A novel artificial variable-free simplex algorithm for solving neutrosophic linear programming problems

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Abstract. In this article, we propose a novel fuzzy artificial variable-free version of simplex algorithm (FAVFSA) for solving fuzzy linear programming problems. Also, we develop this algorithm as a simplified neutrosophic artificial variable-free version of simplex algorithm (NAVFSA) for solving neutrosophic linear programming problems. A computational comparison between the proposed algorithm and the traditional two-phase algorithm is introduced. The computational results show that the size and the running time of the new algorithm is less than the size and the running time of the traditional two-phase algorithm. We also use numerical examples to compare between the fuzzy approach and the neutrosophic approach. The results show that the neutrosophic approach is more accurate than the fuzzy approach.

Keywords: fuzzy number, neutrosophic number, artificial variable-free-version of simplex method, two-phase simplex method.

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1. Introduction

One of the serious issues with linear programming is that modeling needs accurate and well defined data. In fact, this is a task that is almost impossible to accomplish in many cases, due to risk or uncertainty in some data. The classic LP problem can be defined using vague numbers. Fuzzy linear programming explains the mathematical model in more realistic way. Fuzzy set does not represent vague and imprecise information efficiently, because it considers only the truthiness function. The concept of intuitionistic fuzzy set is introduced to handle vague and imprecise information, by considering both the truth and falsity function, also intuitionistic fuzzy set does not simulate human decision-making process. Because the proper decision is fundamentally a problem of arranging and explicate facts the concept of neutrosophic set theory was presented by Smarandache to handle vague, imprecise and inconsistent information. Neutrosophic Set (NS) [20] is a generalization of the fuzzy set [29] and intuitionistic fuzzy set [3]. It can deal with uncertain, indeterminate, and incongruous information where the indeterminacy is quantified explicitly and truth membership, indeterminacy membership and falsity membership are completely independent. The simplex method is the common tool for solving linear programs. It is an iterative method that was developed by Dantzig [10, 14, 15]. It is known that the application of the simplex algorithm requires at least one basic feasible solution. The common techniques that are used for determining an initial feasible basis are the Two-Phase and Big-M methods. In 1997, Arsham [4] presented the simplex method without using artificial variables. In the first step, the basic feasible variable set (BVS) is determined to be the empty set. Then, the non-basic variable is chosen to be the basic variable one by one until the BVS is full. After the problem has complete BVS, the simplex method is performed. However, this method has a mistake as shown by Enge and Huhn [21] in 1998. In 2000, Pan [23] proposed the simplex method by avoiding artificial variables. The algorithm starts when the initial basis gives primal and dual infeasible solutions by adjusting negative reduced costs to a single positive value. Then, the dual solution is feasible and the dual simplex method is performed. After the optimal solution is found in this step, the original reduced costs are restored and the simplex method is performed.

Later, in 2006, Corley et.al [13] constructed the relaxed problem for improving the simplex method without using artificial variables. The cosine criterion is used for choosing the suitable constraints to construct the relaxed problem. Then, it is solved by the simplex method. After the optimal solution of the relaxed problem is found, the relaxed constraints will be restored into the current tableau, and the dual simplex method will be performed until the optimal solution is found. However, this algorithm can solve only the problem which has all positive coefficients. Boonperm and Sinapiromsaran, 2014 [12] proposed the non-acute constraint relaxation technique that improves the simplex method without using artificial variables; it can reduce the start-up time to solve the initial relaxation problem. The algorithm starts by relaxing the non-acute constraints which can guarantee that the relaxed problem is always feasible. So, the relaxed problem can be solved without using artificial variables. After the optimal solution of the relaxed problem is found, the relaxed constraints are restored, and the dual simplex method is used to solve it. Since the relaxed problem can reduce variables or constraints, the computation can be reduced.

For more details about the linear programming, the reader can refer to [5, 8, 9, 15, 24, 25, 26]. On the other hand, for more details about the fuzzy linear programming, the reader is referred to [6, 7]. Finally, for more details about the neutrosophic linear programming, the reader may refer to [2, 16, 19].

In this paper, instead of developing a new method for solving general linear programming problems, we are going to present a novel version of simplex algorithm (fuzzy and neutrosophic artificial variable free simplex algorithm) which has several advantages, for instance, it could start with any feasible or infeasible basis of linear programming problem. It also deals with the artificial variables in an implicit manner therefore, as long as a variable is infeasible its corresponding slack variable would be invisible but implicitly after leaving the basis it would be replaced by the corresponding invisible slack variable.

Also, we compare between the fuzzy approach and the neutrosophic approach by using the same rank function to make the comparison fair.

The remaining parts of this work are organized as follows: In sec. 2, the fundamental concepts of fuzzy and neutrosophic sets have been presented, and a new technique which converts the fuzzy representation to the neutrosophic representation has been proposed. Akanksha Singh et al.'s modifications and a new neutrosophic artificial variable-free simplex algorithm (NAVFSA) are proposed in Section 3. In Section 4, a numerical examples that show the importance of the proposed modification for primal neutrosophic simplex method and they show the superiority of the proposed algorithm (NAVFSA) on the primal neutrosophic simplex method has been shown. Finally, we introduce conclusions and the future work in Section 5.

2. Preliminaries

In this section, three subsections have been introduced. First one is a representation of the fuzzy numbers. Second, the representation of the neutrosophic numbers is shown. Finally, we show how to convert the fuzzy numbers to neutrosophic numbers.

2.1 Fuzzy representation

We review the fundamental notions of fuzzy set theory, initiated by Bellman and Zadeh [11].

Definition 2.1.1. *A convex fuzzy set $\tilde{A}$ on $\mathbb{R}$ is a fuzzy number if the following conditions hold:*

1. Its membership function is piecewise continuous.

2. There exist three intervals $[a, b]$, $[b, c]$, $[c, d]$ such that $\mu_{\tilde{a}}$ is increasing on $[a, b]$, equal to 1 on $[b, c]$, decreasing on $[c, d]$ and equal to 0 elsewhere.

Definition 2.1.2. *Let $\tilde{a} = (a^L, a^U, \alpha, \beta)$ denote the trapezoidal fuzzy number, where $(a^L - \alpha, a^U + \beta)$ is the support of a and $[a^L, a^U]$ its core.*

Remark 1. We denote the set of all trapezoidal fuzzy numbers by $F(\mathbb{R})$ as shown as in figure 1.

