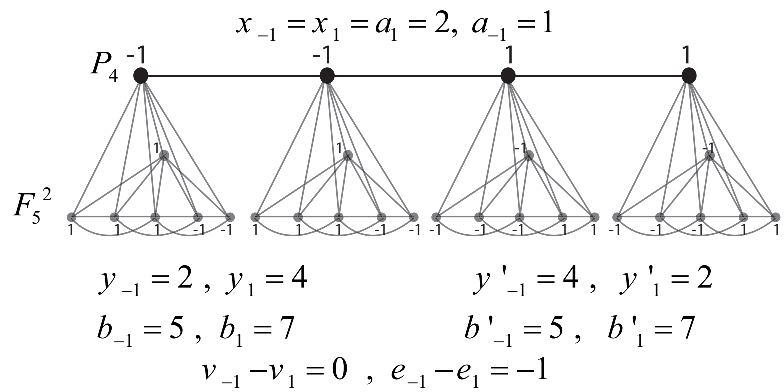

Figure 2: $P_4 \bigodot F_5^2$ is signed product cordial

$M'_{4s-4}, 11_3(-1)_2M_{4s-4}]$ for $P_{4r+2} \bigodot F^2_{4s+1}$. Therefore $x_{-1} = x_1 = a_{-1} = 2r, a_1 = 2r+1, y_{-1} = 2s+2, y_1 = 2s, b_{-1} = 6s+1, b_1 = 6s-1, y'_{-1} = 2s, y'_1 = 2s+2$ and $b'_{-1} = 6s+1, b'_1 = 6s-1$. Hence one can easily show that $v_{-1} - v_1 = 0$ and $e_{-1} - e_1 = -1$. Thus $P_{4r+2} \bigodot F^2_{4s+1}, s \geq 1$, is signed product cordial.

<u>Case (4).</u> $k \equiv 3(mod4)$.

Let $k = 4r+3, r \geq 0$. Then, one can choose the labeling $[L_{4r}1(-1)1 : 1_4(-1)_2M_{4s-4} \;, 1_4(-1)_2M_{4s-4}, (-1)_4 1_2 M'_{4s-4}, (-1)_4 1_2 M'_{4s-4}, \ldots (r - times), (-1)_4 \; 1_2 M'_{4s-4}, 1_4 \; (-1)_2 M_{4s-4}, (-1)_3 1_3 M'_{4s-4}]$ for $P_{4r+3} \bigodot F^2_{4s+1}$. Therefore $x_{-1} = a_1 = a_{-1} = 2r+1, x_1 = 2r+2, y_{-1} = 2s, y_1 = 2s+2, b_{-1} = 6s-1, b_1 = 6s+1, y'_{-1} = 2s+2, y'_1 = 2s, b'_{-1} = 6s-1, b'_1 = 6s+1, y''_{-1} = y''_1 = 2s+1$, and $b''_{-1} = b''_1 = 6s$. Hence, one can easily show that $v_{-1} - v_1 = -1$ and $e_{-1} - e_1 = 0$. Thus $P_{4r+3} \bigodot F^2_{4s+1}, s \geq 1$, is signed product cordial.

Lemma 3.3. *$P_k \bigodot F^2_m$ is signed product cordial for all $k \geq 1$ and $\equiv 2(mod4)$.*

Proof of Lemma 3.3. We need to study the following cases:

<u>Case (1).</u> $k \equiv 0(mod4)$.

Let $k = 4r, r \geq 1$. Then, one can choose the labeling $[L_{4r} : (-1)M'_{4s+2}, (-1) M'_{4s+2}, 1M_{4s+2}, 1M_{4s+2}, \ldots (r - times)]$ for $P_{4r} \bigodot F^2_{4s+2}$. Therefore $x_{-1} = x_1 = 2r, a_{-1} = 2r-1, a_1 = 2r, y_{-1} = 2s+2, y_1 = 2s+1, b_{-1} = 6s+1, b_1 = 6s+2, y'_{-1} = 2s+1, y'_1 = 2s+2, b'_{-1} = 6s+1$ and $b'_1 = 6s+2$. Hence, one can easily show that $v_{-1} - v_1 = 0$ and $e_{-1} - e_1 = 1$. Thus, $P_{4r} \bigodot F^2_{4s+2}, r \geq 1$ is signed product cordial. As an example, Figure 3 illustrates this case $P_4 \bigodot F^2_6$.

<u>Case (2).</u> $k \equiv 1(mod4)$.

