Coefficient estimates for new subclass of pseudo-type meromorphic bi-univalent functions

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Abstract. In the present article, we define a new subclass of pseudo-type meromorphic bi-univalent functions defined on $\mathbb{U}^* = \{z \mid : z \in \mathbb{C} \text{ and } 1 < |z| < \infty\}$, and investigate the initial coefficient estimates $|b_0|$ and $|b_1|$. Further we mention several new or known consequences of our results.

Keywords: analytic functions, univalent functions, meromorphic univalent functions, Bi-univalent functions, meromorphic bi-univalent functions, pseudo functions.

1. Introduction

Let A denote the class of functions $f(z)$ of the form:

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n \tag{1}$$

which are analytic in the open unit open disk $\mathbb{U} = \{z : z \in \mathbb{C}, |z| < 1\}$.

Also, let $\mathbb{S}$ be class of all functions in A which are univalent and normalized by the conditions

$$f(0) = 0 = f^{'}(0) - 1$$

in $\mathbb{U}$. Some of the important and well-investigated subclasses of the univalent function class $\mathbb{S}$ includes the class $S^*(\alpha)(0 \le \alpha < 1)$ of starlike functions of order α in $\mathbb{U}$ and the class $K(\alpha)(0 \le \alpha < 1)$ of convex functions of order α.

Let Σ denote the class of meromorphic univalent functions g of the form

$$g(z) = z + \sum_{n=0}^{\infty} \frac{b_n}{z^n} \tag{2}$$

defined on the domain $\mathbb{U}^* = \{z : z \in \mathbb{C}, 1 < |z| < \infty\}$. It is well known that every function $g \in \Sigma$ has an inverse $g^{-1} = h$, defined by

$$g^{-1}(g(z)) = z, \quad (z \in \mathbb{U}^*),$$

and

$$g^{-1}(g(w)) = w, \quad (M < |w| < \infty,\ M > 0),$$

where

$$g^{-1}(w) = h(w) = w + \sum_{n=0}^{\infty} \frac{B_n}{w^n}$$

$$= w - b_0 - \frac{b_1}{w} - \frac{b_1 b_0 + b_2}{w^2} - \frac{b_1^2 + b_1 b_0^2 + 2b_0 b_2 + b_3}{w^3} + \cdots . \tag{3}$$

A simple computation shows that

$$w = g(h(w)) = (b_0 + B_0) + w + \frac{b_1 + B_1}{w} + \frac{B_2 - b_1 B_0 + b_2}{w^2}$$

$$+ \frac{B_3 - b_1 B_1 + b_1 B_0^2 - 2b_2 B_0 + b_3}{w^3} + \cdots . \tag{4}$$

Comparing the initial coefficients in (4), we find that

$$b_0 + B_0 = 0 \Rightarrow \quad B_0 = -b_0,$$
$$b_1 + B_1 = 0 \Rightarrow \quad B_1 = -b_1,$$
$$B_2 - b_1 B_0 + b_2 = 0 \Rightarrow B_2 = -(b_2 + b_1 b_0),$$
$$B_3 - b_1 B_1 + b_1 B_0^2 - 2b_2 B_0 + b_3 = 0 \Rightarrow B_3 = -(b_3 + 2b_0 b_1 + b_1 b_0^2 + b_1^2).$$

A function $f \in \Sigma$ is said to be meromorphic bi-univalent if $f^{-1} \in \Sigma$. The family of all meromorphic bi-univalent functions is denoted by Σ'. Estimates on the coefficient of meromorphic univalent functions were widely investigated in the literature; for example, Schiffer [1] obtained the estimate $|b_2| < \frac{3}{2}$ for meromorphic univalent functions $f \in \mathbb{S}$ with $b_0 = 0$. In 1983, Duren [2] obtained the inequality $|b_2| < \frac{2}{n+1}$ for $f \in \mathbb{S}$ with $b_k = 0, 1 \leq k \leq \frac{n}{2}$. For the coefficients of inverses of meromorphic univalent functions Springer [3] proved that

$$|B_3| < 1 \text{ and } |B_3 + \frac{1}{2} B_1^2| < \frac{1}{2},$$

and conjectured that

$$|B_{2n-1}| \leq \frac{(2n-2)!}{n!(n-1)!} \ (n = 1, 2, ...).$$

In 1977, Kubota [4] has proved that the Springer conjecture is true for n = 3;4;5; and subsequently Schober [5] obtained sharp bounds for $|B_{2n-1}|$ if $1 \leq n \leq 7$. In 2007. Kapoor and Mishra [6] found the coefficient estimates for the inverse of meromorphic starlike univalent functions of order α in $\mathbb{U}^*$.

