The projective character tables of the maximal subgroups of M_{22} and its automorphism group M_{22}:2

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Abstract. In this paper, a routine written in the computational algebra system GAP is presented to compute the irreducible projective characters IrrProj(G, α_i) with associated factor sets α_i for the maximal subgroups of the sporadic simple Mathieu group M_{22} and its automorphism group M_{22}:2. The said routine derived its fundamentals from the theory of finding all the irreducible projective characters of a finite group G from the ordinary irreducible characters of a so-called representation group $R = M(G).G$ of G, where $M(G)$ denotes the Schur multiplier of G.

Keywords: projective character table, factor set, Schur multiplier, representation group, GAP routine.

1. Introduction

It is a well-known fact that all the sets of irreducible projective characters IrrProj(G, α_i), i=$1, 2, \ldots, m$, of a finite group G with factor sets α_i can be obtained from the ordinary irreducible characters of a so-called representation group $R \cong M(G).G$ of G, where $M(G)$ denotes the Schur multiplier of the group G and m the number of cohomology classes $[\alpha_i]$ in $M(G)$. Using this theory, a routine written in GAP [5] is discussed in Section 3 of this paper to compute the sets IrrProj(G, α_i) for a finite group G. In particular, this said GAP routine will be used to compute the sets IrrProj(G, α_i) for each maximal subgroup G of the sporadic simple Mathieu group M_{22} and its automorphism group $Aut(M_{22}) = M_{22}$:2, except for the ones whose projective character information already appear in the ATLAS [3]. The groups M_{22} and Aut(M_{22}) have Schur multipliers of cyclic orders 12 and 2 respectively (see [3], [7], [11]). Also, their irreducible projective character tables are available in [3] or GAP library [5]. This paper will form part of a series of articles by the current author on the computation of the irreducible projective characters IrrProj(G, α_i) of the maximal subgroups of the sporadic simple Mathieu groups M_{11}, M_{12}, M_{22}, M_{23}, M_{24}

and their automorphism groups. In this regard, the sets IrrProj(G, α_i) for M_{11}, M_{12} and Aut(M_{12}) are computed in [13].

The aforementioned GAP routine has resulted from a question posed by the current author in [18] about the number $|\text{IrrProj}(G, \alpha)|$ of irreducible projective characters IrrProj(G, α) with factor set α of a finite group G. Then G. Robinson [20] provided a proof under which conditions the relation $|\text{IrrProj}(G, \alpha)| \leq |\text{Irr}(G)|$ are satisfied. From this proof, a GAP routine was written by J. Schmidt [21] to determine the number $|\text{IrrProj}(G, \alpha_i)|$ of irreducible projective characters of a finite group G associated with any factor set α_i. The results mentioned above were used in the papers [14], [15],[16] and [17] by the current author to compute irreducible projective characters of a so-called inertia factor group of an extension group $\overline{G} = N.G$. These projective characters were needed to construct the ordinary character table of $\overline{G}$ using the technique of Fischer matrices [4]. Further on, this GAP routine by [21] was modified and extended by the current author in [13] to compute all the sets IrrProj(G, α_i) for any given finite group G on condition that one can compute the ordinary character table of a representation group R (Schur cover) of G. In the current paper, the extended GAP routine in [13] (see Section 3) is slightly adjusted to compute all the sets IrrProj(G, α_i) for each maximal subgroup G of M_{22} and $Aut(M_{22})$. Especially, when the size of a maximal subgroup of M_{22} or its automorphism group $Aut(M_{22})$ becomes too large for the GAP routine in [13] to handle, then the full cover group of G is first converted to a permutation group in GAP or MAGMA [2] before its projective character tables are computed with the said GAP routine.

In Section 2, some definitions and results on projective character theory are given and in Section 4 all the results of the sets IrrProj(G, α_i) for each maximal subgroup G of M_{22} and $Aut(M_{22})$ are summarized in table format. Computations are done in GAP and MAGMA and notations in both GAP and the ATLAS [3] are followed.

2. Preliminary results on projective characters

In this section, some of the basic ideas of projective character theory will be discussed and for this purpose, the outline given in [12] is followed closely. In what will follows, it will be understood that G is a finite group, $\mathbb{C}$ the field of complex numbers, $\mathbb{C}^*$ the nonzero complex numbers, $GL(n, \mathbb{C})$ the group of non-singular $n \times n$ matrices over the complex numbers $\mathbb{C}$, $Z(G)$ the center of G, G' the derived subgroup of G, Irr(G) the set of ordinary irreducible characters of G and IrrProj(G, α) the irreducible projective characters of G with associated factor set α. Interested readers are referred to [6], [8], [9], [10] and [19] for a detailed treatment on ordinary and projective character theory.

Definition 2.1. *A projective representation of a group G of degree n over the complex numbers is a map $P{:}G \to GL(n, \mathbb{C})$, such that* (*i*) $P(1) = I_n$, *and*

(*ii*) *given* $x, y \in G$, *there exists* $\alpha(x, y) \in \mathbb{C}^*$ *such that* $P(x)P(y) = \alpha(x, y)P(xy)$.

Since multiplication in G and $GL(n, \mathbb{C})$ is associative it follows that $\alpha(xy, z)\alpha(x, y) = \alpha(x, yz)\alpha(y, z)$ for all $x, y, z \in G$. In addition, a map $\alpha : G \times G \to \mathbb{C}^*$ that satisfies this condition is called a factor set (or 2-cocycle) α of G in $\mathbb{C}$. We say that P is a projective representation with factor set α. Define $\xi(g) = Trace(P(g))$ for all $g \in G$, then ξ is called a projective character of G with factor set α. We say that ξ is irreducible if P is irreducible. An irreducible projective representation P of a group G is essentially defined in a similar way then an ordinary irreducible representation of G.

Definition 2.2. *Two projective representations* P_1 *and* P_2 *of* G *of degree* n *with factor sets* α_1 *and* α_2 *respectively are said to be projectively equivalent if there exists a mapping* $\phi : G \to \mathbb{C}^*$ *and a matrix* $T \in GL(n, \mathbb{C})$ *such that*

$$P_1(x) = \phi(x)T^{-1}P_2(x)T, \ \ \forall x \in G.$$

If P_1 and P_2 are projectively equivalent, then it follows from Definition 2.2 that, $\forall x, y \in G$,

$$\alpha_2(x, y) = \phi(x)\phi(y)(\phi(xy))^{-1}\alpha_1(x, y)$$

This is an equivalence relation and the equivalence class of the factor set α_1 is denoted by $[\alpha_1]$. The set of all equivalence classes of factor sets of G has a finite abelian group structure, and is called the Schur multiplier $M(G)$ (also known as the second cohomology group $H^2(G, \mathbb{C}^*)$ of G.

Definition 2.3. *A group* $C = A.G$ *is a central extension for* G *if there exists a homomorphism* π *from* C *onto* G *such that* $A = ker(\pi) \leq Z(R) \cap R'$. *In addition, if* $A \cong M(G)$, *then we called the central extension* C *a representation group* R *of* G.

A description on how to obtain the irreducible projective representations of a group G with factor set α from the ordinary irreducible representations of a central extension $C = A.G$ of G, will follow. Let $C = A.G$ be a central extension of the group G with $A = ker(\pi)$. Let $X = \{x_g | g \in G\}$ be a set of coset representatives of A in C, such that $\pi(x_g) = g$ (one-to-one correspondence of elements of X with the elements of G). Therefore, $C = \bigcup_{g \in G} Ax_g$. Then, for all $g, h \in G$, let $a(g, h)$ be the unique element in A such that $x_g x_h = a(g, h)x_{gh}$. Since the product operation to combine two elements in C and G is associative, then it follows that $a(g, h)a(gh, k) = a(g, hk)a(h, k)$ for all $g, h, k \in G$. Now, let λ be a linear character of the abelian group A and put $\alpha(g, h) = \lambda(a(g, h))$ for all $g, h \in G$, then it follows from the relation in the previous sentence that α is a factor set of G. Now, let T be an ordinary irreducible representation of C of degree n and let $P(g) = T(x_g)$ for all $g \in G$, then P is a irreducible projective representation of G with factor set α, i.e., $P(g)P(h) = \lambda(a(g, h))P(gh)$ for all $g, h \in G$. Hence, we can formulate the following definition:

Definition 2.4. *A projective representation P of G constructed from an ordinary irreducible representation T of C in the above manner is said to be linearized by the ordinary representation T (or lifted to C). Furthermore, P is irreducible if and only if T is irreducible.*

Each irreducible projective representation of G with corresponding factor set α can be linearized by an ordinary irreducible representation of a representation group R of G. So the problem of constructing all irreducible projective characters of a finite group G reduces to that of finding the ordinary irreducible characters of a representation group R of G.

Definition 2.5. *A covering group D for G will normally be a quotient $D \cong R/B$ of a representation group $R = M(G).G$ of G by a subgroup B of $M(G)$. If $M(G)/B$ has order n we sometimes refer to the covering group as a n-fold cover of G.*

Projective representations of G are found in the representation group R for all the equivalence classes of factors sets in $M(G)$ but however in a n-fold cover D of G only the n equivalence classes which D covers will be represented [6].

Definition 2.6. *An element $x \in G$ is said to be α-regular if $\alpha(x, g) = \alpha(g, x)$ for all $g \in C_G(x)$. Furthermore, it is well known that $g \in G$ is α-*regular *if and only if $\xi(g) \neq 0$ for some $\xi \in IrrProj(G, \alpha)$ or equivalently that g is α-irregular if and only if $\xi(g) = 0$ for all $\xi \in IrrProj(G, \alpha)$.*

Now, if $x \in G$ is α-regular, then so is every conjugate of x and therefore it is meaningful to speak about α-regular classes of G. The number of IrrProj(G, α) equals the number of α-regular classes of a group G. Projective characters also satisfy the usual orthogonality relations and have analogues to ordinary characters.

