Fekete-Szegö inequality for analytic and bi-univalent functions subordinate to Horadam polynomials

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Abstract. In the present paper, a subclass of analytic and bi-univalent functions by means of Horadam Polynomials is introduced. Certain coefficients bounds for functions belonging to this subclass are obtained. Furthermore, the Fekete-Szegö problem for this subclass is solved.

Keywords: Horadam polynomials, bi-univalent functions, analytic functions, Fekete-Szegö problem.

1. Introduction

Let $\mathcal{A}$ denote the class of all analytic functions f defined in the open unit disk $\mathbb{U} = \{z \in \mathbb{C} : |z| < 1\}$ and normalized by the conditions $f(0) = 0$ and $f'(0) = 1$. Thus each $f \in \mathcal{A}$ has a Taylor-Maclaurin series expansion of the form:

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n, \quad (z \in \mathbb{U}). \tag{1}$$

Further, let $\mathcal{S}$ denote the class of all functions $f \in \mathcal{A}$ which are univalent in $\mathbb{U}$ (for details, see [17]). See, also, some of the recent investigations [1, 2, 7, 9, 10, 11, 28]).

Two of the important and well-investigated subclasses of the analytic and univalent function class $\mathcal{S}$ are the class $\mathcal{S}^*(\alpha)$ of starlike functions of order α in $\mathbb{U}$ and the class $\mathcal{K}(\alpha)$ of convex functions of order α in $\mathbb{U}$. By definition, we have

$$\mathcal{S}^*(\alpha) := \left\{ f : \ f \in \mathcal{S} \ \text{ and } \ \mathrm{Re}\left\{ \frac{zf'(z)}{f(z)} \right\} > \alpha, \quad (z \in \mathbb{U}; 0 \leq \alpha < 1) \right\}, \tag{2}$$

and

$$\mathcal{K}(\alpha) := \left\{ f : \ f \in \mathcal{S} \ \text{ and } \ \mathrm{Re}\left\{ 1 + \frac{zf''(z)}{f'(z)} \right\} > \alpha, \quad (z \in \mathbb{U}; 0 \leq \alpha < 1) \right\}. \tag{3}$$

It is clear from the definitions (2) and (3) that $\mathcal{K}(\alpha) \subset \mathcal{S}^*(\alpha)$. Also we have

$$f(z) \in \mathcal{K}(\alpha) \;\text{ iff }\; zf'(z) \in \mathcal{S}^*(\alpha),$$

and

$$f(z) \in \mathcal{S}^*(\alpha) \;\text{ iff }\; \int_0^z \frac{f(t)}{t} dt = F(z) \in \mathcal{K}(\alpha).$$

It is well-known that, if $f(z)$ is an univalent analytic function from a domain $\mathbb{D}_1$ onto a domain $\mathbb{D}_2$, then the inverse function $g(z)$ defined by

$$g(f(z)) = z, \quad (z \in \mathbb{D}_1),$$

is an analytic and univalent mapping from $\mathbb{D}_2$ to $\mathbb{D}_1$. Moreover, by the familiar Koebe one-quarter theorem (for details, see [17]), we know that the image of $\mathbb{U}$ under every function $f \in \mathcal{S}$ contains a disk of radius $\frac{1}{4}$.

According to this, every function $f \in \mathcal{S}$ has an inverse map f^{-1} that satisfies the following conditions:

$$f^{-1}(f(z)) = z, \;\; (z \in \mathbb{U}),$$

and

$$f\left(f^{-1}(w)\right) = w, \;\; \left(|w| < r_0(f); r_0(f) \geq \frac{1}{4}\right).$$

In fact, the inverse function is given by

$$f^{-1}(w) = w - a_2w^2 + (2a_2^2 - a_3)w^3 - (5a_2^3 - 5a_2a_3 + a_4)w^4 + \cdots . \tag{4}$$

