Models and theories for the choice of teaching strategies in mathematics

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Abstract. This paper aims to highlight the role of the subject, not only as a school discipline but also a tool to educate students to rational thinking and a means for the development of social skills. The intention is to dispel the myth of a cold and prepackaged mathematics, accessible to the chosen few: the idea of a mathematics for all is supported through an experience with a student with specific learning disorders, with which a personalized teaching strategy has allowed the achievement of set goals. A case study of a student with specific reading and writing disorders attended the mathematical analysis course 1 of the degree course in architecture and the personalized teaching strategy created for him are described. Finally, some mathematical models for the choice of better teaching strategies are exhibited.

Keywords: inquiry model, social skills, personalized didactic strategy, decision making, AHP.

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1. Introduction

Today society is characterized by its complexity and flexibility, markets and communication are based on globalization and internationalization. The high unpredictability of the social context in which we find ourselves urges the educational systems to a great responsibility, that of stimulating all students to an autonomy of thought and action through the construction of a personality that can respond to the demands of a reality that, now more than ever, is characterized by a general state of uncertainty. The objective of fostering the pupil freedom of choice and reaction provides the school with an orienting character where the term "orientation" indicates a continuous and personal process that involves awareness, learning and education for choice ([4]). In particular, to put orientation as the main purpose of teaching "means to develop appropriate strategies, methodologies and targeted content from the acquisition of awareness to the understanding of the complex society and the mechanisms that govern the world of studies and work" ([7]).

The current historical context requires that the school recognizes as fundamental the equality of educational opportunities and promotes the achievement of appropriate levels of knowledge and skills by all in order to "ensure to all students high levels of mastery, or at least fully appropriate, of the basic skills in the educational curricula" ([2], p. 12). Therefore, a convergent personalization (ibid.) is fundamental, with respect to which the differentiation of training paths and the programmed and opportunely diversified technical solutions allow everyone to achieve common training goals. However, the enhancement of personal differences is also essential, i.e. promoting different types of individual potential, in relation to the different kinds of talent involved. Therefore, a divergent personalization (ibid.) is necessary, that is the diversification of the training goals, according to the promotion of individual potential. The personalization of learning, then, could represent an effective strategy in the mathematics teaching and learning, in particular for students with specific learning disorders (SLD), for whom ad-hoc educational choices are required.

Therefore, the teacher has to provide, by selecting the best teaching strategy, cognitive tools necessary to start the processes of personalization, self-learning and self-orientation. From an operational point of view, teaching thus becomes a decision making problem and must therefore be addressed as such. Properly, in fact, one can speak of decision when in a situation are present: the alternatives, understood as the possibility of being able to act in several different ways; the probability, seen as the possibility that the outcomes related to each alternative will be realized; the consequences associated with the outcomes. In the school sector, the three elements mentioned (alternatives-probability-consequences) are characteristics of the environment and therefore choosing a teaching strategy is fully equivalent to making a decision as a process that goes from the representation of the problem to the selection of the options through the estimation of expectations and the evaluation of outcomes.

This research considers the teaching process as a decision-making problem. It aims to provide the mathematics teacher with a model that allows him/her to evaluate, case by case, the educational alternatives and choose the "best teaching strategies", i.e. those best suited to the context being analyzed, particularly with reference to students with SLD, for whom the teacher (not always specialized) often has to make decisions in situations of complete uncertainty.

Our research question is: "Can decision theory, applied to mathematics teaching/learning, be an effective tool for the development of personalized educational strategies for students with SLD?". In order to answer this question, we tried to develop a decision-making model that would also take into account relevant results in mathematics education research. More in detail, we started from the hierarchical analytical method of Saaty ([18], [19]), which sets a general objective and takes into account some criteria and alternatives. We identified the Specific General Objective, Specific Criteria and Specific Alternatives according to our personal beliefs and results in mathematics education research. Finally, we customized this model by applying it to a case study of a student with SLD attending the mathematical analysis course 1 of the degree course in architecture. According to the model, we assigned weights to each alternative starting from an observation of the student's behaviors and learning styles. The model provided us with a personalized teaching strategy, which we carried out in the specific case study.

In the following sections we describe the decision-making model from a mathematical standpoint (Sections 2.1 and 2.2), identify the overall objective, criteria and alternatives for the model (Section 2.3), and assign weights to the alternatives based on observation of the specific case study (Section 3). Then, we apply the educational strategy provided by the model to the case study (Section 3.1) and try to answer our research question (Section 4).

