Optimality analysis of fuzzy critical path for projects (an applied study)

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Abstract. The Critical Path Method (CPM) is a crucial agent for the control of complex let alone the planning projects. The prominent achievement of CPM for each activity needs the existence of a valid identified time duration. However, in applicable situations this requirement is usually difficult to fulfill however a great number of activities will be fulfilled for the first time. Thus, there is always a doubt in the network planning about the time durations of activities, a matter that leads to the advancement and development of fuzzy (CPMs.). Rather than employing objective probability measures only A rational approach toward decision making should take into account human subjectivity. We used a method to schedule and control project of extending railway (Basra - Faw) using a fuzzy critical path method. Time is considered to be the major element and the real cost in every project. Thus, estimating activity time methods affect the calculations of total project time and finally the total cost. The total time of the project was calculated with the crispy time defined by $\alpha - Cut$. The researchers determined the fuzzy critical path, fuzzy critical activities, and fuzzy float for the rest of the activities. The total expected time to complete the project will be between (54.4, 68, 84) weeks, which was calculated by using the fuzzy critical path method, which is identical to the length of the first fuzzy critical path when calculating the lengths of all paths, and the estimate of expected time with this method It enables the decision-maker to follow two policies in the completion of the project that accommodate all the conditions that can be encountered without making mistakes, and it also allows him to obtain a clear value in the amount determined by the level of the cut(α).

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1. Introduction

Over the past 30 years, extensively the fuzzy set theory has been studied. In fuzzy set theory pertained were most of the early interest to representing uncertainty in human cognitive processes. Gupta and Kaufmann (1988) mention that over 7,000 research papers, monographs, reports, and books on applications the fuzzy set theory, these have been in publication since 1965 **??**. It does not require any argument that most projects are difficult to be tackled and they encompass risk, due to the existence of many activities joined in a complex way. As a result, to determine the identification of the so-called critical path (CP), many network techniques which determine the project completion time have been developed. The easiest procedure suitable for modeling the fulfillment of a project, and Critical path method (CPM) has been managed to various industrial fields. The (CPM) and its development to probable environment, the Program Evaluation and Review Technique (PERT) are the mostly used tools for tackling and predicting the various short time or long time projects. There is a great gap and distance between the conjugated probability and the real implementation probabilities. The program evaluation and review technique (PERT) can be employed to deal quantitatively with imprecise data, Regardless of this, there is the criticism of PERT, Substitutional procedure for tackling the inaccurate data is to apply the concept of fuzziness, with this, the ambiguous activity times can be expressed by the fuzzy sets. The case where activity times in a project are approximately known has been studied by various studies and these activity times are more appropriately acted rather than clear numbers by fuzzy sets.

2. Fuzzy sets

Fuzzy means ambiguity and lack of clarity, and it is a kind of uncertainty, and in the science of traditional management and engineering means probabilistic uncertainty with a random characteristic, but from the point of view of fuzzy theory is the uncertainty of meanings and description of things such as: excellent, large, small, good, high ... The formation of a conceptual framework that creates a balance among the various aspects. The framework has been used in the case of normal sets, but is more common than the latter. It is the fuzzy set and, essentially, may be found to possess a much wider scope of applicability, mainly in the information processing fields and of pattern classification. Basically, such a framework introduces a normal way of tackling the problems in which the inaccuracy source is the unavailability of sharply definite criteria of class membership rather than the presence of random variables. If the function ($\mu_{\tilde{A}}$) is an affiliation function that gives to each element a degree of belonging to the set ($\tilde{A}$), then the value of the affiliation function is (1) for the elements belonging to the subset ($\mu_{\tilde{A}}(x) = 1$) and the value of the function (0) is for the elements that do not belong to the subset ($\mu_{\tilde{A}}(x) = 0$)*i.e.* :

$$\mu_{\tilde{A}}(X) = \begin{cases} 1, & \text{if } x \in A, \\ 0, & \text{if } x \notin A. \end{cases} \tag{1}$$

As for the fuzzy sets, its elements are not clearly defined because it allows the elements to partially belong to it and it has a affiliation function ranging from (0, 1) Similar points to the concept expressed by the fuzzy set. The fuzzy number ($\tilde{A}$) is a fuzzy set whose membership function $\mu_{\tilde{A}}(X)$ satisfies the following conditions:

i. $\mu_{\tilde{A}}(X)$ is piecewise continuous.

ii. $\mu_{\tilde{A}}(X)$ Fuzzy subset is a convex.

iii. $\mu_{\tilde{A}}(X)$ is the normality for a fuzzy subset, implying that for at least one element x_0 the membership degree must be1, i.e. $\mu_{\tilde{A}}(x_0) = 1$.

