(p,q)-φ relative Gol'dberg order and (p,q)-φ relative Gol'dberg type oriented certain growth properties of entire functions of several complex variables

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Abstract. In this paper our concern is to study some growth properties based on different growth indicators such as (p,q)-φ relative Gol'dberg order, (p,q)-φ relative Gol'dberg type etc. of entire functions of several complex variables.

Keywords: entire functions of several complex variables, growth, (p,q)-φ relative Gol'dberg order, (p,q)-φ relative Gol'dberg type, (p,q)-φ relative Gol'dberg weak type.

1. Introduction, definitions and notations

In theory of Mathematical analysis, we generally indicate the complex and real n-spaces by the symbols $\mathbb{C}^n$ and $\mathbb{R}^n$, respectively. Considering I as the set of non-negative integers, if someone presumes that the points $(z_1, z_2, \cdots, z_n)$, $(m_1, m_2, \cdots, m_n)$ of $\mathbb{C}^n$ or I^n are represented by the respective unsuffixed symbols z, m then the modulus of z, denoted by $|z|$, is defined to be $|z| = (|z_1|^2 + \cdots + |z_n|^2)^{\frac{1}{2}}$. Further if the coordinates of the vector m are non-negative integers $m_1, m_2, \ldots, m_n$, then the expression $z_1^{m_1} \cdots z_n^{m_n}$ may be presented by the symbol z^m where $\|m\| = m_1 + \cdots + m_n$.

Suppose that $D \subseteq \mathbb{C}^n$ is an arbitrary bounded complex n-circular domain with the center at the origin. Then, for any entire function $f(z)$ of n complex variables and $R > 0$, the maximum modulus function $M_{f,D}(R)$ may be defined to be $M_{f,D}(R) = \sup_{z \in D_R} |f(z)|$ where a point $z \in D_R \Leftrightarrow \frac{z}{R} \in D$. It is noticeable from the way of defining the maximum modulus function that $M_{f,D}(R)$ is

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strictly increasing when $f(z)$ is non-constant and the inverse of $M_{f,D}(R)$ i.e., $M_{f,D}^{-1}:(|f(0)|,\infty)\rightarrow(0,\infty)$ exists such that $\lim\limits_{R\rightarrow\infty}M_{f,D}^{-1}(R)=\infty$.

For $k\in[0,\infty)$ and $k\in\mathbb{N}$ where $\mathbb{N}$ be the set of all positive integers, we define iterations of the exponential and logarithmic functions as $\exp^{[k]}R=\exp\left(\exp^{[k-1]}R\right)$ and $\log^{[k]}R=\log(\log^{[k-1]}R)$, with convention that $\log^{[0]}R=R$, $\log^{[-1]}R=\exp R$, $\exp^{[0]}R=R$ and $\exp^{[-1]}R=\log R$. Further we assume that throughout the present paper p, q, m and n always denote positive integers. Further let $L(R)$ be a class of non-decreasing unbounded functions $\varphi(R)$: $[0,+\infty)\rightarrow(0,+\infty)$. Also, in this paper, by an entire function $f(z)$ of n-complex variables we mean an entire function $f(z)$ for any bounded complete n-circular domain D having the center at origin in $\mathbb{C}^n$. Considering this let us recall the definitions of (p,q)-φ relative Gol'dberg order and the (p,q)-φ relative Gol'dberg lower order of an entire function $f(z)$ of n-complex variables which are as follows:

Definition 1.1 ([3])**.** *Let $\varphi(R)\in L(R)$. Also, let $f(z)$ and $g(z)$ be any two entire functions of n-complex variables. The (p,q)-φ relative Gol'dberg order and the (p,q)-φ relative Gol'dberg lower order of $f(z)$ with respect to $g(z)$ are defined as*

$$\begin{matrix}\rho_{g,D}^{(p,q)}(f,\varphi)\\ \lambda_{g,D}^{(p,q)}(f,\varphi)\end{matrix}=\lim_{R\rightarrow\infty}\begin{matrix}\sup\\ \inf\end{matrix}\frac{\log^{[p]}M_{g,D}^{-1}(M_{f,D}(R))}{\log^{[q]}\varphi(R)}.$$