Let $k = 4r+1, r \geq 0$. Then, one can choose the labeling $[L_{4r}1 : (-1)M'_{4s+2}, (-1)M'_{4s+2}, 1M'_{4s+2}, 1M'_{4s+2}, \ldots, (r - times), (-1)M_{4s+2}]$ for $P_{4r+1} \bigodot F^2_{4s+2}$. Therefore $x_{-1} = 2r, x_1 = 2r+1, a_{-1} = 2r-1, a_1 = 2r+1, y_{-1} = 2s+2, y_1 = 2s+1, b_{-1} = 6s+2, b_1 = 6s+1, y'_{-1} = 2s+1, y'_1 = 2s+2, b'_{-1} = 6s+2, b'_1 = 6s+1, y"_{-1} = 2s+2, y''_1 = 2s+1, b''_{-1} = 6s+2$ and $b"_1 = 6s+1$. Hence, one

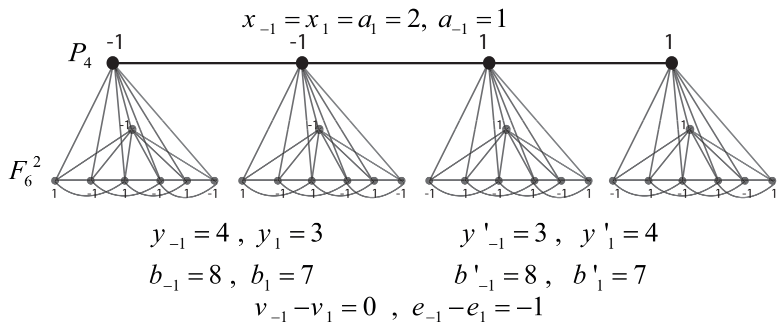

Figure 3: $P_4 \bigodot F_6^2$ is signed product cordial

can easily show that $v_{-1} - v_1 = 0$ and $e_{-1} - e_1 = 0$. Thus$P_{4r+1} \bigodot F_{4s+2}^2, s \geq 1$ is signed product cordial.

<u>Case (3).</u> $k \equiv 2(mod4)$.

Let $k = 4r + 2, r \geq 0$. Then, one can choose the labeling $[L_{4r}1(-1) : (-1)\ M'_{4s+2}, (-1)M'_{4s+2}, 1M_{4s+2}, 1M_{4s+2}, \ldots, (r - times), 1M_{4s+2}, (-1)M'_{4s+2}]$ for $P_{4r+2} \bigodot F_{4s+2}^2$. Therefore $x_{-1} = x_1 = a_1 = 2r + 1, a_{-1} = 2r, y_{-1} = 2s+2, y_1 = 2s+1, b_{-1} = 6s+2, b_1 = 6s+1, y'_{-1} = 2s+1, y'_1 = 2s+2, b'_{-1} = 6s+2$ and $b'_1 = 6s+1$. Hence, one can easily show that $v_{-1} - v_1 = 0$ and $e_{(-1)} - e_1 = -1$. Thus $P_{4r+2} \bigodot F_{4s+2}^2, s \geq 1$ is signed product cordial.

<u>Case (4).</u> $k \equiv 3(mod4)$.

Let $k = 4r + 3, r \geq 0$. Then, one can choose the labeling $[L_{4r}1_2(-1) : (-1)M'_{4s+2}, (-1)M'_{4s+2}, 1M_{4s+2}, 1M_{4s+2}, \ldots, (r - times), 1M_{4s+2}, (-1)_3 1_2 M'_{4s-2}, (-1)M'_{4s+2}]$ for $P_{4r+3} \bigodot F_{4s+2}^2$. Therefore $x_{-1} = 2r+1, x_1 = a_1 = 2r+2, y_{-1} = 2s+2, a_{-1} = 2r, y_{-1} = 2s+2, y_1 = 2s+1, b_{-1} = 6s+2, b_1 = 6s+1, y'_{-1} = 2s+1, y'_1 = 2s+2, b'_{-1} = 6s+2, b'_1 = 6s+1, y''_{-1} = 2s+2, y''_1 = 2s+1, b''_{-1} = 6s+2$ and $b''_1 = 6s+1$. Hence, one can easily show that $v_{-1} - v_1 = 0$ and $e_{-1} - e_1 = 0$. Thus $P_{4r+3} \bigodot F_{4s+2}^2, s \geq 1$ is signed product cordial.

Lemma 3.4. *$P_k \bigodot F_m^2$ is signed product cordial for all $k \geq 1$ and $m \equiv 3(mod4)$.*

Proof of Lemma 3.4. Will be examined following cases:

<u>Case (1).</u> $k \equiv 0(mod4)$.

Let $k = 4r, r \geq 1$. Then, one can choose the labeling $[L_{4r} : 1(-1)_3 M'_{4s}, 1(-1)_3\ M'_{4s}, 1(-1)1_2 M_{4s}, 1(-1)1_2 M_{4s}, \ldots, (r - times)]$ for $P_{4r} \bigodot F_{4s+3}^2$. Therefore $x_{-1} = x_1 = a_1 = 2r, a_{-1} = 2r - 1, y_{-1} = 2s + 3, y_1 = 2s + 1, b_{-1} = 6s + 4, b_1 = 6s + 2, y'_{-1} = 2s + 1, y'_1 = 2s + 3$ and $b'_{-1} = 6s + 4, b'_1 = 6s + 2$. Hence, one can easily show that $v_{-1} - v_1 = 0$ and $e_{-1} - e_1 = -1$. Thus $P_{4r} \bigodot F_{4s+3}^2, s \geq 1$ is signed product cordial. As an example, Figure 4 illustrates this case $P_4 \bigodot F_7^2$.