Definition. A fuzzy number with membership function in the form:

$$\mu_{\tilde{A}}(x; a, m, b) = \begin{cases} \frac{x-a}{m-a}, & x \in [a, m), \\ 1, & x = m, \\ \frac{b-x}{b-m}, & x \in (m, b], \\ 0, & x \notin [a, b], \end{cases} \tag{2}$$

$\tilde{A} = (a, m, b)$ is called a triangular fuzzy number

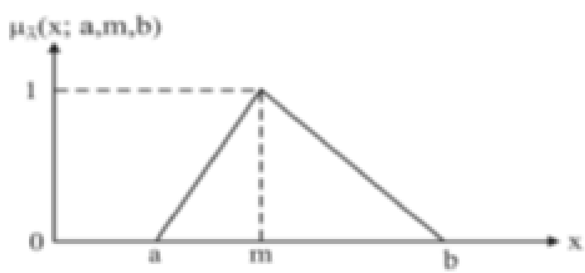

Figure 1: Triangular Fuzzy Number A

3. Basic algebraic operations

For the *union* $(A \cup B)$of fuzzy sets A,B he considered the definition:

$$\mu_{A \cup B}(X) = def Max(\mu_A(x), \mu_B(x)), \ \ \forall x \in X.$$

For the intersection $(A \cap B)$ of fuzzy sets A,B the definition:

$$\mu_{A\cap B}(X) = defMin(\mu_A(x), \mu_B(x)), \forall x \in X.$$

For the fuzzy numbers addition; let $\tilde{A} = (a_1, m_1, b_1)$ and $\tilde{B} = (a_2, m_2, b_2)$ he considered the definition:

$$\tilde{A} + \tilde{B} = (a_1 + a_2, m_1 + m_2, b_1 + b_2). \tag{3}$$

For the fuzzy numbers subtraction; let $\tilde{A} = (a_1, m_1, b_1)$ and $\tilde{B} = (a_2, m_2, b_2)$ he considered the definition:

$$\tilde{A} - \tilde{B} = (a_1 - b_2, m_1 - m_2, b_1 - a_2). \tag{4}$$

4. Fuzzy critical path analysis

When estimating time for activities in the business network marred by uncertainty, we move from the statistical theory that is adopted in the style of review and evaluation PERT to fuzzy theory and follow the fuzzy critical path method for estimating the total project time where each activity is estimated for triangular fuzzy times (a, m, b), when use the fuzzy critical path analysis we need an outline theorem used in the proposed and the important properties following:

Property (I): $\widetilde{FES}_j = Max\{\widetilde{FES}_i + \widetilde{FAT}_{ij}\} j \neq 1, i < j,$

Property (II): $FLC_i = Min\{FLC_j - FAT_{ij}\} i \neq n, i < j,$

Property (III): $FCP(P_k) = \sum 1 \leq i < j \leq nFAT_{ij}, P_k \in P.$

5. Algorithm the fuzzy critical path analysis

1. Identify project activities.
2. For all activities establish precedence relationships.
3. With respect to each activity estimate the fuzzy time.
4. Building the project network.
5. Find all the possible paths for the network by using property (III).
6. Calculate FAT_{ij} with respect in a network to each activity.
7. Let FES_1=$(0, 0, 0, 0)$ by using property (I), Calculate FES_j, j=$2, 3, \ldots, n$.
8. Let $FLC_n = FES_n$ and calculate FLC_i by using the property (II).
9. Get it the fuzzy critical path.
10. Get it the range of membership that the project can be finished at a specific time.

Notations

N: All nodes of the set in a network.
A_{ij} : The activity between nodes i and j.
FAT: The time of A_{ij} for the fuzzy activity.
FES_j: The earliest fuzzy time for node j.
FLC_j: The total slack fuzzy time for A_{ij}.
P: The whole paths found in a network.
FCP(P_k): The fuzzy completion time for path P_k in a network.

