Deformation problem for a double porous viscoelastic medium using state space approach

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Abstract. A deformation problem for a homogeneous, isotropic, double porous, viscoelastic medium has been studied by considering a new mathematical model. To solve the problem, state space approach has been used. A normal force has been applied to describe the problem as an illustration. The solution is obtained in the frequency domain and hence the deformation described by the components of normal stress and equilibrated stress has been found in the form of frequency. Then, these components have been converted into physical domain and computed numerically for a particular material. The numerical results thus obtained have been discussed graphically to show the viscous effect on deformation. A Particular case has also been given to show the generalization of the model taken for the problem.

Keywords: double porous, deformation, equilibrated stress, normal force, normal stress, state space approach, viscoelasticity.

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1. Introduction

The porous medium plays a significant role in different fields of science, engineering and technology. A general three-dimensional consolidation problem for this material has been given by Biot [1] and this is known as Biot theory of poroelasticity. Bowen [2] de Boer and Ehlers [3] have described another theory for these materials with incompressible constituents. Abbas [4] employed Biot's theory to obtain the frequency equation for radial vibrations of a poroelastic cylinder for a porous solid. Aizahrani and Abbas [5] provided a method to study the effect of porosity in a porothermoelastic medium by the finite element technique. A generalization of Biot's theory of poroelasticity describing double porosity model has been given by Barenblatt et al. [6] to express the fluid flow in hydrocarbon reservoirs and aquifers. The investigation of double porosity model has given the chance to represent many important aspects of different problems related to civil engineering. Thus, many researchers have applied this model to solve different kinds of research problems.

The linear theory of viscoelasticity describes the linear conduct of both elastic and an elastic materials. Bland [7] has given description of three dimensional linear viscoelasticity theory. According to Bland, in a perfectly elastic isotropic medium, under the assumption of small displacements, two types of waves can propagate in an isotropic, viscoelastic medium in absence of body forces. Iesan and Scalia [8] has given and solved some theorems in the theory of thermoviscoelasticity. Svanadze [9] described the problem of plane waves and steady vibrations in the theory of viscoelasticity for Kelvin-Voigt materials with double porosity. Kumar et al. [10] studied the wave propagation in anisotropic viscoelastic medium in the context of the theory of three phase lag model of thermoelasticity. Sharma et al. [11] studied the wave propagation in anisotropic thermoviscoelastic medium in the context of Green-Naghdi theories of type-II and type-III. Sharma et al. [12] considered electro–microstretch viscoelastic solids for investigating the propagation of plane waves and fundamental solution. A few problems with double porosity model in elastic solids and thermoelastic solids have been studied by Svanadze [13, 14, 15].

The state space description of linear systems has been used extensively during recent past to avoid the difficulties of the traditional linear model approach. Bahar and Hetnarski [16, 17] described the state space approach to thermoelasticity. Sharma [18] used the state space approach to study the one dimensional problem for generalized theories of thermoelasticity subjected to heat source and body force. Ezzat et al. [19] used state space approach for a problem of generalized thermoviscoelasticity with two relaxation times. Palmeri et al. [20] used state space formulation for the study of linear viscoelastic dynamic system with memory. Menon and Tang [21] applied state space approach to the dynamic analysis of viscoelastic system. Youssef and Al-Lehaibi [22] with the help of state space approach, obtained the general solution for any set of boundary conditions for a half-space filled with an elastic material. Youssef and Harby

[23] considered an infinite elastic body with a spherical cavity and applied state space technique to obtain the general solution for any set of boundary conditions. Ezzat and Karamany [24] established state space approach for two temperature magneto-viscoelasticity theory with thermal relaxation in a medium of perfect conductivity. Kumar et al. [25] used state space approach to discuss a problem for thermoelastic material with double porosity. Kumar and Vohra [26] used the state space approach to look into the plane deformation problem in elastic materials with double porosity. Ezzat and EL-Bary [27] applied state space approach to two-dimensional magneto-thermoelasticitywith fractional order heat transfer in a medium of perfect conductivity.

The problems have been solved by researchers by taking different mathematical models, methods and materials. Some of them are listed here for reference purposes. Abbas and Youssef [28] proposed a general finite element method (FEM) to analyze transient phenomena in a thermoelastic model in the context of the theory of generalized thermoelasticity with one relaxation time. Kumar and Abbas [29] investigated two dimensional problems of micropolarthermoelastic material with two temperature in the context of Lord-Shulman theory. Marin and Vlase [30] studied the effect of internal state variables in thermoelasticity of microstretch bodies. Vlase et al. [31] studied the vibration of mechanical bars systems with symmetries. Marin et al. [32] proposed a mixed initial-boundary value problem in modeling a three-phaselag dipolar thermoelastic body.

In this paper, a boundary value problem of a double porous viscoelastic medium has been formulated and solved using state space approach. The components of deformation, normal stress and equilibrated stresses, have been found in closed form, after getting the expressions in frequency domain. The values for these components have been obtained numerically for a particular medium. These are depicted graphically to show the effect of viscosity. A particular case of interest has also been deduced.