3. A GAP routine to compute IrrProj(G, α_i)

In this section, the GAP routine given in [21] which has its origin in Proposition 3.1 below is extended to give a GAP routine that can computes directly all the distinct sets IrrProj(G, α_i) of G from a representation group R of G.

Proposition 3.1. *Let $R = M(G).G$ be a representation group of a finite group G, where $M(G)$ denotes the Schur multiplier of G. Then the number of irreducible characters $\chi_j \in Irr(R)$ of R which lies over a linear character λ of $M(G)$ is less or equal to $|Irr(G)|$.*

Proof. (see [14] , [15] or [20]). □

From the proof of Propositon 3.1 it is mentioned that the number of irreducible characters $\chi_j \in \mathrm{Irr}(R)$ of a representation group $R = M(G).G$ of G which lies over a linear character $\lambda \in \mathrm{Irr}(M(G))$ (in other words $\lambda \in \mathrm{Irr}(M(G)$

is an irreducible constituent of $\chi_j \downarrow_{M(G)}$, that is, $< \chi_j \downarrow_{M(G)}, \lambda > \neq 0$) is given by $\sum_{\chi \in \mathrm{Irr}(R)} \frac{<\chi \downarrow_{M(G)}, \lambda>}{\chi(1)}$. Using this fact (see the line of the GAP routine starting with "n"), we are able to compute all the sets $\mathrm{IrrProj}(G, \alpha_i)$, i=1, 2, ..., $|\mathrm{Irr}(M(G)|$, of G. In addition, it is shown in the proof of Proposition 3.1 that the quantity $\sum_{\chi \in \mathrm{Irr}(R)} \frac{<\chi \downarrow_{M(G)}, \lambda>}{\chi(1)} \leq |\mathrm{Irr}(G)|$ and the inequality becomes strict if there is a non-identity element $x \in M(G) \setminus \ker(\lambda)$ which is a commutator in R. That means that the number $|\mathrm{IrrProj}(G, \alpha_i)|$ of irreducible projective characters of G with factor set α_i is always less or equal to the number |Irr(G)| of ordinary irreducible characters of G. In the below GAP routine the output "N" gives $|\mathrm{Irr}(M(G))|$ blocks coming from a representation group R (denoted as "Source(f)" in the below routine), where each block will contains one of the sets $\mathrm{IrrProj}(G, \alpha_i)$, $i = 1, 2, .., |\mathrm{Irr}(M(G)|$. Whereas the output "Display(ct)" will display each individual irreducible projective character table of G with factor set α_i in GAP.

Now, the below GAP routine can computes directly all the distinct sets $\mathrm{IrrProj}(G, \alpha_i)$ of G from a suitable representation group R (full covering group) of G. Especially, if the finite group G has a relatively small order then the representation group (Schur cover) of G can be computed easily in GAP (as it was in the case of the groups in [13]). But if the group G becomes too large then GAP experiences difficulties to compute the Schur cover of G and then we have to employ additional techniques in computing these sets $\mathrm{IrrProj}(G, \alpha_i)$. This was the case with most of the maximal subgroups of M_{22} and $\mathrm{Aut}(M_{22})$. To overcome this constraint, we compute the representation group R of Perm := G using the GAP command "S:=SchurCover(Perm)" and then use the GAP commands "iso:=IsomorphismPermGroup(S)" and "x:=Image(iso)" to convert R into a permutation group x. Then to find the normal subgroup z of the group x such that x/z $\cong G$ the command "Nor:=Filtered(NormalSubgroups(x),h1−> Size(h1)=Size(x)/Size(Perm))" is used. Furthermore, the command "f:= NaturalHomomorphismByNormalSubgroup(x,z)" is employed and then we can follow the rest of the GAP routine below from the input line "'I1:=ImagesSource(f)" to compute the sets $\mathrm{IrrProj}(G, \alpha_i)$ of G successfully. The MAGMA routine find in [1] was used to convert the Schur cover of the group $L_3(4){:}2_2$ (see Table 5) into a permutation group before applying the below GAP routine.

```
gap> Perm := G;; (Permutation group with generators found in [22] or can be generated in GAP.)
gap> f := EpimorphismSchurCover(Perm);;
gap> z := Kernel(f);;
gap> x := Source(f);;
gap> I1:=ImagesSource(f);; (Quotient group I1 ≅ G)
gap> t:=Irr(I1);;
gap> 2t:=Irr(x);;
gap> F:=FusionConjugacyClassesOp(f);
gap> map:=ProjectionMap(F);
```

```
>Cen:=SizesCentralizers(CharacterTable(I1));
>Cl:=OrdersClassRepresentatives(CharacterTable(I1));
gap> N:=[];
gap> for i in [1..Size(Irr(z))] do
> n:= Filtered(Irr(x),chi- >not IsZero(ScalarProduct
                    (RestrictedClassFunction(chi,z),Irr(z)[i])));
> s:=List(n,x- >x{map});
> Add(N,s);
> ct:=function()local ct ;ct:=rec();
>ct.SizesCentralizers:=Cen;;
>ct.OrdersClassRepresentatives:=Cl;;
> ct.Irr:=N[i];;
>ct.UnderlyingCharacteristic:=0;ct.Id:="G";
> ConvertToLibraryCharacterTableNC(ct);return ct;end;ct:=ct();
>SetInfoLevel(InfoCharacterTable,2);
>Display(ct);
>od;
gap>N;
```

4. The sets IrrProj(G, α_i) for the maximal subgroups of M_{22} and Aut(M_{22}).

In this section, the Schur multipliers $M(G)$ and the sets IrrProj(G, α_i) for each maximal subgroup G of M_{22} and Aut(M_{22}) are computed using the method discussed in Section 3 except for the ones of the groups $L_3(4)$, A_7, M_{10}, $L_2(11)$, M_{22}, Aut(A_6) and $L_2(11)$:2 which are found in the ATLAS or GAP library. All of the above-mentioned information is summarized in the tables below.

In Table 1, a summary of all the information concerning the structures of the Schur multipliers $M(G)$ and the number $|$IrrProj$(G, \alpha_i)|$ of sets IrrProj(G, α_i) for all the maximal subgroups of M_{22} and Aut(M_{22}) is given. Note that in the last column of Table 1 the number $|$IrrProj$(G, \alpha_i)|$ of irreducible projective characters with factor set α_i for each set IrrProj(G, α_i) of a maximal subgroup G of M_{22} and Aut(M_{22}) is given even if some of the sets contain the same information. But for the groups $L_3(4)$, A_7 and M_{22} only the number of projective characters in each distinct set IrrProj(G, α_i) is listed as it appeared in the ATLAS. Note that the structure of the Schur Multiplier $M(G)$ for each group G can also be determined by the GAP command $AbelianInvariantsMultiplier(G)$. The ordinary irreducible characters Irr(G) of the maximal subgroups of M_{22} and Aut(M_{22}) are always listed in the first block of the the below tables which contain the information of the sets IrrProj(G, α_i) of each maximal subgroup G.

We will consider briefly some of the groups whose Schur multiplier has order greater or equal to 4. The maximal subgroup $G = 2^4{:}A_6$ of M_{22} has a Schur multiplier $M(G) \cong 2 \times 4 \times 3$ of order 24. Therefore, $M(G)$ will have 24 cohomology classes $[\alpha_i]$ which consist out of the the trivial class [1], 3 classes of order 2, 2 classes of order 3, 4 classes of order 4, 6 classes of order 6 and 8 classes of order 12. The number of disctinct sets $\text{IrrProj}(G, \alpha_i)$ of G associated with these classes will be the ordinary irreducible characters of G, three sets with factor sets $\alpha_i^2 = 1$, one set with factor set $\alpha_i^3 = 1$, two sets with factor sets $\alpha_i^4 = 1$, three sets with factor sets $\alpha_i^6 = 1$ and four sets with factor sets $\alpha_i^{12} = 1$. All the sets $\text{IrrProj}(G, \alpha_i)$ of $2^4{:}A_6$ with factor sets α_i of order 3, 4, 6 and 12 can be recovered by applying all possible algebraic conjugation to the classes and irreducible projective characters of G with factor sets α_i of order 3, 4, 6 and 12 which appear in Table 2. For example, there are two projective character tables with factor sets α_i of order 3, but the entries of the first table (see Table 2) will be just the complex entries of the second table. Similar situations arose for the four sets $\text{IrrProj}(G, \alpha_i)$ of $2^4{:}S_5$ with factor sets α_i of order 4. Here we will have two distinct sets $\text{IrrProj}(G, \alpha_i)$ with $\alpha_i^4 = 1$ (see the bottom two blocks of Table 3), where the other two are just the complex entries of these two tables. In addition, the three sets $\text{IrrProj}(G, \alpha_i)$ of $2^4{:}S_5$ with factor sets $\alpha_i^2 = 1$ are all inequivalent. The Schur Multiplier $M(G)$ of $2^3{:}L_3(2)$ is the Klein four-group 2^2 and thus $2^3{:}L_3(2)$ will have 3 $\text{IrrProj}(G, \alpha_i)$ sets with non-trivial factor sets α_i of order 2 (see Table 4). Notice that the two sets in the bottom of Table 4 are the same except for the entries in the last column which just differ with a minus sign. In [21] it is mentioned that the sets $\text{IrrProj}(G, \alpha_i)$ of G are only defined universally up to sign and it is possible that one can obtain different signs if the sets $\text{IrrProj}(G, \alpha_i)$ are re-calculated using a different representation group $R = M(G).G$. But these signs are calculated consistently with a "special factor set" (as explained in [6] or Section 2 of this paper) and so the inner product and conjugacy results of [6] apply to the sets $\text{IrrProj}(G, \alpha_i)$. For this reason, the two above-mentioned sets $\text{IrrProj}(G, \alpha_i)$, $\alpha_i^2 = 1$, in Table 4 just carry the same information. The maximal subgroup $(2^3{:}L_3(2)) \times 2$ of $\text{Aut}(M_{22})$ is a direct product of $2^3{:}L_3(2)$ with the cyclic group of order 2 and therefore we have that the three sets $\text{IrrProj}(G, \alpha_i)$ of $(2^3{:}L_3(2)) \times 2$ with nontrivial factor sets of order two will just be twice the number of irreducible projective characters of that of $2^3{:}L_3(2)$ (see Tables 4 and 8). Since the 2 sets $\text{IrrProj}(G, \alpha_i)$ of $(2^3{:}L_3(2)) \times 2$ with ten irreducible projective characters each contain the same information, only one set at the bottom of Table 8 is listed. The group A_7 has a Schur multiplier of cyclic order six and hence there will be three distinct sets of irreducible projective characters with factor sets of order 2, 3 and 6 (see [3]).