A function $f \in \mathcal{A}$ is said to be bi-univalent in $\mathbb{U}$ if both $f(z)$ and $f^{-1}(z)$ are univalent in $\mathbb{U}$. Let Σ denote the class of bi-univalent functions in $\mathbb{U}$ given by (1). Examples of functions in the class Σ are

$$\frac{z}{1-z}, \; -\log(1-z), \; \frac{1}{2}\log\left(\frac{1+z}{1-z}\right), \cdots .$$

It is worth noting that the familiar Koebe function is not a member of Σ, since it maps the unit disk $\mathbb{U}$ univalently onto the entire complex plane except the part of the negative real axis from $-1/4$ to $-\infty$. Thus, clearly, the image of the domain does not contain the unit disk $\mathbb{U}$. For a brief history and some intriguing examples of functions and characterization of the class Σ, see Srivastava et al. [24] and Yousef et al. [25, 26, 27, 29, 12, 13, 14, 15].

In 1967, Lewin [22] investigated the bi-univalent function class Σ and showed that $|a_2| < 1.51$. Subsequently, Brannan and Clunie [16] conjectured that $|a_2| \leq \sqrt{2}$. The coefficient estimate problem for each of the following Taylor-Maclaurin coefficients $|a_n|$ $(n \in \mathbb{N}\backslash\{1, 2\})$ for each $f \in \Sigma$ given by (1) is presumably still an open problem.

In 2009, Horzum and Kocer [20] considered the Horadam polynomials $h_n(x)$, which are given by the following recurrence relation:

$$h_n(x) = pxh_{n-1}(x) + qh_{n-2}(x), \quad (n \in \mathbb{N} \setminus \{1,2\}), \tag{5}$$

with

$$h_1(x) = a,\ h_2(x) = bx \text{ and } h_3(x) = pbx^2 + aq \tag{6}$$

for some real constants a, b, p and q.

First off, we present some special cases of the polynomials $h_n(x)$ (see, [20] and [19]):

1. For $a = b = p = q = 1$, we get the Fibonacci polynomials $F_n(x)$.
2. For $a = 2$ and $b = p = q = 1$, we get the Lucas polynomials $L_n(x)$.
3. If $a = 1$, $b = p = 2$ and $q = 1$, then we get the Chebyshev polynomials $U_n(x)$ of the second kind.

The Fibonacci polynomials, the Lucas polynomials, the Chebyshev polynomials and the families of orthogonal polynomials and other special polynomials also their generalizations are very important in different disciplines in the mathematical, physical, statistical, and engineering sciences. These kind of polynomials have been studied in several papers from a theoretical point of view (see, [3, 4, 5, 6, 8, 18, 21, 23]).

Theorem 1.1 ([20])**.** *Let $\Omega(x,z)$ be the generating function of the Horadam polynomials $h_n(x)$. Then,*

$$\Omega(x,z) = \sum_{n=1}^{\infty} h_n(x) z^{n-1} = \frac{a + (b - ap)xz}{1 - pxz - qz^2}. \tag{7}$$

2. The class $\mathfrak{B}_{\Sigma}^{\mu}(a,\lambda,\delta)$

We begin this section by defining the class $\mathfrak{B}_{\Sigma}^{\mu}(a,\lambda,\delta)$ as follows:

Definition 2.1. *For $\lambda \geq 1$, $\mu \geq 0$ and $\delta \geq 0$, a function $f \in \Sigma$ given by (1) is said to be in the class $\mathfrak{B}_{\Sigma}^{\mu}(a,\lambda,\delta)$ if the following subordinations are satisfied:*

$$(1-\lambda)\left(\frac{f(z)}{z}\right)^{\mu} + \lambda f'(z)\left(\frac{f(z)}{z}\right)^{\mu-1} + \xi\delta z f''(z) \prec \Omega(x,z) + 1 - a$$

and

$$(1-\lambda)\left(\frac{f^{-1}(w)}{w}\right)^{\mu} + \lambda\left(f^{-1}(w)\right)'\left(\frac{f^{-1}(w)}{w}\right)^{\mu-1} + \xi\delta z\left(f^{-1}(w)\right)'' \prec \Omega(x,w) + 1 - a,$$

where f^{-1} is given by (4) and $\xi = \frac{2\lambda+\mu}{2\lambda+1}$.

