Optimality criterion for three proposed methods to unbalansed transportation models

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Abstract. The idea of balancing the transportation table starts by adding a quantity to the supply or demand to fill the shortage and achieve the requirement of equality between the quantities of demand and supply with making the costs of the row (column) added zeroes, on the other hand what is the profit desired from this addition as it cannot display a need made or created effort and money without Price or for any zero cost and vice versa as no need or commodity can be ordered without a small price, and accordingly has been proposed three methods to address the unbalanced transportation table and agencies without resorting to adding a zero-cost column (row) with application work to comparing the three proposed methods with the classical methods.
Keywords: transportation problem, linear programming, optimization problems, unbalanced transportation model, proposed methods.

1. Introduction

In the life, the facility usually encounters a scarcity of the resources it requires in its production process. Or it seems a decline in the demand for its products, which creates surplus to it and this leads to difficulties on the ability of the marketing device in the discharge of this surplus or, conversely, an increase in demand for products above their capacity or production lines.

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In order to deal with this phenomenon, the transport models tend to carry out a kind of balance, in light of which the excess quantities of supply or demand are distributed, and the costs or profits are calculated at a later stage.

This balance between supply and demand requires a dummy column to address the amount of shortfall in demand by a certain amount by finding the difference between total supply and demand. In the case where the demand is more than the supply, it is necessary to add a dummy row to address this imbalance, and in terms of costs that are placed within the small squares must be zero. But from another perspective, in the practical and life side, we cannot make the costs of the dummy column or row zeroes, if so what is the desired profit of this addition, as well as when the demand for a particular commodity cannot be promoted without any cost zero, and also cannot offer a need made or From this point, we proposed three methods to balance the transportation model without resorting to adding a row or column with zero costs, but we used some calculation methods to balance the model and then compare these methods with the classic methods.

2. Transportation model [3]

In a transportation problem the points may represent factories to produce items, and to supply a required quantity of the products to a certain number of destinations. This process must be done successfully in such a way as to maximize the profit or minimize the cost transportation. Therefore, the places of production and supply are collected as the original points and the destinations respectively. Sometimes the original and destinations points are also termed as sources and sinks. However, to illustrate a typical transportation model, suppose that m factories supply certain items to n warehouses. As well as, let factory $i, (i = 1, 2, \dots, m)$ produces ai units, and the warehouse $j, (j = 1, 2, \dots, n)$ requires bj units. Furthermore, suppose the cost of transportation from factory i to warehouse j is c_{ij}. The decision variables x_{ij} is being the transported amount from the factory i to the warehouse j.

3. Unbalanced transportation problem [1, 2, 4, 5]

There are two different forms of imbalance in the transportation table

Increase in supply $\sum a_i > \sum b_j$;

$$Min(Z) = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} X_{ij},$$

$$\sum_{j=1}^{n} X_{ij} < ai; \quad i = 1, 2, \ldots, m,$$

$$\sum_{i=1}^{m} X_{ij} = bj; \quad j = 1, 2, \ldots, n \quad \forall X_{ij} \geqslant 0.$$

Increase in demand $\sum a_i < \sum b_j$;

$$Min(Z) = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} X_{ij},$$

$$\sum_{j=1}^{n} X_{ij} = ai; \quad i = 1, 2, \ldots, m,$$

$$\sum_{i=1}^{m} X_{ij} < bj; \quad j = 1, 2, \ldots, n \quad \forall X_{ij} \geqslant 0.$$

4. Proposed methods

4.1 Added difference method (1)

Calculate the difference between total supply and demand or vice versa.

$$\sum_{j=1}^{n} D_j - \sum_{i=1}^{m} S_i = Q. \tag{1}$$

Add this difference to the quantity of the row that contains the least cost if the total supply is less than the total demand.

$$S_i = a_{i(min\ cost)} + Q. \tag{2}$$

Or add the difference of the column that has the least cost if the total demand is less than the total supply.

$$D_j = b_{j(min\ cost)} + Q. \tag{3}$$

We complete the solution depending on the methods adopted in finding the first basic solution.

