On Kαc-spaces

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Abstract. That research is submitted to introduce new type of supra Kc-spaces, it is supra αK (αc)-space, supra L (αc)-space, supra (αL)c–space and supra αL(αc)-space. We showed the relation between these types, also we provided some theorems, propositions and examples about the subjects.

Keywords: supra αK (αc)-space, supra L(αc)-space, supra (αL)c –space, supra αL(αc)-space, supra α-Lindelöf space.

1. Introduction

Wilansky was the first author presented the notion of Kc-spaces [1], after that many authors treated with this space such as in [2]. Mashhour in [3] provided the notion of supra spaces. In [3], [4] the researchers presented the definition of the supra closure and the supra interior to any set in the supra space. In [5] the researcher defined the supra continuous function as (whenever $g^(- 1)(V)$ is supra open subset of the supra space (X, μ_X) to any supra open subset V of the supra space (Y, μ_Y), then $g : (X, \mu_X) \to (Y, \mu_Y)$ is supra continuous function). The researcher in [6] introduced the definition of supra open function

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as ($g : (X, \mu_X) \to (Y, \mu_Y)$ is supra open function, whenever for any supra open subset S of X, $g(S)$ is supra open subset of Y). As well as he introduced the definition of the supra homeomorphism functions. In [7] Mukherji and Sarkar presented the concept of Lc-space (when any Lindelöf subset of a space X is closed, thus X is Lc-space). After that [8] the researchers introduced a kind of Lc-space which is stronger than the Kc-space. In this our work we connect between K(αc)-space and the supra space. Also we connect αK(αc)-space and the supra space. By the same way we defined the rest Kind of spaces that we provided in this paper.

2. On K (αc)-spaces

In the next section of the research we will introduce a new types of supra Kc-spaces, we will begin with some definitions that are benefit us in our next topics.

Definition 2.1 ([9]). *Consider (X, μ) is a supra space and $S \subseteq X$, when $S \subseteq Int^{\mu}(cl^{\mu}(int^{\mu})(S))$, so S is supra α-open set. S^c is called supra α-closed.*

Definition 2.2 ([9]). *A supra α-open cover $\{S_\alpha\}_{\alpha \in \Lambda}$ to (X, μ) is a family of supra α-open subsets of X, where $X \subseteq \bigcup_{\alpha \in \Lambda} S_\alpha$*

Definition 2.3 ([9]). *Whenever each supra α-open cover for a supra space (X, μ) owns a finite (countable) subcover, then X is supra α-compact (supra α-Lindelöf).*

Definition 2.4. *When each compact subset of (X, μ) is supra α-closed, so X is supra K (αc)-space.*

Example 2.1. The discrete supra space, is supra K (αc)-space.

Remark 2.1. The supra α-compact space is supra α-Lindelöf.

Example 2.2. The supra co-countable space (R, μ_{coc}) is supra α-Lindelöf, while it is not supra α-compact.

Definition 2.5 ([10]). *If $X \subseteq \bigcup_{\alpha \in \Lambda} U_{\alpha\alpha}$, where $\{U_\alpha\}_{\alpha \in \Lambda}$ is a family of supra open subsets from (X, μ), then $\{U_\alpha\}_{\alpha \in \Lambda}$ is a supra open cover for X.*

Definition 2.6 ([10]). *When each supra open cover to a supra space X owns a finite (or able to be counted) sub cover, then X is supra compact (supra Lindelöf) space.*

Remark 2.2 ([10]). Suppose X is Supra α-compact (supra α-Lindelöf) space, so it will be supra compact (supra Lindelöf).

Example 2.3. The discrete supra topology on infinite countable set is a supra Lindelöf space as well as a supra α-Lindelöf space but neither supra compact nor supra α-compact.