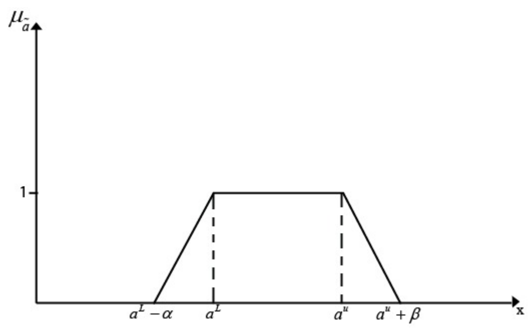

Figure 1: Truth membership function of trapezoidal fuzzy numbers $\tilde{a}$

Definition 2.1.3. *Let $\tilde{a} = (a^L, a^U, \alpha, \beta)$ and $\tilde{b} = (b^L, b^U, \gamma, \theta)$ be two trapezoidal fuzzy numbers, the arithmetic operation on the trapezoidal fuzzy number are defined as:*

$$
\begin{aligned}
x\tilde{a} &= (xa^L, xa^U, x\alpha, x\beta); x > 0, x \in \mathbb{R},\\
x\tilde{a} &= (xa^U, xa^L, -x\beta, -x\alpha); x < 0, x \in \mathbb{R},\\
\tilde{a} + \tilde{b} &= (a^L, a^U, \alpha, \beta) + (b^L, b^U, \gamma, \theta)\\
&= [a^L + b^L, a^U + b^U, \alpha + \gamma, \beta + \theta].
\end{aligned}
$$

2.2 Neutrosophic representation

In this subsection, some basic definitions in the neutrosophic set theory are introduced:

Definition 2.2.1 ([1])**.** *the trapezoidal neutrosophic number $\tilde{A}$ is a neutrosophic set in $\mathbb{R}$ with the following truth (T), indeterminacy (I) and falsity (F) membership functions as shown in figure 2:*

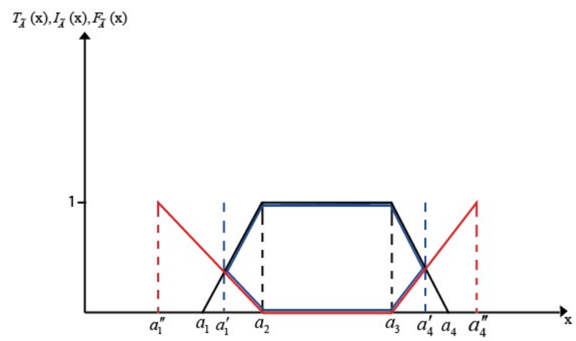

Figure 2: Truth, indeterminacy and falsity membership functions of trapezoidal neutrosophic numbers $\tilde{A}$

$$T_{\tilde{A}}(x) = \begin{cases} \dfrac{\alpha_{\tilde{A}}(x-a_1)}{a_2-a_1}, & a_1 \le x \le a_2 \\ \alpha_{\tilde{A}}, & a_2 \le x \le a_3 \\ \alpha_{\tilde{A}}(\dfrac{x-a_3}{a_4-a_3}), & a_3 \le x \le a_4 \\ 0, & \text{otherwise} \end{cases}$$

$$I_{\tilde{A}}(x) = \begin{cases} \dfrac{(a_2-x+\theta_{\tilde{A}}(x-a_1'))}{a_2-a_1'}, & a_1' \le x \le a_2 \\ \theta_{\tilde{A}}, & a_2 \le x \le a_3 \\ \dfrac{(x-a_3+\theta_{\tilde{A}}(a_4'-x))}{a_4'-a_3}, & a_3 \le x \le a_4' \\ 1, & \text{otherwise} \end{cases}$$

$$F_{\tilde{A}}(x) = \begin{cases} \dfrac{(a_2-x+\beta_{\tilde{A}}(x-a_1''))}{a_2-a_1''}, & a_1'' \le x \le a_2 \\ \beta_{\tilde{A}}, & a_2 \le x \le a_3 \\ \dfrac{(x-a_3+\beta_{\tilde{A}}(a_4''-x))}{a_4''-a_3}, & a_3 \le x \le a_4'' \\ 1, & \text{otherwise} \end{cases}$$

where $\alpha_{\tilde{A}}$, $\theta_{\tilde{A}}$ and $\beta_{\tilde{A}}$ represent the maximum degree of truthiness, minimum degree of indeterminacy and minimum degree of falsity, respectively, $\alpha_{\tilde{A}}$, $\theta_{\tilde{A}}$ and $\beta_{\tilde{A}} \in [0,\ 1]$.

Definition 2.2.2 ([1])**.** *The mathematical operations on two trapezoidal neutrosophic numbers.* $\tilde{A} =< a_1, a_2, a_3, a_4; \alpha_{\tilde{A}}, \theta_{\tilde{A}}, \beta_{\tilde{A}} >$ *and* $\tilde{B} =< b_1, b_2, b_3, b_4; \alpha_{\tilde{B}}, \theta_{\tilde{B}}, \beta_{\tilde{B}} >$ *are as follows:*

$$\tilde{A} + \tilde{B} =< (a_1+b_1, a_2+b_2, a_3+b_3, a_4+b_4); \alpha_{\tilde{A}} \wedge \alpha_{\tilde{A}}, \theta_{\tilde{A}} \vee \theta_{\tilde{A}}, \beta_{\tilde{A}} \vee \beta_{\tilde{A}} >,$$
$$\tilde{A} - \tilde{B} =< (a_1-b_4, a_2-b_3, a_3-b_2, a_4-b_1); \alpha_{\tilde{A}} \wedge \alpha_{\tilde{A}}, \theta_{\tilde{A}} \vee \theta_{\tilde{A}}, \beta_{\tilde{A}} \vee \beta_{\tilde{A}} >,$$

$$\tilde{A}^{-1} =< (\frac{1}{a_4}, \frac{1}{a_3}, \frac{1}{a_2}, \frac{1}{a_1}); \alpha_{\tilde{A}}, \theta_{\tilde{A}}, \beta_{\tilde{A}} > \textit{where } \tilde{A} \neq 0),$$