4.2 Added difference method (2)

Calculate the difference between total supply and demand or vice versa.

$$\sum_{j=1}^{n} D_j - \sum_{i=1}^{m} S_i = Q. \tag{4}$$

Add this difference to the quantity of the row that contains the highest cost if the total supply is less than the total demand.

$$S_i = a_{i(max\ cost)} + Q. \tag{5}$$

Or add the difference of the column that has the highest cost if the total demand is less than the total supply.

$$D_j = b_{j(max\ cost)} + Q. \tag{6}$$

We complete the solution depending on the methods adopted in finding the primary solution.

4.3 Supply Max, demand Min method

Calculate the difference between total supply and demand or vice versa

$$\sum_{j=1}^{n} D_j - \sum_{i=1}^{m} S_i = Q. \tag{7}$$

Determine which row has the highest cost (if the total demand is the lowest) to achieve the highest profit to demand.

$$S_i = a_{i(max\ cost)} + Q. \tag{8}$$

Determine which column has the lowest cost (if the total supply is the lowest) to achieve the highest profit to supply.

$$D_j = b_{j(max\ cost)} + Q. \tag{9}$$

The difference is added to the selected row (column) to achieve equilibrium status in the transportation table.

We complete the solution depending on the methods adopted in finding the primary solution.

5. Empirical side

Introduction

Many algorithms have been developed to solve transportation problem because of their wide application in logistics and supply chain when cost and supply and demand quantities are well known to reduce total cost. Transportation problem is said to be balanced if total supply from all sources equals total demand in all destinations It is called otherwise unbalanced. The following is an unbalanced digital example containing three sources of supply and three demand sites in order to show the application of the idea of the proposed methods

Table (1) unbalance transportation table

		Destinations			Supply
		b_1	b_2	b_3	
Sources	a_1	6	7	8	7
	a_2	6	10	9	5
	a_3	5	4	3	10
Demand		10	20	30	22 60

Based on the steps to solve the Added Difference Method (1) is calculated the difference between supply and demand based on equation (1), since the total supply quantities are the lowest is determined row that has the lowest cost and then add the difference resulting from equation (1) to the amount of that row on equation (2)

$$\sum_{j=1}^{n} D_j - \sum_{i=1}^{m} S_i = 60 - 22 = 38 = Q$$

$S_3 = a_{3(min\ cost)} + Q = 10 + 38 = 48.$

Table (2) balance transportation table

		Destinations			Supply
		b_1	b_2	b_3	
Sources	a_1	6	7	8	7
	a_2	6	10	9	5
	a_3	5	4	3	10+38=48
Demand		10	20	30	60 60

Based on the steps of the solution of the Added Difference Method (2)is calculated the difference between supply and demand based on equation (4), since the total supply quantities are the lowest is determined row with the highest cost and then add the difference resulting from equation (4) to the quantity of that row on equation (5)

$$\sum_{j=1}^{n} D_j - \sum_{i=1}^{m} S_i = 60 - 22 = 38 = Q,$$

$S_2 = a_{2(max\ cost)} + Q = 5 + 38 = 43.$

Based on the steps of solving the method of Supply quantity is larger than demand quantity is calculated the difference between supply and demand based

Table (3) balance transportation table

		Destinations			Supply
		b1	b2	b3	
Sources	a1	6	7	8	7
	a2	6	10	9	5+38=43
	a3	5	4	3	10
Demand		10	20	30	60 60

on equation (7), since the total supply quantities are the lowest is determined row with the highest cost and then add the difference resulting from equation (7) to the quantity of that row on equation (8). Otherwise, we move to a situation where the sum of the demand is the least to determine which column has the least cost to add the difference to which to achieve the equilibrium state in the transportation table.

6. Applied side

Testing example

The above three methods were applied to (Brisbol) company as this company distributes medicines from its main locations located in four governorates (Baghdad, Diwaniyah, Najaf, Karbala) by transferring them to a group of Medicine repository cooperating with the company located in the four governorates. Quantities of supply and demand and transportation costs were measured in Iraqi dinars.

