Comparison of two types of rough approximation via grill

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Abstract. Rough set theory and topology are now branching far into applied areas, such as economics, data processing, imaging and chemistry. As a consequence of this importance gained from topology after the advent of a rough set theory which helps to quantify things that were previously difficult to measure. It was important to work on the extension of the topological space with new concepts such as grill and ideal. In this paper, we present new approximations of rough sets via a grill concept which has helped to extend the topological spaces. In addition, the topology created by the present method is finer than other methods. Finally, grill topological spaces will be obtained in terms of relations and grills aimed at minimizing the boundary regions.
Keywords: rough sets, grill topological space, lower approximation, upper approximation.

1. Introduction

Since the advent of the rough sets theory, which aims at reduce the boundary region of a universe set, it has attracted the attention of many mathematicians, computer specialists, and others, especially those interested in topology. As a consequence of this interest in the theory of rough sets. This theory has many applications in many fields that are used to process control [17], computer [10], data analysis and classification [38], economics [6], such as medical diagnosis, chemistry, psychology, finance, marketing, biochemistry, environmental science, intelligent agents, image analysis, biology, telecommunication and other fields (see [4], [7], [18], [19], ([20]-[24]), ([28]-[33]), [34], [41], [42], [43], ([44]-[48]) and the bibliography throughout these article). The emergence of the value of topological space and its applications besides the theory of rough sets, it had to be expanded by identifying terms such as ideals and grills. Many approaches of rough sets depended on the relation on a universe set. Although the topological grill structures will be used in this article. So, the purpose of this paper is to add new approximations to the rough set theory using the grill definition, the eliminates the difference between the upper and lower approximations compared to previous approximations. We also contrasted this new definition in terms of grills with the upper and lower approximations of [25], [40] and [5], as shown in Theorem 3.1 below. Finally, we strengthened the relation with a variety of different instances.

2. Preliminaries

Two Φ and Ψ mappings that are the basis of the grill concept were first introduced in [9]. There's a resemblance expansion of the topological space between the concept of grills, ideals, filters and nets. There have been some hypotheses and features run by [30, 6, 38, 36]. It was helping to general the topological structure used to understand description rather than the quantity,e.g. diet, intelligence,appearance, standard of education and so on. It also extended topological layout by using the idea of grill adjustments approaches of rough sets and the boundary regions. Throughout this article, if (Y, ζ) is a topological space with a grill g on Y. Then we must call it a grill topological space, denote it by (Y, ζ, G) and abbreviate it by GTS.

Definition 2.1 ([9]). *A nonempty collection G of subsets of a set Y is said to be a grill on space Y, if satisfying the following conditions:*

(1) $\Phi \notin G$,
(2) $B \in G$ *and* $B \subseteq C \subseteq Y \Rightarrow C \in G$,
(3) *if* $B \cup C \in G$ *for* $B, C \subseteq Y$, *then* $B \in G$ *or* $C \in G$.

Remark 2.1. (1) The minimal grill is $G = \{Y\}$ in any space Y which caries the topology ζ.

(2) The maximal grill is $G = P(Y)\backslash\{\Phi\}$ in any topological space (Y, ζ).

Definition 2.2 ([30]). *Let (Y, ζ, G) be a GTS. Considering the operator $\Phi : P(Y) \to P(Y)$ which is denoted by $\Phi_G(B)$ for $B \subseteq Y$, which defined as $\Phi_G(B) = \Phi_G(B, \zeta) = \{y \in Y : B \cap U \in G$ for every open set U containing $y\}$ for each $B \in P(Y)$.*

Definition 2.3 ([30]). *Let (Y, ζ, G) be a GTS. Considering the operator $\Psi : P(Y) \to P(Y)$ by $\Psi(B) = B \cup \Phi(B)$ for all $B \in P(Y)$. The operator Φ is a Kuratowski closure axioms. Based on a grill G on a topological space (Y, ζ), there exists a unique topology ζ_G on Y given by $\zeta_G = \{U \subseteq Y : \Psi(Y \setminus U) = Y \setminus U\}$, where for any $B \subseteq Y$, $\Psi(B) = B \cup \Phi(B) = \zeta_G - Cl(B)$. For any grill G on a topological space (Y, ζ), we have $\zeta \subseteq \zeta_G$.*

Definition 2.4 ([25]). *If R is an equivalence relation on a miss-null set Y, $[y]_R$ is an equivalence classes of y, then for $B \subseteq Y$, a pair of upper and lower*

approximations, $\overline{R}(B)$ *and* $\underline{R}(B)$*, are defined respectively as:*
$\overline{R}(B) = \{y \in Y : [y]_R \cap B \neq \Phi\}$;
$\underline{R}(A) = \{y \in Y : [y]_R \subseteq B\}$.