2. Basic equations

The basic equations in absence of body forces and extrinsic equilibrated body forces for a double porous homogeneous isotropic elastic medium has been taken, following Ieşan and Quintanilla [33], as Constitutive Relations:

$$t_{ij} = \lambda e_{rr}\delta_{ij} + 2\mu e_{ij} + b\delta_{ij}\varphi + d\delta_{ij}\psi, \tag{1}$$

$$\sigma_i = \alpha\varphi_{,i} + b_1\psi_{,i}, \tag{2}$$

$$\tau_i = b_1\varphi_{,i} + \gamma\psi_{,i}\,. \tag{3}$$

Equation of motion:

$$\mu\nabla^2\vec{u} + (\lambda+\mu)\nabla\nabla\cdot\vec{u} + b\nabla\varphi + d\nabla\psi = \rho\frac{\partial^2 u}{\partial t^2}\,, \tag{4}$$

Equilibrated stress equations of motion:

$$\alpha\nabla^2\varphi + b_1\nabla^2\psi - b\nabla\vec{u} - \alpha_1\varphi - \alpha_3\psi = \kappa_1\frac{\partial^2\varphi}{\partial t^2}\,, \tag{5}$$

$$b_1\nabla^2\varphi + \nabla^2\psi - d\nabla\vec{u} - \alpha_3\varphi - \alpha_2\psi = \kappa_2\frac{\partial^2\psi}{\partial t^2}\,, \tag{6}$$

where λ and μ are the Lame's constants; ρ is the mass density; u_i $(i = 1, 2, 3)$ are the displacement components; t_{ij} is the stress tensor; κ_1 and κ_2 are the coefficients of equilibrated inertia; φ and ψ are the volume fraction fields corresponding to pores v_1 and fissures v_2, respectively; σ_i is the equilibrated stress corresponding to pores; τ_i is the equilibrated stress corresponding to fissures; b, d, b_1, α, γ, α_1, α_2, α_3 are the constitutive coefficients; δ_{ij} is the Kronecker's delta; Δ is the Laplacian operator; and a superposed dot represents differentiation with respect to time variable 't'.

Following the Voigt model of linear viscoelasticity, the porous elastic constants λ, μ, b, d, α_1, α_2, α_3, α, γ, b_1 are replaced by the following constants for viscoelastic nature of the material λ_ε, μ_ε, b_ε, d_ε, $\alpha_{1\varepsilon}$, $\alpha_{2\varepsilon}$, $\alpha_{3\varepsilon}$, α_ε, γ_ε, $b_{1\varepsilon}$, respectively.

Therefore, equations (1)-(6) for viscoelastic nature of the medium take the following form of equations:

$$t_{ij} = \lambda_\varepsilon e_{rr}\delta_{ij} + 2\mu_\varepsilon e_{ij} + b_\varepsilon\delta_{ij}\varphi + d_\varepsilon\delta_{ij}\psi, \tag{7}$$

$$\sigma_i = \alpha_\varepsilon\varphi_{,i} + b_{1\varepsilon}\psi_{,i}, \tag{8}$$

$$\tau_i = b_{1\varepsilon}\varphi_{r_i} + \gamma_\varepsilon\psi_{,i}, \tag{9}$$

$$\mu_\varepsilon\nabla^2\vec{u} + (\lambda_\varepsilon + \mu_\varepsilon)\nabla\nabla\vec{u} + b_\varepsilon\nabla\varphi + d_\varepsilon\nabla\psi = \rho\frac{\partial^2\vec{u}}{\partial t^2}, \tag{10}$$

$$\alpha_\varepsilon\nabla^2\varphi + b_{1\varepsilon}\nabla^2\psi - b_\varepsilon\nabla\vec{u} - \alpha_{1\varepsilon}\varphi - \alpha_{3\varepsilon}\psi = \kappa_1\frac{\partial^2\varphi}{\partial t^2}, \tag{11}$$

$$b_{1\varepsilon}\nabla^2\varphi + \gamma_\varepsilon\nabla^2\psi - d_\varepsilon\nabla\vec{u} - \alpha_{3\varepsilon}\varphi - \alpha_{2\varepsilon}\psi = \kappa_2\frac{\partial^2\psi}{\partial t^2}, \tag{12}$$

where

$$\begin{cases} \lambda_\varepsilon = \lambda + \lambda_\nu\dfrac{\partial}{\partial t}, & \mu_\varepsilon = \mu + \mu_\nu\dfrac{\partial}{\partial t}, \\ b_\varepsilon = b + b_\nu\dfrac{\partial}{\partial t}, & d_\varepsilon = d + d_\nu\dfrac{\partial}{\partial t}, \\ \alpha_{1\varepsilon} = \alpha_1 + \alpha_{1\nu}\dfrac{\partial}{\partial t}, & \alpha_{2\varepsilon} = \alpha_2 + \alpha_{2\nu}\dfrac{\partial}{\partial t}, \\ \alpha_{3\varepsilon} = \alpha_3 + \alpha_{3\nu}\dfrac{\partial}{\partial t}, & \alpha_\varepsilon = \alpha + \alpha_\nu\dfrac{\partial}{\partial t}, \\ \gamma_\varepsilon = \gamma + \gamma_\nu\dfrac{\partial}{\partial t}, & b_{1\varepsilon} = b_1 + b_{1\nu}\dfrac{\partial}{\partial t}, \end{cases} \tag{13}$$

and

$$\lambda_\nu, \mu_\nu, b_\nu, d_\nu, \alpha_{1\nu}, \alpha_{2\nu}, \alpha_{3\nu}, \alpha_\nu, \gamma_\nu, b_{1\nu}$$

are the viscoelastic constants.