Proposition 2.1 ([9]). *Each supra α-closed subset from a supra α-compact space will be supra α-compact set.*

Proposition 2.2 ([9]). *Suppose X is supra α-Lindelöf space, and T is supra α-closed subset of X, hence T is supra α-Lindelöf.*

Definition 2.7. *Consider the supra space (X, μ) and x, y is non-equal points in X, when we can find supra α-open sets V_1 in the supra space X, in which $x \in V_1$, $y \notin V_1$, so X is supra αT_1space.*

Example 2.4. The supra co-finite topology on R is supra αT$_1$-space.

Definition 2.8 ([11]). *Suppose (X, μ) is a supra space, x, y are non-equal points in X, we can find are supra α-open sets G_1, G_2 in the supra space X, where $x \in G_1$, $y \in G_2$ and $G_1 \cap G_2 = \emptyset$ so X is supra αT_2-space.*

Example 2.5. The discrete supra topology on R is supra αT$_2$-space.

Theorem 2.1. *Consider (X, μ) is a supra αT_2-space, and T is supra α-compact subset of it, then T is a supra α-closed.*

Proof. Take $d \in$ T^c then for any point $b \in$ T, $d \neq b$. There exist disjoint supra α-open sets G, L in the supra space X containing b, d respectively. We get $H = \{G_b\}_{b \in T}$ is supra α-open cover to T, since T is supra α-compact set, hence H owns a finite sub cover in which $T \subseteq \cup_{i=1}^{n} G_{b_i}$. Suppose $\tilde{L} = \cap_{i=1}^{n} L_{b_i}$ which is supra α-open set having d in it, implies $\tilde{G} = \cup_{i=1}^{n} G_{b_i}$, then $\tilde{G} \cap \tilde{L} = \emptyset$, so that $T \cap \tilde{L} = \emptyset$, $\tilde{L} \subseteq T^c$, therefore T^c is supra α-open. Hence T will be supra α-closed set. □

Definition 2.9. *Suppose T is a supra α-compact subset of the supra space X, whenever T supra α-closed, hence the space X is called supra αK (αc)-space.*

Example 2.6.

1-The co-countable supra space (R, μ_{coc}) is supra αK (αc)-space, also it is Kc-space.

2- (R, μ_D) is supra αK (αc)-space.

Proposition 2.3. *Consider the supra spade (X, μ) and $T \subseteq X$. T is supra α-closed set iff there is a supra closed set L with $cl^{\mu}(int(L)) \subseteq T \subseteq L$.*

Proof. From $cl^{\mu}(int^{\mu}(T)) \subseteq T$ $(cl^{\mu}(T)) \subseteq T$ and $\subseteq cl^{\mu}(T)$, we have $cl^{\mu}(int^{\mu}(cl^{\mu}(T))) \subseteq T \subseteq cl^{\mu}(T)$, pick $cl^{\mu}(T) = L$, hence $cl^{\mu}(int^{\mu}(L)) \subseteq T \subseteq L$.

Conversely, since $cl^{\mu}(int^{\mu}(L)) \subseteq T \subseteq L$, so $cl^{\mu}(cl^{\mu}(int^{\mu}(L))) \subseteq cl^{\mu}(T) \subseteq cl^{\mu}(L) = L$, then $cl^{\mu}(int^{\mu}(T))(L) \subseteq cl^{\mu}(T) \subseteq L$, that implies, $int^{\mu}(cl^{\mu}(int^{\mu}(L)))$ $\subseteq int^{\mu}(cl^{\mu}(T)) \subseteq int^{\mu}(L)$, then $cl^{\mu}(int^{\mu}(cl^{\mu}(int^{\mu}(L)))) \subseteq cl^{\mu}(int^{\mu}(cl^{\mu}(T))) \subseteq cl^{\mu}(int^{\mu}(L))$ (because $cl^{\mu}(int^{\mu}(L)) \subseteq T \subseteq L$), therefore $cl^{\mu}(int^{\mu}(cl^{\mu}(T))) \subseteq T$, therefore T is supra α-closed. □

Lemma 2.1. *Let T be supra α-closed set in the supra space X which is a domain of a supra homeomorphism function h and the supra space Y its co-domain, then $h(T)$ is a supra α-closed set in Y.*