2. Teaching as a decision problem

The study of decisions can be made in terms of absolute rationality or limited rationality whose difference lies in the fact that, while the former model is based on an ideal combination of rationality and information preferring the best alternative, the latter recognizes the objective narrowness of the human mind by proposing the selection of the most satisfactory alternative (Lanciano, [9]). In this regard, it must be stressed that the consequences of a decision derive not only from the chosen course of action but also from the state of nature that represents the context in which the decision making process develops. On the basis of the knowledge that the decision maker has of the state of nature, they differ:

- decisions in a situation of certainty: the decision maker knows the state of nature;

- decisions in risk situations: the decision maker has a measure of probability for each state of nature even though he does not know it directly;
- decisions in situations of uncertainty: the decision maker has neither information regarding the state of nature nor the probabilities associated with it.

The attitude used by decision makers towards situations determines the distinction between the normative approach, which bases the choice with reference to rational decision-making ideals, and the descriptive approach, which analyses how to make a decision according to the context.

Based on the objectives and context the teacher must then consider what alternatives he can follow and for each alternative what the various consequences might be. For each pair (alternative, circumstance) the teacher obtains a result in accordance with a utility function: the decision is rational if it is based on the criterion of obtaining a maximum value for the utility function. This decision is however subjective depending on the objectives and utility functions of the teacher and therefore depends on the culture, experience and preferences. Moreover, even if the choice is rational, it is made in terms of limited rationality because, in general, there are few alternatives, but it increases as the teacher expands his/her culture and experience (Delli Rocili, Maturo, [6]; Maturo, Zappacosta, [12]).

2.1 A model for evaluating educational alternatives

Multi-Criteria Decision Analysis (MCDA) provides support to the decision maker, or a group of decision makers, when many conflicting assessments have to be considered, especially in data synthesis phase while working with complex and heterogeneous pieces of information.

Let $\mathcal{A} = \{A_1, A_2, \dots, A_m\}$ be the set of the alternatives, i. e. the possible educational strategies.

Let $\mathcal{O} = \{O_1, O_2, \dots, O_n\}$ be the set of the objectives that we want to achieve.

Let $\mathcal{D} = \{D_1, D_2, \dots, D_k\}$ be the set of the decision making processes.

In the first phase decision makers must establish a procedure to assign to each pair (alternative A_i, objective O_j) a score p_{ij} that measures the degree in which the alternative A_i satisfies the objective O_j.

Assume that $p_{ij} \in [0, 1]$, where:

- $p_{ij} = 0$ if the objective O_j is not at all satisfied by A_i;
- $p_{ij} = 1$ if the objective O_j is completely satisfied by A_i

At the end of the procedure we obtain a matrix $P = (p_{ij})$ of the scores which is the starting point of the elaborations that lead to the choice of the alternative, or at least to their ordering, possibly even partial (Maturo, Ventre, [10, 11]).

There may be constraints. It could be necessary to establish for each objective O_j a threshold $j > 0$, with the constraint $p_{ij} \geq j$, for each $i = 1, \ldots, m$.

Furthermore, consideration should be given to mixed strategies that is a convex linear combinations of alternatives A_i. So, a mixed strategy has the form:

$$A(h_1, h_2, \ldots, h_m) = h_1 A_1 + h_2 A_2 + \cdots + h_m A_m$$

with

- $h_1, h_2, \ldots, h_m$ non–negative real numbers;
- the h_i's are such that $h_1 + h_2 + \cdots + h_m = 1$.

The number h_i can represent the fraction of time in which the teaching strategy A_i is adopted.

If we consider also the mixed strategies, then the single alternatives A_i are called pure strategies.

The need to consider mixed strategies also arises in particular if there are "at risk" alternatives: this kind of alternatives have high scores for certain objectives and low for others (possibly below the threshold).

It is appropriate to construct a ranking of the alternative educational plans, i. e., a linear ordering of the alternatives that takes into account the objectives which contribute to the most suitable formation of the student. Such a ranking can be usefully obtained by means of the application of the Analytic Hierarchy Process (Saaty [18], [19]).

2.2 The Analytic Hierarchy Process: attributions of weights and scores

The Analytic Hierarchy Process (AHP) is both a method and a technique that allows to compare alternatives of different qualitative and quantitative nature, not easily comparable in a direct way, through the assignment of numerical values that specify their priority. The first step is the construction of the hierarchical structure that represents the elements to be considered in the decision making problem. The Analytic Hierarchy Process is based on the representation of a decision problem in terms of a directed graph $G = (V, A)$. Let us recall that (Knuth [8]):

- a *directed graph*, or *digraph*, is a pair $G = (V, A)$, where V is a non-empty set whose elements are called *vertices* and A is a set of ordered pairs of vertices, called *arcs*;
- a vertex is indicated with a latin letter; for every arc (u, v), u is called the *initial vertex* and v the *final vertex* or *end vertex*;
- an ordered n-tuple of vertices $(v_1, v_2, \ldots, v_n), n > 1$, is called a *path* with length $n - 1$, if, and only if, every pair $(v_i, v_{i+1}), i = 1, 2, \ldots, n - 1$, is an arc of G.