Ranking of fuzzy numbers

One of the most important methods used to compare the fuzzy numbers is the rank function $R : F(R) \to R$, in which F(R) It is the fuzzy set numbers defined on the real numbers. There are several functions, the most important of which is the ranking of trigonometric fuzzy numbers:

$$R\tilde{A} = \frac{a + 2m + b}{4} \tag{5}$$

6. α-cut

When we want to display an element $(x \in X)$that typically belongs to a fuzzy set A, we may demand its membership number to be greater than some threshold $\alpha \in [0, 1]$. The ordinary set of such element is the $\alpha - Cut\ A_\alpha$ of A,$A_\alpha = \{x \in X, \mu_A(x) \geqslant \alpha\}$. On also defines *strong* $\alpha - Cut\ A_\alpha = \{x \in X, \mu_A(x) > \alpha\}$. When the $\tilde{A} = (a, m, b)$ is triangular fuzzy number then $\alpha - Cut\ \alpha \in [0, 1][A_i(\alpha), A_u(\alpha)]$

$$A_i(\alpha) = a + (m - a)\alpha \tag{6}$$

$$A_u(\alpha) = b + (m - b)\alpha \tag{7}$$

$$= b - (b - m)\alpha \tag{8}$$

7. Application

Extend railway (Basra - Faw) project: This project is among the future projects of the General Iraqi Railways Company, as the importance of the project lies in:

1. Part of the dry canal that connects Europe with East Asia through (Turkey-Zakho-Dohuk-Mosul-Erbil-Kirkuk-Baquba-Baghdad-Kut-Amara-Basra-Faw).

2. Connecting Al-Fao Port to Abu Flus Port.

The investment company that presented the study for this project is the German company (Doorsh group), as it divided the project into three sectors, and each sector is subdivided into small projects, and thus we will get eight sub-projects distributed among three sectors and the following table represents the sectors and projects branching from them.

Table I: represents the sectors and projects branching

Sectors		Projects	Time \ Week	Sector length / km
Sector (1)	1-1	Tunnel construction work	224	10
	1-2	Earth works and railway construction	66	
Sector (2)	2-1	Ground works for the railway	68	45
	2-2	Ground works and building housing complexes	176	
	2-3	Extending a railway	60	
Sector (3)	3-1	Ground works for the railway	68	40
	3-2	Ground works and building housing complexes	350	
	3-3	Extending a railway	66	

Selection of the third sector, the first stage (ground works for the railway), which is at the time of the completion of this stage (68) weeks, as shown in the above table:

Table II: The priority of activities for Project (3-1)

No	Activities		*Prior Activity*	Time\ week		
				a	*m*	*b*
1	Clean the site and weed out	A		16	20	23
2	Tear down buildings and concrete structures	B		12.8	16	20
3	Removing old paved roads	C		12.8	16	20
4	Removal and transmission of electrical and communication cables	D	A,B,C	12.8	16	21
5	Scrape the natural ground with a thickness of 30 cm	E	A,B,C	6.4	8	10
6	Cutting, straightening and leveling	F	C	9.6	12	15
7	Dig rainwater drainage channels	G	E	4.8	6	7
8	Vertical channels for rain water drainage	H	D,F	12.8	16	19
9	Supplying and installing mesh for earthen dams	I	D,F	12.8	16	20
10	Create Earth dam on the line	J	I	12.8	16	20
11	Lighting and electrical work	K	G,H	6.4	8	10

Step (1) Construct the project network.

Step (2) The possible paths for the network by using property (III) and equation (3).