Table (4) unbalance transportation table

		Destinations				Supply
		Medicine repository1	Medicine repository2	Medicine repository3	Medicine repository4	
sources	Baghdad	109	209	150	129	125
	Diwaniyah	200	119	145	134	205
	Najaf	151	110	166	116	134
	Karbala	108	130	149.5	140	150
Demand		125	175	109	168	614 577

By using Added Difference Method (1). Based on the steps to solve the first proposed method is calculated the difference between supply and demand based on equation (1), since the total demand quantities are the lowest is determined column that has the lowest cost and then add the difference resulting from equation (1) to the amount of that column on equation (2), If there is more than one column that has the lowest cost, the selection will be randomly

$$\sum_{i=1}^{m} S_i - \sum_{j=1}^{n} D_j = 614 - 577 = 37 = Q$$

$D_1 = b_{1(min\ cost)} + Q = 125 + 37 = 162.$

Table (5) balance transportation table

		Destinations				
		Medicine repository1	Medicine repository2	Medicine repository3	Medicine repository4	Supply
Sources	Baghdad	109	209	150	129	125
	Diwaniyah	200	119	145	134	205
	Najaf	151	110	166	116	134
	Karbala	108	130	149.5	140	150
Demand		125+37=162	175	109	168	614

By using Added Difference Method (2) and back to Table 4. Based on the steps of the solution of the second proposed method is calculated the difference between supply and demand based on equation (4), since the total demand quantities are the lowest is determined column with the highest cost and then add the difference resulting from equation (4) to the quantity of that column on equation (5)

$$\sum_{i=1}^{m} S_i - \sum_{j=1}^{n} D_j = 614 - 577 = 37 = Q,$$

$D_2 = b_{2(max\ cost)} + Q = 175 + 37 = 212.$

Table (6) balance transportation table

		Destinations				
		Medicine repository1	Medicine repository2	Medicine repository3	Medicine repository4	Supply
sources	Baghdad	109	209	150	129	125
	Diwaniyah	200	119	145	134	205
	Najaf	151	110	166	116	134
	Karbala	108	130	149.5	140	150
Demand		125	175+37=212	109	168	614

By using method of Supply max, demand min and back to Table 4.

Based on the steps of the solution of the third proposed method is calculated the difference between supply and demand based on equation (7), since the total supply quantities are the lowest is determined row with the highest cost and then add the difference resulting from equation (7) to the quantity of that row on equation (8)

$$\sum_{i=1}^{m} S_i - \sum_{j=1}^{n} D_j = 614 - 577 = 37 = Q,$$

$D_1 = b_{1(min\ cost)} + Q = 125 + 37 = 162.$

Table (7) balance transportation table

		Destinations				Supply
		Medicine repository1	Medicine repository2	Medicine repository3	Medicine repository4	
Sources	Baghdad	109	209	150	129	125
	Diwaniyah	200	119	145	134	205
	Najaf	151	110	166	116	134
	Karbala	108	130	149.5	140	150
Demand		125+37=162	175	109	168	614

After balancing the transportation table by using the three proposed solution methods, we will use the software WINQSP and compare the results of the proposed methods with the classic Vogel approximation method.

Table (8) Results of the Added Difference Method (1)

12-25-2019	From	To	Shipment	Unit Cost	Total Cost	Reduced Cost
1	Baghdad	Repository 1	12	109	1308	0
2	Baghdad	Repository 4	113	129	14577	0
3	Diwaniyah	Repository 2	96	119	11424	0
4	Diwaniyah	Repository 3	109	145	15805	0
5	Najaf	Repository 2	79	110	8690	0
6	Najaf	Repository 4	55	116	6380	0
7	Karbala	Repository 1	150	108	16200	0
	Total	Objective	Function	Value =	74384	