In this paper, we assume that the nondecreasing unbounded function $\varphi(R)$: $[0,+\infty)\rightarrow(0,+\infty)$ always satisfies $\lim_{R\rightarrow+\infty}\frac{\log^{[q]}\varphi(\alpha R)}{\log^{[q]}\varphi(R)}=1$ for all $\alpha>0$. Since, Biswas et al. [3] have already shown that $\rho_{g,D}^{(p,q)}(f,\varphi)$ and $\lambda_{g,D}^{(p,q)}(f,\varphi)$ are independent of the choice of the domain D when $\varphi(R):[0,+\infty)\rightarrow(0,+\infty)$ is a nondecreasing unbounded function and satisfies $\lim_{R\rightarrow+\infty}\frac{\log^{[q]}\varphi(\alpha R)}{\log^{[q]}\varphi(R)}=1$ for all $\alpha>0$, so here we shall always use the notations $\rho_{g}^{(p,q)}(f,\varphi)$ and $\lambda_{g}^{(p,q)}(f,\varphi)$ instead of $\rho_{g,D}^{(p,q)}(f,\varphi)$ and $\lambda_{g,D}^{(p,q)}(f,\varphi)$, respectively.

Now, for the development of such growth indicators, one may introduce (p,q)-φ relative Gol'dberg type $\sigma_{g,D}^{(p,q)}(f,\varphi)$ and (p,q)-φ relative Gol'dberg weak type $\tau_{g,D}^{(p,q)}(f,\varphi)$ in the following way:

Definition 1.2 ([6])**.** *Let $\varphi(R)\in L(R)$. Let $f(z)$ and $g(z)$ be any two entire functions of n-complex variables such that $0<\rho_{g}^{(p,q)}(f,\varphi)<\infty$. Then, the (p,q)-φ relative Gol'dberg type $\sigma_{g,D}^{(p,q)}(f,\varphi)$ and the (p,q)-φ relative Gol'dberg lower type $\overline{\sigma}_{g,D}^{(p,q)}(f,\varphi)$ of $f(z)$ with respect to $g(z)$ are defined as:*

$$\begin{matrix}\sigma_{g,D}^{(p,q)}(f,\varphi)\\ \overline{\sigma}_{g,D}^{(p,q)}(f,\varphi)\end{matrix}=\lim_{R\rightarrow\infty}\begin{matrix}\sup\\ \inf\end{matrix}\frac{\log^{[p-1]}M_{g,D}^{-1}(M_{f,D}(R))}{\left(\log^{[q-1]}\varphi(R)\right)^{\rho_{g}^{(p,q)}(f,\varphi)}}.$$

Definition 1.3 ([6]). *Let $\varphi(R) \in L(R)$. Let $f(z)$ and $g(z)$ be any two entire functions of n-complex variables such that $0 < \lambda_g^{(p,q)}(f,\varphi) < \infty$. Then, the (p,q)-φ relative Gol'dberg weak type $\tau_{g,D}^{(p,q)}(f,\varphi)$ and (p,q)-φ relative Gol'dberg upper weak type $\overline{\tau}_{g,D}^{(p,q)}(f,\varphi)$ of $f(z)$ with respect to $g(z)$ are defined as:*

$$\begin{matrix} \overline{\tau}_{g,D}^{(p,q)}(f,\varphi) \\ \tau_{g,D}^{(p,q)}(f,\varphi) \end{matrix} = \lim_{R\to\infty} \begin{matrix} \sup \\ \inf \end{matrix} \frac{\log^{[p-1]} M_{g,D}^{-1}(M_{f,D}(R))}{\left(\log^{[q-1]}\varphi(R)\right)^{\lambda_g^{(p,q)}(f,\varphi)}}.$$

As Gol'dberg has shown that (see [8]) Gol'dberg type depends on the domain D, therefore in general all the growth indicators defined in Definition 1.2 and Definition 1.3 also depend on D.