Recently, several researchers (for example [7], [9], [10], [11], [12], [13], [14], [15], [16], [18], [19]) introduced various subclasses of bi-univalent functions and meromorphically bi-univalent functions. Estimates on the initial coefficients for functions in each of these subclasses are obtained.

In 2013, Babalola [20] defined a new subclass λ-pseudo starlike function of order $0 \leq \beta < 1$ satisfying the analytic condition

$$\Re\left\{\frac{z(f(z)')^{\lambda}}{f(z)}\right\} > \beta \ (\lambda \geq 1,\ z \in \mathbb{U}). \tag{5}$$

In particular, Babalola [20] proved that all λ-pseudo-starlike functions are Bazilevic of type $1 - \frac{1}{\lambda}$ and order $\beta^{\frac{1}{\lambda}}$ and are univalent in open unit disk $\mathbb{U}$.

In this paper, motivated by the previous works, we introduce two new subclasses of pseudo-type of meromorphically bi-univalent functions and obtained the estimates for the initial coefficients $|b_0|$ and $|b_1|$ of functions in these subclasses.

In order to derive our main results, we first recall the following lemma.

Carathéodory's Lemma (see, for example, [21]). If $p \in P$, then

$$|p_j| \leq 2, \qquad (j \in \mathbb{N}),$$

where P is the family of all functions $p(z)$, analytic in $\mathbb{U}^*$, for which

$$\Re(p(z)) > 0, \qquad (z \in \mathbb{U}^*),$$

where

$$p(z) = 1 + \frac{p_1}{z} + \frac{p_2}{z_2} + \frac{p_3}{z_3} + ... \qquad (z \in \mathbb{U}^*),$$

$\mathbb{N}$ being the set of positive integers.

2. Coefficient bounds for the function class $\Sigma'_{\lambda,\beta}(\alpha)$

We begin by introducing the function class $\Sigma'_{\lambda,\beta}(\alpha)$ by means of the following definition.

Definition 2.1. *A function $g(z) \in \Sigma'$ given by (2) is said to be in the class $\Sigma_{\lambda,\beta}(\alpha)$ if the following conditions are satisfied:*

$$\left|\arg\left[(1-\beta)\left(\frac{g(z)}{z}\right)^{\lambda} + \beta\left(\frac{z(g(z)')^{\lambda}}{g(z)}\right)\right]\right| < \frac{\alpha\pi}{2}, \tag{6}$$

$(0 < \beta \leq 1, 0 < \alpha \leq 1, \lambda \geq 1, z \in \mathbb{U}^*)$, *and*

$$\left|\arg\left[(1-\beta)\left(\frac{h(w)}{w}\right)^{\lambda} + \beta\left(\frac{w(h(w)')^{\lambda}}{h(w)}\right)\right]\right| < \frac{\alpha\pi}{2}, \tag{7}$$

$(0 < \beta \leq 1, 0 < \alpha \leq 1, \lambda \geq 1, w \in \mathbb{U}^*)$, *where the function h is given by (3).*

We denote by $\Sigma'_{\lambda,\beta}(\alpha)$ the class of meromorphically strongly (λ, β)-pseudo-starlike bi-univalent functions of order $\alpha \in \mathbb{U}^*$.

We begin this section by finding the estimates on the Coefficients $|b_1|$ and $|b_2|$ for functions in the class $\Sigma'_{\lambda,\beta}(\alpha)$.