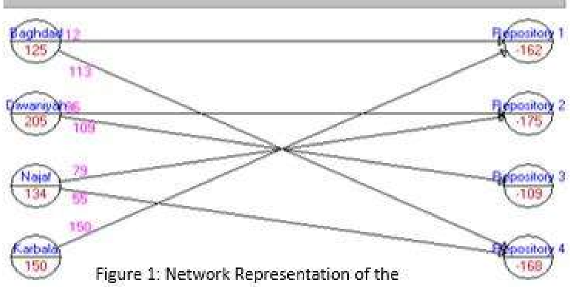

Figure 1: Network Representation of the Transportation Problem

Table (9) Limits of Optimization and Sensitivity Analysis

12-25-2019 22:20:04	From	To	Unit Cost	Reduced Cost	Basis Status	Allowable Min. Cost	Allowable Max. Cost
1	Baghdad	Repository 1	109	0	basic	108	164
2	Baghdad	Repository 2	209	86	at bound	123	M
3	Baghdad	Repository 3	150	1	at bound	149	M
4	Baghdad	Repository 4	129	0	basic	125	130
5	Diwaniyah	Repository 1	200	95	at bound	105	M
6	Diwaniyah	Repository 2	119	0	basic	118	123
7	Diwaniyah	Repository 3	145	0	basic	-4	146
8	Diwaniyah	Repository 4	134	9	at bound	125	M
9	Najaf	Repository 1	151	55	at bound	96	M
10	Najaf	Repository 2	110	0	basic	106	111
11	Najaf	Repository 3	166	30	at bound	136	M
12	Najaf	Repository 4	116	0	basic	115	120
13	Karbala	Repository 1	108	0	basic	-M	109
14	Karbala	Repository 2	130	8	at bound	122	M
15	Karbala	Repository 3	149.50	1.50	at bound	148	M
16	Karbala	Repository 4	140	12	at bound	128	M

It is noted that from the results of table no. (8)special of the Added Difference Method (1) that the required quantities of (614) tons from the cities equipped with the transport to the sub repositories need for the orders at the lowest possible transport cost, for example was equipped medicine repository4 from the cities of Baghdad and Najaf, where the quantity supplied from the city of Baghdad 113 The cost of transporting 14,577 and the quantity supplied from Najaf city was 55 and the cost of transporting 6,380, which was divided the quantities transported on the basis of reducing the total costs of transportation. But the extent of optimization and sensitivity analysis and the possibility of change in demand and supply and the cost of transporting per unite this is shown in table (9).

It is noted that there is a reduction in costs on some of the variables that were not transported from the cities to the repository of medicines containing low costs as they increase the value of the objective function, for example, the main site located in Baghdad city was not transported to repository of medicines (2, 3) due to reduced costs by (86, 1) respectively and so on for the other non-basic variables. With regard to the upper and lower limits that make the transportation problem within the limits of optimization, for example, the main site in Najaf city has been transported to the repositories (2, 4) at a transport cost per unit of (110, 116) respectively which are within Upper and lower limits of that province.

Table (10) Results of the Added Difference Method (2)

12-25-2019	From	To	Shipment	Unit Cost	Total Cost	Reduced Cost
1	Baghdad	Repository 4	125	129	16125	0
2	Diwaniyah	Repository 2	121	119	14399	0
3	Diwaniyah	Repository 3	84	145	12180	0
4	Najaf	Repository 2	91	110	10010	0
5	Najaf	Repository 4	43	116	4988	0
6	Karbala	Repository 1	125	108	13500	0
7	Karbala	Repository 3	25	149.50	3,737.50	0
	Total	Objective	Function	Value =	74,939.50	