During the past decades, several authors (see [1] to [10]) made closed investigations on the growth properties of entire functions of several complex variables using different growth indicator such as Gol'dberg order, (p,q)-th Gol'dberg order, relative Gol'dberg order, etc. In the present paper our concern is to study some growth properties based upon the dicussion of several growth indicators such as (p,q)-φ relative Gol'dberg type, (p,q)-φ relative Gol'dberg weak type etc. of entire functions of several complex variables.

2. Main results

In this section we present the main results of this paper. Here we always suppose that $\varphi_1(R)$, $\varphi_2(R)$ both belong to $L(R)$ with $\lim_{R\to+\infty} \frac{\log^{[q]}\varphi_1(\alpha R)}{\log^{[q]}\varphi_1(R)} = 1$ and $\lim_{R\to+\infty} \frac{\log^{[n]}\varphi_2(\alpha R)}{\log^{[n]}\varphi_2(R)} = 1$ for all $\alpha > 0$.

Theorem 2.1. *Let $f(z)$, $g(z)$, $h(z)$ and $k(z)$ be any four entire functions of n- complex variables and D be a bounded complete n-circular domain with center at origin in $\mathbb{C}^n$. Also, let $0 < \lambda_h^{(p,q)}(f,\varphi_1) \le \rho_h^{(p,q)}(f,\varphi_1) < \infty$ and $0 < \lambda_k^{(m,n)}(g,\varphi_2) \le \rho_k^{(m,n)}(g,\varphi_2) < \infty$. Then*

$$\frac{\lambda_h^{(p,q)}(f,\varphi_1)}{\rho_k^{(m,n)}(g,\varphi_2)} \le \liminf_{R\to+\infty} \frac{\log^{[p]} M_{h,D}^{-1}\left(M_{f,D}\left(\varphi_1^{-1}(\exp^{[q]} R)\right)\right)}{\log^{[m]} M_{k,D}^{-1}\left(M_{g,D}\left(\varphi_2^{-1}(\exp^{[n]} R)\right)\right)}$$
$$\le \min\left\{\frac{\lambda_h^{(p,q)}(f,\varphi_1)}{\lambda_k^{(m,n)}(g,\varphi_2)}, \frac{\rho_h^{(p,q)}(f,\varphi_1)}{\rho_k^{(m,n)}(g,\varphi_2)}\right\} \le \max\left\{\frac{\lambda_h^{(p,q)}(f,\varphi_1)}{\lambda_k^{(m,n)}(g,\varphi_2)}, \frac{\rho_h^{(p,q)}(f,\varphi_1)}{\rho_k^{(m,n)}(g,\varphi_2)}\right\}$$
$$\le \limsup_{R\to+\infty} \frac{\log^{[p]} M_{h,D}^{-1}\left(M_{f,D}\left(\varphi_1^{-1}(\exp^{[q]} R)\right)\right)}{\log^{[m]} M_{k,D}^{-1}\left(M_{g,D}\left(\varphi_2^{-1}(\exp^{[n]} R)\right)\right)} \le \frac{\rho_h^{(p,q)}(f,\varphi_1)}{\lambda_k^{(m,n)}(g,\varphi_2)}.$$

Proof of Theorem 2.1. From the definitions of $\lambda_k^{(m,n)}(g,\varphi_2)$, $\rho_k^{(m,n)}(g,\varphi_2)$, $\lambda_h^{(p,q)}(f,\varphi_1)$and $\rho_h^{(p,q)}(f,\varphi_1)$ we have for arbitrary positive ε and for all sufficiently large values of R that

$$\log^{[p]} M_{h,D}^{-1}\left(M_{f,D}\left(\varphi_1^{-1}(\exp^{[q]} R)\right)\right) \geqslant (\lambda_h^{(p,q)}(f,\varphi_1) - \varepsilon)R \tag{1}$$