Proof. Because of T is a supra α-closed subset of X, so $cl^\mu(int^\mu(L)) \subseteq T \subseteq L$, where L is a supra closed subset of X, so $cl^\mu h(int^\mu(L)) = h(cl^\mu(int^\mu(L))) \subseteq h(T) \subseteq h(L)$, but $h(L)$ is supra closed subset of Y, which implies $h(T)$ is supra α-closed. □

Definition 2.10. *If $g(L)$ is supra α-open (or supra α-closed) set in (Y, μ_Y) for each supra open (supra closed) set L in the supra space (X, μ_X), hence $g : (X, \mu_X) \to (Y, \mu_Y)$ is supra α-open (supra α-closed) function.*

Definition 2.11. *In Definition 2.10 if L is supra α-open (or supra α-closed) subset of (X, μ_X) and its image is supra open (supra closed) subset of (Y, μ_Y), then g is supra α^*-open (supra α^*-closed) function.*

Definition 2.12. *In Definition 2.10 whenever L and $g(L)$ are supra α-open (or supra α-closed) subsets of the supra spaces (X, μ_X) and (Y, μ_Y) respectively, hence g is called supra α^{**}-open (supra α^{**}-closed) function.*

Definition 2.13 ([12]). *The function $h : (X, \mu_X) \to (Y, \mu_Y)$ is supra α-continuous, whenever $h^{-1}(S)$ is supra α-open subset of X for any supra open subset S of Y .*

Definition 2.14. *In Definition 2.13 if S is supra α-open subset and its inverse image is supra open (supra α-open), then g is supra α^*-continuous (supra α^{**}-continuous).*

Definition 2.15. *If any supra compact subset in (X, μ_X) is supra closed set, so X is supra (αK)c-space.*

Example 2.7. (R, mu_D) is supra (αK)c-space.

Theorem 2.2. *Let the supra space (X, μ_X) be supra (αK)c-space and h from (X, μ_X) onto the supra space (Y, μ_Y) is injective, supra α^*-open and supra closed function, hence the space Y is supra (αK)c-space.*

Proof. Consider B is a supra α-compact set in Y. Also $\{H_i, i \in I\}$ be a supra α-open cover for $h^{-1}(B) \Longrightarrow h^{-1}(B) \subseteq \cup_{i\in I} H_i \Longrightarrow h(h^{-1}(B) \subseteq h(\cup_{i\in I} H_i) \Longrightarrow B \subseteq \cup_{i\in I} h(H_i)$ where all $h(H_i), i \in I$ are supra α-open subsets of Y, so there is a finite collection from the sets $h(H_i)$ in which $B \subseteq \cup_{i=1}^n h(H_i) \Longrightarrow h^{-1}(B) \subseteq h^{-1}(\cup_{i=1}^n h(H_i) \Longrightarrow h^{-1}(B) \subseteq \cup_{i=1}^n h^{-1}(h(H_i) \Longrightarrow h^{-1}(B) \subseteq \cup_{i=1}^n H_i \Longrightarrow h^{-1}(B)$ is supra α-compact subset of X, therefore $h^{-1}(B)$ is a supra α-closed (because X is supra (αK)c-space). So $h(h^{-1}(B)) = B$ is supra closed subset of Y, there upon B is a supra α-closed, so (Y, μ) will be a supra (αK) c-space. □

Corollary 2.1. *Let the supra space (X, μ_X) be supra (αK)c-space and g from (X, μ_X) onto the supra space (Y, μ_Y) is injective, supra open (supra open) and supra α^*-closed (supra α^{**}-closed) function, hence the space Y is supra (αK)c-space.*

Proposition 2.4. *Let the supra space (X, μ_X) be supra αK(αc)-space and h from (X, μ_X) onto the supra space (Y, μ_Y) is injective, supra α-open and supra α^{**}-closed function, hence the space Y is supra αK(αc)-space.*