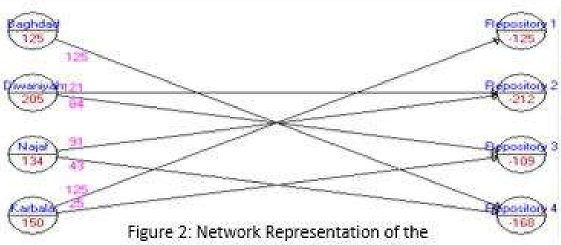

Figure 2: Network Representation of the Transportation Problem

Table (11) Limits of Optimization and Sensitivity Analysis

12-25-2019 22:23:41	From	To	Unit Cost	Reduced Cost	Basis Status	Allowable Min. Cost	Allowable Max. Cost
1	Baghdad	Repository 1	109	1.50	at bound	107.50	M
2	Baghdad	Repository 2	209	86	at bound	123	M
3	Baghdad	Repository 3	150	1	at bound	149	M
4	Baghdad	Repository 4	129	0	basic	-M	129.50
5	Diwaniyah	Repository 1	200	96.50	at bound	103.50	M
6	Diwaniyah	Repository 2	119	0	basic	118.50	125.50
7	Diwaniyah	Repository 3	145	0	basic	138.50	145.50
8	Diwaniyah	Repository 4	134	9	at bound	125	M
9	Najaf	Repository 1	151	56.50	at bound	94.50	M
10	Najaf	Repository 2	110	0	basic	101	110.50
11	Najaf	Repository 3	166	30	at bound	136	M
12	Najaf	Repository 4	116	0	basic	115.50	125
13	Karbala	Repository 1	108	0	basic	0	109.50
14	Karbala	Repository 2	130	6.50	at bound	123.50	M
15	Karbala	Repository 3	149.50	0	basic	149	156
16	Karbala	Repository 4	140	10.50	at bound	129.50	M

It is noted from the results of table no. (10) special of the Added Difference Method (2) that the required quantities of (614) tons from the cities equipped with the transport to the sub repositories need for the orders at the lowest possible transport cost, for example was equipped medicine repository2 from the cities of Diwaniyah and Najaf, where the quantity supplied from the city of Diwaniyah 121 The cost of transporting 14399 and the quantity supplied from Najaf city was 91 and the cost of transporting 10010, which was divided the quantities transported on the basis of reducing the total costs of transportation. As for the extent of optimization and sensitivity analysis and the possibility of change in demand and supply and the cost of transporting, this is shown in table (11).

It is noted that there is a reduction in costs on some of the variables that were not transported from the cities to the repository of medicines containing low costs as they increase the value of the objective function, for example, the main site located in Najaf city was not transported to repository of medicines (1, 3) due to reduced costs by (56.50, 30) respectively and so on for the other non-basic variables.

With regard to the upper and lower limits that make the transportation problem within the limits of optimization, for example, the main site in Karbala city has been transported to the repositories (1, 3) at a transport cost per unit of (108, 149.50) respectively which are within Upper and lower limits of that province.

Table (12) Results of Supply max demand min method

12-25-2019	From	To	Shipment	Unit Cost	Total Cost	Reduced Cost
1	Baghdad	Repository 1	12	109	1308	0
2	Baghdad	Repository 4	113	129	14577	0
3	Diwaniyah	Repository 2	96	119	11424	0
4	Diwaniyah	Repository 3	109	145	15805	0
5	Najaf	Repository 2	79	110	8690	0
6	Najaf	Repository 4	55	116	6380	0
7	Karbala	Repository 1	150	108	16200	0
	Total	Objective	Function	Value =	74384	