Theorem 2.1. *Let $g(z)$ be given by (2) be in the class $\Sigma'_{\lambda,\beta}(\alpha)$. Then*

$$|b_0| \leq \frac{2\alpha}{(\beta + \beta\lambda - \lambda)} \tag{8}$$

and

$$|b_1| \leq 2\alpha^2 \left(\frac{\sqrt{[\lambda(\lambda-1)(1-\beta)+2\beta]^2 + (\lambda - \lambda\beta - \beta)^4}}{(\beta + 2\lambda\beta - \lambda)(\lambda - \lambda\beta - \beta)^2} \right). \tag{9}$$

Proof. Let $g \in \Sigma'_{\lambda,\beta}(\alpha)$. Then, by Definition 2.1 of meromorphically bi-univalent function class $\Sigma'_{\lambda,\beta}(\alpha)$, the conditions (6) and (7) can be rewritten as follows:

$$(1-\beta)\left(\frac{g(z)}{z}\right)^{\lambda} + \beta\left(\frac{z(g(z)')^{\lambda}}{g(z)}\right) = [p(z)]^{\alpha} \tag{10}$$

and

$$(1-\beta)\left(\frac{h(w)}{w}\right)^{\lambda} + \beta\left(\frac{w(h(w)')^{\lambda}}{h(w)}\right) = [q(w)]^{\alpha}, \tag{11}$$

respectively. Here, and in what follows,the functions $p(z) \in P$ and $q(w) \in P$ have the following forms:

$$p(z) = 1 + \frac{p_1}{z} + \frac{p_2}{z^2} + \frac{p_3}{z^3} + \cdots \qquad (z \in \mathbb{U}^*) \tag{12}$$

and

$$q(w) = 1 + \frac{q_1}{w} + \frac{q_2}{w^2} + \frac{q_3}{w^3} + \cdots \qquad (w \in \mathbb{U}^*). \tag{13}$$

Clearly, we have

$$\begin{aligned}[p(z)]^{\alpha} &= 1 + \frac{\alpha p_1}{z} + \frac{\frac{1}{2}\alpha(\alpha-1)p_1^2 + \alpha p_2}{z^2} \\ &\quad + \frac{\frac{1}{6}\alpha(\alpha-1)(\alpha-2)p_1^3 + \alpha(\alpha-1)p_1p_2 + \alpha p_3}{z^3} + \cdots\end{aligned}$$

and

$$\begin{aligned}[q(w)]^{\alpha} &= 1 + \frac{\alpha q_1}{w} + \frac{\frac{1}{2}\alpha(\alpha-1)q_1^2 + \alpha q_2}{w^2} \\ &\quad + \frac{\frac{1}{6}\alpha(\alpha-1)(\alpha-2)q_1^3 + \alpha(\alpha-1)q_1q_2 + \alpha q_3}{w^3} + \cdots \quad .\end{aligned}$$

We also find that

$$\begin{aligned}&(1-\beta)\left(\frac{g(z)}{z}\right)^{\lambda} + \beta\left(\frac{z(g(z)')^{\lambda}}{g(z)}\right) \\ &= 1 + \frac{(\lambda - \lambda\beta - \beta)b_0}{z} + \frac{\frac{1}{2}(\lambda(\lambda-1)(1-\beta)+2\beta)b_0^2 + (\lambda - \beta - 2\lambda\beta)b_1}{z^2} \\ &\quad + \frac{\begin{array}{c}\frac{1}{6}(\lambda(\lambda-1)(\lambda-2)(1-\beta) - 6\beta)b_0^3 \\ + (\lambda(\lambda-1)(1-\beta) + 2\beta + \lambda\beta)b_0b_1 + (\lambda - \beta - 3\lambda\beta)b_2\end{array}}{z^3} + \cdots\end{aligned} \tag{14}$$

and

$$(1-\beta)\left(\frac{h(w)}{w}\right)^{\lambda}+\beta\left(\frac{w(h(w)')^{\lambda}}{h(w)}\right)$$
$$=1-\frac{(\lambda-\lambda\beta-\beta)b_0}{w}+\frac{\frac{1}{2}(\lambda(\lambda-1)(1-\beta)+2\beta)b_0^2+(\beta-\lambda+2\lambda\beta)b_1}{w^2}$$
$$+\frac{\frac{1}{6}(6\beta-\lambda(\lambda-1)(\lambda-2)(1-\beta))b_0^3+(\lambda(\lambda-1)(1-\beta)-\lambda(1-\beta)+3\beta+3\lambda\beta)b_0b_1+(\beta-\lambda+3\lambda\beta)b_2}{w^3}+\cdots \quad . \tag{15}$$