$$\lambda\tilde{A} = \begin{cases} < \lambda a_1, \lambda a_2, \lambda a_3, \lambda a_4; \alpha_{\tilde{A}}, \theta_{\tilde{A}}, \beta_{\tilde{A}} >, \lambda > 0 \\ < \lambda a_4, \lambda a_3, \lambda a_2, \lambda a_1; \alpha_{\tilde{A}}, \theta_{\tilde{A}}, \beta_{\tilde{A}} >, \lambda < 0 \end{cases}$$

$$\tilde{A}\tilde{B} \begin{cases} < (a_1b_1, a_2b_2, a_3b_3, a_4b_4), \alpha_{\tilde{A}} \wedge \alpha_{\tilde{A}}, \theta_{\tilde{A}} \vee \theta_{\tilde{A}}, \beta_{\tilde{A}} \vee \beta_{\tilde{A}} > \\ \textit{if } (a_4 > 0, b_4 > 0) \\ < (a_1b_4, a_2b_3, a_3b_2, a_4b_1), \alpha_{\tilde{A}} \wedge \alpha_{\tilde{A}}, \theta_{\tilde{A}} \vee \theta_{\tilde{A}}, \beta_{\tilde{A}} \vee \beta_{\tilde{A}} > \\ \textit{if } (a_4 < 0, b_4 > 0) \\ < (a_4b_4, a_3b_3, a_2b_2, a_1b_1), \alpha_{\tilde{A}} \wedge \alpha_{\tilde{A}}, \theta_{\tilde{A}} \vee \theta_{\tilde{A}}, \beta_{\tilde{A}} \vee \beta_{\tilde{A}} > \\ \textit{if } (a_4 < 0, b_4 < 0) \end{cases}$$

$$\frac{\tilde{A}}{\tilde{B}} \begin{cases} < (\frac{a_1}{b_4}, \frac{a_2}{b_3}, \frac{a_3}{b_2}, \frac{a_4}{b_1}), \alpha_{\tilde{A}} \wedge \alpha_{\tilde{A}}, \theta_{\tilde{A}} \vee \theta_{\tilde{A}}, \beta_A \vee \beta_{\tilde{A}} > \\ \textit{if } (a_4 > 0, b_4 > 0) \\ < (\frac{a_4}{b_4}, \frac{a_3}{b_3}, \frac{a_2}{b_2}, \frac{a_1}{b_1}), \alpha_{\tilde{A}} \wedge \alpha_{\tilde{A}}, \theta_{\tilde{A}} \vee \theta_{\tilde{A}}, \beta_{\tilde{A}} \vee \beta_{\tilde{A}} > \\ \textit{if } (a_4 < 0, b_4 > 0) \\ < (\frac{a_4}{b_1}, \frac{a_3}{b_2}, \frac{a_2}{b_3}, \frac{a_1}{b_4}), \alpha_{\tilde{A}} \wedge \alpha_{\tilde{A}}, \theta_{\tilde{A}} \vee \theta_{\tilde{A}}, \beta_{\tilde{A}} \vee \beta_{\tilde{A}} > \\ \textit{if } (a_4 < 0, b_4 < 0) \end{cases}$$

2.3 Transfer from fuzzy representation to neutrosophic representation

The goal of this section is to explain how to convert fuzzy numbers representation into neutrosophic numbers representation. This transformation is used for simplicity and to make the comparison fair. There are many types of techniques to transfer from fuzzy to neutrosophic representation such as, rank functions and $\propto$-cut technique.

Definition 2.3.1. *Ranking function is a viable approach for ordering fuzzy numbers and neutrosophic numbers. It is known that there are many rank functions for ordering the fuzzy numbers and neutrosophic numbers.*

In this subsection, we explain how to apply technique to convert from fuzzy number to neutrosophic number:

From Figure 1 and Figure 2 we can illustrate the following relations between the two representations

$$a_1 = a_2 - \alpha, a_2 = a^L, a_3 = a^U \text{ and } a_4 = a_3 + \beta. \tag{1}$$

Assuming that the rank function is used for ordering the fuzzy numbers as follows:

$$R(\tilde{a}) = \frac{a^L + a^U}{2} + \frac{\beta - \alpha}{4}. \tag{2}$$

From (1) we can express the used rank function for ordering the neutrosophic numbers as follows:

$$R(\tilde{a}) = \frac{1}{4}\sum_{i=1}^{4}\tilde{a}_i + (T_{\tilde{a}} - I_{\tilde{a}} - F_{\tilde{a}}). \tag{3}$$

From (1), we can convert fuzzy numbers representation into neutrosophic numbers representation. On the other hand, from (2) and (3), we can use the same function for both fuzzy numbers and neutrosophic numbers to obtain a fair comparison between them.

Assuming that $T_{\tilde{A}} = 1$, $I_{\tilde{A}} = 0$, $F_{\tilde{A}} = 0$, then the TrNN $\tilde{a} =< a_1, a_2, a_3, a_4; T_{\tilde{A}}, I_{\tilde{A}}, F_{\tilde{A}} >$ will be transformed into a trapezoidal neutrosophic number $\tilde{a} = < a_1, a_2, a_3, a_4; 1, 0, 0 >$ and hence, in this case,

1. The expression

$$R(\tilde{a}) = \frac{1}{4}\sum_{j=1}^{4} a_i + (T_{\tilde{a}} - I_{\tilde{a}} - F_{\tilde{a}})$$

is equivalent to the expression $R(\tilde{a}) = \frac{1}{4}\sum_{j=1}^{4} a_i + 1$.

Furthermore, it is well known that if $a_1 = a_2 = a_3 = a_4$ then the trapezoidal neutrosophic number $\tilde{A} =< a_1, a_2, a_3, a_4; 1, 0, 0 >$ will be transformed into a real number $A = (a, a, a, a; 1, 0, 0)$ and hence, in this case. Furthermore, it is well known that if $a_1 = a_2 = a_3 = a_4$ then the trapezoidal neutrosophic number $\tilde{A} =< a_1, a_2, a_3, a_4; 1, 0, 0 >$ will be transformed into a real number $A = (a, a, a, a; 1, 0, 0)$ and hence, in this case.