Furthermore, in our context, we assume the following conditions be satisfied from a directed graph:

1. the vertices are distributed in a fixed integer number $n \geq 2$ of levels, each level is indexed from 1 to n;
2. there is only one vertex of level 1, called the *root* of the directed graph;
3. for every vertex v different from the root there is at least one path having the root as the initial vertex and v as the final vertex;
4. every vertex u of level $i < n$ is the initial vertex of at least one arc and there are no arcs with the initial vertex of level n;
5. if an arc has the initial vertex of level $i < n$, then it has the end vertex in the level $i + 1$.

To this aim it is worth to describe the functional aspects of each level. If $n = 3$ we consider the digraph in which the level 1 vertex is called the *General Objective* and denoted by GO; the vertices of level 2 are called *Criteria* and vertices of level 3 are called *Alternatives*. Briefly, the general objective indicates the choice or the goal of the entire decision-making process; the Alternatives are the different ways we consider in order to reach the GO or the different options we must choose and the criteria are the parameters used to evaluate the alternatives.

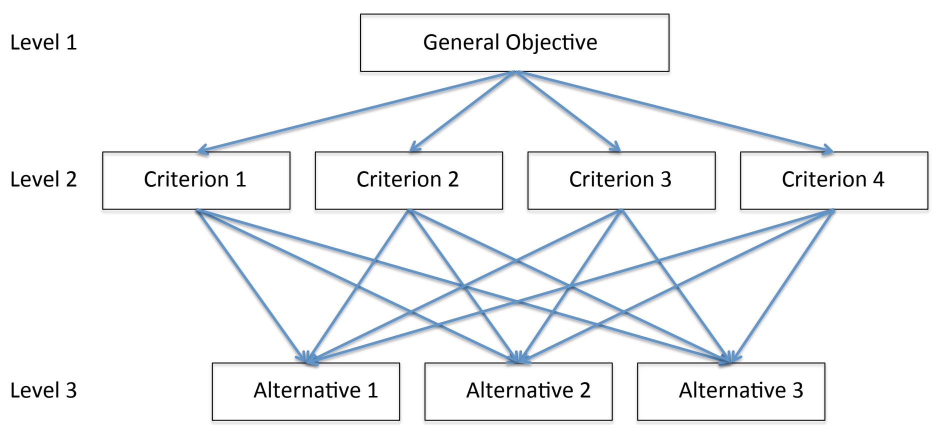

Figure 1: The digraph of the functional aspects

A decision maker assigns a score to each arc following the AHP procedure (Maturo, Ventre, [10], [11]). The second step consists into collecting data to determine the ratios of preference of the elements of a level over any of the higher, i.e. previous, level. The third step implements the estimation of local assessments, i.e. the weightings that express the relative importance of the elements of a hierarchical level over any element of the next higher level. We therefore compare alternatives $A1, A2$ and $A3$ with respect to each criterion and the individual criteria with respect to the general objective. In order to determine the value of each comparison the following scale of evaluations is used:

Value a_{ij}	**Interpretation**
1	i and j are equally important
3	i is a slightly more important than j
5	i is quite more important than j
7	i is definitely more important than j
9	i is absolutely more important than j
1/3	i is a slightly less important than j
1/5	i is quite less important than j
1/7	i is definitely less important than j
1/9	i is absolutely less important than j

Table 1 – Scale of evaluations

Let us remark the following when assigning values to the comparisons:

1. if alternative i assumes the value x in comparison with alternative j with respect to a criterion, then the comparison of alternative j with alternative i with respect to the same criterion assumes the value $1/x$. Analogous is the procedure to assign values when comparing couples of criteria;

2. since equally important alternatives correspond to value 1, the diagonal of the matrices are composed entirely of unit values.

Matrices satisfying claims 1 and 2 above are called *pairwise comparison matrices*. Comparison matrices represent quantitative preferences between criteria or between alternatives.