Now, equating the Coefficients in (10) and (11), we get

$$(\lambda-\lambda\beta-\beta)b_0=\alpha p_1, \tag{16}$$
$$\frac{1}{2}(\lambda(\lambda-1)(1-\beta)+2\beta)b_0^2+(\lambda-\beta-2\lambda\beta)b_1=\frac{1}{2}\alpha(\alpha-1)p_1^2+\alpha p_2, \tag{17}$$
$$-(\lambda-\lambda\beta-\beta)b_0=\alpha q_1, \tag{18}$$
$$\frac{1}{2}(\lambda(\lambda-1)(1-\beta)+2\beta)b_0^2+(\beta-\lambda+2\lambda\beta)b_1=\frac{1}{2}\alpha(\alpha-1)q_1^2+\alpha q_2. \tag{19}$$

From (16) and (18), we find that

$$p_1=-q_1 \tag{20}$$

and

$$2(\lambda-\lambda\beta-\beta)^2b_0^2=\alpha^2(p_1^2+q_1^2) \tag{21}$$

that is,

$$b_0^2=\frac{\alpha^2(p_1^2+q_1^2)}{2(\lambda-\lambda\beta-\beta)^2}.$$

Applying Carathéodory's Lemma for the Coefficients p_1 and q_1, we immediately have

$$|b_0|\leq\frac{2\alpha}{(\beta+\lambda\beta-\lambda)}.$$

This gives the bound on$|b_0|$ as asserted in (8).

Next, in order to find the bound on $|b_0|$, by using the equation (17) and the equation (19),we get

$$\left(\frac{1}{2}(\lambda(\lambda-1)(1-\beta)+2\beta)b_0^2+(\lambda-\beta-2\lambda\beta)b_1\right)$$
$$\cdot\left(\frac{1}{2}(\lambda(\lambda-1)(1-\beta)+2\beta)b_0^2+(\beta-\lambda+2\lambda\beta)b_1\right)$$

$$= \left(\frac{1}{2}\alpha(\alpha-1)p_1^2+\alpha p_2\right)\cdot\left(\frac{1}{2}\alpha(\alpha-1)q_1^2+\alpha q_2\right),$$

$$\frac{1}{4}(\lambda(\lambda-1)(1-\beta)+2\beta)^2b_0^4-(\beta-\lambda+2\lambda\beta)^2b_1^2$$

$$=\frac{1}{4}\alpha^2(\alpha-1)^2p_1^2q_1^2+\frac{1}{2}\alpha^2(\alpha-1)\left(p_2q_1^2+p_1^2q_2\right)+\alpha^2p_2q_2,$$

$$(\beta-\lambda+2\lambda\beta)^2b_1^2=\left(\frac{1}{2}(\lambda(\lambda-1)(1-\beta)+2\beta)b_0^2\right)^2$$

$$-\frac{1}{4}\alpha^2(\alpha-1)^2p_1^2q_1^2-\frac{1}{2}\alpha^2(\alpha-1)\left(p_2q_1^2+p_1^2q_2\right)-\alpha^2p_2q_2$$

and

$$(\beta-\lambda+2\lambda\beta)^2b_1^2=\left(\frac{1}{4}(\lambda(\lambda-1)(1-\beta)+2\beta)^2\frac{\alpha^4(p_1^2+q_1^2)^2}{4(\lambda-\lambda\beta-\beta)^4}\right)$$

$$-\frac{1}{4}\alpha^2(\alpha-1)^2p_1^2q_1^2-\frac{1}{2}\alpha^2(\alpha-1)\left(p_2q_1^2+p_1^2q_2\right)-\alpha^2p_2q_2.$$