2. The expression

$$R(\tilde{a}) = \frac{1}{4}\sum_{j=1}^{4} a_i + (T_{\tilde{a}} - I_{\tilde{a}} - F_{\tilde{a}})$$

is equivalent to the expression $R(A) = a + 1 \neq a$.

The following table represents the fuzzy rank function, the corresponding neutrosophic rank function and the corresponding real rank function.

Table 1: fuzzy rank function and this corresponding neutrosophic rank function

No	Fuzzy Rank Function	Corresponding Neutrosophic Rank Function	Corresponding Real Rank Function
1	$R(\tilde{a}) = (\frac{a^l + a^u}{2} + \frac{\beta - \alpha}{4})$	$R(\tilde{a}) = \frac{1}{4}\sum_{j=1}^{4} a_i + (T_{\tilde{a}} - I_{\tilde{a}} - F_{\tilde{a}})$	$R(a) = a + 1$

3. Algorithms

In this section; we first present Akanksha Singh *et al.*'s modifications [28] and the proposed modification about the mathematical incorrect assumptions, considered by Abdel-Basset *et al.* [1] in their proposed method to convert from neutrosophic numbers into real numbers. Second, we propose a new fuzzy artificial variable- free simplex algorithm. Finally, we develop this algorithm in order to solve linear programming with neutrosophic numbers (neutrosophic artificial variable-free simplex algorithm).

3.1 Akanksha Singh *et al.*'s modifications [28]

The following table presents Akanksha Singh et al.'s modifications to convert from neutrosophic number to deterministic number.

Table 2: Akanksha Singh et al.'s modifications

no	NLPP-(Type)	NLPP- (Form)	Exact Crisp LPP
1	The coefficients of the objective function are represented by trapezoidal neutrosophic numbers	$\max\setminus\min[\sum_{j=1}^{n}=\tilde{c}_j x_j]$ $s.t.$ $\sum_{j=1}^{n} a_{ij}x_j \leq,\geq,= b_j,$ $i=1,2,\ldots\ldots,m;$ $x_j\geq 0, j=1,2,\ldots\ldots,n$	$\max\setminus\min[\sum_{j=1}^{n} R(\tilde{c}_j x_j)-\sum_{j=1}^{n} T_{\tilde{c}_j}x_j$ $+\sum_{j=1}^{n} I_{\tilde{c}_j}x_j+\sum_{j=1}^{n} F_{\tilde{c}_j}x_j+\min_{1\leq j\leq n}\{T_{\tilde{c}_j}x_j\}-$ $\max_{1\leq j\leq n}\{I_{c_j}x_j\}-\max_{1\leq j\leq n}\{F_{\tilde{c}_j}x_j\}]$ $s.t.$ $\sum_{j=1}^{n} a_{ij}x_j\leq,\geq,= b_j, i=1,2,\ldots\ldots,m;$ $x_j\geq 0, j=1,2,\ldots\ldots,n.$
2	The coefficients of constraints variables and right hand side are represented by trapezoidal neutrosophic numbers	$\max\setminus\min[\sum_{j=1}^{n}=\tilde{c}_j x_j]$ $s.t.$ $\sum_{j=1}^{n} \tilde{a}_{ij}x_j \leq,\geq,= \tilde{b}_j,$ $i=1,2,\ldots\ldots,m;$ $x_j\geq 0, j=1,2,\ldots\ldots,n$	$\max\setminus\min\sum_{j=1}^{n} c_j x_j$ $s.t.[\sum_{j=1}^{n} R(\tilde{a}_{ij}x_j)-\sum_{j=1}^{n} T_{\tilde{a}_{ij}}x_j+\sum_{j=1}^{n} I_{\tilde{a}_{ij}}x_j$ $+\sum_{j=1}^{n} F_{\tilde{a}_{ij}}x_j+\min_{1\leq j\leq n}\{T_{\tilde{a}_{ij}}x_j\}-\max_{1\leq j\leq n}$ $\{I_{\tilde{a}_{ij}}x_j\}-\max_{1\leq j\leq n}\{F_{\tilde{a}_{ij}}x_j\}]\leq,\geq,= R(\tilde{b}_i)$ $x_j\geq 0, j=1,2,\ldots\ldots,n.$
3	All parameters are represented by trapezoidal neutrosophic numbers, except variables are exemplified only by real values	$\max\setminus\min[\sum_{j=1}^{n}=\tilde{c}_j x_j]$ $s.t.$ $\sum_{j=1}^{n} \tilde{a}_{ij}x_j \leq,\geq,= \tilde{b}_j,$ $i=1,2,\ldots\ldots,m;$ $x_j\geq 0, j=1,2,\ldots\ldots,n$	$\max\setminus\min[\sum_{j=1}^{n} R(\tilde{c}_j x_j)-\sum_{j=1}^{n} T_{\tilde{c}_j}x_j+$ $\sum_{j=1}^{n} I_{\tilde{c}_j}x_j+\sum_{j=1}^{n} F_{\tilde{c}_j}x_j+\min_{1\leq j\leq n}\{T_{\tilde{c}_j}x_j\}-$ $\max_{1\leq j\leq n}\{I_{\tilde{c}_j}x_j\}-\max_{1\leq j\leq n}\{F_{\tilde{c}_j}x_j\}]$ $s.t.$ $(\sum_{j=1}^{n} R(\tilde{a})_{ij}x_j)\leq,\geq,=(R\tilde{b}_j),\ i=1,2,\ldots\ldots,m;$ $x_j\geq 0, j=1,2,\ldots\ldots,n.$
4	The coefficients of objective function and constraints variables are represented by real numbers and right hand side are represented by trapezoidal neutrosophic numbers	$\max\setminus\min[\sum_{j=1}^{n}=c_j x_j]$ $s.t.$ $\sum_{j=1}^{n} a_{ij}x_j \leq,\geq,= \tilde{b}_j,$ $i=1,2,\ldots\ldots,m;$ $x_j\geq 0, j=1,2,\ldots\ldots,n$	$\max\setminus\min\sum_{j=1}^{n} c_j x_j$ $s.t.$ $R[\sum_{j=1}^{n}(a_{ij}x_j)\leq,\geq,= R(\tilde{b}_j),$ $i=1,2,\ldots\ldots,m;$ $x_j\geq 0, j=1,2,\ldots\ldots,n.$

Remark 2.
- If $R(a)=a+1$ and the coefficients of the objective function & constraints variables are real, then the fuzzy linear programming problem is equivalent to the neutrosophic linear programming problem.