After constructing the pairwise comparison matrices, we have to fix the weights of the elements in each level that assess the relevance in a scale of values ranging from 0 to 1. These weights must meet the normality condition:

$$w_1 + w_2 + \cdots + w_n = 1.$$

The way to determine the weights is based on the consideration that, if the decision maker knew all the actual weights of the elements of the pairwise comparison matrix, then it would be:

$$A = \left(\frac{w_i}{w_j}\right) = \begin{pmatrix} \frac{w_1}{w_1} & \cdots & \frac{w_1}{w_n} \\ \vdots & \ddots & \vdots \\ \frac{w_n}{w_1} & \cdots & \frac{w_n}{w_n} \end{pmatrix}.$$

In this case the weights would be obtained from any of the rows which are all multiple of the same row $\left(\frac{1}{w_1}, \frac{1}{w_2}, \ldots, \frac{1}{w_n}\right)$. It follows that the matrix A has rank 1.

Being $\boldsymbol{w} = (w_1, w_2, \ldots, w_n)^T$, we get:

$$A\boldsymbol{w} = n\boldsymbol{w}.$$

Thus, from the equation above, n is an eigenvalue of A and $\boldsymbol{w}$ is one of the eigenvectors associated with n. Since the elements on the diagonal are all 1, denoted with $\lambda_1, \lambda_2, \ldots, \lambda_n = n$ the eigenvalues of A, the value of the trace of A is:

$$tr(A) = \lambda_1 + \lambda_2 + \cdots + \lambda_n = n.$$

As n is an eigenvalue of A the other $n-1$ eigenvalues of A must be zero.

The matrix A satisfies the condition $a_{ij}a_{jk} = a_{ik}$ for every i, j, k, called *consistency condition*, that implies transitivity of the preferences, and A is said to be *consistent*.

In practice the decision maker does not know the vector $\boldsymbol{w}$: the a_{ij} values that he assigns according to his judgement may deviate from the unknown $\frac{w_i}{w_j}$. So the decision maker may produce inconsistent pairwise comparison matrices. However the closer the a_{ij} values are to $\frac{w_i}{w_j}$, the closer the maximum eigenvalue is to n and the closer the other eigenvalues are to zero.

Therefore the vector of the weights $\boldsymbol{w'}^T = (w'_1, w'_2, \ldots, w'_n)$ associated to the maximum eigenvalue (among the infinite $\boldsymbol{w'}^T$ we choose the one for which $w'_1 + w'_2 + \cdots + w'_n = 1$) will be an estimate of the vector $\boldsymbol{w}^T = (w_1, w_2, \ldots, w_n)$) the more precise the more the maximum eigenvalue λ_{max} of A is close to n what is due to the continuity of the involved operations (Ventre, [21]).

With these premises, the ratio

$$CI = \frac{\lambda_{max} - n}{n - 1}$$

is defined as the *consistency index* of A and reveals how far the matrix is from consistency.

2.3 A decision-making model for mathematical education

Mathematics is one of the disciplines in which many students manifest difficulties that compromise the relationship with the subject. A student who comes out of secondary school has a long series of "failures" accompanied by the conviction that she can never do mathematics because she is not good at it. Mathematics, still today, is often taught using a transmissive model, derived from behavioral theories (e.g. Skinner, [20]). According to this model, a mathematics lesson is usually developed through the following phases: correction of the assigned tasks, resolution by the teacher or a student of more complex ones, explanation by the teacher of a part of the program, carrying out exercises on the new topic. A transmissive model leads to the assumption that, in mathematics, the absence of errors certifies the absence of difficulties and on the other hand the absence of difficulties guarantees the absence of errors (Zan, [24]). This identification leads to the didactic objective of obtaining the greatest number of correct answers and, inevitably, to the fear of making mistakes and the conviction that mathematics is not for everyone (Zan and Di Martino, [25]). Many researchers (e.g. Piaget, [16]; Von Glasersfeld, [23]) showed that the student has to actively build mathematical concepts and techniques in order to achieve meaningful learning. This led first to the development of constructivist models and then to the socio-constructivist models (e.g. Vygotskij, [22] and Radford, [17]), which consider learning as a necessarily social process, which takes place first among people and only later within the student. Among the socio-constructivist models, the collaborative group and peer tutoring (Pesci, [14] and [15]) that take on both the disciplinary dimension and the affective and social dimension and facilitate discussion in the classroom. Another constructivist model is the inquiry model (Borasi, [5]) that proposes a positive and fundamental role of errors in mathematics teaching. This model sees knowledge as a dynamic process of investigation where cognitive conflict and doubt represent the motivations to continuously search for a more and more refined understanding. Therefore, instead of eliminating ambiguities and contradictions to avoid confusion or errors, these elements must be highlighted to stimulate and give shape to ideas and discussions. Questions such as "what would happen if this result were true?" or "under what circumstances could this error be corrected?" lead to a reformulation of the problem where error is only the starting point for a deeper understanding. Communication in the classroom plays a fundamental role and so does the conception of mathematics as a humanistic discipline: the teacher provides the necessary support for the student's autonomous search for understanding, who in turn is an active member of a research community (Pascucci, [13]).