The Schur multiplier of the maximal subgroup $L_3(4){:}2_2$ of $\text{Aut}(M_{22})$ is cyclic of order four and will have 3 sets $\text{IrrProj}(L_3(4){:}2_2, \alpha_i)$ with nontrivial factor sets α_2, α_2^2 and α_2^3 of order 4, 2 and 4, respectively. The trivial factor set $\alpha_1 = \alpha_2^4 = 1$ is associated with the ordinary irreducible characters $\text{Irr}(L_3(4){:}2_2)$ of $L_3(4){:}2_2$. Since $\alpha_2^3 = \alpha_2^{-1} = \overline{\alpha_2}$, the entries of the set $\text{IrrProj}(L_3(4){:}2_2, \alpha_2)$ of $L_3(4){:}2_2$ in the third block of Table 5 are just the complex conjugates of the entries of the set $\text{IrrProj}(L_3(4){:}2_2, \alpha_2^3)$. Therefore, only the set $\text{IrrProj}(L_3(4){:}2_2, \alpha_2)$ of

$L_3(4){:}2_2$ is listed at the bottom of Table 5 since IrrProj($L_3(4){:}2_2, \alpha_2^3$) can be easily deduced from it.

Table 1: $M(G)$ **and** $|$**IrrProj**$(G, \alpha_i)|$ **of maximal subgroups** G **of** M_{22} **and Aut**(M_{22})

Maximal subgroups of M_{22}	$\|G\|$	$[M_{22}:G]$	$M(G)$	$\|$IrrProj$(G, \alpha_i)\|$
$L_3(4)$	$20160 = 2^6.3^2.5.7$	22	$4 \times 4 \times 3$	[10,8,6,7,9,7,5,6]
$2^4{:}A_6$	$5760 = 2^7.3^2.5$	77	$2 \times 4 \times 3$	[12,9,11,10,10,10,7,7,6,6,7,7, 9,9,8,8,5,5,5,5,4,4,4,4]
A_7	$2520 = 2^3.3^2.5.7$	176	6	[9,7,7,5]
A_7	$2520 = 2^3.3^2.5.7$	176	6	[9,7,7,5]
$2^4{:}S_5$	$1920 = 2^7.3.5$	231	4×2	[12,11,8,7,7,7,5,5]
$2^3{:}L_3(2)$	$1344 = 2^6.3.7$	330	2^2	[11,8,5,5]
$M_{10} \cong A_6.2_3$	$720 = 2^4.3^2.5$	616	3	[8,7,7]
$L_2(11)$	$660 = 2^2.3.5.11$	672	2	[8,7]
Maximal subgroups of Aut(M_{22})	$\|G\|$	[Aut(M_{22}):G]	$M(G)$	$\|$IrrProj$(G, \alpha_i)\|$
M_{22}	$443520 = 2^7.3^2.5.7.11$	2	12	[12,11,8,11,10,7]
$L_3(4){:}2_2$	$40320 = 2^7.3^2.5.7$	22	4	[14,13,11,11]
$2^4{:}S_6$	$11520 = 2^8.3^2.5$	77	2^2	[21,9,16,11]
$2^5{:}S_5$	$3840 = 2^8.3.5$	231	2^3	[24,10,13,13,16,14,5,9]
$(2^3{:}L_3(2)) \times 2$	$2688 = 2^7.3.7$	330	2^2	[22,16,10,10]
$Aut(A_6) \cong A_6.2^2$	$1440 = 2^5.3^2.5$	616	2	[13,10]
$L_2(11){:}2$	$1320 = 2^3.3.5.11$	672	2	[13,11]

Table 2: **IrrProj(G,**α_i**) for** $2^4{:}A_6$**,** $\alpha_1 = 1$

$[g]_G$	1a	2a	4a	4b	2b	3a	6a	8a	4a	5a	5b	3b
$\|C_G(g)\|$	5760	384	16	32	32	36	12	8	8	5	5	9
χ_1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	5	5	1	1	1	-1	-1	-1	-1	0	0	2
χ_3	5	5	1	1	1	2	2	-1	-1	0	0	-1
χ_4	8	8	0	0	0	-1	-1	0	0	T	*T	-1
χ_5	8	8	0	0	0	-1	-1	0	0	*T	T	-1
χ_6	9	9	1	1	1	0	0	1	1	-1	-1	0
χ_7	10	10	-2	-2	-2	1	1	0	0	0	0	1
χ_8	15	-1	-1	3	-1	3	-1	1	-1	0	0	0
χ_9	15	-1	-1	-1	3	3	-1	-1	1	0	0	0
χ_{10}	30	-2	-2	2	2	-3	1	0	0	0	0	0
χ_{11}	45	-3	1	1	-3	0	0	-1	1	0	0	0
χ_{12}	45	-3	1	-3	1	0	0	1	-1	0	0	0

Table 2 (continued): **IrrProj(G,α_i) for** $2^4{:}A_6$

$\alpha_2^2 = \alpha_3^2 = \alpha_4^2 = \alpha_5^3 = \alpha_6^4 = \alpha_7^4 = 1$

$[g]_G$	1a	2a	4a	4b	2b	3a	6a	8a	4a	5a	5b	3b
$\lvert C_G(g)\rvert$	5760	384	16	32	32	36	12	8	8	5	5	9
χ_1	4	4	0	0	0	-1	-1	0	0	-1	1	-2
χ_2	4	4	0	0	0	2	2	0	0	-1	1	1
χ_3	8	8	0	0	0	1	1	0	0	T	-*T	-1
χ_4	8	8	0	0	0	1	1	0	0	*T	-T	-1
χ_5	10	10	0	0	0	-1	-1	O	O	0	0	1
χ_6	10	10	0	0	0	-1	-1	-O	-O	0	0	1
χ_7	30	-2	0	0	0	3	-1	-O	O	0	0	0
χ_8	30	-2	0	0	0	3	-1	O	-O	0	0	0
χ_9	60	-4	0	0	0	-3	1	0	0	0	0	0
χ_1	6	-2	0	-2	-2	3	-1	0	0	1	1	0
χ_2	10	2	0	2	-2	1	1	0	-2	0	0	1
χ_3	10	2	0	2	-2	1	1	0	2	0	0	1
χ_4	10	2	0	-2	2	1	1	K	0	0	0	1
χ_5	10	2	0	-2	2	1	1	-K	0	0	0	1
χ_6	18	-6	0	2	2	0	0	0	0	T	*T	0
χ_7	18	-6	0	2	2	0	0	0	0	*T	T	0
χ_8	24	-8	0	0	0	3	-1	0	0	-1	-1	0
χ_9	30	-10	0	-2	-2	-3	1	0	0	0	0	0
χ_{10}	40	8	0	0	0	-2	-2	0	0	0	0	1
χ_{11}	40	8	0	0	0	1	1	0	0	0	0	-2
χ_1	10	2	K	0	0	-1	-1	P	O	0	0	1
χ_2	10	2	K	0	0	-1	-1	-P	-O	0	0	1
χ_3	10	2	-K	0	0	-1	-1	P	-O	0	0	1
χ_4	10	2	-K	0	0	-1	-1	-P	O	0	0	1
χ_5	12	-4	0	0	0	3	-1	0	0	-*T	T	0
χ_6	12	-4	0	0	0	3	-1	0	0	-T	*T	0
χ_7	24	-8	0	0	0	-3	1	0	0	-1	1	0
χ_8	36	-12	0	0	0	0	0	0	0	1	-1	0
χ_9	40	8	0	0	0	-1	-1	0	0	0	0	-2
χ_{10}	40	8	0	0	0	2	2	0	0	0	0	1
χ_1	3	A	-1	$\overline{D}$	D	0	0	1	-D	T	U	0
χ_2	3	A	-1	$\overline{D}$	D	0	0	1	-D	*T	V	0
χ_3	6	B	2	-$\overline{F}$	-F	0	0	0	0	1	-$\overline{D}$	0
χ_4	9	C	1	-$\overline{D}$	-D	0	0	1	-D	-1	$\overline{D}$	0
χ_5	15	D	-1	$\overline{A}$	D	0	-$\overline{F}$	1	D	0	0	0
χ_6	15	E	1	$\overline{D}$	D	0	0	-1	D	0	0	0
χ_7	15	D	-1	$\overline{D}$	A	0	-$\overline{F}$	-1	-D	0	0	0
χ_8	30	F	-2	-$\overline{F}$	-F	0	$\overline{F}$	0	0	0	0	0
χ_9	45	-A	1	-$\overline{D}$	-A	0	0	-1	-D	0	0	0
χ_{10}	45	-A	1	-$\overline{A}$	-D	0	0	1	D	0	0	0
χ_1	4	0	0	K	0	2	0	Q	0	-1	1	1
χ_2	20	0	0	K	0	-2	0	-Q	0	0	0	2
χ_3	20	0	0	K	0	4	0	-Q	0	0	0	-1
χ_4	32	0	0	0	0	-2	0	0	0	-*T	T	-1
χ_5	32	0	0	0	0	-2	0	0	0	-T	*T	-1
χ_6	36	0	0	K	0	0	0	Q	0	1	-1	0
χ_7	40	0	0	L	0	2	0	0	0	0	0	1
χ_1	16	0	0	0	0	-2	0	0	0	1	1	-2
χ_2	16	0	0	0	0	4	0	0	0	1	1	1
χ_3	32	0	0	0	0	2	0	0	0	-*T	-T	-1
χ_4	32	0	0	0	0	2	0	0	0	-T	-*T	-1
χ_5	40	0	0	0	0	-2	0	R	0	0	0	1
χ_6	40	0	0	0	0	-2	0	-R	0	0	0	1