- $\sim$: *represents the presence* of neutrosophic numbers within the matrices or vectors.
- NLPP: neutrosophic linear programming problem.

3.2 A novel neutrosophic artificial variable-free simplex algorithm

Nayatullah et al [22] proposed a simple direct artificial variable free simplex algorithm (AVFSM) for solving the linear programming problems with real numbers. In this subsection, we propose a new algorithm which solves linear programming with neutrosophic numbers (Neutrosophic artificial variable-Free Simplex Algorithm). The proposed algorithm overcame traditional neutrosophic simplex algorithms.

Algorithm 1 Neutrosophic Artificial variable -Free Simplex Algorithm (NAVFSA)

Step 0: (Initialization)

Step 5: Converting fuzzy numbers into neutrosophic numbers (see section 2.3.1)
Apply Akanksha Singh et al.'s modifications according to (see section 3.1)

Step 1: Let $\tilde{L}$ be a maximal subset of $\tilde{B}$ such that $\tilde{L} = \{i : d_{i0} < 0, i \in B\}$. **if** $\tilde{L} = \varphi$ **then** $D(\tilde{B})$ is primal feasible. Exit.

Step 2: Denote the basic variables x_L by x_L^- and compute infeasibility objective vector $W(\tilde{B}) \in R^{\tilde{N}}$ such that $w_j = \sum_{i \in L} d_{ij}$, $j \in \tilde{N}$
Place w to the top of the reduced short table $D(\tilde{B})$.

Step 3: Let $\tilde{K} \subseteq \tilde{N}$ such that $\tilde{K} = j : w_j < 0, j \in \tilde{N}$. **if** $\tilde{K} = \varphi$ **then** $D(\tilde{B})$ is primal inconsistent. Exit.

Step 4: (Choice of entering variable)
Choose $k \in \tilde{K}$ such that $w_k \leq w_h$, $\forall h \in \tilde{K}$

Step 5: (Choice of leaving variable)

Step 5: Choose $r_1 \in \tilde{L}$ and $r_1 \in \tilde{B} \backslash \tilde{L}$ such that:

$$r_1 = \arg\min\{\{\frac{d_{i0}}{d_{ik}} \mid (d_{i0} \leq 0, d_{ik} < 0)\}, i \in \tilde{L}\}$$

$r_2 = \arg\max\{\{\frac{d_{i0}}{d_{ik}} \mid (d_{i0} \geq 0, d_{ik} > 0)\}, i \in \tilde{B}\backslash\tilde{L}\} Setr := \arg\min\{\frac{d_{r_1 0}}{d_{r_1 k}}, \frac{d_{r_2 0}}{d_{r_2 k}}\}$

Step 6: Make a pivot on (r, k) ($\Rightarrow$ Set $\tilde{B} := (\tilde{B} \cup \tilde{k}) \backslash r$, $\tilde{N} := (\tilde{N} \cup r) \backslash k$ and update $D(\tilde{B})$ along with the supplement $w(\tilde{B})$).

Step 7: If $r \in \tilde{L}$, set $\tilde{L} := \tilde{L} \backslash r$ and $w_r := w_r + 1$, substitute notation of $-x_r^-$ by x_r.

Step 8: Pivot-operation

Step 8: For $r \in B$, $k \in \tilde{N}$ and (r, k) being the position of the pivot element d_{rk} $(\neq 0)$ of D, then one can obtain an updated equivalent short table $D(\tilde{B}')$ with a new basis $\tilde{B}' := (\tilde{B} \cup k) \backslash r$ and the new non-basis $\tilde{N}' := (\tilde{N} \cup r) \backslash k$ by performing the following operations on $D(\tilde{B})$

$$d'_{rk} := \frac{1}{d_{rk}}$$

$$d'_{rj} := \frac{d_{rj}}{d_{rk}}, j \in \tilde{N} \backslash k$$

$$d'_{ik} := -\frac{d_{ik}}{d_{rk}}, i \in \tilde{N} \backslash r$$

$$d'_{ij} := d_{ij} - d_{rj} \times \frac{d_{ik}}{d_{rk}}, i \in \tilde{B} \backslash r, j \in \tilde{N} \backslash k$$

The above replacement is known as a pivot operation on (r, k).

Step 9: if $\tilde{L} = \varphi$ **then** $D(\tilde{B})$ is primal feasible. Exit.

Step 10: If phase 1 ends with an objective value equal to zero, it implies that all artificial variables have attained a value zero (means all infeasibilities have been removed) and our current basis is feasible to the original problem, then we return to the original objective and proceed with simplex phase 2. Otherwise, we conclude that the problem has no solution.

Remark 3. The proposed algorithm can be applied on the fuzzy artificial variable free simplex algorithm.

4. Numerical examples

In this section, we solve some well-known fuzzy and neutrosophic linear programming.

Problems presented in [27] with the traditional simplex method and the proposed method.

4.1 Example 1: case study

A farmer who raises chickens would like to determine the amounts of the available ingredients that would meet certain nutritional requirements. The available ingredients and the nutrients in the ingredients are summarized below (Table 3).