3. Formulation and solution of the problem

A one dimensional problem of a homogeneous, isotropic, double porous, viscoelastic half space medium has been considered. The state variables of the problem depend only on the space variable, distance x and on time t. Therefore, the displacement components u_i, volume fractions φ and ψ are taken as

$$\{u_i, \varphi, \psi\} = \{u_i(x,t), \varphi(x,t), \psi(x,t)\}. \tag{14}$$

A normal force is considered to be acted upon the bounding surface $x = 0$. Now, to find the solution, dimensionless parameters are defined as

$$\begin{cases} x_1' = \dfrac{\omega_1}{c_1}x_1, \quad x_3' = \dfrac{\omega_1}{c_1}x_3, \quad u_1' = \dfrac{\omega_1}{c_1}u_1, \\ u_3' = \dfrac{\omega_1}{c_1}u_3, \quad t_{ij}' = \dfrac{t_{ij}}{\lambda_\varepsilon}, \quad \varphi' = \dfrac{\kappa_1\omega_1^2}{\alpha_{1\varepsilon}}\varphi, \\ \psi' = \dfrac{\kappa_1\omega_1^2}{\alpha_{1\varepsilon}}\psi, \quad t' = \omega_1 t, \quad \sigma_1' = \left(\dfrac{c_1}{\alpha_\varepsilon\omega_1}\right)\sigma_1, \\ \tau_1' = \left(\dfrac{c_1}{\alpha_\varepsilon\omega_1}\right)\tau_1, \end{cases} \tag{15}$$

where

$$\omega_1 = \frac{\lambda}{\kappa_1} \quad \text{and} \quad c_1^2 = \frac{\lambda_\varepsilon + 2\mu_\varepsilon}{\rho}. \tag{16}$$

Here, we consider

$$Q_i = \omega_1\left(\frac{\lambda_\nu}{\lambda}, \frac{\mu_\nu}{\mu}, \frac{b_\nu}{b}, \frac{d_\nu}{d}, \frac{\alpha_\nu}{\alpha}, \frac{\gamma_\nu}{\gamma}, \frac{\alpha_{1\nu}}{\alpha_1}, \frac{\alpha_{2\nu}}{\alpha_2}, \frac{\alpha_{3\nu}}{\alpha_3}, \frac{b_{1\nu}}{b_1}\right), i = 1, 2, \ldots, 10, \tag{17}$$

and hence

$$\begin{aligned} &(\lambda_\varepsilon, \mu_\varepsilon, b_\varepsilon, d_\varepsilon, \alpha_\varepsilon, \alpha_{1\varepsilon}, \alpha_{2\varepsilon}, \alpha_{3\varepsilon}, \gamma_{1\varepsilon}, \gamma_{2\varepsilon}, b_{1\varepsilon}) \\ &\quad = (\lambda, \mu, b, d, \alpha_1, \alpha_2, \alpha_3, \gamma_1, \gamma_2, b)\left(1 + Q_i\frac{\partial}{\partial t'}\right); \; i = 1, 2 \ldots, 10. \end{aligned} \tag{18}$$

Using (15) in equations (10)-(12), and then using (14), we get

$$\frac{\partial^2 u}{\partial x^2} + \delta_1\frac{\partial\varphi}{\partial x} + \delta_2\frac{\partial\psi}{\partial x} = \frac{\partial^2 u}{\partial t^2}, \tag{19}$$

$$\delta_3\frac{\partial^2\varphi}{\partial x^2} + \delta_4\frac{\partial^2\psi}{\partial x^2} - \delta_5\frac{\partial u}{\partial x} - \delta_6\varphi - \delta_7\psi = \frac{\partial^2\varphi}{\partial t^2}, \tag{20}$$

$$\delta_8\frac{\partial^2\varphi}{\partial x^2} + \delta_9\frac{\partial^2\psi}{\partial x^2} - \delta_{10}\frac{\partial u}{\partial x} - \delta_{11}\varphi - \delta_{12}\psi = \frac{\partial^2\psi}{\partial t^2}, \tag{21}$$