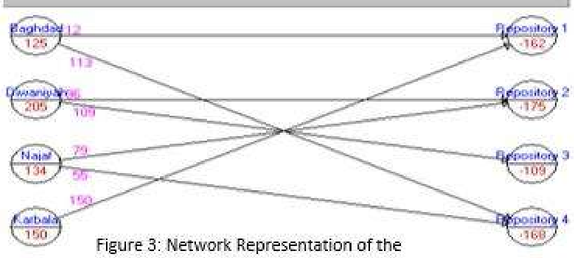

Figure 3: Network Representation of the Transportation Problem

Table (13) Limits of Optimization and Sensitivity Analysis

12-25-2019 22:20:04	From	To	Unit Cost	Reduced Cost	Basis Status	Allowable Min. Cost	Allowable Max. Cost
1	Baghdad	Repository 1	109	0	basic	108	164
2	Baghdad	Repository 2	209	86	at bound	123	M
3	Baghdad	Repository 3	150	1	at bound	149	M
4	Baghdad	Repository 4	129	0	basic	125	130
5	Diwaniyah	Repository 1	200	95	at bound	105	M
6	Diwaniyah	Repository 2	119	0	basic	118	123
7	Diwaniyah	Repository 3	145	0	basic	-4	146
8	Diwaniyah	Repository 4	134	9	at bound	125	M
9	Najaf	Repository 1	151	55	at bound	96	M
10	Najaf	Repository 2	110	0	basic	106	111
11	Najaf	Repository 3	166	30	at bound	136	M
12	Najaf	Repository 4	116	0	basic	115	120
13	Karbala	Repository 1	108	0	basic	-M	109
14	Karbala	Repository 2	130	8	at bound	122	M
15	Karbala	Repository 3	149.50	1.50	at bound	148	M
16	Karbala	Repository 4	140	12	at bound	128	M

It is noted from the results of table no. (12) special of Supply max demand min method that the required quantities of (614) tons from the cities equipped with the transport to the sub repositories need for the orders at the lowest possible transport cost, for example was equipped medicine repository1 from the cities of Baghdad and Karbala, where the quantity supplied from the city of Baghdad 12 The cost of transporting 1308 and the quantity supplied from Karbala city was 150 and the cost of transporting 16200, which was divided the quantities transported on the basis of reducing the total costs of transportation. As for the extent of optimization and sensitivity analysis and the possibility of change in demand and supply and the cost of transporting the unit and one, this is shown in Table 13.

It is noted that there is a reduction in costs on some of the variables that were not transported from the cities to the repository of medicines containing low costs as they increase the value of the objective function, for example, the main site located in Baghdad city was not transported to repository of medicines (2, 3) due to reduced costs by (86, 1) respectively and so on for the other non-basic variables. With regard to the upper and lower limits that make the transportation problem within the limits of optimization, for example, the main site in Diwaniyah city has been transported to the repositories (2, 3) at a transport cost per unit of (119, 145) respectively which are within Upper and lower limits of that province.

Table (14) Results of the Vogel approximation method

12-26-2019	From	To	Shipment	Unit Cost	Total Cost	Reduced Cost
1	Baghdad	Repository 4	113	129	14577	0
2	Baghdad	Repository 5	12	0	0	0
3	Diwaniyah	Repository 2	96	119	11424	0
4	Diwaniyah	Repository 3	109	145	15805	0
5	Najaf	Repository 2	79	110	8690	0
6	Najaf	Repository 4	55	116	6380	0
7	Karbala	Repository 1	125	108	13500	0
8	Karbala	Repository 5	25	0	0	0
	Total	Objective	Function	Value =	70376	