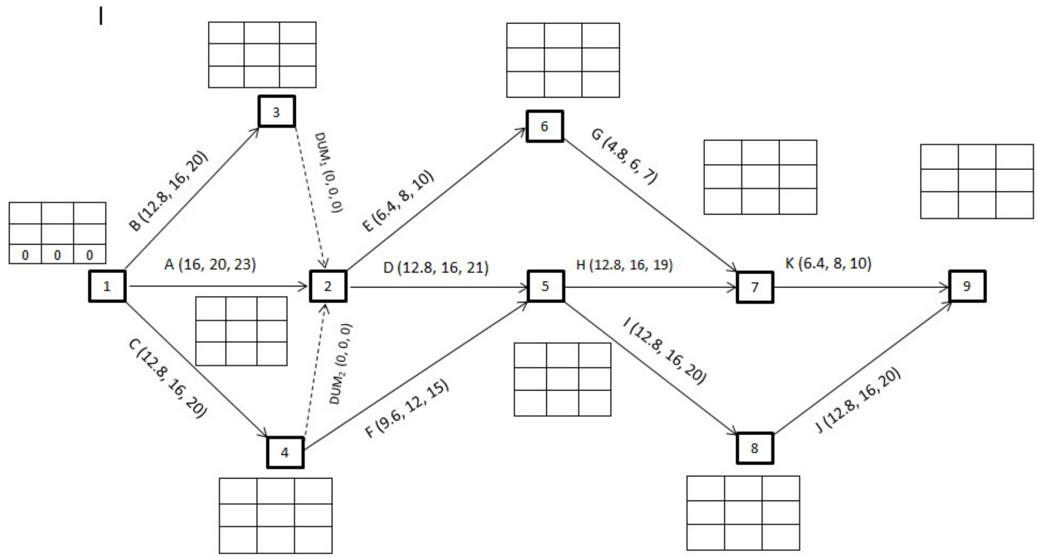

Figure 2: Network Project

$P_1 = FAT_{12}+FAT_{25}+FAT_{57}+FAT_{79} = (16, 20, 23)+(12.8, 16, 21)+(12.8, 16, 19)+ (6.4, 8, 10) = (48, 60, 73)$
$P_2 = FAT_{12}+FAT_{26}+FAT_{67}+FAT_{79} = (16, 20, 23)+(6.4, 8, 10)+(4.8, 6, 7)+ (6.4, 8, 10) = (33.6, 42, 50)$
$P_3 = FAT_{12}+FAT_{25}+FAT_{58}+FAT_{89} = (16, 20, 23)+(12.8, 16, 21)+(12.8, 16, 20)+ (12.8, 16, 20) = (54.4, 68, 84)$
$P_4 = FAT_{13} + FAT_{32} + FAT_{25} + FAT_{57} + FAT_{79} = (12.8, 16, 20) + (0, 0, 0) + (12.8, 16, 21) + (12.8, 16, 19) + (6.4, 8, 10) = (44.8, 56, 70)$
$P_5 = FAT_{13} + FAT_{32} + FAT_{26} + FAT_{67} + FAT_{79} = (12.8, 16, 20) + (0, 0, 0) + (6.4, 8, 10) + (4.8, 6, 7) + (6.4, 8, 10) = (30.4, 38, 47)$
$P_6 = FAT_{13} + FAT_{32} + FAT_{25} + FAT_{58} + FAT_{89} = (12.8, 16, 20) + (0, 0, 0) + (12.8, 16, 21) + (12.8, 16, 20) + (12.8, 16, 20) = (51.2, 64, 81)$
$P_7 = FAT_{14} + FAT_{42} + FAT_{25} + FAT_{57} + FAT_{79} = (12.8, 16, 20) + (0, 0, 0) + (12.8, 16, 21) + (12.8, 16, 19) + (6.4, 8, 10) = (44.8, 56, 70)$
$P_8 = FAT_{14} + FAT_{42} + FAT_{25} + FAT_{58} + FAT_{89} = (12.8, 16, 20) + (0, 0, 0) + (12.8, 16, 21) + (12.8, 16, 20) + (12.8, 16, 20) = (51.2, 64, 81)$
$P_9 = FAT_{14} + FAT_{42} + FAT_{26} + FAT_{67} + FAT_{79} = (12.8, 16, 20) + (0, 0, 0) + (6.4, 8, 10) + (4.8, 6, 7) + (6.4, 8, 10) = (30.4, 38, 47)$
$P_{10} = FAT_{14} + FAT_{45} + FAT_{58} + FAT_{89} = (12.8, 16, 20) + (9.6, 12, 15) + (12.8, 16, 20) + (12.8, 16, 20) = (48, 60, 75)$
$P_{11} = FAT_{14} + FAT_{45} + FAT_{57} + FAT_{79} = (12.8, 16, 20) + (9.6, 12, 15) + (12.8, 16, 19) + (6.4, 8, 10) + (6.4, 8, 10) = (41.6, 52, 64)$
Step (3) By using property (I) and equation (3) find the earliest fuzzy time for node j.

$FES_{12} = (0,0,0),\ FES_{13} = (0,0,0),\ FES_{14} = (0,0,0)$

$$FES_2 = Max \left\{ \begin{array}{c} FES_{12} + FAT_{12} = (0,0,0) + (16,20,23) \\ FES_{13} + FAT_{13} = (0,0,0) + (12.8,16,20) \\ FES_{14} + FAT_{14} = (0,0,0) + (16,20,23) \end{array} \right\} = (16,20,23)$$

$FES_3 = FES_{13} + FAT_{13} = (0,0,0) + (12.8,16,20) = (12.8,16,20)$

$FES_4 = FES_{14} + FAT_{14} = (0,0,0) + (12.8,16,20) = (12.8,16,20)$

$$FES_5 = Max \left\{ \begin{array}{c} FES_2 + FAT_{25} = (16,20,23) + (12.8,16,21) \\ FES_4 + FAT_{45} = (12.8,16,20) + (9.6,12,15) \end{array} \right\} = (28.8,36,44)$$