where

$$\begin{cases} \delta_1 = \dfrac{b_\varepsilon \alpha_{1\varepsilon}}{\rho c_1^2 k_1 \omega_1^2}, \quad \delta_2 = \dfrac{d_\varepsilon \alpha_{1\varepsilon}}{\rho c_1^2 k_1 \omega_1^2}, \quad \delta_3 = \dfrac{\alpha_\varepsilon}{c_1^2 k_1}, \delta_7 = \dfrac{\alpha_{3\varepsilon}}{k_1 \omega_1^2}, \\ \delta_8 = \dfrac{b_{1\varepsilon}}{c_1^2 k_2}, \quad \delta_9 = \dfrac{\gamma_\varepsilon}{c_1^2 k_2}, \quad \delta_4 = \dfrac{b_{1\varepsilon}}{c_1^2 k_1}, \\ \delta_5 = \dfrac{b_\varepsilon}{\alpha_{1\varepsilon}}, \quad \delta_6 = \dfrac{\alpha_{1\varepsilon}}{k_1 \omega_1^2}, \quad \delta_{10} = \dfrac{d_\varepsilon k_1}{\alpha_1 k_2}, \\ \delta_{11} = \dfrac{\alpha_{3\varepsilon}}{k_2 \omega_1^2}, \quad \delta_{12} = \dfrac{\alpha_{2\varepsilon}}{k_2 \omega_1^2}. \end{cases} \tag{22}$$

Assuming harmonic form of solution of equations (19)-(21) as

$$\{u(x,t), \varphi(x,t), \psi(x,t)\} = (\bar{u}, \bar{\varphi}, \bar{\psi}) e^{-i\omega t}. \tag{23}$$

Equations (19)-(21) with the help of (23) give

$$\bar{u}_{,11} = N_1 \bar{u} + N_2 \bar{\phi}_{,1} + N_3 \bar{\psi}_{,1}, \tag{24}$$

$$\bar{\phi}_{,11} = M_1 \bar{\psi}_{,11} + M_2 \bar{u}_{,1} + M_3 \bar{\phi} + M_4 \bar{\psi}, \tag{25}$$

$$\bar{\psi}_{,11} = M_5 \bar{\phi}_{,11} + M_6 \bar{u}_{,1} + M_7 \bar{\phi} + M_8 \bar{\psi}, \tag{26}$$

where

$$\begin{cases} N_1 = -\omega^2, \quad N_2 = -\delta_1, \quad N_3 = -\delta_2, \\ M_1 = \dfrac{-\delta_4}{\delta_3}, \quad M_2 = \dfrac{\delta_5}{\delta_3}, \quad M_3 = \dfrac{\delta_6 - \omega^2}{\delta_3}, \\ M_4 = \dfrac{\delta_7}{\delta_3}, \quad M_5 = \dfrac{-\delta_8}{\delta_9}, \quad M_6 = \dfrac{\delta_{10}}{\delta_9}, \\ M_7 = \dfrac{\delta_{11}}{\delta_9}, \quad M_8 = \dfrac{\delta_{12} - \omega^2}{\delta_9}. \end{cases} \tag{27}$$

Equations (24)-(26) can be re-written as

$$\bar{u}_{,11} = N_1 \bar{u} + N_2 \bar{\phi}_{,1} + N_3 \bar{\psi}_{,1}, \tag{28}$$

$$\bar{\phi}_{,11} = N_4 \bar{u}_{,1} + N_5 \bar{\phi} + N_6 \bar{\psi}, \tag{29}$$

$$\bar{\psi}_{,11} = N_7 \bar{u}_{,1} + N_8 \bar{\phi} + N_9 \bar{\psi}, \tag{30}$$

where

$$\begin{cases} N_4 = \dfrac{M_1 M_6 + M_2}{M_9}, \quad N_5 = \dfrac{M_1 M_7 + M_3}{M_9}, \quad N_6 = \dfrac{M_1 M_8 + M_4}{M_9}, \\ N_7 = M_5 N_4 + M_6, \quad N_8 = M_5 N_5 + M_7, \quad N_9 = M_5 N_6 + M_8. \end{cases} \tag{31}$$

4. State-space formulation

The equations (28)-(30) can be written in the matrix form as

$$\frac{dV(x,\omega)}{dx} = A(\omega)V(x,\omega), \tag{32}$$

where the values of $A(\omega)$, $V(x,\omega)$ are given in the appendix. Taking the displacement $\bar{u}$ and volume fractions $\bar{\varphi}$ and $\bar{\psi}$ as state variables, the solution of the matrix form of equation (32) can be taken as

$$V(x,\omega) = \exp[A(\omega)x]V(0,\omega), \tag{33}$$

where the value of $V(0,\omega)$ is given in the appendix. To get the matrix $\exp[A(\omega)x]$, Cayley-Hamilton theorem is used.

The characteristic equation of the matrix $A(\omega)$ is obtained as

$$\lambda^6 + D_1\lambda^4 + D_2\lambda^2 + D_3 = 0, \tag{34}$$

where

$$\begin{cases} D_1 = -N_1 - N_5 - N_9 - N_2N_4 - N_3N_7, \\ D_2 = N_1N_5 + N_1N_9 + N_5N_9 - N_6N_8 + N_2N_4N_9 \\ \qquad -N_2N_6N_7 + N_3N_5N_7 - N_3N_4N_8, \\ D_3 = N_1N_6N_8 - N_1N_5N_9. \end{cases} \tag{35}$$

Equation (34) is cubic in λ^2, yield three roots say λ_1^2, λ_2^2, λ_3^2.