The average cost of each unit of ingredients corn, lime and alfalfa are close to 6, 5 and 3 dollars, respectively. The minimum daily requirement of nutrients protein, carbohydrates and vitamins are approximately 8, 5 and 4 units,

Table 3: The data of Example

Nutrient	Ingredient		
	Corn	Lime	Alfalfa
Protein	3	4	2
Carbohydrates	4	2	1
Vitamins	2	1	3

respectively. The aim is to find the optimal mix. This problem is evidently an uncertain optimization problem due to variations in costs and in minimum daily requirements. So the amount of each unit of ingredients will be uncertain. Hence, the problem is described as an ambiguous linear programming problem. Symmetrically fuzzy numbers are used in the form of Trapezoidal for each uncertain value. The minimum daily requirement of nutrients protein, carbohydrates and vitamins which are approximately 8, 5 and 4 units, is modeled as (6, 10, 3, 3), (4, 6, 2, 2) and (2, 6, 1, 1) respectively. In a similar way, the other parameters also modeled as symmetric trapezoidal fuzzy numbers taking into account the nature of the problem and the other requirements. So the problem is formulated as follows:

$$\begin{array}{lll} (\mathbf{P_1}) & \text{Min}\,\tilde{z} & = (4,8,3,3)x_1 + (4,6,2,2)x_2 + (2,4,1,1)x_3 \\ & s.t. & \\ & & 3x_1 + 4x_2 + 2x_3 \geq (6,10,3,3) \\ & & 4x_1 + 2x_2 + x_3 \geq (4,6,2,2) \\ & & 2x_1 + x_2 + 3x_3 \geq (2,6,1,1) \\ & & x_1, x_2, x_3 \geq 0. \end{array}$$

We will solve this problem by using fuzzy and neutrosophic artificial variable-free simplex method uses the same rank function and compare between the results.

1. **Solving the case study (P_1) by using Fuzzy artificial variable-Free Simplex Method**

Putting the last formula into the standard form, we have:

$$\begin{array}{rcl} \text{Min}\,\tilde{z} & = & (4,8,3,3)x_1 + (4,6,2,2)x_2 + (2,4,1,1)x_3 \\ & s.t. & \\ & - & 3x_1 - 4x_2 - 2x_3 + x_4 = (-10,6,3,3) \\ & - & 4x_1 - 2x_2 - x_3 + x_5 \leq (-6,-4,2,2) \\ & - & 2x_1 - x_2 - 3x_3 + x_6 \leq (-6,-2,1,1) \\ & & x_1, x_2, x_3, x_4, x_5, x_6 \geq 0. \end{array}$$

By adding non-negative slack variables x_4, x_5, x_6, we can construct the associated reduced short table along with row vector w (sum of rows of infeasible

basic variables) of the above problem as
Initial table:

$$\begin{array}{c} \\ w \\ x_4 \\ x_5 \\ x_6 \end{array} \begin{array}{c} \begin{array}{cccc} b & x_1 & x_2 & x_3 \end{array} \\ \begin{bmatrix} (-22,-12,6,6) & -9 & -7 & -6 \\ (-10,-6,3,3) & -3 & -4 & -2 \\ (-6,-4,2,2) & -4 & -2 & -1 \\ (-6,-2,1,1) & -2 & -1 & -3 \end{bmatrix} \end{array}$$

Clearly, current basic solution is infeasible.
Here $L = 4, 5, 6$, replace $-x_L^- \to x_L$

$$\begin{array}{c} \\ w \\ -x_4- \\ -x_5- \\ -x_6- \end{array} \begin{array}{c} \begin{array}{cccc} b & x_1 & x_2 & x_3 \end{array} \\ \begin{bmatrix} (-22,-12,6,6) & -9 & -7 & -6 \\ (-10,-6,3,3) & -3 & -4 & -2 \\ (-6,-4,2,2) & -4* & -2 & -1 \\ (-6,-2,1,1) & -2 & -1 & -3. \end{bmatrix} \end{array}$$

Here, $B = \{4, 5, 6\}$ and $N = \{1, 2, 3\}$, according to most negative coefficient rule $k = 1$, so entering basic variable is x_1 and according to new minimum ratio test $r = 5$, the leaving basic variable is '$-x_5^-$'. Perform the pivot operation on (5, 1).
Replace $-x_5^- \to x_5, L = 4, 5, 6\backslash 5 = 4, 6, w_5 := w_5 + 1.$

$$\begin{array}{c} \\ w \\ x_4- \\ x_1 \\ x_6- \end{array} \begin{array}{c} \begin{array}{cccc} b & x_5 & x_2 & x_3 \end{array} \\ \begin{bmatrix} (-13,3/2,21/2,21/2) & -5/4 & -5/2 & -15/4 \\ (-7,-3/2,9/2,9/2) & -3/4 & -5/2 & -5/4 \\ (1,6/4,1/2,1/2) & -1/4 & 1/2 & 1/4 \\ (-4,1,2,2) & -1/2 & 0 & -5/2 \end{bmatrix} \end{array}$$

It can be seen that now a single infeasibility is removed.
Iteration 2. Here, $k = 3$ and $r = 6$ perform pivot operation on (6, 3).
Since $r \in L$, replace $-x_6^- \to x_6; L = 4, 6\backslash 6 = 4, w_6 := w_6 + 1.$

$$\begin{array}{c} \\ w \\ -x_4^- \\ x_1 \\ x_3 \end{array} \begin{array}{c} \begin{array}{cccc} b & x_5 & x_2 & x_6 \end{array} \\ \begin{bmatrix} (\frac{-29}{2},\frac{15}{2},\frac{27}{2},\frac{27}{2}) & \frac{-1}{2} & \frac{-5}{2} & \frac{-1}{2} \\ (\frac{-15}{2},\frac{1}{2},\frac{11}{2},\frac{11}{2}) & \frac{-1}{2} & \frac{-5}{2} & \frac{-1}{2} \\ (\frac{3}{5},\frac{8}{5},\frac{7}{10},\frac{7}{10}) & \frac{-3}{10} & \frac{1}{2} & \frac{1}{10} \\ (\frac{-2}{5},\frac{8}{5},\frac{4}{10},\frac{4}{10}) & \frac{1}{5} & 0 & \frac{-2}{5}. \end{bmatrix} \end{array}$$

Iteration 3. Here, $k = 2$ and $r = 4$ perform pivot operation on (4, 2).
Since $r \in L$, replace $-x_4^- \to x_4$; $L = \{4\}\backslash\{4\} = \{\}$, $w_4 := w_4 + 1$.

$$\begin{array}{c} \\ w \\ x_2 \\ x_1 \\ x_3 \end{array} \begin{array}{c} \begin{array}{cccc} b & x_5 & x_4 & x_6 \end{array} \\ \begin{bmatrix} (-15,15,19,19) & 0 & 0 & 0 \\ (\frac{-1}{5},3,\frac{11}{5},\frac{11}{5}) & \frac{1}{5} & \frac{-2}{5} & \frac{1}{5} \\ (\frac{-9}{10},\frac{17}{10},\frac{9}{5},\frac{9}{5}) & \frac{-2}{5} & \frac{1}{5} & 0 \\ (\frac{-2}{5},\frac{8}{5},\frac{4}{5},\frac{4}{5}) & \frac{1}{5} & 0 & \frac{-2}{5}. \end{bmatrix} \end{array}$$

Now, all the infeasibilities have been removed. Primal feasibility is achieved; the feasible solution is $(x_1,\, x_2,\, x_3)$, $x_1 = (\frac{-9}{10}, \frac{17}{10}, \frac{9}{5}, \frac{9}{5})$, $x_2 = (\frac{-1}{5}, 3, \frac{11}{5}, \frac{11}{5})$ and $x_3 = (\frac{-2}{5}, \frac{8}{5}, \frac{4}{5}, \frac{4}{5})$.