3. Initial coefficient estimates

In this section, we propose to find the estimates on the Taylor-Maclaurin coefficients $|a_2|$ and $|a_3|$ for functions in the class $\mathfrak{B}_{\Sigma}^{\mu}(a,\lambda,\delta)$, which we introduced in Definition 2.1.

We first state Theorem 3.1.

Theorem 3.1. *For $\lambda \geq 1$, $\mu \geq 0$ and $\delta \geq 0$, let $f \in \mathcal{A}$ belongs to the class $\mathfrak{B}_{\Sigma}^{\mu}(a,\lambda,\delta)$. Then*

$$|a_2| \leq \frac{2\,|bx|\,\sqrt{|bx|}}{\sqrt{\left|(\mu+2\lambda)\left[1+\mu+\frac{12\delta}{2\lambda+1}\right](bx)^2 - 2\,(\mu+\lambda+2\xi\delta)^2\,(pbx^2+aq)\right|}},$$

and

$$|a_3| \leq \frac{(bx)^2}{(\mu+\lambda+2\xi\delta)^2} + \frac{|bx|}{(\mu+2\lambda)\left(1+\frac{6\delta}{2\lambda+1}\right)}.$$

Proof. Let $f \in \mathfrak{B}_{\Sigma}^{\mu}(a,\lambda,\delta)$. From Definition 2.1, for some analytic functions ϕ, ψ such that $\phi(0)=\psi(0)=0$ and $|\phi(z)|<1$, $|\psi(w)|<1$ for all $z,w \in \mathbb{U}$, then we can write

$$\begin{aligned}(8)\qquad &(1-\lambda)\left(\frac{f(z)}{z}\right)^{\mu} + \lambda f'(z)\left(\frac{f(z)}{z}\right)^{\mu-1} + \xi\delta z f''(z)\\ &= 1 + h_1(x) - a + h_2(x)\phi(z) + h_3(x)\phi^2(z) + \cdots\end{aligned}$$

and

$$\begin{aligned}(9)\qquad &(1-\lambda)\left(\frac{f^{-1}(w)}{w}\right)^{\mu} + \lambda\left(f^{-1}(w)\right)'\left(\frac{f^{-1}(w)}{w}\right)^{\mu-1} + \xi\delta z\left(f^{-1}(w)\right)''\\ &= 1 + h_1(x) - a + h_2(x)\psi(w) + h_3(x)\psi^2(w) + \cdots .\end{aligned}$$

From the equalities (8) and (9), we obtain that

$$\begin{aligned}(10)\qquad &(1-\lambda)\left(\frac{f(z)}{z}\right)^{\mu} + \lambda f'(z)\left(\frac{f(z)}{z}\right)^{\mu-1} + \xi\delta z f''(z)\\ &= 1 + h_2(x)r_1 z + \left[h_2(x)r_2 + h_3(x)r_1^2\right]z^2 + \cdots\end{aligned}$$

and

$$\begin{aligned}(11)\qquad &(1-\lambda)\left(\frac{f^{-1}(w)}{w}\right)^{\mu} + \lambda\left(f^{-1}(w)\right)'\left(\frac{f^{-1}(w)}{w}\right)^{\mu-1} + \xi\delta z\left(f^{-1}(w)\right)''\\ &= 1 + h_2(x)s_1 w + \left[h_2(x)s_2 + h_3(x)s_1^2\right]w^2 + \cdots .\end{aligned}$$