The Taylor series expansion for $\exp[A(\omega)x]$ is taken as

$$\exp[A(\omega)x] = \sum_{n=0}^{\infty}\left\{\frac{[A(\omega)x]^n}{n!}\right\}. \tag{36}$$

Truncating this infinite series and by using Cayley-Hamilton theorem, we get

$$\exp[A(\omega)x] = a_0 I + a_1 A + a_2 A^2, \tag{37}$$

where a_0, a_1, a_2 are the parameters depending on x and ω. Now, the eigen values $-\lambda_1$, $-\lambda_2$, $-\lambda_3$ of the matrix A satisfy equation (37) as per the Cayley-Hamilton theorem. Thus, we have

$$\begin{cases} \exp[-\lambda_1 x] = a_0 - a_1\lambda_1 + a_2\lambda_1^2\,, \\ \exp[-\lambda_2 x] = a_0 - a_1\lambda_2 + a_2\lambda_2^2\,, \\ \exp[-\lambda_3 x] = a_0 - a_1\lambda_3 + a_2\lambda_3^2\,. \end{cases} \tag{38}$$

Solving equations (38), the values of a_0, a_1 and a_2 are obtained as given in appendix. Using these values of a_0, a_1 and a_2 in (37), we obtain

$$\exp[A(\omega)x] = L(x,\omega), \tag{39}$$

and hence equation (33) can be written as

$$V(x,\omega) = L(x,\omega)V(0,\omega), \tag{40}$$

where $L(x,\omega)$ is a 6×6 matrix.

Thus, the solution in the frequency domain is obtained as,

$$\begin{bmatrix} \bar{u} \\ \bar{\varphi} \\ \bar{\psi} \end{bmatrix} = \begin{bmatrix} l_{11} & l_{12} & l_{13} \\ l_{21} & l_{22} & l_{23} \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ A_3 \end{bmatrix}, \tag{41}$$

where

$$\begin{cases} l_{11} = a_0 + a_2 N_1, \;\; l_{12} = 0, l_{13} = 0, \;\; l_{21} = 0, \;\; l_{22} = a_0 + a_2 N_5, \\ l_{23} = a_2 N_6, \;\; l_{31} = 0, \;\; l_{32} = a_2 N_8, \;\; l_{33} = a_0 + a_2 N_9 \,. \end{cases} \tag{42}$$

5. Boundary conditions

As per the formulation of the problem, a normal force is considered to be acted upon the bounding surface $x = 0$. Therefore, the boundary conditions at $x = 0$ are written as,

$$\text{(i)} t_{11} = -F_1 \exp[-i\omega t], \tag{43}$$

$$\text{(ii)} \sigma_1 = -F_2 \exp[-i\omega t], \tag{44}$$

$$\text{(iii)} \tau_1 = -F_3 \exp[-i\omega t], \tag{45}$$

where F_t; $(i = 1, 2, 3)$ are the magnitudes of the force applied on the boundary.

Substituting the values of $\bar{u}$, $\bar{\varphi}$, $\bar{\psi}$ from equation (41), with the help of (14), (15) and (23), in equations (7)-(9) and then the resulting expressions in the boundary conditions (43)-(45), we obtain

$$\begin{bmatrix} Q_1 & Q_2 & Q_3 \\ Q_4 & Q_5 & Q_6 \\ Q_7 & Q_8 & Q_9 \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ A_3 \end{bmatrix} = \begin{bmatrix} -F_1 \\ -F_2 \\ -F_3 \end{bmatrix}. \tag{46}$$

The values of $Q_1, Q_2, \ldots, Q_9$, are given in appendix.

Solving the above system of equations for A_1, A_2, A_3, we obtain

$$A_1 = \frac{\Gamma_1}{\Gamma}, \quad A_2 = \frac{\Gamma_2}{\Gamma}, \quad A_3 = \frac{\Gamma_3}{\Gamma}, \tag{47}$$

where

$$\begin{cases} \Gamma = \begin{vmatrix} Q_1 & Q_2 & Q_3 \\ Q_4 & Q_5 & Q_6 \\ Q_7 & Q_8 & Q_9 \end{vmatrix}, \qquad \Gamma_1 = \begin{vmatrix} -F_1 & Q_2 & Q_3 \\ -F_2 & Q_5 & Q_6 \\ -F_3 & Q_8 & Q_7 \end{vmatrix}, \\ \Gamma_2 = \begin{vmatrix} Q_1 & -F_1 & Q_3 \\ Q_4 & -F_2 & Q_6 \\ Q_7 & -F_3 & Q_9 \end{vmatrix}, \quad \Gamma_3 = \begin{vmatrix} Q_1 & Q_2 & -F_1 \\ Q_4 & Q_5 & -F_2 \\ Q_7 & Q_8 & -F_3 \end{vmatrix}. \end{cases} \tag{48}$$