Proof. Consider $\{H_{<}i > | i \in I\}$ is a supra α-open covering for the inverse image of $B \Longrightarrow h^{-1}(B) \subseteq \cup_{i\in I} H_i \Longrightarrow h(h^{-1}(B)) \subseteq h(\cup_{i\in I} H_i) \Longrightarrow B \subseteq \cup_{i\in I} h(H_i)$, in which each $h(H_i)$ is supra α-open set in Y, but B is supra α-compact, so there is a finite subfamily of the sets $h(H_i)$ in which $B \subseteq \cup_{i=1}^{n} h(H_i) \Longrightarrow h^{-1}(B) \subseteq \cup_{i=1}^{n} H_i \Longrightarrow h^{-1}(B)$ is supra α-compact subset from the supra space X, therefore it is a supra α-closed (since X is a supra αK(αc)-space). Hence $h(h^{-1}(B)) = B$ is supra α-closed subset from the supra space Y, so Y is a supra αK (αc)-space. □

Proposition 2.5. *When $h : (X, \mu_X) \to (Y, \mu_Y)$ is onto supra α^*-continuous function, where X is supra α-compact (or supra α-Lindelöf) space, hence Y is also supra α-compact (supra α-Lindelöf).*

Proof. Consider the supra α-open cover $\{C_i | i \in I\}$ to the supra space Y, so $Y \subseteq \cup_{i\in I} C_i \Longrightarrow h^{-1}(Y) \subseteq h^{-1}(\cup_{i\in I} C_i) \Longrightarrow X \subseteq \cup_{i\in I} h^{-1}(C_i)$ (since h is onto), where each of $h^{-1}(C_i), i \in I$ is supra α-open subset from the supra space X, hence the collection of $h^{-1}(C_i), i \in I$ is supra α-open cover to the supra space X which is supra α-compact, so there is a finite sub collection from $h^{-1}(C_i) | i \in I$ in which $X \subseteq \cup_{i=1}^{n} h^{-1}(C_i) \Longrightarrow h(X) \subseteq h(\cup_{i=1}^{n} h^{-1}(C_i)) \to Y \subseteq \cup_{i=1}^{n} h(h^{-1}(C_i)) \Longrightarrow Y \subseteq \cup_{i=1}^{n} C_i$, which means Y is supra α-compact space.
The other possibility can be proved by the same way. □

Proposition 2.6 ([9])**.** *A finite supra space, is supra α-compact space.*

Proposition 2.7. *Suppose (X, μ_X) is supra K(αc)-space and $h : (X, \mu_X) \to (Y, \mu_Y)$ is supra homeomorphism function, hence (Y, μ_Y) is also supra K(αc)-space.*

Proof. Suppose S is a supra compact subset from the supra space (Y, μ_Y), $h^{-1}(S)$ is supra compact sub of (X, μ_X), but X is supra K(αc)-space, thus $h^{-1}(S)$ is supra α-closed. Now $h(h^{-1}(S)) = S$ is supra α-closed subset from Y (h is surjective function), and this lead us to the required. □

Definition 2.16. *Whenever any supra Lindelöf subset from the supra space (X, μ_X) is supra α-closed, thus X is supra L(αc)-space.*

Example 2.8. (Z, μ_{ind}) is supra L (αc)-space.

Remark 2.3. Any supra Lc-space is supra $L(\alpha c)$-space. (Z, μ_{ind}) is an example about the converse,

Remark 2.4. Suppose the supra space X is supra L(αc)-space, hence it will be supra K(αc)-space, because when B is supra compact subset from the space X, it will be supra Lindelöf, Since X is supra L(αc)-space, B will be supra α-closed set, then X is supra K(αc)-space.

Remark 2.5. Suppose X is supra Lindelöf space and Y is a subspace of X, so Y is not necessary supra Lindelöf. But if any sub space of supra Lindelöf (supra α-Lindelöf) is supra Lindelöf (supra α-Lindelöf) sub space we called it a supra hereditary Lindelöf (α-Lindelöf).

Example 2.9. The excluded point supra space (R, μ_{EX}) is supra Lindelöf space but $(R - \{x_0\}, \mu_{EX})$ is not supra Lindelöf.

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