$$\log^{[m]} M_{k,D}^{-1}\left(M_{g,D}\left(\varphi_2^{-1}(\exp^{[n]} R)\right)\right) \leq (\rho_k^{(m,n)}(g,\varphi_2)+\varepsilon)R \tag{2}$$

$$\log^{[m]} M_{k,D}^{-1}\left(M_{g,D}\left(\varphi_2^{-1}(\exp^{[n]} R)\right)\right) \geqslant (\lambda_k^{(m,n)}(g,\varphi_2)-\varepsilon)R \tag{3}$$

and

$$\log^{[p]} M_{h,D}^{-1}\left(M_{f,D}\left(\varphi_1^{-1}(\exp^{[q]} R)\right)\right) \leq (\rho_h^{(p,q)}(f,\varphi_1)+\varepsilon)R. \tag{4}$$

Again, we get for a sequence of positive numbers of R tending to infinity that

$$\log^{[p]} M_{h,D}^{-1}\left(M_{f,D}\left(\varphi_1^{-1}(\exp^{[q]} R)\right)\right) \leq (\lambda_h^{(p,q)}(f,\varphi_1)+\varepsilon)R \tag{5}$$

$$\log^{[m]} M_{k,D}^{-1}\left(M_{g,D}\left(\varphi_2^{-1}(\exp^{[n]} R)\right)\right) \leq (\lambda_k^{(m,n)}(g,\varphi_2)+\varepsilon)R \tag{6}$$

$$\log^{[m]} M_{k,D}^{-1}\left(M_{g,D}\left(\varphi_2^{-1}(\exp^{[n]} R)\right)\right) \geqslant (\rho_k^{(m,n)}(g,\varphi_2)-\varepsilon)R \tag{7}$$

and

$$\log^{[p]} M_{h,D}^{-1}\left(M_{f,D}\left(\varphi_1^{-1}(\exp^{[q]} R)\right)\right) \geqslant (\rho_h^{(p,q)}(f,\varphi_1)-\varepsilon)R. \tag{8}$$

Now, from (1) and (2), it follows for all sufficiently large positive numbers of R that

$$\frac{\log^{[p]} M_{h,D}^{-1}\left(M_{f,D}\left(\varphi_1^{-1}(\exp^{[q]} R)\right)\right)}{\log^{[m]} M_{k,D}^{-1}\left(M_{g,D}\left(\varphi_2^{-1}(\exp^{[n]} R)\right)\right)} \geqslant \frac{(\lambda_h^{(p,q)}(f,\varphi_1)-\varepsilon)R}{(\rho_k^{(m,n)}(g,\varphi_2)+\varepsilon)R}.$$

As $\varepsilon\,(>0)$ is arbitrary, we obtain that

$$\liminf_{R\to+\infty}\frac{\log^{[p]} M_{h,D}^{-1}\left(M_{f,D}\left(\varphi_1^{-1}(\exp^{[q]} R)\right)\right)}{\log^{[m]} M_{k,D}^{-1}\left(M_{g,D}\left(\varphi_2^{-1}(\exp^{[n]} R)\right)\right)} \geqslant \frac{\lambda_h^{(p,q)}(f,\varphi_1)}{\rho_k^{(m,n)}(g,\varphi_2)}. \tag{9}$$

Combining (5) and (3), we get for a sequence of positive numbers of R tending to infinity that

$$\frac{\log^{[p]} M_{h,D}^{-1}\left(M_{f,D}\left(\varphi_1^{-1}(\exp^{[q]} R)\right)\right)}{\log^{[m]} M_{k,D}^{-1}\left(M_{g,D}\left(\varphi_2^{-1}(\exp^{[n]} R)\right)\right)} \leq \frac{(\lambda_h^{(p,q)}(f,\varphi_1)+\varepsilon)R}{(\lambda_k^{(m,n)}(g,\varphi_2)-\varepsilon)R}.$$