Using these expressions in (41) and then using (7)-(9) with the help of (23), we get the normal stress and the components of equilibrated stress as

$$t_{11} = \left(S_1 \frac{\Gamma_1}{\Gamma} + S_2 \frac{\Gamma_2}{\Gamma} + S_3 \frac{\Gamma_3}{\Gamma} \right) e^{-i\omega t}, \tag{49}$$

$$\sigma_1 = \left(S_4 \frac{\Gamma_1}{\Gamma} + S_5 \frac{\Gamma_2}{\Gamma} + S_6 \frac{\Gamma_3}{\Gamma} \right) e^{-i\omega t}, \tag{50}$$

$$\tau_1 = \left(S_7 \frac{\Gamma_1}{\Gamma} + S_8 \frac{\Gamma_2}{\Gamma} + S_9 \frac{\Gamma_3}{\Gamma} \right) e^{-i\omega t}, \tag{51}$$

where $S_1, S_2, \ldots, S_9$ are given in appendix.

6. Particular case

Taking $F_3 = 0$ in equations (49)-(51), we get the resulting expressions for the corresponding problem for an elastic porous medium.

7. Numerical results and discussion

To illustrate the theoretical results of the problem numerically, we take the material as copper material. Sherief and Saleh [34] has given the physical parameters for this material as

$$\begin{aligned}
&\lambda = 7.76 \times 10^{10} Nm^{-2}, && C^* = 3.831 \times 10^{3} m^2 s^{-2} K^{-1},\\
&\mu = 3.86 \times 10^{10} Nm^{-2}, && t = 0.1s,\\
&\alpha_2 = 2.4 \times 10^{10} Nm^{-2}, && \alpha_3 = 2.5 \times 10^{10} Nm^{-2},\\
&\rho = 8.954 \times 10^{3} Kgm^{-3}, && \gamma = 1.1 \times 10^{-5} N,\\
&\alpha = 1.3 \times 10^{-5} N, && b_1 = 0.12 \times 10^{-5} N,\\
&d = 0.1 \times 10^{10} Nm^{-2}, && \kappa_1 = 0.1456 \times 10^{-12} Nm^{-2} s^2,\\
&b = 0.9 \times 10^{10} Nm^{-2}, && \alpha_1 = 2.3 \times 10^{10} Nm^{-2},\\
&\kappa_2 = 0.1546 \times 10^{-12} Nm^{-2} s^2
\end{aligned}$$

$$\begin{aligned}
&Q_1 = 0.005, \quad Q_2 = 0.003, \quad Q_3 = 0.004, \quad Q_4 = 0.005,\\
&Q_5 = 0.001, \quad Q_6 = 0.004, \quad Q_7 = 0.002, \quad Q_8 = 0.008,\\
&Q_9 = 0.005, \quad Q_{10} = 0.006
\end{aligned}$$

The variation with respect to distance x in the values of normal stress and equilibrated stresses are obtained using Matlab software and are shown in Figures 1-3, respectively. To show the effect of viscosity, in all these figures, the graphs for viscoelastic double porous material (VDP) and elastic double porous material (DP) are drawn.

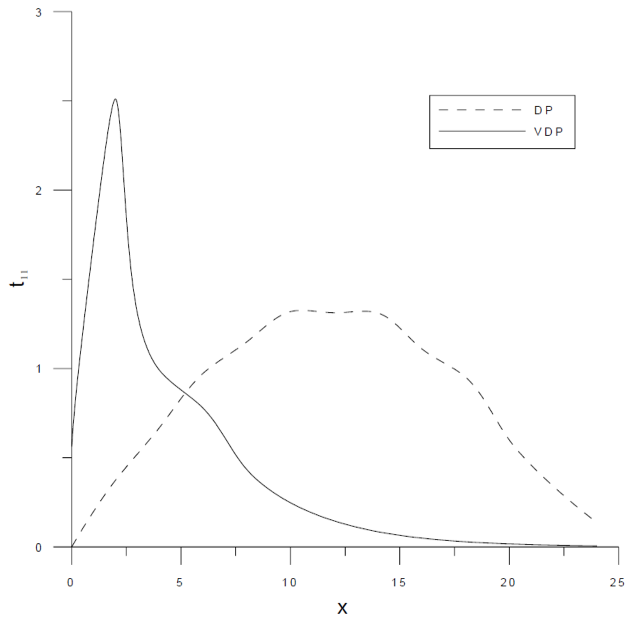

Figure 1: Variation of normal stress t_{11} with respect to distance x

Figure 1 shows the variation of t_{11} with respect to distance x. It is found that for VDP, the value of t_{11} increases for the range $0 < x \leq 3$ and then decreases for $x > 3$, whereas for DP, the value increases for the range $0 < x \leq 13$ and then decreases for the remaining range. Also, the magnitude value is more for DP in comparison to VDP except for the region $0 < x \leq 5$.