Starting from the above premises and our personal beliefs, we set the general objective, criteria and alternatives for the decisional model.

- General objective: to guarantee a training that, through education, ensures the formation of knowledge and the development of action and communication skills;

- Criterion 1: acquisition of the objectives set out in the educational plan. This criterion considers how much a teaching strategy allows you to address all the topics set out from the course of study through good time management;

- Criterion 2: ability to measure oneself against peers in a safe and rational way while respecting the ideas of others in accordance with the Indications for the curriculum (2007) (see [27]) where mathematics "contributes to the development of ability to communicate and discuss, to argue correctly, to understand the points of view and arguments of others";

- Criterion 3: internalization of the objectives set by the didactic plan, intended as the ability to re-elaborate knowledge from a critical and personal point of view because the most effective way to bring students closer to mathematics is therefore the image of a "method to face problems, a language, a box of tools that allow us to enhance our intuition" (Baccaglini Frank et al., [1]);

- Criterion 4: ability to cope with trials, planned or not, without negative moods, anxiety and terror of judgement; This criterion is inspired by the need to build learning teaching models that take into account the emotions: knowing how to manage situations by overcoming the fear of failure is a skill that the school must teach with the same care with which teach disciplinary skills;

- Alternative 1: new didactics, inspired by the inquiry model that is a model of teaching-learning that proposes a positive and fundamental role of errors in mathematics teaching and that puts the pupil and her emotions at the center of the context with cooperative learning experiences;

- Alternative 2: traditional didactic, that prefers frontal lessons under the assumption that, in terms of learning, the acquisition and internalization take place at the same time and that the error is the manifestation of the failure to complete one of the two processes;

- Alternative 3: distance learning, characterized by virtual interactions among teacher and students, mediated through the use of technological tools.

These alternatives were chosen because Alternative 2 is a "passive old strategy" based on the transmissive method which is the most widespread in Italian schools, while Alternatives 1-3 are "new active strategies", developed in the field of research in mathematics teaching, based on the idea that knowledge is a dynamic process of investigation. The comparison between Alternative 1 and Alternative 3 allows to understand if the teaching can be conducted to an online practice, given the recent diffusion of telematic courses.

3. A case study

In this section we describe a case study to which the model described above has been applied to build a customized teaching strategy for a student with specific learning disorders.

To this end, indeed, the Regional School Offices are committed to promote the issuance of detailed certifications that allow as much as possible, together with parents and figures who follow the student in school activities, a Personalized Didactic Plan (PDP) (MIUR, [29]).

A common trait to the different SLDs, which is particularly interesting from a didactical point of view, is the problem these students encounter with short term memory. In the field of cognitive psychology, the concept is extended with that of "Working memory" (Baddeley and Hitch, [3]) where the term specifically indicates that set of notions necessary for written and oral productions that remain short in the student's mind. As this capacity is reduced, temporary storage and the first management/manipulation of data is compromised, leading to the difficulty of taking notes, difficulty of attention and the need for longer periods of time.

It is therefore necessary to reduce the information load, leaving more space for working time to store the fundamental concepts. For this reason, students with SLDs learn better through observation and experimentation, supported by visual aids.

From [26], some indications for the didactic strategy are:

- Use of schemes and concept maps;
- Dispense with reading aloud and mnemonic study;
- Privileging learning from experience and laboratory teaching;
- Encourage students to self-assess their own learning processes;
- Encourage peer tutoring and promote collaborative learning;
- Guarantee longer times for written tests and study;
- Take an encouraging attitude to improve self-esteem;
- Evaluate according to progress and difficulties;
- Use of calculator and digital devices.

Since learning disorders are various and create different deficits that require different compensatory tools and dispensation measures, to approach the case and to identify objectives, strategies and criteria it was essential to see how the student's specific disorders, dyslexia and dysgraphia, interfere in the learning of mathematics. Briefly:

- **Dyslexia:** it leads to difficulties in reading which is slow and incorrect, making it difficult to understand the text and to distinguish useful parts of it from those containing incidental information. In mathematics, these deficits therefore involve difficulty in recognizing the hypotheses and elements used in a problem making even the simplest ones complex. In addition, students affected by this disorder, have difficulty in studying the theory from the book and therefore, to develop the full potential of these students, psychologists suggest the constant use of mind maps that allow you to fix and rework ideas slowly until the acquisition of the argument. This type of maps is the one that best suits the way of learning of students with specific reading disorders because, through images and colors, it aims to stimulate visual memory through the inclusion of mental associations (see [28]).