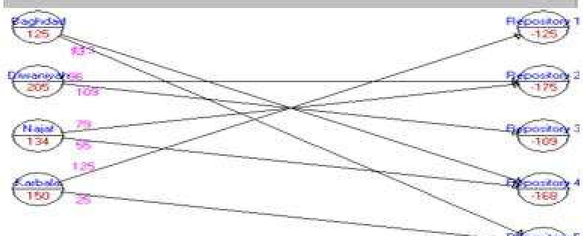

Figure 4: Network Representation of the Transportation Problem

Table (15) Limits of Optimization and Sensitivity Analysis

12-26-2019 00:34:33	From	To	Unit Cost	Reduced Cost	Basis Status	Allowable Min. Cost	Allowable Max. Cost
1	Baghdad	Repository 1	109	1	at bound	108	M
2	Baghdad	Repository 2	209	86	at bound	123	M
3	Baghdad	Repository 3	150	1	at bound	149	M
4	Baghdad	Repository 4	129	0	basic	125	129.50
5	Baghdad	Repository 5	0	0	basic	0	1
6	Diwaniyah	Repository 1	200	96	at bound	104	M
7	Diwaniyah	Repository 2	119	0	basic	118.50	123
8	Diwaniyah	Repository 3	145	0	basic	-4	145.50
9	Diwaniyah	Repository 4	134	9	at bound	125	M
10	Diwaniyah	Repository 5	0	4	at bound	-4	M
11	Najaf	Repository 1	151	56	at bound	95	M
12	Najaf	Repository 2	110	0	basic	106	110.50
13	Najaf	Repository 3	166	30	at bound	136	M
14	Najaf	Repository 4	116	0	basic	115.50	120
15	Najaf	Repository 5	0	13	at bound	-13	M
16	Karbala	Repository 1	108	0	basic	0	109
17	Karbala	Repository 2	130	7	at bound	123	M
18	Karbala	Repository 3	149.50	0.50	at bound	149	M
19	Karbala	Repository 4	140	11	at bound	129	M
20	Karbala	Repository 5	0	0	basic	-1	0

It is noted from the results of table no. (14) special of Vogel approximation method that the required quantities of (614) tons from the cities equipped with the transport to the sub repositories need for the orders at the lowest possible transport cost, for example was equipped medicine repository2 from the cities of Diwaniyah and Najaf, where the quantity supplied from the city of Diwaniyah 96 The cost of transporting 11424 and the quantity supplied from Najaf city was 79 and the cost of transporting 8690, which was divided the quantities transported on the basis of reducing the total costs of transportation. As for the extent of optimization and sensitivity analysis and the possibility of change in demand and supply and the cost of transporting the unit and one, this is shown in table (15).

It is noted that there is a reduction in costs on some of the variables that were not transported from the cities to the repository of medicines containing low costs as they increase the value of the objective function, for example, the main site located in Baghdad city was not transported to repository of medicines (1, 2, 3) due to reduced costs by (1, 86, 1) respectively and so on for the other non-basic variables. With regard to the upper and lower limits that make the transportation problem within the limits of optimization, for example, the main site in Diwaniyah city has been transported to the repositories (2, 3) at a transport cost per unit of (119, 145) respectively which are within Upper and lower limits of that province.

7. Discussion

Based on the use and application of the proposed methods, the total transportation costs are presented in the following table:

No	Methods	Total Transportation Costs
1.	Added Difference (1)	74 384
2.	Added Difference (2)	74 939.50
3.	Supply max demand min	74 384
4.	Vogel Approximation Method	70 376

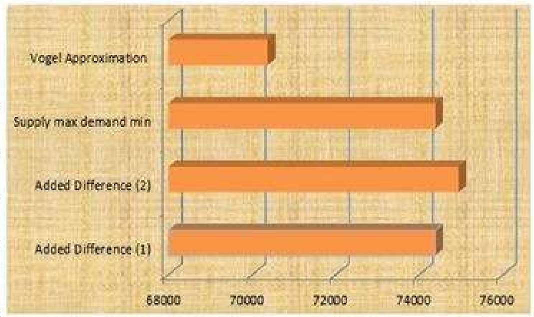

Figure 5: Total Transportation Cost

Note from the Empirical example after balancing the transportation problem using the three solution methods proposed for the above example, and using WINQSP program we have shown the total cost for the added difference methods (1) and the Supply quantity is larger than demand quantity by (236) and the total transportation cost of the added difference method (2) by (449). And the Vogel approximation method had shown the total cost of transportation (104).

1. As for the application side, we find that the added difference (1)has been equal to the Supply max demand min method in terms of the total cost equivalent of 74384.

2. As for the added difference (2) method, it recorded the total costs equivalent 74939.50.

3. The best method is the normal method, which used the method of approximation Vogel, where the total cost of 70376, which is the lowest total transportation costs of these methods.

4. Point (3) shows us that the total transportation costs reached (70376) according to the Vogel approximation method, which is less than the methods remained by a large difference because of the balance of the transportation problem by adding a row or column with an amount equal to the amount of difference between supply and demand at zero costs, while In the other proposed methods, the transportation problem was balanced without adding a row or column with zero costs.

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Accepted: May 18, 2020