Therefore, to obtain the optimal solution we solve **(SM) phase 2**.

Iteration 4.

$$\begin{array}{c} \\ z \\ x_2 \\ x_1 \\ x_3 \end{array}
\begin{array}{c}
\begin{array}{cccc} b & x_5 & x_4 & x_6 \end{array} \\
\left[\begin{array}{cccc}
\frac{1}{5} & (\frac{-38}{5}, 30, \frac{121}{5}, \frac{121}{5}) & \frac{4}{5} & \frac{4}{5} \\
(\frac{-1}{5}, 3, \frac{11}{5}, \frac{11}{5}) & \frac{1}{5} & \frac{-2}{5} & \frac{1}{5} \\
(\frac{-9}{10}, \frac{17}{10}, \frac{9}{5}, \frac{9}{5}) & \frac{-2}{5} & \frac{1}{5} & 0 \\
(\frac{-2}{5}, \frac{8}{5}, \frac{4}{5}, \frac{4}{5}) & \frac{1}{5} & 0 & \frac{-2}{5}.
\end{array}\right]
\end{array}$$

The optimal solution is $(x_1,\, x_2,\, x_3)$, $x_1 = 0$, $x_2 = \frac{1}{5}$ and $x_3 = \frac{-2}{5}$

$$\text{Min}\, \tilde{z} \;=\; (4,8,3,3)x_1 + (4,6,2,2)x_2 + (2,4,1,1)x_3 \;=\; 11.2.$$

We will solve the same example by using neutrosophic artificial variable-free simplex Method uses the same rank function and we will compare between them.

4.1.1 Solving the same example (case study) by using neutrosophic artificial variable-free simplex method

First: We will convert the fuzzy numbers into neutrosophic numbers. After that, we apply the following rank function:

$$R(\breve{a}) = \frac{1}{4}\sum_{i=1}^{4} \tilde{a}_i + (T_a - I_{\tilde{a}} - F_{\tilde{a}})$$

$$R(a) = a + 1$$

$$\begin{aligned}
\text{Min}\, \tilde{z} \;&=\; R[(1,4,8,11)]x_1 + R[(2,4,6,8)]x_2 + R[(1,2,4,5)]x_3 \\
s.t.\; & \\
- \;& R[3x_1 + 4x_2 + 2x_3] \geq R[(3,6,10,13)] \\
- \;& R[4x_1 + 2x_2 + 1x_3] \geq R[(2,4,6,8)] \\
- \;& R[2x_1 + 1x_2 + 3x_3] \geq R[(1,2,6,7)] \\
& x_1, x_2, x_3 \geq 0.
\end{aligned}$$

$$\begin{aligned}
\text{Min}\, \tilde{z} \;&=\; 7x_1 + 6x_2 + 4x_3 - 2 \\
s.t.\; & \\
& 3x_1 + 4x_2 + 2x_3 \geq 8 \\
& 4x_1 + 2x_2 + x_3 \geq 5 \\
& 2x_1 + x_2 + 3x_3 \geq 4 \\
& x_1, x_2, x_3 \geq 0.
\end{aligned}$$

Putting the last formula into the standard form, we have:

$$\begin{aligned}\text{Min } \tilde{z} &= 7x_1 + 6x_2 + 4x_3 - 2\\ s.t. &\\ &-3x_1 - 4x_2 - 2x_3 + x_4 = -8\\ &-4x_1 - 2x_2 - x_3 + x_5 = -5\\ &-2x_1 - x_2 - 3x_3 + x_6 = -4\\ &x_1, x_2, x_3, x_4, x_5, x_6 \geq 0.\end{aligned}$$

Then, the first tableau is as follows:

$$\begin{array}{c c} & \begin{array}{cccc} b & x_1 & x_2 & x_3 \end{array}\\ \begin{array}{c} w\\ x_4\\ x_5\\ x_6 \end{array} & \begin{bmatrix} -17 & -9 & -7 & -6\\ -8 & -3 & -4 & -2\\ -5 & -4 & -2 & -1\\ -4 & -2 & -1 & -3 \end{bmatrix} \end{array}$$

Clearly, current basic solution is infeasible.
Here L = {4, 5, 6}, replace $-x_L^- \to x_L$

$$\begin{array}{c c} & \begin{array}{cccc} b & x_1 & x_2 & x_3 \end{array}\\ \begin{array}{c} w\\ -x_4^-\\ -x_5^-\\ -x_6^- \end{array} & \begin{bmatrix} -17 & -9 & -7 & -6\\ -8 & -3 & -4 & -2\\ -5 & -4 & -2 & -1\\ -4 & -2 & -1 & -3 \end{bmatrix} \end{array}$$

Here, $B = \{4, 5, 6\}$ and $N = \{1, 2, 3\}$, according to most negative coefficient rule $k = 1$, so entering basic variable is x_1 and according to new minimum ratio test r = 5, the leaving basic variable is '$-x_5^-$'. Perform the pivot operation on (5, 1). Replace $-x_5^- \to x_5$, $L = \{4, 5, 6\}\backslash\{5\} = \{4, 6\}$, $w_5 := w_5 + 1$.