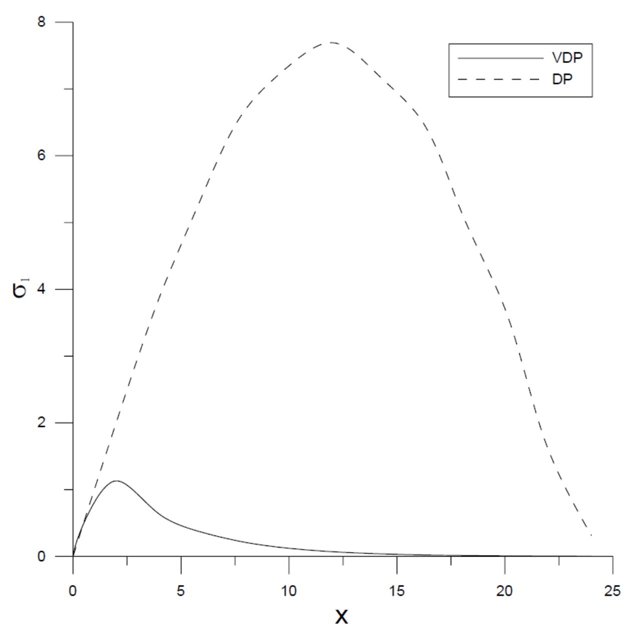

Figure 2: Variation of equilibrated stress σ_1 with respect to distance x

Figure 2 represents the variation of σ_1 with respect to distance x. It is found that for VDP, the value of σ_1 increases for the values $0 < x \leq 3$ and then decreases for the remaining region with the increase in the value of x, whereas for DP the value increases for the region $0 < x \leq 13$ and then decreases for the remaining region. Also, the magnitude value is more for DP in comparison to VDP.

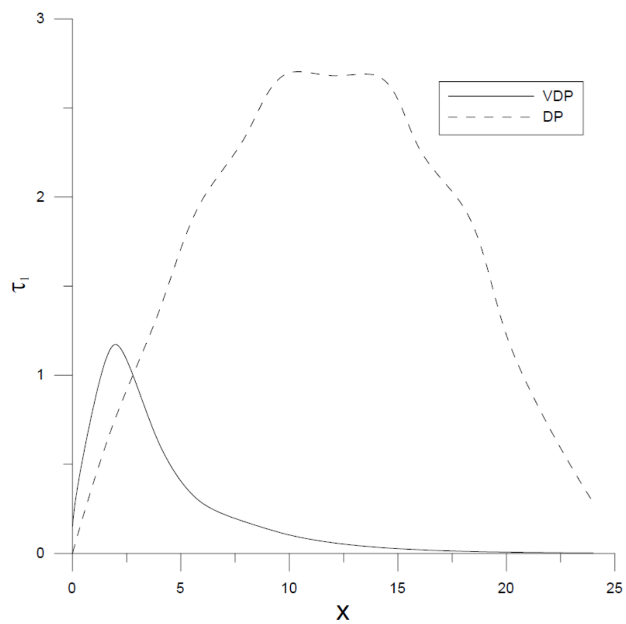

Figure 3: Variation of equilibrated stress τ_1 with respect to x

Figure 3 depicts the variation of τ_1 with respect to distance x. It is found that for VDP, the value of τ_1 increase for region $0 < x \leq 1$ and then decreases as $x > 1$ with the increase in the value of x whereas for DP, the values increase for the region $0 < x \leq 13$ and then decreases for the remaining region. Also, the magnitude value is more for DP in comparison to VDP.

8. Conclusion

State space approach has been used to solve a deformation problem for a homogeneous isotropic double porous viscoelastic medium. The components of normal stress and equilibrated stress have been found and computed numerically for a particular material, i.e., copper. The numerical results obtained for these components have been discussed graphically to show the viscous effect. A particular case has been deduced to show the generalization of the model taken for the problem.

Appendix

$$A(\omega)=\begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 0 & 1\\ N_1 & 0 & 0 & 0 & N_2 & N_3\\ 0 & N_5 & N_6 & N_4 & 0 & 0\\ 0 & N_8 & N_9 & N_7 & 0 & 0 \end{bmatrix},$$

$$V(x,\omega)=\begin{bmatrix} \bar{u}(x,\omega)\\ \bar{\varphi}(x,\omega)\\ \bar{\psi}(x,\omega)\\ (\bar{u}(x,\omega))_{,1}\\ (\bar{\varphi}(x,\omega))_{,1}\\ (\bar{\psi}(x,\omega))_{,1} \end{bmatrix},$$

$$V(0,w)=\begin{bmatrix} \bar{u}(0,\omega)\\ \bar{\varphi}(0,\omega)\\ \bar{\psi}(0,\omega)\\ (\bar{u}(0,\omega))_{,1}\\ (\bar{\varphi}(0,\omega))_{,1}\\ (\bar{\psi}(0,\omega))_{,1} \end{bmatrix},$$