Theorem 2.2 ([25])**.** *If R is an equivalence relation on a miss-null set Y, and $B, C \subseteq Y$, then the following properties are satisfied concerning for the upper approximation:*

(1) $\overline{R}(\Phi) = \Phi$;
(2) $B \subseteq C \Rightarrow \overline{R}(B) \subseteq \overline{R}(C)$;
(3) $B \subseteq \overline{R}(B)$;
(4) $\overline{R}(B \cup C) = \overline{R}(B) \cup \overline{R}(C)$;
(5) $\overline{R}(B \cap C) \subseteq \overline{R}(B) \cup \overline{R}(C)$;
(6) $\overline{R}(\overline{R}(B)) = \overline{R}(B)$;
(7) $\overline{R}(B) = [\overline{R}(B^c)]^c$, *where B^c denote the complement of B.*

Corollary 2.1 ([25])**.** *If R is an equivalence relation on a miss-null set Y. Then, the operator $\overline{R}$ on $P(Y)$ satisfies the Kuratowski's axioms and deduces the topology on Y called τ_R is given by $\zeta_R = \{B \subseteq Y : \overline{R}(B^c) = B^c\}$.*

Definition 2.5 ([39])**.** *Let R be a binary relation on a nonempty set Y. For $B \subseteq Y$, we notice that, the rough approximations of the lower and upper are defined as, $\underline{R}(B) = \{y \in Y : yR \cap B \subseteq B\}$, $\overline{R}(B) = \{y \in Y : yR \cap B \neq \Phi\}$. Where $yR = \{c \in Y : yRc\}$ is called after set.*

Theorem 2.3 ([25])**.** *If R is a pre-order relation on Y, then the upper approximation satisfies the properties of Theorem 2.6.*

Definition 2.6 ([2])**.** *If R is a reflexive relation on a nonempty set Y. Then, $B \subseteq Y$. $\underline{R}(B)$ and $\overline{R}(B)$ are defined by $\underline{R}(B) = \{y \in Y :< y >_R \subseteq B\}$, $\overline{R}(B) = \{y \in Y :< y >_R \cap B \neq \Phi\}$. Where, $< y >_R = \cap\{pR : y \in pR\}$.*

Proposition 2.1 ([2])**.** *If R is a binary relation on a nonempty set Y and $z \in < y >_R$ then, $< z >_R \subseteq < y >_R$.*

3. Novel approximation rough sets via a grill

The main aim of this section is to develop some notions and characterizations of the rough sets theory in terms of the grill. The new definitions of the lower approximation and upper approximation aim at increasing the lower approximation of any set $B \subseteq Y$ and reducing the boundary region. Also, we extend the results of new definitions to compare with the definitions of [26, 27], [39, 40] and [3, 2].

Definition 3.1. *If R is a reflexive relation on a $GTS.$, $B \subseteq Y$ and G is a grill on Y, then the R^G-upper and R_G-lower approximations of B are defined respectively by:*

$$R^G(B) = \{y \in Y :< y >_R \cap B \in G\};$$
$$R_G(B) = \{y \in Y :< y >_R \cap B^c \notin G\}.$$

Theorem 3.1. *Let $G = P(Y)\backslash\{\Phi\}$ be a grill,*

(1) *If R is an equivalence relation, then Definition 3.1 coincides with Pawlak's definition.*

(2) *If R is a pre-order relation, then Definition 3.1 corresponds with Yao's definition.*

(3) *If R is a reflexive relation, then Definition 3.1 coincides with Allam's definition.*

Proof. (1) $\underline{R}(B) = \{y \in Y : [Y]_R \cap B \subseteq B\}$, that is $< y >_R \cap B^c = \Phi$ and $< y >_R \cap B^c \notin G$, i.e. $R_G(B) = \{y \in Y :< y >_R \cap B^c \notin G\}.\overline{R}(B) = \{y \in Y : [Y]_R \cap B \neq \Phi\}$, that is $< y >_R \cap B \in G$, i.e. $R^G(B) = \{y \in Y :< y >_R \cap B \in G\}$.