It is fairly well known that if

$$|\phi(z)| = \left|r_1 z + r_2 z^2 + r_3 z^3 + \cdots\right| < 1, \quad (z \in \mathbb{U})$$

and

$$|\psi(w)| = \left|s_1 w + s_2 w^2 + s_3 w^3 + \cdots\right| < 1, \quad (w \in \mathbb{U}),$$

then

$$|r_k| < 1 \text{ and } |s_k| < 1 \text{ for } k \in \mathbb{N}. \tag{12}$$

Thus, upon comparing the corresponding coefficients in (10) and (11), we have

$$(\mu + \lambda + 2\xi\delta)\, a_2 = h_2(x) r_1, \tag{13}$$

$$(\mu + 2\lambda)\left[\left(\frac{\mu - 1}{2}\right) a_2^2 + \left(1 + \frac{6\delta}{2\lambda + 1}\right) a_3\right] = h_2(x) r_2 + h_3(x) r_1^2, \tag{14}$$

$$-(\mu + \lambda + 2\xi\delta)\, a_2 = h_2(x) s_1, \tag{15}$$

and

$$(\mu + 2\lambda)\left[\left(\frac{\mu + 3}{2} + \frac{12\delta}{2\lambda + 1}\right) a_2^2 - \left(1 + \frac{6\delta}{2\lambda + 1}\right) a_3\right] = h_2(x) s_2 + h_3(x) s_1^2. \tag{16}$$

It follows from (13) and (15) that

$$r_1 = -s_1 \tag{17}$$

and

$$2(\mu + \lambda + 2\xi\delta)^2 a_2^2 = h_2^2(x)\left(r_1^2 + s_1^2\right). \tag{18}$$

If we add (14) and (16), we get

$$(\mu + 2\lambda)\left[1 + \mu + \frac{12\delta}{2\lambda + 1}\right] a_2^2 = h_2(x)(r_2 + s_2) + h_3(x)\left(r_1^2 + s_1^2\right). \tag{19}$$

Substituting the value of $\left(r_1^2 + s_1^2\right)$ from (18) in the right hand side of (19), we deduce that

$$\left[(\mu + 2\lambda)\left[1 + \mu + \frac{12\delta}{2\lambda + 1}\right] h_2^2(x) - 2(\mu + \lambda + 2\xi\delta)^2 h_3(x)\right] a_2^2 = h_2^3(x)(r_2 + s_2). \tag{20}$$

Moreover, computations using (6), (12) and (20), we find that

$$|a_2| \le \frac{2|bx|\sqrt{|bx|}}{\sqrt{\left|(\mu + 2\lambda)\left[1 + \mu + \frac{12\delta}{2\lambda+1}\right](bx)^2 - 2(\mu + \lambda + 2\xi\delta)^2 (pbx^2 + aq)\right|}}.$$

Moreover, if we subtract (16) from (14), we obtain

$$2\left(\mu+2\lambda\right)\left(1+\frac{6\delta}{2\lambda+1}\right)\left(a_3-a_2^2\right)=h_2(x)\left(r_2-s_2\right)+h_3(x)\left(r_1^2-s_1^2\right). \tag{21}$$

Then, in view of (17) and (18), Eq. (21) becomes

$$a_3=\frac{h_2^2(x)}{2\left(\mu+\lambda+2\xi\delta\right)^2}\left(r_1^2+s_1^2\right)+\frac{h_2(x)}{2\left(\mu+2\lambda\right)\left(1+\frac{6\delta}{2\lambda+1}\right)}\left(r_2-s_2\right).$$

Thus applying (6), we conclude that

$$|a_3|\leq\frac{(bx)^2}{\left(\mu+\lambda+2\xi\delta\right)^2}+\frac{|bx|}{\left(\mu+2\lambda\right)\left(1+\frac{6\delta}{2\lambda+1}\right)}.$$

So, the proof of Theorem 3.1 is complete. □

By setting $\mu=\delta=0$ and $\lambda=1$ in Theorem 3.1, we obtain the following consequence.