- **Dysgraphia:** a deficit of a motor nature and refers to the graphic aspects of handwriting. The dysgraphers produce poorly readable texts (even by themselves) with words often misaligned and characterized by letters of different sizes. The elaboration of a written text is a difficult and long process with obvious repercussions in the study of mathematics. Mathematics, in fact, has its own language, characterized by symbols, signs and letters of the Greek and Latin alphabet. Inaccurate writing may cause errors in the resolution of algebraic expressions, for example by confusing the letter "z" and the number "2", or in the resolution of a linear system which requires many transcription steps. Another difficulty due to alignment is found in the case of powers where it confuses base and exponent with a multi-digit number (28 instead of 2^8).

The student, at the beginning of the course, presented his certificate and was immediately interested in possible recovery activities. After almost a month, when the first difficulties began to emerge, he constantly benefited from the hours of tutoring available during which a personalized teaching strategy was put in place that led the boy to the acquisition of his own method of study which proved to be successful.

3.1 The model applied to the case study

Let us specify the meanings of the GO, the criteria and the alternatives in our context.

- Specific General Objective: to promote the achievement of the GO through a personalized didactic plan.

About alternatives we have:

- Specific Alternative 1;
- Specific Alternative 2;
- Specific Alternative 3.

They are based on the educational models of Alternative 1, Alternative 2 and Alternative 3 but considering the compensatory instruments and dispensation measures.

The criteria become:

- Specific Criterion 1: acquisition of the objectives set by the educational plan that in quantitative terms are reduced compared to those considered in Criterion 1. In fact, a student with SLDs, although having an intelligence in the norm, unlike his peers, learns at a slower pace because during the study he dissipates most of his energy to compensate for his disorders;

- Specific Criterion 2: ability to measure oneself against peers in a safe and rational way while respecting the ideas of others as much as possible also increasing the level of self-esteem and personal gratification;
- Specific Criterion 3: see Criterion 3;
- Specific Criterion 4: see Criterion 4 with the difference that, in this case, in the process of evaluation of written tests the content rather than the form is taken into account, not penalizing errors when the concept expressed is clear and giving more weight to oral checks.

At this moment we can build the first matrix of the pair comparisons by assigning to each Specific Criterion a score that indicates the weight of the criterion in the achievement of the Specific General Objective.

Let us consider the following:

- SC2 is slightly more important than SC1 because working with a student with SLDs it is preferable to reduce the quantity and ensure the quality of the study topics giving more weight to the human aspect of mathematics even if the learning objectives may have flexible size;
- SC3 and SC4 are more important than SC2 because the school must guarantee the training of a citizen who is able to react in the face of trials and with the ability to put the "thinking" and the "doing" in close relationship.

SGO	SC1	SC2	SC3	SC4
SC1	1	3	1	1
SC2	1/3	1	1/3	1/5
SC3	1	3	1	1
SC4	1	5	1	1

M1: Comparison of criteria by the ratio of preference with respect to the general objective

The following matrices are obtained by comparing each specific alternative with the individual specific criteria and the number represents the extent to which the alternatives meet the criteria.

About SC1 let us consider:

- SA2 satisfies SC1 less than SA1 and SA3 because with a traditional didactics the student's understanding is not taken up by the method but is delegated to the individual act of the subject.

SC1	SA1	SA2	SA3
SA1	1	3	1
SA2	1/3	1	1/3
SA3	1	3	1

M2: Comparison of alternatives by the ratio of preference with respect to the criterion 1

About SC2 let us consider:

- traditional didactic is far from the social character of human learning so SA1 satisfies more SC2;
- sharing the same space with others is a fundamental thing for personal and social development, necessary for the student to learn to respect rules and roles. So SA2 satisfies more SC2 than SA3.

SC2	SA1	SA2	SA3
SA1	1	5	7
SA2	1/5	1	3
SA3	1/7	1/3	1

M3: Comparison of alternatives by the ratio of preference with respect to the criterion 2

About SC3 let us consider:

- distance learning offers insufficient physical interaction between student-teacher and student-student: expressions and gestures make the difference in the learning process; so SA3 is a slightly less important than SA1 to satisfy SC3;
- to overcome mechanical learning an "active" teaching can make the student able to master the knowledge and use it in other contexts. For this reason, the SA1 and SA3 are more appropriate than SA2 for building skills.