$$\begin{aligned} a_0 &= e^{-\lambda_1 x}\left[\frac{\lambda_2\lambda_3}{(\lambda_1-\lambda_2)(\lambda_1-\lambda_3)}\right]+e^{-\lambda_2 x}\left[\frac{\lambda_1\lambda_3}{(\lambda_2-\lambda_1)(\lambda_2-\lambda_3)}\right]\\ &\quad + e^{\lambda_3 x}\left[\frac{\lambda_1\lambda_2}{(\lambda_3-\lambda_1)(\lambda_3-\lambda_2)}\right],\\ a_1 &= e^{-\lambda_1 x}\left[\frac{(\lambda_2+\lambda_3)}{(\lambda_1-\lambda_2)(\lambda_1-\lambda_3)}\right]+e^{-\lambda_2 x}\left[\frac{(\lambda_1+\lambda_3)}{(\lambda_2-\lambda_1)(\lambda_2-\lambda_3)}\right]\\ &\quad + e^{-\lambda_3 x}\left[\frac{(\lambda_1+\lambda_2)}{(\lambda_3-\lambda_1)(\lambda_3-\lambda_2)}\right],\\ a_2 &= e^{-\lambda_1 x}\left[\frac{1}{(\lambda_1-\lambda_2)(\lambda_1-\lambda_3)}\right]+e^{-\lambda_2 x}\left[\frac{1}{(\lambda_2-\lambda_1)(\lambda_2-\lambda_3)}\right]\\ &\quad + e^{-\lambda_3 x}\left[\frac{1}{(\lambda_3-\lambda_1)(\lambda_3-\lambda_2)}\right], \end{aligned}$$

$$Q_1=P_1\left(Z_1+N_1Z_3\right),\quad Q_2=P_2\left(a_0^0+a_2^0N_5\right)+P_3\left(a_2^0N_8\right),$$

$$Q_3=P_2\left(a_2^0N_6\right)+P_3\left(a_0^0+a_2^0N_9\right),$$

$$Q_4=0,\quad Q_5=P_4\left(Z_1+N_5Z_3\right)+P_5N_8Z_3,\quad Q_6=P_4N_6Z_3+P_5\left(Z_1+N_9Z_3\right),$$

$$Q_7=0,\quad Q_8=P_5\left(Z_1+N_5Z_3\right)+P_6N_8Z_3,\quad Q_9=P_4N_6Z_3+P_6\left(Z_1+N_9Z_3\right),$$

where

$$P_1=\frac{\lambda+2\mu}{\lambda}, P_2=\frac{b\alpha_1}{\lambda k_1\omega^2}, P_3=\frac{d\alpha_1}{\lambda k_1\omega^2}, P_4=\frac{\alpha_1}{k_1\omega^2}, P_5=\frac{b_1\alpha_1}{\alpha k_1\omega^2},$$
$$P_6=\frac{\gamma\alpha_1}{\alpha k_1\omega}, Z_1=-\lambda_1 D_{11}-\lambda_2 D_{12}-\lambda_3 D_{13}, Z_2=-\lambda_1 D_{21}-\lambda_2 D_{22}-\lambda_3 D_{23},$$
$$Z_3=-\lambda_1 D_{31}-\lambda_2 D_{32}-\lambda_3 D_{33},$$

and

$$D_{11}=\frac{\lambda_2\lambda_3}{(\lambda_1-\lambda_2)(\lambda_1-\lambda_3)}, D_{12}=\frac{\lambda_1\lambda_3}{(\lambda_2-\lambda_1)(\lambda_2-\lambda_3)},$$
$$D_{13}=\frac{\lambda_1\lambda_2}{(\lambda_3-\lambda_1)(\lambda_3-\lambda_2)}, D_{31}=\frac{1}{(\lambda_1-\lambda_2)(\lambda_1-\lambda_3)},$$
$$D_{32}=\frac{1}{(\lambda_2-\lambda_1)(\lambda_2-\lambda_3)}, D_{33}=\frac{1}{(\lambda_3-\lambda_1)(\lambda_3-\lambda_2)},$$

and

$$\text{at } x=0; a_0=a_0^0, a_2=a_2^0.$$

Further,

$$S_1=P_1(Y_1+N_1Y_3), S_2=0, S_3=0, S_4=0, S_5=P_4(Y_1+N_5Y_3)+P_5N_8Y_3,$$
$$S_6=P_4N_6Y_3+P_5(Y_1+N_9Y_3), S_7=0, S_8=P_5(Y_1+N_5Y_3)+P_6N_8Y_3,$$
$$S_9=P_5N_6Y_3+P_6(Y_1+N_9Y_3),$$

where

$$Y_1=-\lambda_1 D_{11}e^{-\lambda_1 x}-\lambda_2 D_{12}e^{-\lambda_2 x}-\lambda_3 D_{13}e^{-\lambda_3 x},$$
$$Y_2=-\lambda_1 D_{11}e^{-\lambda_1 x}-\lambda_2 D_{12}e^{-\lambda_2 x}-\lambda_3 D_{13}e^{-\lambda_3 x},$$
$$Y_3=-\lambda_1 D_{21}e^{-\lambda_1 x}-\lambda_2 D_{22}e^{-\lambda_2 x}-\lambda_3 D_{23}e^{-\lambda_3 x}.$$

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