Table 2 (continued): **IrrProj(G,α_i) for** $2^4{:}A_6$

$\alpha_8^6 = \alpha_9^6 = \alpha_{10}^6 = \alpha_{11}^{12} = \alpha_{12}^{12} = \alpha_{13}^{12} = \alpha_{14}^{12} = 1$

$[g]_G$	1a	2a	4a	4b	2b	3a	6a	8a	4a	5a	5b	3b
$\lvert C_G(g)\rvert$	5760	384	16	32	32	36	12	8	8	5	5	9
χ_1	6	B	0	0	0	0	0	-O	S	1	$\overline{D}$	0
χ_2	6	B	0	0	0	0	0	O	-S	1	$\overline{D}$	0
χ_3	12	G	0	0	0	0	0	0	0	-*T	V	0
χ_4	12	G	0	0	0	0	0	0	0	-T	U	0
χ_5	30	F	0	0	0	0	-$\overline{F}$	-O	-S	0	0	0
χ_6	30	F	0	0	0	0	-$\overline{F}$	O	S	0	0	0
χ_7	60	H	0	0	0	0	$\overline{F}$	0	0	0	0	0
χ_1	6	F	0	$\overline{F}$	F	0	-$\overline{F}$	0	0	1	-$\overline{D}$	0
χ_2	18	-B	0	-$\overline{F}$	-F	0	0	0	0	T	U	0
χ_3	18	-B	0	-$\overline{F}$	-F	0	0	0	0	*T	V	0
χ_4	24	I	0	0	0	0	-$\overline{F}$	0	0	-1	$\overline{D}$	0
χ_5	30	J	0	$\overline{F}$	F	0	$\overline{F}$	0	0	0	0	0
χ_6	30	B	0	$\overline{F}$	-F	0	0	0	F	0	0	0
χ_7	30	B	0	$\overline{F}$	-F	0	0	0	-F	0	0	0
χ_8	30	B	0	-$\overline{F}$	F	0	0	K	0	0	0	0
χ_9	30	B	0	-$\overline{F}$	F	0	0	-K	0	0	0	0
χ_1	12	$\overline{H}$	0	0	0	0	-F	0	0	-*T	$\overline{V}$	0
χ_2	12	$\overline{H}$	0	0	0	0	-F	0	0	-T	$\overline{U}$	0
χ_3	24	$\overline{I}$	0	0	0	0	F	0	0	-1	-D	0
χ_4	30	$\overline{B}$	K	0	0	0	0	P	$\overline{S}$	0	0	0
χ_5	30	$\overline{B}$	K	0	0	0	0	-P	-$\overline{S}$	0	0	0
χ_6	30	$\overline{B}$	-K	0	0	0	0	P	-$\overline{S}$	0	0	0
χ_7	30	$\overline{B}$	-K	0	0	0	0	-P	$\overline{S}$	0	0	0
χ_8	36	- $\overline{G}$	0	0	0	0	0	0	0	1	D	0
χ_1	12	0	0	M	0	0	0	Q	0	-*T	V	0
χ_2	12	0	0	M	0	0	0	Q	0	-T	U	0
χ_3	24	0	0	N	0	0	0	0	0	-1	-$\overline{D}$	0
χ_4	36	0	0	-M	0	0	0	Q	0	1	$\overline{D}$	0
χ_5	60	0	0	M	0	0	0	-Q	0	0	0	0
χ_1	12	0	0	- M	0	0	0	$\overline{Q}$	0	-*T	V	0
χ_2	12	0	0	-M	0	0	0	$\overline{Q}$	0	-T	U	0
χ_3	24	0	0	-N	0	0	0	0	0	-1	-$\overline{D}$	0
χ_4	36	0	0	M	0	0	0	$\overline{Q}$	0	1	$\overline{D}$	0
χ_5	60	0	0	-M	0	0	0	-$\overline{Q}$	0	0	0	0
χ_1	24	0	0	0	0	0	0	-R	0	-1	D	0
χ_2	24	0	0	0	0	0	0	R	0	-1	D	0
χ_3	48	0	0	0	0	0	0	0	0	T	$\overline{U}$	0
χ_4	48	0	0	0	0	0	0	0	0	*T	$\overline{V}$	0
χ_1	24	0	0	0	0	0	0	-R	0	-1	$\overline{D}$	0
χ_2	24	0	0	0	0	0	0	R	0	-1	$\overline{D}$	0
χ_3	48	0	0	0	0	0	0	0	0	T	U	0
χ_4	48	0	0	0	0	0	0	0	0	*T	V	0

where A = = $\frac{-3+3\sqrt{3}\mathrm{i}}{2}$ = 3b3, B = $-3+3\sqrt{3}\mathrm{i}$ = 6b3,
C = $\frac{-9+9\sqrt{3}\mathrm{i}}{2}$ = 9b3, D = $\frac{1-\sqrt{3}\mathrm{i}}{2}$ = -b3, E = $\frac{-15+15\sqrt{3}\mathrm{i}}{2}$ = 15b3,
F = $1-\sqrt{3}\mathrm{i}$ = 1-i3, G = $-6+6\sqrt{3}\mathrm{i}$ = 12b3, H = $2-2\sqrt{3}\mathrm{i}$ = -4b3,
I = $4-4\sqrt{3}\mathrm{i}$ = -8b3, J = $5-5\sqrt{3}\mathrm{i}$ = 5-5i3, K = -2i,
L = 4i, M = 2E$(12)^{11}$, N = -4E$(12)^{11}$, O = -$\sqrt{2}$ = -r2,
P = $\sqrt{2}\mathrm{i}$ = -i2, Q = -1-i, R = -2E(8), S = E$(24)^{11}$-E$(24)^{17}$,
T = $\frac{1-\sqrt{5}}{2}$ = -b5, U = -E(15)-E$(15)^4$, V = -E$(15)^7$-E$(15)^{13}$