Corollary 3.1. *If f belongs to the class $\mathfrak{B}_\Sigma(a,1)=\mathcal{S}^*_\Sigma$ of bi-starlike functions, then*

$$|a_2|\leq\frac{2\,|bx|\,\sqrt{|bx|}}{\sqrt{\left|2\left(bx\right)^2-2\left(pbx^2+aq\right)\right|}},$$

and

$$|a_3|\leq(bx)^2+\frac{|bx|}{2}.$$

4. Fekete-Szegö problem for the function class $\mathfrak{B}^\mu_\Sigma(a,\lambda,\delta)$

In this section, we aim to provide Fekete-Szegö inequalities for functions in the class $\mathfrak{B}^\mu_\Sigma(a,\lambda,\delta)$. These inequalities are given in the following theorem.

Theorem 4.1. *For $\lambda\geq1$, $\mu\geq0$ and $\delta\geq0$, let $f\in\mathcal{A}$ belongs to the class $\mathfrak{B}^\mu_\Sigma(a,\lambda,\delta)$. Then*

$$\left|a_3-\upsilon a_2^2\right|\leq\begin{cases}\dfrac{|bx|}{\left(\mu+2\lambda\right)\left(1+\frac{6\delta}{2\lambda+1}\right)}, & |\upsilon-1|\leq\dfrac{1}{2\left(\mu+2\lambda\right)\left(1+\frac{6\delta}{2\lambda+1}\right)}\times|\Upsilon(x)|,\\[2ex] \dfrac{2\,|bx|\,|1-\upsilon|}{|\Upsilon(x)|}, & |\upsilon-1|\geq\dfrac{1}{2\left(\mu+2\lambda\right)\left(1+\frac{6\delta}{2\lambda+1}\right)}\times|\Upsilon(x)|,\end{cases}$$

where $\Upsilon(x)=[(\mu+2\lambda)[1+\mu+\frac{12\delta}{2\lambda+1}]-2(\mu+\lambda+2\xi\delta)^2\frac{(pbx^2+aq)}{(bx)^2}]$.

Proof. From (20) and (21)

$$
\begin{aligned}
a_3 - \upsilon a_2^2 &= (1-\upsilon)\,\frac{h_2^3(x)\,(r_2+s_2)}{\left[(\mu+2\lambda)\left[1+\mu+\frac{12\delta}{2\lambda+1}\right]h_2^2(x) - 2\,(\mu+\lambda+2\xi\delta)^2\,h_3(x)\right]} \\
&\quad + \frac{h_2(x)}{2\,(\mu+2\lambda)\left(1+\frac{6\delta}{2\lambda+1}\right)}\,(r_2-s_2) \\
&= h_2(x)\left[\left[\varphi(\upsilon,x) + \frac{1}{2\,(\mu+2\lambda)\left(1+\frac{6\delta}{2\lambda+1}\right)}\right] r_2 \right.\\
&\quad \left. + \left[\varphi(\upsilon,x) - \frac{1}{2\,(\mu+2\lambda)\left(1+\frac{6\delta}{2\lambda+1}\right)}\right] s_2\right],
\end{aligned}
$$

where

$$
\varphi(\upsilon,x) = \frac{h_2^2(x)\,(1-\upsilon)}{\left[(\mu+2\lambda)\left[1+\mu+\frac{12\delta}{2\lambda+1}\right]h_2^2(x) - 2\,(\mu+\lambda+2\xi\delta)^2\,h_3(x)\right]},
$$