(2) It is evident.

(3) Similarly proof(1). □

Lemma 3.1. *If R is a binary relation on a miss-null set Y then the following condition holds:*

(1) *If $G = \{Y\}$ then $R^G(B) = \Phi$,*

(2) *If $G = P(Y)\backslash\{\Phi\}$ then $R^G(B) = \overline{R}(B)$.*

Proof. (1)If $G = \{Y\}$ hence $< y >_R \cap B \neq Y$ and so $R^G(B) = \{y \in Y :< x >_R \cap B \notin G\}$ and $R^G(B) = \Phi$.

(2) If $G = P(Y)\backslash\{\Phi\}$ then $R^G(B) = \overline{R}(B)$ for $\overline{R}^G(B)$, Pawlak's, Yao's and Allam's definition. □

Theorem 3.2. *Let $G = P(X)\backslash\{\Phi\}$ and R be a reflexive relation on ζ_R^G. Then, the R^G-upper approximation in Definition 3.1 satisfies the following properties:*

1. $R^G(Y) = Y$,
2. $B \subseteq C \Rightarrow R^G(B) \subseteq R^G(C)$,
3. $R^G(B \cup C) = R^G(B) \cup R^G(C)$,
4. $R^G(B \cap C) \subseteq R^G(B) \cap R^G(C)$,
5. $R^G(R^g(B)) \subseteq R^G(B)$,
6. $R_G(B) = [R^G(B^c)]^c$,
7. $B \notin G \Rightarrow R^G(B) = \phi$.

Proof. 1. It is clear from Definition 3.1.

2. Let $y \in R^G(B)$, then $< y >_R \cap B \in G$. Since $< y >_R \cap B \subseteq< y >_R \cap C$, so $y \in R^G(C)$.

3. $R^G(B \cup C) = \{y \in Y :< y >_R \cap(B \cup C) \in G\} = \{y : (< y >_R \cap B) \cup (< y >_R \cap C) \in g\} = \{y :< y >_R \cap B \in G\} \cup \{y :< y >_R \cap C) \in G\} = R^g(B) \cup R^G(C)$.

4. Let $y \in R^G(B \cap C)$ that is $< y >_R \cap(B \cap C) \in G$, i.e. $(< y >_R \cap B) \cap (< y >_R \cap C) \in G$, and $y \in R^G(B) \cap R^G(C)$.

5. Let $Y \in R^G(R^G(B))$, hence $< y >_R \cap R^G(B) \neq \Phi$. Then, there exists $z \in< y >_R \cap R^G(B)$, i.e. $< z >_R \subseteq< y >_R$ and $< z >_R \cap R^G(B) \in G$. It follows that $y \in R^G(B)$.

6. $[R^G(B^c)]^c = \{y \in Y :< y >_R \cap B^c \in G\}^c = \{y \in Y :< y >_R \cap B^c \notin g\} = R_G(B)$.

7. If $B \notin G$, then $\phi_G(B) = \Phi$ [30]. So, $< y >_R \cap B \notin G$ and $R^G(B) = \Phi$. □

The next example shows that $B \nsubseteq \overline{R}^g(B)$, in general. So, Theorem 3.2 is not valid in general, as shown in Example 3.3.

Example 3.3. Let $Y = \{1, 2, 3, 4\}$, $G = \{\{2\}, \{1, 2\}, \{2, 3\}, Y\}$ be a grill on Y and $R = \{(1, 1), (2, 2), (3, 3), (4, 4), (1, 2), (1, 3), (2, 3), (2, 4), (3, 4), (3, 1)\}$ be a relation on Y. Hence, $1R = \{1, 2, 3\}$ and so $< 1 >_R = \{1, 3\}$, $2R = \{2, 3, 4\}$ and so $< 2 >_R = \{2, 3\}$, $3R = \{1, 3, 4\}$ and so $< 3 >_R = \{3\}$, $4R = \{4\}$ and so $< 4 >_R = \{4\}$. Let $B = \{1, 2\}$. Then, $R^G(b) = \{2\}$.