Table 3: **IrrProj(G,α_i) for** $2^4{:}S_5$

$\alpha_1 = \alpha_2^2 = \alpha_3^2 = \alpha_4^2 = \alpha_5^4 = \alpha_6^4 = 1$

$[g]_G$	1a	2a	4a	2b	4b	4c	2c	3a	6a	4d	8a	5a
$\vert C_G(g)\vert$	1920	128	16	48	16	32	32	6	6	8	8	5
χ_1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	1	-1	-1	1	1	1	1	-1	-1	-1	1
χ_3	4	4	2	2	0	0	0	1	-1	0	0	-1
χ_4	4	4	-2	-2	0	0	0	1	1	0	0	-1
χ_5	5	5	-1	-1	1	1	1	-1	-1	1	1	0
χ_6	5	5	1	1	1	1	1	-1	1	-1	-1	0
χ_7	6	6	0	0	-2	-2	-2	0	0	0	0	1
χ_8	15	-1	-1	3	-1	-1	3	0	0	1	-1	0
χ_9	15	-1	-1	3	-1	3	-1	0	0	-1	1	0
χ_{10}	15	-1	1	-3	-1	-1	3	0	0	-1	1	0
χ_{11}	15	-1	1	-3	-1	3	-1	0	0	1	-1	0
χ_{12}	30	-2	0	0	2	-2	-2	0	0	0	0	0
χ_1	6	-2	0	0	0	-2	2	0	0	-2	0	1
χ_2	6	-2	0	0	0	-2	2	0	0	2	0	1
χ_3	6	-2	0	0	0	2	-2	0	0	0	A	1
χ_4	6	-2	0	0	0	2	-2	0	0	0	-A	1
χ_5	10	2	-2	-2	0	-2	-2	1	-1	0	0	0
χ_6	10	2	0	4	0	2	2	1	-1	0	0	0
χ_7	10	2	2	2	0	-2	-2	1	1	0	0	0
χ_8	10	2	0	-4	0	2	2	1	1	0	0	0
χ_9	20	4	2	-2	0	0	0	-1	-1	0	0	0
χ_{10}	20	4	-2	2	0	0	0	-1	1	0	0	0
χ_{11}	24	-8	0	0	0	0	0	0	0	0	0	-1
χ_1	6	-2	0	0	A	0	0	0	0	E	F	1
χ_2	6	-2	0	0	A	0	0	0	0	-E	-F	1
χ_3	6	-2	0	0	-A	0	0	0	0	-E	F	1
χ_4	6	-2	0	0	-A	0	0	0	0	E	-F	1
χ_5	20	4	0	0	0	0	0	-2	0	0	0	0
χ_6	20	4	0	0	0	0	0	1	B	0	0	0
χ_7	20	4	0	0	0	0	0	1	-B	0	0	0
χ_8	24	-8	0	0	0	0	0	0	0	0	0	-1
χ_1	4	4	0	0	0	0	0	2	0	0	0	-1
χ_2	4	4	0	0	0	0	0	-1	B	0	0	-1
χ_3	4	4	0	0	0	0	0	-1	-B	0	0	-1
χ_4	6	6	0	0	0	0	0	0	0	E	E	1
χ_5	6	6	0	0	0	0	0	0	0	-E	-E	1
χ_6	30	-2	0	0	0	0	0	0	0	E	-E	0
χ_7	30	-2	0	0	0	0	0	0	0	-E	E	0
χ_1	4	0	0	2	0	2	0	-1	C	0	G	-1
χ_2	4	0	0	-2	0	2	0	-1	-C	0	-G	-1
χ_3	16	0	0	4	0	0	0	-1	-C	0	0	1
χ_4	16	0	0	-4	0	0	0	-1	C	0	0	1
χ_5	20	0	0	-2	0	2	0	1	-C	0	G	0
χ_6	20	0	0	2	0	2	0	1	C	0	-G	0
χ_7	24	0	0	0	0	-4	0	0	0	0	0	-1
χ_1	16	0	0	0	0	0	0	-2	0	0	0	1
χ_2	16	0	0	0	0	0	0	1	D	0	0	1
χ_3	16	0	0	0	0	0	0	1	-D	0	0	1
χ_4	24	0	0	0	0	0	0	0	0	0	H	-1
χ_5	24	0	0	0	0	0	0	0	0	0	-H	-1

A = -2i, B = $-\sqrt{3}$i = -i3, C = i, D = $\sqrt{3}$ = r3,
E = $\sqrt{2}$i = i2, F = $-\sqrt{2}$ = -r2, G = 1-i, H = 2*E(8)^3

Table 4: **IrrProj(G,α_i) for** $2^3{:}L_3(2)$

$\alpha_1 = \alpha_2^2 = \alpha_3^2 = \alpha_4^2 = 1$

$[g]_G$	1a	2a	2b	2c	4a	3a	6a	4a	4b	7a	7b
$\lvert C_G(g)\rvert$	1344	192	32	32	16	6	6	8	8	7	7
χ_1	1	1	1	1	1	1	1	1	1	1	1
χ_2	3	3	-1	-1	-1	0	0	1	1	C	$\overline{\text{C}}$
χ_3	3	3	-1	-1	-1	0	0	1	1	$\overline{\text{C}}$	C
χ_4	6	6	2	2	2	0	0	0	0	-1	-1
χ_5	7	7	-1	-1	-1	1	1	-1	-1	0	0
χ_6	7	-1	-1	3	-1	1	-1	-1	1	0	0
χ_7	7	-1	3	-1	-1	1	-1	1	-1	0	0
χ_8	8	8	0	0	0	-1	-1	0	0	1	1
χ_9	14	-2	2	2	-2	-1	1	0	0	0	0
χ_{10}	21	-3	-3	1	1	0	0	1	-1	0	0
χ_{11}	21	-3	1	-3	1	0	0	-1	1	0	0
χ_1	4	4	0	0	0	1	1	0	0	-$\overline{\text{C}}$	C
χ_2	4	4	0	0	0	1	1	0	0	-C	$\overline{\text{C}}$
χ_3	6	6	0	0	0	0	0	B	B	-1	1
χ_4	6	6	0	0	0	0	0	-B	-B	-1	1
χ_5	8	8	0	0	0	-1	-1	0	0	1	-1
χ_6	14	-2	0	0	0	-1	1	B	-B	0	0
χ_7	14	-2	0	0	0	-1	1	-B	B	0	0
χ_8	28	-4	0	0	0	1	-1	0	0	0	0
χ_1	8	0	0	0	0	2	0	0	0	1	-1
χ_2	8	0	0	0	0	-1	A	0	0	1	-1
χ_3	8	0	0	0	0	-1	-A	0	0	1	-1
χ_4	24	0	0	0	0	0	0	0	0	C	-$\overline{\text{C}}$
χ_5	24	0	0	0	0	0	0	0	0	$\overline{\text{C}}$	-C
χ_1	8	0	0	0	0	2	0	0	0	1	1
χ_2	8	0	0	0	0	-1	A	0	0	1	1
χ_3	8	0	0	0	0	-1	-A	0	0	1	1
χ_4	24	0	0	0	0	0	0	0	0	C	$\overline{\text{C}}$
χ_5	24	0	0	0	0	0	0	0	0	$\overline{\text{C}}$	C

A = -$\sqrt{3}$i = -i3, B = -$\sqrt{2}$ = -r2, C = $\frac{-1-\sqrt{7}\text{i}}{2}$ = -1-b7

Table 5: **IrrProj(G,α_i) for** $L_3(4){:}2_2$

$\alpha_1 = \alpha_2^2 = \alpha_3^4 = 1$

$[g]_G$	1a	5a	2a	7a	7b	14a	14b	2b	4a	3a	6a	8a	4b	4c
$\lvert C_G(g)\rvert$	40320	5	336	14	14	14	14	128	32	18	6	8	16	16
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	1	-1	1	1	-1	-1	1	1	1	-1	-1	1	-1
χ_3	20	0	-6	-1	-1	1	1	4	0	2	0	0	0	-2
χ_4	20	0	6	-1	-1	-1	-1	4	0	2	0	0	0	2
χ_5	35	0	-7	0	0	0	0	3	3	-1	-1	1	-1	1
χ_6	35	0	7	0	0	0	0	3	3	-1	1	-1	-1	-1
χ_7	45	0	-3	B	$\overline{B}$	-B	-$\overline{B}$	-3	1	0	0	-1	1	1
χ_8	45	0	-3	$\overline{B}$	B	-$\overline{B}$	-B	-3	1	0	0	-1	1	1
χ_9	45	0	3	B	$\overline{B}$	B	$\overline{B}$	-3	1	0	0	1	1	-1
χ_{10}	45	0	3	$\overline{B}$	B	$\overline{B}$	B	-3	1	0	0	1	1	-1
χ_{11}	64	-1	8	1	1	1	1	0	0	1	-1	0	0	0
χ_{12}	64	-1	-8	1	1	-1	-1	0	0	1	1	0	0	0
χ_{13}	70	0	0	0	0	0	0	6	-2	-2	0	0	2	0
χ_{14}	126	1	0	0	0	0	0	-2	-2	0	0	0	-2	0
χ_1	10	0	-4	-$\overline{B}$	-B	$\overline{B}$	-B	2	-2	-1	-1	0	0	0
χ_2	10	0	-4	-B	-$\overline{B}$	B	-$\overline{B}$	2	-2	-1	-1	0	0	0
χ_3	10	0	4	-$\overline{B}$	-B	-$\overline{B}$	B	2	-2	-1	1	0	0	0
χ_4	10	0	4	-B	-$\overline{B}$	-B	$\overline{B}$	2	-2	-1	1	0	0	0
χ_5	36	-1	-6	-1	-1	1	-1	4	0	0	0	0	0	2
χ_6	36	-1	6	-1	-1	-1	1	4	0	0	0	0	0	-2
χ_7	56	-1	0	0	0	0	0	-8	0	-2	0	0	0	0
χ_8	64	1	8	-1	-1	1	-1	0	0	-1	-1	0	0	0
χ_9	64	1	-8	-1	-1	-1	1	0	0	-1	1	0	0	0
χ_{10}	70	0	0	0	0	0	0	-2	-2	2	0	-D	0	0
χ_{11}	70	0	0	0	0	0	0	-2	-2	2	0	D	0	0
χ_{12}	90	0	-6	1	1	1	-1	2	2	0	0	0	0	-2
χ_{13}	90	0	6	1	1	-1	1	2	2	0	0	0	0	2
χ_1	20	0	6	-A	-A	-1	A	0	D	-D	0	F	0	0
χ_2	20	0	-6	-A	-A	1	-A	0	D	-D	0	-F	0	0
χ_3	36	A	6	A	A	-1	A	0	D	0	0	-F	0	0
χ_4	36	A	-6	A	A	1	-A	0	D	0	0	F	0	0
χ_5	56	A	0	0	0	0	0	0	E	-D	0	0	0	0
χ_6	64	-A	-8	A	A	-1	A	0	0	A	1	0	0	0
χ_7	64	-A	8	A	A	1	-A	0	0	A	-1	0	0	0
χ_8	80	0	-4	C	-$\overline{C}$	B	$\overline{C}$	0	0	-A	-1	0	0	0
χ_9	80	0	-4	-$\overline{C}$	C	$\overline{B}$	-C	0	0	-A	-1	0	0	0
χ_{10}	80	0	4	C	-$\overline{C}$	-B	-$\overline{C}$	0	0	-A	1	0	0	0
χ_{11}	80	0	4	-$\overline{C}$	C	-$\overline{B}$	C	0	0	-A	1	0	0	0