Then, in view of (6), we conclude that

$$
\left|a_3 - \upsilon a_2^2\right| \le \begin{cases} \frac{|h_2(x)|}{(\mu+2\lambda)\left(1+\frac{6\delta}{2\lambda+1}\right)} & 0 \le |\varphi(\upsilon,x)| \le \frac{1}{2(\mu+2\lambda)\left(1+\frac{6\delta}{2\lambda+1}\right)}, \\ 2\,|h_2(x)|\,|\varphi(\upsilon,x)| & |\varphi(\upsilon,x)| \ge \frac{1}{2(\mu+2\lambda)\left(1+\frac{6\delta}{2\lambda+1}\right)}. \end{cases}
$$

Which completes the proof of Theorem 4.1. □

Putting $\mu = \delta = 0$ and $\lambda = 1$ in Theorem 4.1, we conclude the following result:

Corollary 4.1. *If f belongs to the class $\mathcal{S}_\Sigma^*$, then*

$$
\left|a_3 - \upsilon a_2^2\right| \le \begin{cases} \dfrac{|bx|}{2}, & |\upsilon - 1| \le \left|\dfrac{(bx)^2 - \left(pbx^2+aq\right)}{2(bx)^2}\right| \\[2ex] \dfrac{2\,|bx|^3\,|1-\upsilon|}{|(bx)^2 - (pbx^2+aq)|}, & |\upsilon - 1| \ge \left|\dfrac{(bx)^2 - \left(pbx^2+aq\right)}{2(bx)^2}\right|. \end{cases}
$$

Putting $\upsilon = 1$ in Theorem 4.1, we conclude the following result:

Corollary 4.2. *If f belongs to the class $\mathcal{S}_\Sigma^*$, then*

$$
\left|a_3 - a_2^2\right| \le \frac{|bx|}{(\mu+2\lambda)\left(1+\frac{6\delta}{2\lambda+1}\right)}.
$$

References

[1] T. Al-Hawary, A. A. Amourah, M. Darus, *Differential sandwich theorems for p-valent functions associated with two generalized differential operator and integral operator*, International Information Institute (Tokyo), Information, 17 (2014), 3559.

[2] T. Al-Hawary, A. A. Amourah, Feras Yousef, M. Darus, *A certain fractional derivative operator and new class of analytic functions with negative coefficients*, International Information Institute (Tokyo), Information, 18 (2015), 4433-4441.

[3] S. Altınkaya, Ş. Yalçın, *On the* $(p,q)-$*Lucas polynomial coefficient bounds of the bi-univalent function class* σ, Boletín de la Sociedad Matemática Mexicana, (2018), 1-9.

[4] Ala Amourah, *Faber polynomial coefficient estimates for a class of analytic bi-univalent functions*, AIP Conference Proceedings, 2096, AIP Publishing LLC, 2019.

[5] Ala Amourah, *Initial bounds for analytic and bi-univalent functions by means of (p,q)-Chebyshev polynomials defined by differential operator*, General Letters in Mathematics, 7 (2019), 45-51.

[6] A. A. Amourah, *Fekete-Szegö inequality for analytic and bi-univalent functions subordinate to* $(p,q)-$*Lucas polynomials*, TWMS Journal of Applied and Engineering Mathematics, 2019, accepted.

[7] A. A. Amourah, M. Darus, *Some properties of a new class of univalent functions involving a new generalized differential operator with negative coefficients*, Indian Journal of Science and Technology, 9 (2016), 1-7.

[8] A. A. Amourah, M. Illafe, *A Comprehensive subclass of analytic and bi-univalent functions associated with subordination*, Palestine Journal of Mathematics, 9 (2020), 187-193.

[9] A. A. Amourah, F. Yousef, T. Al-Hawary, M. Darus, *On a class of p-valent non-Bazilevič functions of order* $\mu + i\beta$, International Journal of Mathematical Analysis, 10 (2016), 701-710.

[10] A. A. Amourah, F. Yousef, T. Al-Hawary, M. Darus, *On* $H_3(p)$ *Hankel determinant for certain subclass of p-valent functions*, Italian Journal of Pure and Applied Mathematics Math., 37 (2017), 611-618.