SC3	SA1	SA2	SA3
SA1	1	5	3
SA2	1/5	1	1/3
SA3	1/3	3	1

M4: Comparison of alternatives by the ratio of preference with respect to the criterion 3

About SC4 let us consider:

- exercising young people to face tests in a lucid way is a fundamental aspect for the construction of a personality that faces in a competitive way the working challenges of a competitive society. Distance learning facilitates the examination phase because the student does not have direct contact with those who have to judge him/her. For this reason, the student may not learn how to manage emotions and therefore SA1 and SA2 are more appropriate than SA3 to implement SC4;

- the Inquiry model is a teaching-learning model where error is only the starting point for a deeper understanding. So, the evaluation phase becomes a new opportunity to get involved and not a scary moment of judgment. With more serenity the student will face the challenges in the best way.

SC4	SA1	SA2	SA3
SA1	1	3	7
SA2	1/3	1	5
SA3	1/7	1/5	1

M5: Comparison of alternatives by the ratio of preference with respect to the criterion 4

In our case, with the help of MATLAB, we get:

	M1	M2	M3	M4	M5
λ_{max}	4,0328	3,0000	3,0649	3,0385	3,0649
CI	0,0109	0	0,0324	0,0193	0,0193

We can therefore write the local priorities obtained by proceeding with the last step of the AHP method which, through the aggregation of the relative weights of each level, provides a weighted ranking of the alternatives.

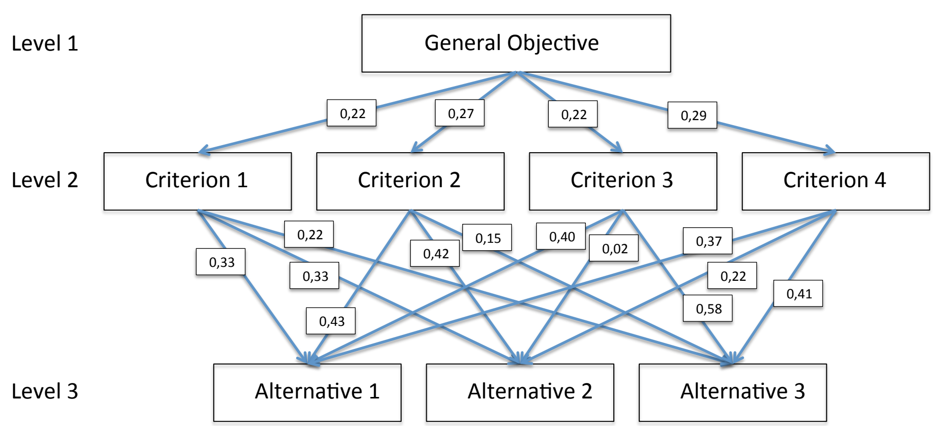

Figure 2: The digraph of the functional aspects, with weights

We observe that scores are non-negative real numbers and such that the sum of the scores of the arcs coming out of the same vertex u is equal to 1. The score assigned to an arc (u, v) indicates the extent to which the final vertex v meets the initial vertex u: the score of a path is the product of the scores of the arcs that form the path.

- For every vertex v different from GO the score $p(v)$ of v is the sum of the scores of all the paths that start from GO and arrive in v.

- For every level, the sum of the points of the vertices of level i is equal to 1.

The global properties determined will then be:

- A1=0,22*0,33+0,27*0,43+0,22*0,40+0,29*0,37=0,38;
- A2=0,22*0,33+0,27*0,42+0,22*0,02+0,29*0,22=0,26;
- A3=0,22*0,22+0,27*0,15+0,22*0,58+0,29*0,41=0,36;

We can conclude that, in order to achieve the objective, a new teaching strategy is preferred to a traditional one and direct communication and human contact are factors not to be overlooked. Alternative 2 got the lowest score considering that learning cannot be a process of transmission but of meaning construction. Alternatives 1 and 3 differ for the different weight of the emotional dimension: canceling the physical contact the level of communication decreases and the emotional and social distancing increases. The online lesson would be too impersonal to adapt it to the real needs of the student.

4. Discussion and results

We applied SA2 that emerged from the model to the student.

In our case, the meetings consisted of afternoon lessons lasting two hours, usually held shortly after the lesson held in the classroom in the morning by the teacher.

Most of the meeting was based on the study of the last topics explained by the teacher, of which the student collected notes in a bare and confusing way, characterized by erasures and large empty spaces, common characteristic of the disgraphics.