A = -i, B = $\frac{-1-\sqrt{7}i}{2}$ = -1-b7, C = -E(28)^3-E(28)^{19}-E(28)^{27},
D = 2i, E = -4i, F = 1-i

Table 6: **IrrProj(G,α_i) for $2^4{:}S_6$**

$\alpha_1 = \alpha_2^2 = \alpha_3^2 = \alpha_4^2 = 1$

$[g]_G$	1a	2a	2b	4a	2c	3a	6a	2d	4b	4c	6b	12a	4d	8a	2e	4d	4e	8b	5a	6c	3b
$\lvert C_G(g)\rvert$	11520	768	384	96	128	72	24	64	32	64	12	12	16	16	192	64	16	16	5	6	18
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	1	-1	-1	-1	1	1	1	1	1	-1	-1	1	1	-1	-1	-1	-1	1	-1	1
χ_3	5	5	3	3	3	2	2	1	1	1	0	0	-1	-1	-1	-1	1	1	0	-1	-1
χ_4	5	5	-3	-3	-3	2	2	1	1	1	0	0	-1	-1	1	1	-1	-1	0	1	-1
χ_5	5	5	-1	-1	-1	-1	-1	1	1	1	-1	-1	-1	-1	3	3	1	1	0	0	2
χ_6	5	5	1	1	1	-1	-1	1	1	1	1	1	-1	-1	-3	-3	-1	-1	0	0	2
χ_7	9	9	3	3	3	0	0	1	1	1	0	0	1	1	3	3	-1	-1	-1	0	0
χ_8	9	9	-3	-3	-3	0	0	1	1	1	0	0	1	1	-3	-3	1	1	-1	0	0
χ_9	10	10	-2	-2	-2	1	1	-2	-2	-2	1	1	0	0	2	2	0	0	0	-1	1
χ_{10}	10	10	2	2	2	1	1	-2	-2	-2	-1	-1	0	0	-2	-2	0	0	0	1	1
χ_{11}	15	-1	-7	1	1	3	-1	3	-1	-1	-1	1	1	-1	-3	1	-1	1	0	0	0
χ_{12}	15	-1	5	1	-3	3	-1	-1	-1	3	-1	1	-1	1	-3	1	1	-1	0	0	0
χ_{13}	15	-1	7	-1	-1	3	-1	3	-1	-1	1	-1	1	-1	3	-1	1	-1	0	0	0
χ_{14}	15	-1	-5	-1	3	3	-1	-1	-1	3	1	-1	-1	1	3	-1	-1	1	0	0	0
χ_{15}	16	16	0	0	0	-2	-2	0	0	0	0	0	0	0	0	0	0	0	1	0	-2
χ_{16}	30	-2	-2	2	-2	-3	1	2	-2	2	1	-1	0	0	-6	2	0	0	0	0	0
χ_{17}	30	-2	2	-2	2	-3	1	2	-2	2	-1	1	0	0	6	-2	0	0	0	0	0
χ_{18}	45	-3	9	-3	1	0	0	1	1	-3	0	0	-1	1	-3	1	-1	1	0	0	0
χ_{19}	45	-3	-9	3	-1	0	0	1	1	-3	0	0	-1	1	3	-1	1	-1	0	0	0
χ_{20}	45	-3	-3	-3	5	0	0	-3	1	1	0	0	1	-1	-3	1	1	-1	0	0	0
χ_{21}	45	-3	3	3	-5	0	0	-3	1	1	0	0	1	-1	3	-1	-1	1	0	0	0
χ_1	4	4	0	0	0	-1	-1	0	0	0	A	A	0	0	0	0	0	0	1	0	-2
χ_2	4	4	0	0	0	-1	-1	0	0	0	-A	-A	0	0	0	0	0	0	1	0	-2
χ_3	4	4	0	0	0	2	2	0	0	0	0	0	0	0	0	0	0	0	1	C	1
χ_4	4	4	0	0	0	2	2	0	0	0	0	0	0	0	0	0	0	0	1	-C	1
χ_5	16	16	0	0	0	2	2	0	0	0	0	0	0	0	0	0	0	0	-1	0	-2
χ_6	20	20	0	0	0	-2	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	2
χ_7	60	-4	0	0	0	6	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_8	60	-4	0	0	0	-3	1	0	0	0	-A	A	0	0	0	0	0	0	0	0	0
χ_9	60	-4	0	0	0	-3	1	0	0	0	A	-A	0	0	0	0	0	0	0	0	0
χ_1	6	2	4	2	0	3	-1	-2	0	-2	1	-1	0	0	0	0	-2	0	1	0	0
χ_2	6	2	-4	-2	0	3	-1	-2	0	-2	-1	1	0	0	0	0	2	0	1	0	0
χ_3	10	-2	-4	2	0	1	1	-2	0	2	-1	-1	-2	0	-4	0	0	0	0	-1	-1
χ_4	10	-2	-4	2	0	1	1	-2	0	2	-1	-1	2	0	4	0	0	0	0	1	-1
χ_5	10	-2	4	-2	0	1	1	-2	0	2	1	1	2	0	-4	0	0	0	0	-1	-1
χ_6	10	-2	4	-2	0	1	1	-2	0	2	1	1	-2	0	4	0	0	0	0	1	-1
χ_7	20	-4	0	0	0	2	2	4	0	-4	0	0	0	0	0	0	0	0	0	0	-2
χ_8	24	8	8	4	0	3	-1	0	0	0	-1	1	0	0	0	0	0	0	-1	0	0
χ_9	24	8	-8	-4	0	3	-1	0	0	0	1	-1	0	0	0	0	0	0	-1	0	0
χ_{10}	30	10	-4	-2	0	-3	1	-2	0	-2	-1	1	0	0	0	0	-2	0	0	0	0
χ_{11}	30	10	4	2	0	-3	1	-2	0	-2	1	-1	0	0	0	0	2	0	0	0	0
χ_{12}	36	12	0	0	0	0	0	4	0	4	0	0	0	0	0	0	0	0	1	0	0
χ_{13}	40	-8	8	-4	0	1	1	0	0	0	-1	-1	0	0	0	0	0	0	0	0	2
χ_{14}	40	-8	-8	4	0	1	1	0	0	0	1	1	0	0	0	0	0	0	0	0	2
χ_{15}	40	-8	0	0	0	-2	-2	0	0	0	0	0	0	0	-8	0	0	0	0	1	-1
χ_{16}	40	-8	0	0	0	-2	-2	0	0	0	0	0	0	0	8	0	0	0	0	-1	-1
χ_1	20	-4	0	0	0	-2	-2	0	0	0	0	0	0	B	0	0	0	0	0	0	-2
χ_2	20	-4	0	0	0	-2	-2	0	0	0	0	0	0	-B	0	0	0	0	0	0	-2
χ_3	24	8	0	0	0	6	-2	0	0	0	0	0	0	0	0	0	0	0	1	0	0
χ_4	24	8	0	0	0	-3	1	0	0	0	-A	A	0	0	0	0	0	0	1	0	0
χ_5	24	8	0	0	0	-3	1	0	0	0	A	-A	0	0	0	0	0	0	1	0	0
χ_6	36	12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	B	-1	0	0
χ_7	36	12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-B	-1	0	0
χ_8	40	-8	0	0	0	-1	-1	0	0	0	A	A	0	0	0	0	0	0	0	0	2
χ_9	40	-8	0	0	0	-1	-1	0	0	0	-A	-A	0	0	0	0	0	0	0	0	2
χ_{10}	40	-8	0	0	0	2	2	0	0	0	0	0	0	0	0	0	0	0	0	C	-1
χ_{11}	40	-8	0	0	0	2	2	0	0	0	0	0	0	0	0	0	0	0	0	-C	-1