Since $\varepsilon\,(>0)$ is arbitrary, it follows that

$$\liminf_{R\to+\infty}\frac{\log^{[p]} M_{h,D}^{-1}\left(M_{f,D}\left(\varphi_1^{-1}(\exp^{[q]} R)\right)\right)}{\log^{[m]} M_{k,D}^{-1}\left(M_{g,D}\left(\varphi_2^{-1}(\exp^{[n]} R)\right)\right)} \leq \frac{\lambda_h^{(p,q)}(f,\varphi_1)}{\lambda_k^{(m,n)}(g,\varphi_2)}. \tag{10}$$

Now, from (1) and (6), we obtain for a sequence of positive numbers of R tending to infinity that

$$\frac{\log^{[p]} M_{h,D}^{-1}\left(M_{f,D}\left(\varphi_1^{-1}(\exp^{[q]} R)\right)\right)}{\log^{[m]} M_{k,D}^{-1}\left(M_{g,D}\left(\varphi_2^{-1}(\exp^{[n]} R)\right)\right)} \geq \frac{(\lambda_h^{(p,q)}(f,\varphi_1)-\varepsilon)R}{(\lambda_k^{(m,n)}(g,\varphi_2)+\varepsilon)R}.$$

As $\varepsilon\,(>0)$ is arbitrary, we get from above that

$$\limsup_{R\to+\infty}\frac{\log^{[p]} M_{h,D}^{-1}\left(M_{f,D}\left(\varphi_1^{-1}(\exp^{[q]} R)\right)\right)}{\log^{[m]} M_{k,D}^{-1}\left(M_{g,D}\left(\varphi_2^{-1}(\exp^{[n]} R)\right)\right)}\geq\frac{\lambda_h^{(p,q)}(f,\varphi_1)}{\lambda_k^{(m,n)}(g,\varphi_2)}. \tag{11}$$

Now, it follows from (3) and (4) for all sufficiently large positive numbers of R that

$$\frac{\log^{[p]} M_{h,D}^{-1}\left(M_{f,D}\left(\varphi_1^{-1}(\exp^{[q]} R)\right)\right)}{\log^{[m]} M_{k,D}^{-1}\left(M_{g,D}\left(\varphi_2^{-1}(\exp^{[n]} R)\right)\right)}\leq\frac{(\rho_h^{(p,q)}(f,\varphi_1)+\varepsilon)R}{(\lambda_k^{(m,n)}(g,\varphi_2)-\varepsilon)R}.$$

Since $\varepsilon\,(>0)$ is arbitrary, we obtain that

$$\limsup_{R\to+\infty}\frac{\log^{[p]} M_{h,D}^{-1}\left(M_{f,D}\left(\varphi_1^{-1}(\exp^{[q]} R)\right)\right)}{\log^{[m]} M_{k,D}^{-1}\left(M_{g,D}\left(\varphi_2^{-1}(\exp^{[n]} R)\right)\right)}\leq\frac{\rho_h^{(p,q)}(f,\varphi_1)}{\lambda_k^{(m,n)}(g,\varphi_2)}. \tag{12}$$

Now, from (4) and (7), it follows for a sequence of positive numbers of R tending to infinity that

$$\frac{\log^{[p]} M_{h,D}^{-1}\left(M_{f,D}\left(\varphi_1^{-1}(\exp^{[q]} R)\right)\right)}{\log^{[m]} M_{k,D}^{-1}\left(M_{g,D}\left(\varphi_2^{-1}(\exp^{[n]} R)\right)\right)}\leq\frac{(\rho_h^{(p,q)}(f,\varphi_1)+\varepsilon)R}{(\rho_k^{(m,n)}(g,\varphi_2)-\varepsilon)R}.$$

As $\varepsilon\,(>0)$ is arbitrary, we obtain that

$$\liminf_{R\to+\infty}\frac{\log^{[p]} M_{h,D}^{-1}\left(M_{f,D}\left(\varphi_1^{-1}(\exp^{[q]} R)\right)\right)}{\log^{[m]} M_{k,D}^{-1}\left(M_{g,D}\left(\varphi_2^{-1}(\exp^{[n]} R)\right)\right)}\leq\frac{\rho_h^{(p,q)}(f,\varphi_1)}{\rho_k^{(m,n)}(g,\varphi_2)}. \tag{13}$$