Applying Carathéodory's Lemma once again for the Coefficients p_1, q_1, p_2 and q_2 , we get

$$|b_1|^2\le\alpha^4\left(\frac{(2(\lambda(\lambda-1)(1-\beta)+2\beta))^2+4(\lambda-\lambda\beta-\beta)^4}{(\beta+2\lambda\beta-\lambda)^2(\lambda-\lambda\beta-\beta)^4}\right)\Rightarrow$$

$$|b_1|\le2\alpha^2\left(\frac{\sqrt{[\lambda(\lambda-1)(1-\beta)+2\beta]^2+(\lambda-\lambda\beta-\beta)^4}}{(\beta+2\lambda\beta-\lambda)(\lambda-\lambda\beta-\beta)^2}\right),$$

which evidently completes the proof of Theorem 2.2. □

3. Coefficient bounds for the function class $\Sigma'_{\lambda,\beta}(\mu)$

We first introduce the function class $\Sigma'_{\lambda,\beta}(\mu)$ as follows.

Definition 3.1. *A function $g(z)\in\Sigma'$ given by* (2) *is said to be in the class $\Sigma'_{\lambda,\beta}(\mu)$ if the following conditions are satisfied:*

$$\Re\left((1-\beta)\left(\frac{g(z)}{z}\right)^\lambda+\beta\left(\frac{z(g(z)')^\lambda}{g(z)}\right)\right)>\mu, \tag{22}$$

$(0<\beta\le1,\ 0\le\mu<1,\ \lambda\ge1,\ z\in\mathbb{U}^*)$, *and*

$$\Re\left((1-\beta)\left(\frac{h(w)}{w}\right)^\lambda+\beta\left(\frac{w(h(w)')^\lambda}{h(w)}\right)\right)>\mu, \tag{23}$$

$(0<\beta\le1,\ 0\le\mu<1,\ \lambda\ge1,\ w\in\mathbb{U}^*)$ *where the function h is given by* (3).

We call $\Sigma'_{\lambda,\beta}(\mu)$ the class of meromorphically λ-pseudo-starlike bi-univalent functions of order μ.

Next, we now derive the estimates on the Coefficients $|b_0|$ and $|b_1|$ for the meromorphically bi univalent function class $\Sigma'_{\lambda,\beta}(\mu)$.

Theorem 3.1. *Let $g(z)$ be given by (2) be in the class $\Sigma'_{\lambda,\beta}(\mu)$. Then*

$$|b_0| \leq \frac{2(1-\mu)}{(\beta+\lambda\beta-\lambda)} \tag{24}$$

and

$$|b_1| \leq 2(1-\mu)\left(\frac{\sqrt{(1-\mu)^2(\lambda(\lambda-1)(1-\beta)+2\beta)^2+(\lambda-\lambda\beta-\beta)^2}}{(\beta+\lambda\beta-\lambda)(\beta+2\lambda\beta-\lambda)}\right). \tag{25}$$

Proof. Let $g \in \Sigma'_{\lambda,\beta}(\mu)$. Then, by Definition 2.1 of meromorphically bi-univalent function class $\Sigma'_{\lambda,\beta}(\mu)$, the conditions (22) and (23) can be rewritten as follows:

$$(1-\beta)\left(\frac{g(z)}{z}\right)^{\lambda} + \beta\left(\frac{z(g(z)')^{\lambda}}{g(z)}\right) = \mu + (1-\mu)p(z) \tag{26}$$

and

$$(1-\beta)\left(\frac{h(w)}{w}\right)^{\lambda} + \beta\left(\frac{w(h(w)')^{\lambda}}{h(w)}\right) = \mu + (1-\mu)q(w), \tag{27}$$

respectively. Here, just as in our proof of Theorem 2.2, with the functions $p(z) \in P$ and $q(w) \in P$ have the forms in (12) and (13). Clearly we have

$$\mu + (1-\mu)p(z) = 1 + \frac{(1-\mu)p_1}{z} + \frac{(1-\mu)p_2}{z^2} + \frac{(1-\mu)p_3}{z^3} + \cdots$$

and

$$\mu + (1-\mu)q(w) = 1 + \frac{(1-\mu)q_1}{w} + \frac{(1-\mu)q_2}{w^2} + \frac{(1-\mu)q_3}{w^3} + \cdots \quad .$$