A = $-\sqrt{3}$i = -i3, B = $-2\sqrt{2}$i = -2i2, C = $\sqrt{3}$ = r3

Table 7: **IrrProj(G,α_i) for** $2^5{:}S_5$

$\alpha_1 = \alpha_2^2 = \alpha_3^2 = \alpha_4^2 = 1$

$[g]_G$	1a	2a	2b	2c	2d	4a	4b	2e	2f	4c	4d	4e	4f	2g	6a	3a	6b	6c	4g	8a	8b	4h	10a	5a
$\|C_G(g)\|$	3840	256	384	640	96	32	32	96	64	64	32	32	64	64	12	12	12	12	16	16	16	16	10	10
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	1	-1	-1	1	1	-1	-1	1	1	1	-1	-1	-1	-1	1	1	-1	1	1	-1	-1	-1	1
χ_3	1	1	-1	-1	-1	-1	1	1	1	1	1	-1	-1	-1	-1	1	-1	1	-1	-1	1	1	-1	1
χ_4	1	1	1	1	-1	-1	-1	-1	1	1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	1	1
χ_5	4	4	-4	-4	2	2	-2	-2	0	0	0	0	0	0	-1	1	-1	1	0	0	0	0	1	-1
χ_6	4	4	-4	-4	-2	-2	2	2	0	0	0	0	0	0	-1	1	1	-1	0	0	0	0	1	-1
χ_7	4	4	4	4	-2	-2	-2	-2	0	0	0	0	0	0	1	1	1	1	0	0	0	0	-1	-1
χ_8	4	4	4	4	2	2	2	2	0	0	0	0	0	0	1	1	-1	-1	0	0	0	0	-1	-1
χ_9	5	5	-5	-5	1	1	-1	-1	1	1	1	-1	-1	-1	1	-1	1	-1	-1	-1	1	1	0	0
χ_{10}	5	5	-5	-5	-1	-1	1	1	1	1	1	-1	-1	-1	1	-1	-1	1	1	1	-1	-1	0	0
χ_{11}	5	5	5	5	-1	-1	-1	-1	1	1	1	1	1	1	-1	-1	-1	-1	1	1	1	1	0	0
χ_{12}	5	5	5	5	1	1	1	1	1	1	1	1	1	1	-1	-1	1	1	-1	-1	-1	-1	0	0
χ_{13}	6	6	6	6	0	0	0	0	-2	-2	-2	-2	-2	-2	0	0	0	0	0	0	0	0	1	1
χ_{14}	6	6	-6	-6	0	0	0	0	-2	-2	-2	2	2	2	0	0	0	0	0	0	0	0	-1	1
χ_{15}	15	-1	3	-5	3	-1	-1	3	3	-1	-1	-1	3	-1	0	0	0	0	1	-1	-1	1	0	0
χ_{16}	15	-1	3	-5	-3	1	1	-3	3	-1	-1	-1	3	-1	0	0	0	0	-1	1	1	-1	0	0
χ_{17}	15	-1	-3	5	-3	1	-1	3	3	-1	-1	1	-3	1	0	0	0	0	-1	1	-1	1	0	0
χ_{18}	15	-1	-3	5	3	-1	1	-3	3	-1	-1	1	-3	1	0	0	0	0	1	-1	1	-1	0	0
χ_{19}	15	-1	3	-5	3	-1	-1	3	-1	3	-1	-1	-1	3	0	0	0	0	-1	1	1	-1	0	0
χ_{20}	15	-1	3	-5	-3	1	1	-3	-1	3	-1	-1	-1	3	0	0	0	0	1	-1	-1	1	0	0
χ_{21}	15	-1	-3	5	-3	1	-1	3	-1	3	-1	1	1	-3	0	0	0	0	1	-1	1	-1	0	0
χ_{22}	15	-1	-3	5	3	-1	1	-3	-1	3	-1	1	1	-3	0	0	0	0	-1	1	-1	1	0	0
χ_{23}	30	-2	6	-10	0	0	0	0	-2	-2	2	2	-2	-2	0	0	0	0	0	0	0	0	0	0
χ_{24}	30	-2	-6	10	0	0	0	0	-2	-2	2	-2	2	2	0	0	0	0	0	0	0	0	0	0
χ_1	12	4	0	0	0	0	0	0	0	0	A	0	0	0	0	0	0	0	0	0	0	0	0	2
χ_2	12	4	0	0	0	0	0	0	0	0	-A	0	0	0	0	0	0	0	0	0	0	0	0	2
χ_3	20	-4	8	0	0	0	0	0	0	0	0	0	0	0	2	2	0	0	0	0	0	0	0	0
χ_4	20	-4	-8	0	0	0	0	0	0	0	0	0	0	0	-2	2	0	0	0	0	0	0	0	0
χ_5	20	-4	-8	0	0	0	0	0	0	0	0	0	0	0	1	-1	C	-C	0	0	0	0	0	0
χ_6	20	-4	-8	0	0	0	0	0	0	0	0	0	0	0	1	-1	-C	C	0	0	0	0	0	0
χ_7	20	-4	8	0	0	0	0	0	0	0	0	0	0	0	-1	-1	-C	-C	0	0	0	0	0	0
χ_8	20	-4	8	0	0	0	0	0	0	0	0	0	0	0	-1	-1	C	C	0	0	0	0	0	0
χ_9	24	8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	F	-1
χ_{10}	24	8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-F	-1
χ_1	6	2	0	-4	0	0	0	0	0	0	B	0	-B	-B	0	0	0	0	D	E	-D	-E	-1	1
χ_2	6	2	0	-4	0	0	0	0	0	0	B	0	-B	-B	0	0	0	0	-D	-E	D	E	-1	1
χ_3	6	2	0	-4	0	0	0	0	0	0	-B	0	B	B	0	0	0	0	D	-E	-D	E	-1	1
χ_4	6	2	0	-4	0	0	0	0	0	0	-B	0	B	B	0	0	0	0	-D	E	D	-E	-1	1
χ_5	6	2	0	4	0	0	0	0	0	0	B	0	B	B	0	0	0	0	-D	-E	-D	-E	1	1
χ_6	6	2	0	4	0	0	0	0	0	0	B	0	B	B	0	0	0	0	D	E	D	E	1	1
χ_7	6	2	0	4	0	0	0	0	0	0	-B	0	-B	-B	0	0	0	0	-D	E	-D	E	1	1
χ_8	6	2	0	4	0	0	0	0	0	0	-B	0	-B	-B	0	0	0	0	D	-E	D	-E	1	1
χ_9	20	-4	0	0	0	0	0	0	0	0	0	0	-A	A	0	2	0	0	0	0	0	0	0	0
χ_{10}	20	-4	0	0	0	0	0	0	0	0	0	0	A	-A	0	2	0	0	0	0	0	0	0	0
χ_{11}	24	8	0	-16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-1
χ_{12}	24	8	0	16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1
χ_{13}	40	-8	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	0	0	0	0	0	0	0	0
χ_1	6	2	0	-4	0	0	0	0	2	2	0	2	0	0	0	0	0	0	-2	0	-2	0	-1	-1
χ_2	6	2	0	-4	0	0	0	0	2	2	0	2	0	0	0	0	0	0	2	0	2	0	-1	-1
χ_3	6	2	0	4	0	0	0	0	2	2	0	-2	0	0	0	0	0	0	2	0	-2	0	1	-1
χ_4	6	2	0	4	0	0	0	0	2	2	0	-2	0	0	0	0	0	0	-2	0	2	0	1	-1
χ_5	6	2	0	-4	0	0	0	0	-2	-2	0	-2	0	0	0	0	0	0	0	B	0	B	-1	-1
χ_6	6	2	0	-4	0	0	0	0	-2	-2	0	-2	0	0	0	0	0	0	0	-B	0	-B	-1	-1
χ_7	6	2	0	4	0	0	0	0	-2	-2	0	2	0	0	0	0	0	0	0	-B	0	B	1	-1
χ_8	6	2	0	4	0	0	0	0	-2	-2	0	2	0	0	0	0	0	0	0	B	0	-B	1	-1
χ_9	20	-4	0	0	0	0	0	0	-4	4	0	0	0	0	0	2	0	0	0	0	0	0	0	0
χ_{10}	20	-4	0	0	0	0	0	0	4	-4	0	0	0	0	0	2	0	0	0	0	0	0	0	0
χ_{11}	24	8	0	-16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
χ_{12}	24	8	0	16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	1
χ_{13}	40	-8	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	0	0	0	0	0	0	0	0

Table 7 (continued): **IrrProj(G,α_i)** **for** $2^5{:}S_5$

$\alpha_5^2 = \alpha_6^2 = \alpha_7^2 = \alpha_8^2 = 1$

$[g]_G$	1a	2a	2b	2c	2d	4a	4b	2e	2f	4c	4d	4e	4f	2g	6a	3a	6b	6c	4g	8a	8b	4h	10a	5a
$\|C_G(g)\|$	3840	256	384	640	96	32	32	96	64	64	32	32	64	64	12	12	12	12	16	16	16	16	10	10
χ_1	10	-2	-4	0	4	0	-2	-2	2	-2	0	2	0	0	1	1	-1	-1	0	0	0	0	0	0
χ_2	10	-2	-4	0	2	-2	0	-4	-2	2	0	-2	0	0	1	1	1	1	0	0	0	0	0	0
χ_3	10	-2	-4	0	-2	2	0	4	-2	2	0	-2	0	0	1	1	-1	-1	0	0	0	0	0	0
χ_4	10	-2	-4	0	-4	0	2	2	2	-2	0	2	0	0	1	1	1	1	0	0	0	0	0	0
χ_5	10	-2	4	0	-4	0	-2	-2	2	-2	0	-2	0	0	-1	1	1	-1	0	0	0	0	0	0
χ_6	10	-2	4	0	-2	2	0	-4	-2	2	0	2	0	0	-1	1	-1	1	0	0	0	0	0	0
χ_7	10	-2	4	0	2	-2	0	4	-2	2	0	2	0	0	-1	1	1	-1	0	0	0	0	0	0
χ_8	10	-2	4	0	4	0	2	2	2	-2	0	-2	0	0	-1	1	-1	1	0	0	0	0	0	0
χ_9	12	4	0	0	0	0	0	0	-4	-4	0	0	0	0	0	0	0	0	0	0	0	0	0	-2
χ_{10}	12	4	0	0	0	0	0	0	4	4	0	0	0	0	0	0	0	0	0	0	0	0	0	-2
χ_{11}	20	-4	-8	0	2	2	-2	2	0	0	0	0	0	0	-1	-1	1	1	0	0	0	0	0	0
χ_{12}	20	-4	-8	0	-2	-2	2	-2	0	0	0	0	0	0	-1	-1	-1	-1	0	0	0	0	0	0
χ_{13}	20	-4	8	0	-2	-2	-2	2	0	0	0	0	0	0	1	-1	-1	1	0	0	0	0	0	0
χ_{14}	20	-4	8	0	2	2	2	-2	0	0	0	0	0	0	1	-1	1	-1	0	0	0	0	0	0
χ_{15}	24	8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	F	1
χ_{16}	24	8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-F	1
χ_1	4	4	4	4	0	0	0	0	0	0	0	0	0	0	2	-2	0	0	0	0	0	0	-1	1
χ_2	4	4	-4	-4	0	0	0	0	0	0	0	0	0	0	-2	-2	0	0	0	0	0	0	1	1
χ_3	4	4	-4	-4	0	0	0	0	0	0	0	0	0	0	1	1	-C	-C	0	0	0	0	1	1
χ_4	4	4	-4	-4	0	0	0	0	0	0	0	0	0	0	1	1	C	C	0	0	0	0	1	1
χ_5	4	4	4	4	0	0	0	0	0	0	0	0	0	0	-1	1	C	-C	0	0	0	0	-1	1
χ_6	4	4	4	4	0	0	0	0	0	0	0	0	0	0	-1	1	-C	C	0	0	0	0	-1	1
χ_7	6	6	-6	-6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-D	-D	-D	-D	-1	-1
χ_8	6	6	-6	-6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	D	D	D	D	-1	-1
χ_9	6	6	6	6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	D	D	-D	-D	1	-1
χ_{10}	6	6	6	6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-D	-D	D	D	1	-1
χ_{11}	30	-2	6	-10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-D	D	-D	D	0	0
χ_{12}	30	-2	6	-10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	D	-D	D	-D	0	0
χ_{13}	30	-2	-6	10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	D	-D	-D	D	0	0
χ_{14}	30	-2	-6	10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-D	D	D	-D	0	0
χ_1	4	4	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	0	0	0	0	0	0	F	1
χ_2	4	4	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	0	0	0	0	0	0	-F	1
χ_3	8	8	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	2
χ_4	12	12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2
χ_5	60	-4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_1	2	2	0	0	0	0	0	0	2	2	2	0	0	0	0	2	0	0	0	0	0	0	0	2
χ_2	6	6	0	0	0	0	0	0	-2	-2	-2	0	0	0	0	0	0	0	0	0	0	0	F	1
χ_3	6	6	0	0	0	0	0	0	-2	-2	-2	0	0	0	0	0	0	0	0	0	0	0	-F	1
χ_4	8	8	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	-2
χ_5	10	10	0	0	0	0	0	0	2	2	2	0	0	0	0	-2	0	0	0	0	0	0	0	0
χ_6	30	-2	0	0	0	0	0	0	6	-2	-2	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_7	30	-2	0	0	0	0	0	0	-2	6	-2	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_8	30	-2	0	0	0	0	0	0	-2	-2	2	-4	0	0	0	0	0	0	0	0	0	0	0	0
χ_9	30	-2	0	0	0	0	0	0	-2	-2	2	4	0	0	0	0	0	0	0	0	0	0	0	0