The theory was gradually joined by practice, trying as much as possible to provide constructivist learning and, once the new topic was finished, work was done on the previous ones.

To avoid the use of the book, since reading was slow and incorrect due to dyslexia, sometimes also the theory was approached through exercises or questions to answer, as the didactic strategy provided.

During the lessons we gave the student more time to carry out the activities to transmit calm and serenity for the completion of the SC4.

In order to prevent the pupil from distracting himself, we often drew his attention by questioning him and repeating the concept in different ways.

To stimulate and give shape to ideas and discussions, weight has been given to questions such as "what would happen if this result was true?" or "under what circumstances could this error be corrected?". With this approach we built dialogues during which the student was not afraid to express his difficulties.

Difficulties began to come up from the first topics when, in the numerical set exercises, there was a lot of confusion between the parentheses used to describe the intervals. For example:

- interval from "a" to "b" not including the extremes, $]a, b[$;
- interval from "a" to "b" including the extremes, $[a, b]$.

To cope with this difficulty, with a bit of imagination, the teacher made him imagine drawing two arms "embracing" the number if the number had to be present in the interval or, otherwise, they refused to do so. In this way, together with the use of graphic representations, the pupil acquired the competence to distinguish between the two types of parentheses and rarely found confusion until the end of the course.

In order to facilitate the repetition of the topics dealt with, we have extensively used diagrams and maps, compensatory instruments.

The mind maps, which we built together, were of great help especially in solving equations and inequalities with absolute values. Initially, the student's difficulty consisted in not being able to "visualize" the writing of the systems that came out of the procedure. With the use of a map, he was able, after several lessons, to carry out the simplest exercises correctly but still presenting difficulties for the more complex ones.

The result was satisfactory, however, because we believe that these difficulties were linked not so much to a lack of understanding of the absolute value function, but rather to a lack of ability to concentrate for so long on the same procedure (as pointed out above regarding the Specific Criterion 1 so SA2 requires to "evaluate according to progress and difficulties").

Towards the end of the course, other boys who were more frequent users of teacher's service were added to the tutoring lessons in view of the exam. In this way, the teacher was able to work on the last topics of the course, the derivatives and the study of graphs of functions, experimenting a collective study during which the student with specific learning disorders discussed with his or her classmates to the point of realizing, under teacher's guidance, his or their mistakes.

The dialogue between the students, being free and spontaneous, has favored the concretization of the SC2.

Finally, during the lessons, the teacher often used the calculator to reduce the material to memorize as much as possible. For example, when studying the domain of a function, initially the student made extensive use of tables summarizing the domains of elementary functions. Subsequently, by using the calculator that reports "ERR" if the function is not applied to an element of the domain, he really understood what this procedure meant by obtaining the right conditions for the simple functions such as roots and logarithms.

At the end of the course the boy passed the exam and reached the minimum targets. In fact, he has acquired the fundamental concepts to the point of facing the final test independently. As the tutoring activity progressed, the student also managed to follow the explanations more and more with a more profitable study. The participation of peers during the tutoring activity certainly helped the boy who, with an "equal" comparison, acquired such security that he was able to guide his companions in difficulty during the exercises.

5. Conclusions

The aim of this paper is therefore to emphasize that students with SLD can compensate for their deficits if they are properly followed. Everyone is unique from a biological, neurological and psychological point of view and it is necessary to enhance the alternative ways of learning with scenes that allow the possibility to grow by compensating for the difficulties.

Therefore the design of a learning plan must be planned so that each student (while facing learning at different levels and in different ways) can access the processes proper to the acquisition of skills and knowledge ensuring, in this way, a formative success and the enhancement of all students.

The hierarchical analytical method of Saaty has allowed the formation of a conscious environment able to respect the "different" way of learning of a student with SLDs whose cognitive abilities and physical characteristics are in the norm. The didactic plan has, in fact, allowed a learning on a par with his companions and overcoming problems such as low self-esteem or a sense of ineffectiveness by allowing social skills to grow. About our case the ultimate goal has been to maximize understanding with the means available by bringing the student to the peak of their abilities.

Another consideration is that the affective dimension, understood as everything related to relationships and emotions, both of teachers and students, is crucial for a serene and safe climate in which students find motivations and stimuli that are necessary conditions to achieve specific goals set by the educational plans.

This strategy, having been applied to only one student, cannot be generalized because each case requires different objectives and tools depending on the disturbance and the problems that this entails. However, the suggested indications are a good start to approach starting from a general scheme that must be modified for the different cases, considering new specific criteria and new specific alternatives.

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