$$\begin{array}{c c} & \begin{array}{cccc} b & x_5 & x_2 & x_3 \end{array}\\ \begin{array}{c} w\\ -x_4^-\\ x_1\\ -x_6^- \end{array} & \begin{bmatrix} \frac{-23}{4} & \frac{-5}{4} & \frac{-5}{2} & \frac{-15}{4}\\ \frac{-17}{4} & \frac{-3}{4} & \frac{-5}{2} & \frac{-5}{4}\\ \frac{5}{4} & \frac{-1}{4} & \frac{1}{2} & \frac{1}{4}\\ \frac{-3}{2} & \frac{-1}{2} & 0 & \frac{-5}{2} \end{bmatrix} \end{array}$$

It can be seen that now a single infeasibility is removed.
Iteration 2. Here, $k = 3$ and $r = 6$ perform pivot operation on (6, 3). Since $r \in L$, replace $-x_6^- x_6$; $L = \{4, 6\}\backslash\{6\} = 4$, $w_6 := w_6 + 1$.

$$\begin{array}{c c} & \begin{array}{cccc} b & x_5 & x_2 & x_6 \end{array}\\ \begin{array}{c} w\\ -x_4^-\\ x_1\\ x_3 \end{array} & \begin{bmatrix} \frac{-7}{2} & \frac{-1}{2} & \frac{-5}{2} & \frac{-1}{2}\\ \frac{-7}{2} & \frac{-1}{2} & \frac{-5}{2} & \frac{-1}{2}\\ \frac{11}{10} & \frac{-3}{10} & \frac{1}{2} & \frac{1}{10}\\ \frac{3}{5} & \frac{1}{5} & 0 & \frac{-2}{5} \end{bmatrix} \end{array}$$

Iteration 3. Here, $k = 2$ and $r = 4$ perform pivot operation on (4, 2). Since $r \in L$, replace $-x_4^- \to x_4$; $L = \{4\}\backslash\{4\} = \{\}$, $w_4 := w_4 + 1$.

$$\begin{array}{c} \\ w \\ x_2 \\ x_1 \\ x_3 \end{array} \begin{array}{c} \begin{array}{cccc} b & x_5 & x_4 & x_6 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ \frac{7}{5} & \frac{1}{5} & \frac{-2}{5} & \frac{1}{5} \\ \frac{2}{5} & \frac{-2}{5} & \frac{1}{5} & 0 \\ 0 & \frac{1}{5} & 0 & \frac{-2}{5} \end{bmatrix} \end{array}$$

Now, all the infeasibilities have been removed. Primal feasibility is achieved; the feasible solution is $(x_1, x_2, x_3) = (\frac{2}{5}, \frac{7}{5}, 0)$

Therefore, to obtain the optimal solution we solve **(SM) phase 2**.

Iteration 4.

$$\begin{array}{c} \\ Z \\ x_2 \\ x_1 \\ x_3 \end{array} \begin{array}{c} \begin{array}{cccc} b & x_5 & x_4 & x_6 \end{array} \\ \begin{bmatrix} \frac{1}{5} & \frac{56}{5} & \frac{4}{5} & \frac{4}{5} \\ \frac{1}{5} & \frac{7}{5} & \frac{1}{5} & \frac{-2}{5} \\ 0 & \frac{2}{5} & \frac{-2}{5} & \frac{1}{5} \\ \frac{-2}{5} & 0 & \frac{1}{5} & 0 \end{bmatrix} \end{array}$$

The optimal solution is $(x_1,\ x_2,\ x_3)$, $x_1 = 0$, $x_2 = \frac{1}{5}$ and $x_3 = -2/5$, Min $z = 7x_1 + 6x_2 + 4x_3 - 2 = \frac{56}{5} - 2 = 9.2$.

By the same way as in example1 (P_1), we can solve both example 2 (P_2) and example 3 (P_3) in the same way by using the proposed algorithm (fuzzy and neutrosophic artificial variable-free simplex algorithm).

Table 4: A comparison between two-phase algorithm, Fuzzy and Neutrosophic AVFSA

Problem	Two-Phase Simplex Algorithm				Fuzzy Art-Free Simplex Algorithm				Neutrosophic Art-Free Simplex Algorithm			
	Z	x_1	x_2	x_3	Z	x_1	x_2	x_3	Z	x_1	x_2	x_3
P_1	11.2	$\frac{2}{5}$	$\frac{7}{5}$	0	11.2	$\frac{2}{5}$	$\frac{7}{5}$	0	9.2	$\frac{2}{5}$	$\frac{7}{5}$	0
P_2	5.6	$\frac{11}{5}$	$\frac{2}{5}$	0	5.6	$\frac{11}{5}$	$\frac{2}{5}$	0	2.46	$\frac{69}{5}0$	$\frac{2}{25}$	0
P_3	19.07	$\frac{16}{7}$	$\frac{15}{28}$	-	19.07	$\frac{16}{7}$	$\frac{15}{28}$	-	19.07	$\frac{16}{7}$	$\frac{15}{28}$	-

In table 4, a good comparisons have been made between two-phase simplex algorithm, fuzzy and neutrosophic artificial variable-free simplex algorithm; we noticed that the neutrosophic approach is more accurate than the fuzzy approach as shown as in problem 1 and problem 2. However, there is one case the fuzzy approach is equivalent to the neutrosophic approach according to (remark 3) as shown as in problem 3.

Conclusion

In this work, a new algorithm (Artificial Variable-Free Simplex Algorithm) has been proposed, which solves linear programming problems with fuzzy and neutrosophic numbers. We evaluated the proposed algorithm by using a comparison against the traditional simplex method. We noticed that the artificial variables in the proposed algorithm are virtually present, but their presence is not revealed to the user in the form of extra columns in the simplex table. It also follows the same pivoting sequence as of simplex phase 1 without showing any explicit description of artificial variables or artificial constraints. Therefore, the proposed algorithm reduces the size of the problem and reduces the execution time to solve the problem, then CPU time for the proposed algorithm is also faster than the two-phase simplex method. We also compared between the fuzzy approach and the neutrosophic approach using numerical examples. Finally, the numerical examples show that the neutrosophic approach is more accurate than the fuzzy approach and the proposed method overcomes the traditional simplex method for both the fuzzy and neutrosophic approach. In future work, we can improve this work by proposing a new algorithm (Dual Artificial Variable Free Simplex Algorithm) which solves linear programming problems with fuzzy and neutrosophic numbers.

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