Theorem 3.4. *For the binary relation R on a miss-null set Y and G_1, G_2 being two grills on a GTS Y we get:*

1. $G_1 \subseteq G_2 \Rightarrow R^{G_1} \subseteq R^{G_2}$ and $R_{G_2} \subseteq R_{G_1}$;

2. $R^{G_1 \cap G_2}(B) = R^{G_1}(B) \cap R^{G_2}(B)$;

3. $R^{G_1 \cup G_2}(B) = R^{G_1}(B) \cup R^{G_2}(B)$.

Proof. 1. Let $y \in R^{G_1}$, then $< y >_R \cap B \in G_1$. Since $G_1 \subseteq G_2$, then $< y >_R \cap B \in G_2$ and $y \in R^{G_2}$. i.e. $R^{G_1} \subseteq R^{G_2}$.

Let $y \in R_{G_2}$, then $< y >_R \cap B^c \notin G_2$. That, $< y >_R \cap B^c \notin G_1$ and $y \in R_{G_1}$. i.e. $R_{G_2} \subseteq R_{G_1}$. 2. It is clear. 3. Obvious. □

Theorem 3.5. *Let G be a maximal grill on Y and R being a reflexive relation. Then R_G-lower approximation satisfies the following properties:*

1-$R_G(Y) = Y$,

2- $B \subseteq C \Rightarrow R_G(B) \subseteq R_G(C)$,

3- $R_G(B \cap C) \subseteq R_G(B) \cup R_G(C)$,

4- $R_G(B) \cup R_G(C) \subseteq R_G(B \cap C)$,

5- $R_G(B) \subseteq R_G(R_G(B))$,

6- $R_G(B) = [R^G(B^c)]^c$,

7- $B^c \notin G \Rightarrow R_G(B) = Y$.

Proof. 1. From $R_G(B) = \{y \in Y :< y >_R \cap B^c \notin G\}$. Then $R_G(Y) = \{y \in Y :< y >_R \cap \Phi = \Phi \notin G\} = Y$.

2. If $B \subseteq C$, $y \in R_G(B)$ that $< y >_R \cap B^c \notin G$ and $< y >_R \cap C^c \notin G$. i.e. $y \in R_G(C)$.

3. Let $y \in R_G(B \cap C)$. Then $< y >_R \cap (B \cap C)^c \notin G$, i.e. $y \in R_G(B) \cup R_G(C)$.

4. Let $y \in R_G(B) \cup R_G(C)$. Then $y \in \{< y >_R \cap B^c \notin G\} \cup \{< y >_R \cap C^c \notin G\}$ and $y \in R_G(B \cap C)$.

Proof 5, 6 and 7 are straightforward. □

4. Some topological approaches for rough approximations through grill

Definition 4.1. *Supposing that G is a grill on the non-empty set Y and the relation R is reflexive on the GTS Y, $B \subseteq Y$. the rough approximations $\underline{R}$ and $\overline{R}$ of set B are assigned respectively by:*

$$\underline{R}_G(B) = \{y \in B :< y >_R \cap B^c \notin G\},$$
$$\overline{R}^G(B) = B \cup R^G(B).$$

Definition 4.2. *Let G be a grill on the GTS Y and R be a reflexive relation on the non-empty set Y. For $B \subseteq Y$ the positive, boundary , negative regions and the accuracy measure of B are assigned respectively by: $POS_{R_G}(B)$, $BND_{R_G}(B) = \frac{\overline{R}^G(B)}{\underline{R}_G(B)}$, $NEG_{R_G}(B) = X \backslash \overline{R}^G(B)$ and $\mu_{R_G}(B) = \frac{\underline{R}_G(B)}{\overline{R}^G(B)}$.*

Theorem 4.1. *If R is a binary relation on the miss-null set Y and G is any grill, then $\overline{R}(A) \subseteq \overline{R}^G(A)$.*

Proof. It is obvious. □

Theorem 4.2. *Let G be a grill and R be a reflexive relation on τ_R^g. Then, the rough approximation in Definition 4.1 satisfies the following:*