[11] A. A. Amourah, F. Yousef, *Some properties of a class of analytic functions involving a new generalized differential operator*, Boletim da Sociedade Paranaense de Matemática, 38 (2020), 33-42.

[12] A. Amourah, B. A. Frasin, G. Murugusundaramoorthy, T. Al-Hawary, *Bi-Bazilevič functions of order $\vartheta + i\delta$ associated with $(p,q)-$Lucas polynomials*, AIMS Mathematics, 6 (2021), 4296-4305.

[13] A. Amourah, T. Al-Hawary, B. A. Frasin, *Application of Chebyshev polynomials to certain class of bi-Bazilevič functions of order $\alpha + i\beta$*, Afr. Mat., (2021), 1-8.

[14] A. Amourah, B.A. Frasin, T. Abdeljawad, *Fekete-Szegö inequality for analytic and bi-univalent functions subordinate to Gegenbauer polynomials*, J. Funct. Spaces, 2021, Article ID 5574673, 7 pages.

[15] Ala Amourah, A. Alamoush, M. Al-Kaseasbeh, *Gegenbauer polynomials and bi-univalent functions*, Palestine Journal of Mathematics, 10 (2021).

[16] D.A. Brannan, J.G. Clunie, *Aspects of contemporary complex analysis,* Academic Press, New York and London, 1980.

[17] P.L. Duren, *Univalent functions*, Grundlehren der Mathematischen Wissenschaften, Band 259, Springer-Verlag, New York, 1983.

[18] L. Gwang Yeon, M. Asci, *Some properties of the $(p,q)-$Fibonacci and $(p,q)-$Lucas polynomials*, J. App. Math. 2012 (2012), 264842-1.

[19] A.F. Horadam, J. M. Mahon, *Pell and Pell-Lucas polynomials*, The Fibonacci Quarterly, 23 (1985), 7-20.

[20] T. Horzum, E. G. Kocer, *On some properties of Horadam polynomials*, Int. Math. Forum., 25 (2009).

[21] T. Koshy, *Fibonacci and Lucas numbers with applications*, Wiley, 2019.

[22] M. Lewin, *On a coefficient problem for bi-univalent functions*, Proc. Amer. Math. Soc., 18 (1967), 63-68.

[23] H. M. Srivastava, Ş. Altınkaya, S. Yalçın, *Certain subclasses of bi-univalent functions associated with the Horadam polynomials*, Iranian Journal of Science and Technology, Transactions A: Science, 43 (2019), 1873-1879.

[24] H.M. Srivastava, A.K. Mishra, P. Gochhayat, *Certain subclasses of analytic and bi-univalent functions*, Appl. Math. Lett., 23 (2010), 1188-1192.

[25] F. Yousef, T. Al-Hawary, G. Murugusundaramoorthy, *Fekete-Szegö functional problems for some subclasses of bi-univalent functions defined by Frasin differential operator*, Afrika Matematika, 30 (2019), 495-503.

[26] F. Yousef, S. Alroud, M. Illafe, *A comprehensive subclass of bi-univalent functions associated with Chebyshev polynomials of the second kind*, Boletín de la Sociedad Matemática Mexicana, 26 (2020), 329-339.

[27] F. Yousef, S. Alroud, M. Illafe, *New subclasses of analytic and bi-univalent functions endowed with coefficient estimate problems*, Analysis and Mathematical Physics, 11 (2021), 1-12.

[28] F. Yousef, A. A. Amourah, M. Darus, *Differential sandwich theorems for p-valent functions associated with a certain generalized differential operator and integral operator*, Italian Journal of Pure and Applied Mathematics, 36 (2016), 543-556.

[29] F. Yousef, B. A. Frasin, T. Al-Hawary, *Fekete-Szegö inequality for analytic and bi-univalent functions subordinate to chebyshev polynomials*, Filomat, 32 (2018), 3229-3236.

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