Now, equating the Coefficients in (26) and (27) and using (14) and (15), we get

$$(\lambda-\lambda\beta-\beta)b_0 = (1-\mu)p_1, \tag{28}$$

$$\frac{1}{2}(\lambda(\lambda-1)(1-\beta)+2\beta)b_0^2 + 2(\lambda-\beta-2\lambda\beta)b_1 = (1-\mu)p_2, \tag{29}$$

$$-(\lambda-\lambda\beta-\beta)b_0 = (1-\mu)q_1, \tag{30}$$

$$\frac{1}{2}(\lambda(\lambda-1)(1-\beta)+2\beta)b_0^2 + 2(\beta-\lambda+2\lambda\beta)b_1 = (1-\mu)q_2. \tag{31}$$

From (28) and (30), we obtain

$$p_1 = -q_1 \tag{32}$$

and

$$2(\lambda-\lambda\beta-\beta)^2 b_0^2=(1-\mu)^2(p_1^2+q_1^2) \tag{33}$$

that is,

$$b_0^2=\frac{(1-\mu)^2(p_1^2+q_1^2)}{2(\lambda-\lambda\beta-\beta)^2}.$$

Applying Carathéodory's Lemma for the Coefficients p_1 and q_1 , we immediately have

$$|b_0|\leq \frac{2(1-\mu)}{(\beta+\lambda\beta-\lambda)}.$$

This gives the bound on$|b_0|$ as asserted in (24).

Next, in order to find the bound on $|b_1|$, by using the equation (29) and the equation (31), we get

$$\begin{aligned}
&\left(\frac{1}{2}(\lambda(\lambda-1)(1-\beta)+2\beta)b_0^2+(\lambda-\beta-2\lambda\beta)b_1\right)\\
&\cdot\left(\frac{1}{2}(\lambda(\lambda-1)(1-\beta)+2\beta)b_0^2+(\beta-\lambda+2\lambda\beta)b_1\right)\\
&=((1-\mu)p_2)\,.\,((1-\mu)p_2)\,,\\
&\frac{1}{4}(\lambda(\lambda-1)(1-\beta)+2\beta)b_0^4-(\beta+2\lambda\beta-\lambda))^2 b_1^2=(1-\mu)^2 p_2 q_2,\\
&(\beta+2\lambda\beta-\lambda)^2 b_1^2=\frac{1}{4}\left((\lambda(\lambda-1)(1-\beta)+2\beta)^2 b_0^2\right)^2-(1-\mu)^2 p_2 q_2
\end{aligned}$$

and

$$(\beta+2\lambda\beta-\lambda)^2 b_1^2=\frac{1}{4}(\lambda(\lambda-1)(1-\beta)+2\beta)^2\frac{(1-\mu)^4(p_1^2+q_1^2)}{2(\lambda-\lambda\beta-\beta)^2}-(1-\mu)^2 p_2 q_2.$$

Applying Carathéodory's Lemma once again for the Coefficients $p_1,\ q_1,\ p_2$ and q_2, we get

$$(\beta+2\lambda\beta-\lambda)^2|b_1^2|\leq 4(\lambda(\lambda-1)(1-\beta)+2\beta)^2\frac{(1-\mu)^4}{(\lambda-\lambda\beta-\beta)^2}+4(1-\mu)^2,$$

that is,

$$|b_1|\leq 2(1-\mu)\left(\frac{\sqrt{(1-\mu)^2(\lambda(\lambda-1)(1-\beta)+2\beta)^2+(\lambda-\lambda\beta-\beta)^2}}{(\beta+\lambda\beta-\lambda)(\beta+2\lambda\beta-\lambda)}\right).$$

This completes the proof of Theorem 3.2. □

Remark 3.1. By taking $\beta=1$, in Theorem 2.2 and Theorem 3.2 we can deduce the corresponding coefficient estimates for several simpler meromorphically bi-univalent function classes, (see [19]).

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Accepted: April 22, 2021