1. $\overline{R}^G(Y) = Y$,
2. $B \subseteq \overline{R}^G(B)$,
3. $B \subseteq C \Rightarrow \overline{R}^G(B) \subseteq \overline{R}^G(C)$,
4. $\overline{R}^G(B \cup C) = \overline{R}^G(B) \cup \overline{R}^G(C)$,
5. $\overline{R}^G(B \cap C) \subseteq \overline{R}^G(B) \cap \overline{R}^G(C)$,
6. $\overline{R}^G(\overline{R}^G(B)) \subseteq \overline{R}^G(B)$,
7. $\overline{R}^G(B) = [\overline{R}_G(B^c)]^c$,
8. $B \notin G \Rightarrow \overline{R}^G(B) = B$,
9. $\underline{R}_G(Y) = Y$,
10. $\underline{R}_G(B) \subseteq B$,
11. $B \subseteq C \Rightarrow \underline{R}_G(B) \subseteq \underline{R}_G(C)$,
12. $\underline{R}_G(B) \cup \underline{R}_G(C) \subseteq \underline{R}_G(B \cup C)$,
13. $\underline{R}_G(B \cap C) = \underline{R}_G(B) \cap \underline{R}_G(C)$,
14. $\underline{R}_G(\underline{R}_G(B)) = \underline{R}_G(B)$,
15. $\underline{R}_G(B) = \underline{R}_G[(B^c)]^c$,
16. If $B^c \notin G$ then, $\underline{R}_G(B) = B$.

Proof. The proof is clear and so it is omitted. □

Corollary 4.1. *If R is a reflexive relation on a GTS Y, then the lower approximation in Definition 4.1 satisfies Kuratowski's axioms and induces a topology on Y named τ_R^G. It is defined by $\tau_R^G = \{B \subseteq Y : \underline{R}_G(B) = B\}$.*

Theorem 4.3. *Let (Y, τ_R^G) be a topological space defined in Corollary 4.1. Then,*
1. $Cl_R^G(B) \subseteq Cl_R(B)$ where $Cl_R(B) = B \cup \{y \in Y :< y >_R \cap B \neq \phi\}$,
2. $R^G(B)$ is closed.

Proof. 1- Let $y \in Cl_R^G(B)$. Then, $y \in B$ or $< y >_R \in G$. Hence $y \in Cl_R(B)$.
2- Let $y \in Cl_R^G(R^G(B))$. It is implied that $y \in R^G(B)$ or $y \in R^G(R^G(B))$. Since $R^G(R^G(B)) \subseteq R^G(B)$, it follows the result. □

Theorem 4.4. *If the relation R is reflexive on the GTS Y and G_1, G_2 be two grills on Y. If $G_1 \subseteq G_2$ then $BND_{G_1} \subseteq BND_{G_2}$.*

Proof. Let $y \in BND_{G_1}(B)$. Then, $y \in \overline{R}^{G_1}(A)$ and $y \in (\underline{R}_{G_1})^c$. It follows that $y \in \overline{R}_{G_2}(B)$ and $y \in (\underline{R}_{G_2})^c$. Hence, $y \in BND_{G_2}(B)$. □

In Example 4.5, we show that the rough sets by grill decrease the boundary region compared to Allam's method [5].

Example 4.5. Let $Y = \{1, 2, 3, 4\}$ with $R = \{(1, 1), (2, 2), (3, 3), (4, 4), (1, 2), (1, 4), (2, 3), (3, 2)\}$, $G = \{Y\}$ as Table 1.

Table 1: Comparison between Allam's method and a grill method.