<u>Case (2).</u> $k \equiv 1(mod4)$.

Let $k = 4r+1, r \geq 0$. Then, one can choose the labeling $[L_{4r}1 : 1(-1)_3 M'_{4s}, 1\ (-1)_3 M'_{4s}, 1(-1)1_2 M_{4s}, 1(-1)1_2 M_{4s}, \ldots, (r - times), (-1)1_2(-1)M'_{4s}]$ for P_{4r+1}

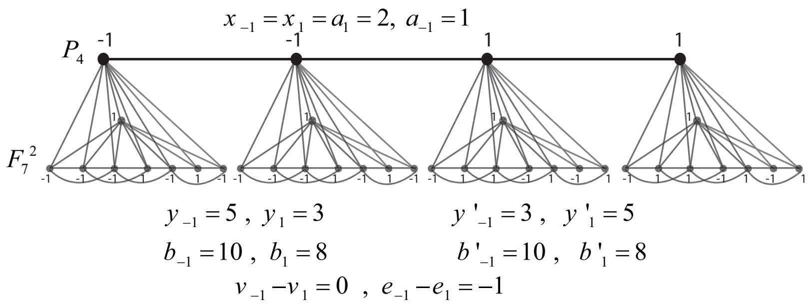

Figure 4: $P_4 \odot F_7^2$ is signed product cordial

$\odot F_{4s+3}^2$. Therefore $x_{-1} = 2r, x_1 = a_{-1} = 2r-1, a_1 = 2r+1, y_{-1} = 2s+3, y_1 = 2s+1, b_{-1} = 6s+2, b_1 = 6s+4, y'_{-1} = 2s+1, y'_1 = 2s+3, b'_{-1} = 6s+2, b'_1 = 6s+4, y''_{-1} = y''_1 = 2s+2, b''_{-1} = 6s+2$ and $b''_1 = 6s+4$. Hence, one can easily show that $v_{-1} - v_1 = -1$ and $e_{-1} - e_1 = 0$. Thus $P_{4r+1} \odot F_{4s+3}^2, s \geq 1$ is signed product cordial.

<u>**Case (3).**</u> $\boldsymbol{k \equiv 2(mod\ 4)}$**.**

Let $k = 4r+2, r \geq 0$. Then, one can choose the labeling $[L_{4r}1(-1) : 1(-1)_3M'_{4s}, 1\,(-1)_3M'_{4s}, 1(-1)1_2M_{4s}, 1(-1)1_2M_{4s}, \ldots, (r{-}times), 1(-1)1_2\ M_{4s}, 1(-1)_3M'_{4s}]$ for $P_{4r+2} \odot F_{4s+3}^2$. Therefore, $x_{-1} = x_1 = a_1 = 2r+1, a_{-1} = 2r, y_{-1} = 2s+3, y_1 = 2s+1, b_{-1} = 6s+4, b_1 = 6s+2, y'_{-1} = 2s+1, y'_1 = 2s+3, b'_{-1} = 6s+4$ and $b'_1 = 6s+2$. Hence, one can easily show that $v_{-1} - v1 = 0$ and $e_{-1} - e_1 = -1$. Thus $P_{4r+2} \odot F_{4s+3}^2$,$s \geq 1$ is signed product cordial.

<u>**Case (4).**</u> $\boldsymbol{k \equiv 3(mod4)}$.

Let $k = 4r+3, r \geq 0$. Then, one can choose the labeling $[L_{4r}1(-1)_2 : 1(-1)_3M'_{4s},\ 1(-1)_3M'_{4s}, 1(-1)1_2M_{4s}, 1(-1)1_2M_{4s}, \ldots, (r-times), 1(-1)1_2M_{4s}, 1(-1)_3M'_{4s},\ (-1)1_2(-1)M'_{4s}]$ for $P_{4r+3} \odot F_{4s+3}^2$. Therefore $x_{-1} = a_1 = 2r+2, x_1 = 2r+1, a_{-1} = 2r, y_{-1} = 2s+3, y_1 = 2s+1, b_{-1} = 6s+4, b_1 = 6s+2, y'_{-1} = 2s+1, y'_1 = 2s+3, b'_{-1} = 6s+4, b'_1 = 6s+2, y''_{-1} = y''_1 = 2s+2, b''_{-1} = 6s+4$ and $b''_1 = 6s+2$. Hence one can easily show that $v_{-1} - v_1 = 1$ and $e_{-1} - e_1 = 0$. Thus $P_{4r+3} \odot F_{4s+3}^2, s \geq 1$ is signed product cordial.

As a consequence of all previous lemmas, we can establish the following theorem.

Theorem 3.1. *The corona between path and fourth power of fan graphs $P_k \odot F_m^2$ is signed product cordial for all k and m.*

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