Again, combining (2) and (8), we get for a sequence of positive numbers of R tending to infinity that

$$\frac{\log^{[p]} M_{h,D}^{-1}\left(M_{f,D}\left(\varphi_1^{-1}(\exp^{[q]} R)\right)\right)}{\log^{[m]} M_{k,D}^{-1}\left(M_{g,D}\left(\varphi_2^{-1}(\exp^{[n]} R)\right)\right)}\geqslant\frac{(\rho_h^{(p,q)}(f,\varphi_1)-\varepsilon)R}{(\rho_k^{(m,n)}(g,\varphi_2)+\varepsilon)R}.$$

Since $\varepsilon\,(>0)$ is arbitrary, it follows that

$$\limsup_{R\to+\infty}\frac{\log^{[p]} M_{h,D}^{-1}\left(M_{f,D}\left(\varphi_1^{-1}(\exp^{[q]} R)\right)\right)}{\log^{[m]} M_{k,D}^{-1}\left(M_{g,D}\left(\varphi_2^{-1}(\exp^{[n]} R)\right)\right)}\geqslant\frac{\rho_h^{(p,q)}(f,\varphi_1)}{\rho_k^{(m,n)}(g,\varphi_2)}. \tag{14}$$

Thus, the theorem follows from (9), (10), (11), (12), (13) and (14).

The proof of the following theorem can be carried out as of the Theorem 2.1, therefore we omit the details.

Theorem 2.2. *Let f, g, h, k be any four entire function such that* $0<\overline{\sigma}_{h,D}^{(p,q)}(f,\varphi_1) \leq \sigma_{h,D}^{(p,q)}(f,\varphi_1) < \infty$, $0 < \overline{\sigma}_{k,D}^{(m,n)}(g,\varphi_2) \leq \sigma_{k,D}^{(m,n)}(g,\varphi_2) < \infty$ *and* $\rho_h^{(p,q)}(f,\varphi_1) = \rho_k^{(m,n)}(g,\varphi_2)$. *Then*

$$\frac{\overline{\sigma}_{h,D}^{(p,q)}(f,\varphi_1)}{\sigma_{k,D}^{(m,n)}(g,\varphi_2)} \leq \liminf_{R\to+\infty} \frac{\log^{[p-1]} M_{h,D}^{-1}\left(M_{f,D}\left(\varphi_1^{-1}(\exp^{[q-1]} R)\right)\right)}{\log^{[m-1]} M_{k,D}^{-1}\left(M_{g,D}\left(\varphi_2^{-1}(\exp^{[n-1]} R)\right)\right)}$$
$$\leq \min\left\{\frac{\overline{\sigma}_{h,D}^{(p,q)}(f,\varphi_1)}{\overline{\sigma}_{k,D}^{(m,n)}(g,\varphi_2)}, \frac{\sigma_{h,D}^{(p,q)}(f,\varphi_1)}{\sigma_{k,D}^{(m,n)}(g,\varphi_2)}\right\} \leq \max\left\{\frac{\overline{\sigma}_{h,D}^{(p,q)}(f,\varphi_1)}{\overline{\sigma}_{k,D}^{(m,n)}(g,\varphi_2)}, \frac{\sigma_{h,D}^{(p,q)}(f,\varphi_1)}{\sigma_{k,D}^{(m,n)}(g,\varphi_2)}\right\}$$
$$\leq \limsup_{R\to+\infty} \frac{\log^{[p-1]} M_{h,D}^{-1}\left(M_{f,D}\left(\varphi_1^{-1}(\exp^{[q-1]} R)\right)\right)}{\log^{[m-1]} M_{k,D}^{-1}\left(M_{g,D}\left(\varphi_2^{-1}(\exp^{[n-1]} R)\right)\right)} \leq \frac{\sigma_{h,D}^{(p,q)}(f,\varphi_1)}{\overline{\sigma}_{k,D}^{(m,n)}(g,\varphi_2)}.$$