B	$\underline{R}(B)$	$\underline{R}_G(B)$	$\overline{R}(B)$	$\overline{R}^G(B)$	$BNDR(B)$	$BNDR_G(B)$
{1}	ϕ	{1}	{1}	{1}	{1}	Φ
{2}	ϕ	{2}	{1, 2, 3}	{2}	{1, 2, 3}	Φ
{3}	ϕ	{3}	{ 2, 3}	{3}	{2, 3}	Φ
{4}	{4}	{4}	{1, 4}	{4}	{1}	Φ
{1, 2}	ϕ	{1, 2}	{1, 2, 3}	{1, 2}	{1, 2, 3}	Φ
{1, 3}	ϕ	{1, 3}	{1, 2, 3}	{1, 3}	{1, 2, 3}	Φ
{1, 4}	ϕ	{1, 4}	{1, 4}	{1, 4}	{1, 4}	Φ
{2, 3}	{2, 3}	{2, 3}	{1, 2, 3}	{2, 3}	{1}	Φ
{2, 4}	{4}	{2, 4}	Y	{2, 4}	{1, 2, 3}	Φ
{3, 4}	{4}	{3, 4}	Y	{3, 4}	{1, 2, 3}	Φ
{1, 2, 3}	{2, 3}	{1, 2, 3}	{1, 2, 3}	{1, 2, 3}	{1}	Φ
{1, 2, 4}	{1, 4}	{1, 2, 4}	Y	{1, 2, 4}	{2, 4}	Φ
{1, 3, 4}	{4}	{1, 3, 4}	Y	{1, 3, 4}	{1, 2, 3}	Φ
$\{2, 3, 4\}$	$\{2, 3, 4\}$	$\{2, 3, 4\}$	Y	$\{2, 3, 4\}$	{1}	Φ
Φ	Φ	Φ	Φ	Φ	Φ	Φ
Y	Y	Y	Y	Y	Φ	Φ

Example 4.6. Let $Y = \{1, 2, 3, 4\}$, $R = \{(1, 1), (2, 2), (3, 3), (4, 4), (1, 2), (1, 4), (2, 3), (3, 2)\}$, $G = \{\{2\}, \{1, 2\}, \{2, 3\}, \{2, 4\}, \{1, 2, 3\}, \{1, 2, 4\}, Y\}$ be a grill as shown in Table 2.

From the previous examples, we notice that a grill method decreased the boundary in comparison to Allam's method.

Table 2: Comparison between Allam's method and a grill method.

B	$\underline{R}(B)$	$\underline{R}_G(B)$	$\overline{R}(B)$	$\overline{R}^G(B)$	$BNDR(B)$	$BNDR_G(B)$
{1}	Φ	Φ	{1}	{1}	{1}	{1}
{2}	Φ	{2}	{1, 2, 3}	{2,3}	{1, 2, 3}	{2}
{3}	Φ	Φ	{ 2, 3}	{3}	{2, 3}	{3}
{4}	{4}	{4}	{1, 4}	{4}	{1}	Φ
{1, 2}	Φ	{1, 2}	{1, 2, 3}	{1, 2, 3}	{1, 2, 3}	{3}
{1, 3}	Φ	Φ	{1, 2, 3}	{1, 3}	{1, 2, 3}	{1, 3}
{1, 4}	Φ	{4}	{1, 4}	{1, 4}	{1, 4}	{1}
{2, 3}	{2, 3}	{2, 3}	{1, 2, 3}	{1, 2, 3}	{1}	{1}
{2, 4}	{4}	{2, 4}	Y	Y	{1, 2, 3}	{1, 3}
{3, 4}	{4}	{4}	Y	{3, 4}	{1, 2, 3}	{3}
{1, 2, 3}	{2, 3}	{1, 2, 3}	{1, 2, 3}	{1, 2, 3}	{1}	Φ
{1, 2, 4}	{1, 4}	{1, 2, 4}	Y	Y	{2, 3}	{3}
{1, 3, 4}	{4}	{4}	Y	{1, 3, 4}	{1, 2, 3}	{1, 3}
{2, 3, 4}	{2, 3, 4}	{2, 3, 4}	Y	Y	{1}	{1}
Φ	Φ	Φ	Φ	Φ	Φ	Φ
Y	Y	Y	Y	Y	Φ	Φ

Theorem 4.7. *If the relation R is reflexive on Y and G be a grill on Y, then $\beta = \{< y >_R \backslash B : y \in Y, B \notin G\}$ is a basis for ζ_R^G.*

Proof. We satisfied the relation $< y >_R \backslash B \subseteq \underline{R}_G(< y >_R \backslash B)$. So, if $z \in < y >_R \backslash B$. Then $< z >_R \subseteq < y >_R$, $< z >_R \cap (< z >_R - B)^c = < z >_R \cap ((< z >_R)^c \cup B) = < z >_R \cap B \subseteq B \notin G$. Thus, $< z >_R \cap (< y >_R - B)^c \notin G$. Hence $z \in \underline{R}_G(< y >_R \backslash B)$. Now, we prove the requirement. Firstly, let $< y >_R \backslash B, < z >_R \backslash C \in \beta$ and $x \in (< y >_R \backslash B \cap < z >_R \backslash C)$. Then, $< x >_R \subseteq < y >_R$ and $< x >_R \subseteq < z >_R$ hence, $(< x >_R \subseteq (B \cup C)) \subseteq < y >_R \backslash B$ and $(< x >_R \subseteq (B \cup C)) \subseteq < z >_R \backslash C$, i.e., $(< x >_R \backslash (B \cup C)) \in \beta$, $x \in (< x >_R \backslash (B \cup C)) \subseteq (< y >_R \backslash B) \cap < z >_R \backslash C$. Secondly, $\cup\{< y >_R \backslash B : y \in Y, B \notin G\} = Y$. □