A = -4i, B = -2i, C = $\sqrt{3}$i = i3, D = $\sqrt{2}$i = i2, E = $\sqrt{2}$ = r2, F = -$\sqrt{5}$= -r5

Table 8: **IrrProj(G,α_i) for** $(2^3{:}L_3(2)) \times 2$

$\alpha_1 = \alpha_2^2 = \alpha_3^2 = 1$

$[g]_G$	1a	2a	2b	2c	2d	2e	2f	2g	4a	4b	4c	4d	4e	4f	3a	6a	6b	6c	7a	14a	14b	7b
$\lvert C_G(g)\rvert$	2688	2688	384	384	64	64	64	64	32	32	16	16	16	16	12	12	12	12	14	14	14	14
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	-1	-1	1	-1	1	1	-1	1	-1	-1	1	1	-1	1	-1	-1	1	1	-1	-1	1
χ_3	3	-3	-3	3	1	-1	-1	1	-1	1	-1	1	1	-1	0	0	0	0	C	-C	-$\overline{C}$	$\overline{C}$
χ_4	3	-3	-3	3	1	-1	-1	1	-1	1	-1	1	1	-1	0	0	0	0	$\overline{C}$	-$\overline{C}$	-C	C
χ_5	3	3	3	3	-1	-1	-1	-1	-1	-1	1	1	1	1	0	0	0	0	C	C	$\overline{C}$	$\overline{C}$
χ_6	3	3	3	3	-1	-1	-1	-1	-1	-1	1	1	1	1	0	0	0	0	$\overline{C}$	$\overline{C}$	C	C
χ_7	6	6	6	6	2	2	2	2	2	2	0	0	0	0	0	0	0	0	-1	-1	-1	-1
χ_8	6	-6	-6	6	-2	2	2	-2	2	-2	0	0	0	0	0	0	0	0	-1	1	1	-1
χ_9	7	7	7	7	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	0	0	0	0
χ_{10}	7	-7	-7	7	1	-1	-1	1	-1	1	1	-1	-1	1	1	-1	-1	1	0	0	0	0
χ_{11}	7	7	-1	-1	3	3	-1	-1	-1	-1	1	1	-1	-1	1	1	-1	-1	0	0	0	0
χ_{12}	7	7	-1	-1	-1	-1	3	3	-1	-1	-1	-1	1	1	1	1	-1	-1	0	0	0	0
χ_{13}	7	-7	1	-1	1	-1	3	-3	-1	1	1	-1	1	-1	1	-1	1	-1	0	0	0	0
χ_{14}	7	-7	1	-1	-3	3	-1	1	-1	1	-1	1	-1	1	1	-1	1	-1	0	0	0	0
χ_{15}	8	8	8	8	0	0	0	0	0	0	0	0	0	0	-1	-1	-1	-1	1	1	1	1
χ_{16}	8	-8	-8	8	0	0	0	0	0	0	0	0	0	0	-1	1	1	-1	1	-1	-1	1
χ_{17}	14	14	-2	-2	2	2	2	2	-2	-2	0	0	0	0	-1	-1	1	1	0	0	0	0
χ_{18}	14	-14	2	-2	-2	2	2	-2	-2	2	0	0	0	0	-1	1	-1	1	0	0	0	0
χ_{19}	21	21	-3	-3	1	1	-3	-3	1	1	-1	-1	1	1	0	0	0	0	0	0	0	0
χ_{20}	21	21	-3	-3	-3	-3	1	1	1	1	1	1	-1	-1	0	0	0	0	0	0	0	0
χ_{21}	21	-21	3	-3	3	-3	1	-1	1	-1	-1	1	-1	1	0	0	0	0	0	0	0	0
χ_{22}	21	-21	3	-3	-1	1	-3	3	1	-1	1	-1	1	-1	0	0	0	0	0	0	0	0
χ_1	4	-4	-4	-4	0	0	0	0	0	0	0	0	0	0	1	1	1	1	-$\overline{C}$	-$\overline{C}$	-C	-C
χ_2	4	-4	-4	-4	0	0	0	0	0	0	0	0	0	0	1	1	1	1	-C	-C	-$\overline{C}$	-$\overline{C}$
χ_3	4	4	4	-4	0	0	0	0	0	0	0	0	0	0	1	-1	-1	1	-$\overline{C}$	$\overline{C}$	C	-C
χ_4	4	4	4	-4	0	0	0	0	0	0	0	0	0	0	1	-1	-1	1	-C	C	$\overline{C}$	-$\overline{C}$
χ_5	6	-6	-6	-6	0	0	0	0	0	0	A	-A	-A	A	0	0	0	0	-1	-1	-1	-1
χ_6	6	-6	-6	-6	0	0	0	0	0	0	-A	A	A	-A	0	0	0	0	-1	-1	-1	-1
χ_7	6	6	6	-6	0	0	0	0	0	0	A	A	A	A	0	0	0	0	-1	1	1	-1
χ_8	6	6	6	-6	0	0	0	0	0	0	-A	-A	-A	-A	0	0	0	0	-1	1	1	-1
χ_9	8	8	8	-8	0	0	0	0	0	0	0	0	0	0	-1	1	1	-1	1	-1	-1	1
χ_{10}	8	-8	-8	-8	0	0	0	0	0	0	0	0	0	0	-1	-1	-1	-1	1	1	1	1
χ_{11}	14	14	-2	2	0	0	0	0	0	0	-A	-A	A	A	-1	1	-1	1	0	0	0	0
χ_{12}	14	14	-2	2	0	0	0	0	0	0	A	A	-A	-A	-1	1	-1	1	0	0	0	0
χ_{13}	14	-14	2	2	0	0	0	0	0	0	-A	A	-A	A	-1	-1	1	1	0	0	0	0
χ_{14}	14	-14	2	2	0	0	0	0	0	0	A	-A	A	-A	-1	-1	1	1	0	0	0	0
χ_{15}	28	28	-4	4	0	0	0	0	0	0	0	0	0	0	1	-1	1	-1	0	0	0	0
χ_{16}	28	-28	4	4	0	0	0	0	0	0	0	0	0	0	1	1	-1	-1	0	0	0	0
χ_1	8	-8	0	0	0	0	0	0	0	0	0	0	0	0	2	-2	0	0	1	-1	-1	1
χ_2	8	8	0	0	0	0	0	0	0	0	0	0	0	0	2	2	0	0	1	1	1	1
χ_3	8	-8	0	0	0	0	0	0	0	0	0	0	0	0	-1	1	B	-B	1	-1	-1	1
χ_4	8	-8	0	0	0	0	0	0	0	0	0	0	0	0	-1	1	-B	B	1	-1	-1	1
χ_5	8	8	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	B	B	1	1	1	1
χ_6	8	8	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	-B	-B	1	1	1	1
χ_7	24	-24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	C	-C	-$\overline{C}$	$\overline{C}$
χ_8	24	-24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	$\overline{C}$	-$\overline{C}$	-C	C
χ_9	24	24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	C	C	$\overline{C}$	$\overline{C}$
χ_{10}	24	24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	$\overline{C}$	$\overline{C}$	C	C

A = -$\sqrt{2}$ = -r2, B = -$\sqrt{3}$i = -i3, C = $\frac{-1+\sqrt{7}i}{2}$ = b7

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