Remark 2.1. If we take "$0<\tau_{h,D}^{(p,q)}(f,\varphi_1)\leq\overline{\tau}_{h,D}^{(p,q)}(f,\varphi_1)<\infty$", "$0<\tau_{k,D}^{(m,n)}(g,\varphi_2) \leq\overline{\tau}_{k,D}^{(m,n)}(g,\varphi_2) < \infty$" and "$\lambda_h^{(p,q)}(f,\varphi_1)=\lambda_k^{(m,n)}(g,\varphi_2)$" instead of "$0<\overline{\sigma}_{h,D}^{(p,q)}(f,\varphi_1) \leq \sigma_{h,D}^{(p,q)}(f,\varphi_1) < \infty$", "$0 < \overline{\sigma}_{k,D}^{(m,n)}(g,\varphi_2) \leq \sigma_{k,D}^{(m,n)}(g,\varphi_2) < \infty$" and "$\rho_h^{(p,q)}(f,\varphi_1) = \rho_k^{(m,n)}(g,\varphi_2)$", respectively and other conditions remain same, the conclusion of Theorem 2.2 remains true with "$\overline{\sigma}_{h,D}^{(p,q)}(f,\varphi_1)$", "$\sigma_{h,D}^{(p,q)}(f,\varphi_1)$", "$\overline{\sigma}_{k,D}^{(m,n)}(g,\varphi_2)$" and "$\sigma_{k,D}^{(m,n)}(g,\varphi_2)$" replaced by "$\tau_{h,D}^{(p,q)}(f,\varphi_1)$", "$\overline{\tau}_{h,D}^{(p,q)}(f,\varphi_1)$", "$\tau_{k,D}^{(m,n)}(g,\varphi_2)$" and "$\overline{\tau}_{k,D}^{(m,n)}(g,\varphi_2)$", respectively.

Remark 2.2. If we take "$0<\tau_{k,D}^{(m,n)}(g,\varphi_2)\leq\overline{\tau}_{k,D}^{(m,n)}(g,\varphi_2)<\infty$" and "$\rho_h^{(p,q)}(f,\varphi_1) = \lambda_k^{(m,n)}(g,\varphi_2)$" instead of "$0<\overline{\sigma}_{k,D}^{(m,n)}(g,\varphi_2)\leq\sigma_{k,D}^{(m,n)}(g,\varphi_2)<\infty$" and "$\rho_h^{(p,q)}(f,\varphi_1) = \rho_k^{(m,n)}(g,\varphi_2)$", respectively and other conditions remain same, the conclusion of Theorem 2.2 remains true with "$\overline{\sigma}_{k,D}^{(m,n)}(g,\varphi_2)$" and "$\sigma_{k,D}^{(m,n)}(g,\varphi_2)$" replaced by "$\tau_{k,D}^{(m,n)}(g,\varphi_2)$" and "$\overline{\tau}_{k,D}^{(m,n)}(g,\varphi_2)$", respectively.

Remark 2.3. If we take "$0 < \tau_{h,D}^{(p,q)}(f,\varphi_1) \leq \overline{\tau}_{h,D}^{(p,q)}(f,\varphi_1) < \infty$" and "$\lambda_h^{(p,q)}(f,\varphi_1) = \rho_k^{(m,n)}(g,\varphi_2)$" instead of "$0<\overline{\sigma}_{h,D}^{(p,q)}(f,\varphi_1)\leq\sigma_{h,D}^{(p,q)}(f,\varphi_1)<\infty$" and "$\rho_h^{(p,q)}(f,\varphi_1) = \rho_k^{(m,n)}(g,\varphi_2)$", respectively and other conditions remain same, the conclusion of Theorem 2.2 remains true with "$\overline{\sigma}_{h,D}^{(p,q)}(f,\varphi_1)$" and "$\sigma_{h,D}^{(p,q)}(f,\varphi_1)$" replaced by "$\tau_{h,D}^{(p,q)}(f,\varphi_1)$" and "$\overline{\tau}_{h,D}^{(p,q)}(f,\varphi_1)$", respectively.

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