Theorem 4.8. *If the relation R is the reflexive on Y and G_f be an grill of finite subsets of Y then the topological space (Y, ζ_R^G) is a T_1-space.*

Proof. Since $R^G(\{y\}) = \Phi$ for all$y \in Y$. It follows that $\overline{R}^G(\{y\}) = \{y\}$. □

5. Rough membership functions via grill

In the fifth section, we introduce a new approximation to the membership function for any relation R that is reflexive on a miss-null finite set Y.

Definition 5.1. *Let Y be a miss-null finite set, $B \subseteq Y$ and G be a grill on Y. Then a membership function can be defined as $\mu_B^G(y) = \max\{\frac{|(G^y - <y>_R) \cap B|}{|G^y - <y>_R|} : G^y \in G\}$, $y \in Y$ and G^y belongs to G containing y.*

Example 5.1. Let $Y = \{1,2,3,4\}$, $R = \{(1,2),(1,4),\ (4,3),\ (1,3),\ (3,4)\}$, $(1,1),\ (2,2),(3,3),(4,4)\}$ and $G = \{\{1\},\{1,2\},\ \{1,3\},\{1,4\},\ \{1,2,3\},\{1,2,4\}$, $\{1,3,4\},Y\}$ be a grill on Y. $< 1 >_R = Y$; $< 2 >_R = \{2\}$; $< 3 >_R = \{3\}$; $< 4 >_R = \{3,4\}$. Thus, $\{G^1 - < 1 >_R : G^1 \in G\} = \Phi$; $\{G^2 - < 2 >_R : G^2 \in G\} = \{\{1\},\{1,3\},\ \{1,4\},\{1,3,4\}\}$; $\{G^3 - < 3 >_R : G^3 \in G\} = \{\{1\},\ \{1,2\},\{1,4\}, \{1,2,4\}\}$; $\{G^4 - < 4 >_R : G^4 \in G\} = \{\{1\},\ \{1,2\}\}$. Let $B = \{1,2\}$, then $\mu_B^G(1) = 0$, $\mu_B^G(2) = 0.25$, $\mu_B^G(3) = 0.5$, $\mu_B^G(4) = 1$.

Remark 5.2. By Example 5.1, we note that $\mu_{B^c}^G(y) \neq 1 - \mu_B^G(y)$.

Theorem 5.3. *Let Y be a miss-null set, R be a reflexive relation on Y, $B, C \subseteq Y$ and G be a grill on Y. Then the membership function, defined in, satisfies the following:*

1. $\mu_B^G(y) = 0 \Rightarrow y \in \underline{R}_G(B)$,
2. $\mu_B^G(y) = 1 \Rightarrow y \in (\overline{R}_G(B))^c$,
3. *If* $R = \{(y,y) : y \in Y\}$, *then* μ_B^G *is the characteristic function of* B,
4. *If* yRz, *then* $\mu_B^G(y) = \mu_B^G(z)$ *provided that* R *is an equivalence relation,*
5. $B \subseteq C \Rightarrow \mu_B^G(y) = \mu_C^G(y)$.

Proof. Straightforward. □

6. Conclusion

Pawlak's, Yao's, and Allam's rough set depend on the equivalence relation for Pawlak and general relation for Yao and Allam. General topology has many applications in different fields that were far from it before the emergence of rough set theory. In this study, we give another approach of rough sets by using a grill topological structure. We introduce new definitions of the approximations space and the rough membership function via the grill. By comparing new definitions with Pawlak's, Yao's, and Allam's, we have shown that the current definitions are more general. Also, it is shown that the present method is used to decrease the boundary region in the rough set theory.

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