$FES_6 = FES_2 + FAT_{26} = (16,20,23) + (6.4,18,10) = (22.4,28,33)$

$$FES_7 = Max \left\{ \begin{array}{c} FES_5 + FAT_{57} = (28.8,36,44) + (12.8,16,19) \\ FES_6 + FAT_{67} = (22.4,28,33) + (4.8,6,7) \end{array} \right\} = (41.6,52,63)$$

$FES_8 = FES_5 + FAT_{58} = (28.8,36,44) + (12.8,16,20) = (41.6,52,64)$

$$FES_9 = Max \left\{ \begin{array}{c} FES_7 + FAT_{79} = (41.6,52,63) + (6.4,8,10) \\ FES_8 + FAT_{89} = (41.6,52,64) + (12.8,16,20) \end{array} \right\} = (54.4,68,84)$$

Step (4) By using property (II) and equation (4) find the fuzzy latest compilation of node i:

$FLC_9 = FES_9 = (54.4,68,84)$

$FLC_8 = FLC_9 - FAT_{89} = (54.4,68,84) - (12.8,16,20) = (34.4,52,71.2)$

$FLC_7 = FLC_9 - FAT_{79} = (54.4,68,84) - (6.4,8,10) = (44.4,60,77.6)$

$FLC_6 = FLC_7 - FAT_{67} = (44.4,60,77.6) - (4.8,6,7) = (37.4,54,72.8)$

$$FLC_5 = Min \left\{ \begin{array}{c} FLC_7 - FAT_{57} = (44.4,60,77.6) - (12.8,16,19) \\ FLC_8 - FAT_{58} = (34.4,52,71.2) - (12.8,16,20) \end{array} \right\}$$
$$= (14.4,36,58.4)$$

$$FLC_2 = Min \left\{ \begin{array}{c} FLC_6 - FAT_{26} = (37.4,54,72.8) - (6.4,8,10) \\ FLC_5 - FAT_{25} = (14.4,36,58.4) - (12.8,16,21) \end{array} \right\}$$
$$= (-6.6,20,45.6)$$

$$FLC_4 = Min \left\{ \begin{array}{c} FLC_2 - FAT_{42} = (-6.6,20,45.6) - (0,0,0) \\ FLC_5 - FAT_{45} = (14.4,36,58.4) - (9.6,12,15) \end{array} \right\}$$
$$= (-6.6,20,45.6)$$

$FLC_3 = FLC_2 - FAT_{32} = (-6.6,20,45.6) - (0,0,0) = (-6.6,20,45.6)$

$$FLC_1 = Min \left\{ \begin{array}{c} FLC_2 - FAT_{12} = (-6.6,20,45.6) - (16,20,23) \\ FLC_3 - FAT_{13} = (-6.6,20,45.6) - (12.8,16,20) \\ FLC_4 - FAT_{14} = (-6.6,20,45.6) - (12.8,16,20) \end{array} \right\}$$
$$= (-29.6,0,29.6)$$

$FF_1 = FLC_1 - FES_1 = (-29.6,0,29.6) - (0,0,0) = (-29.6,0,29.6)$

$FF_2 = FLC_2 - FES_2 = (-6.6,20,45.6) - (16,20,23) = (-29.6,0,29.6)$

$FF_3 = FLC_3 - FES_3 = (-6.6,20,45.6) - (12.8,16,20) = (-26.6,4,32.8)$

$FF_4 = FLC_4 - FES_4 = (-6.6,20,45.6) - (12.8,16,20) = (-26.6,4,32.8)$

$FF_5 = FLC_5 - FES_5 = (14.4,36,58.4) - (28.8,36,44) = (-29.6,0,29.6)$

$FF_6 = FLC_6 - FES_6 = (37.4,54,72.8) - (22.4,28,33) = (4.4,26,50.4)$

$FF_7 = FLC_7 - FES_7 = (44.4,60,77.6) - (41.6,52,63) = (-18.6,8,36)$

$FF_8 = FLC_8 - FES_8 = (34.4,52,71.2) - (41.6,52,64) = (-29.6,0,29.6)$

$FF_9 = FLC_9 - FES_9 = (54.4,68,84) - (54.4,68,84) = (-29.6,0,29.6)$

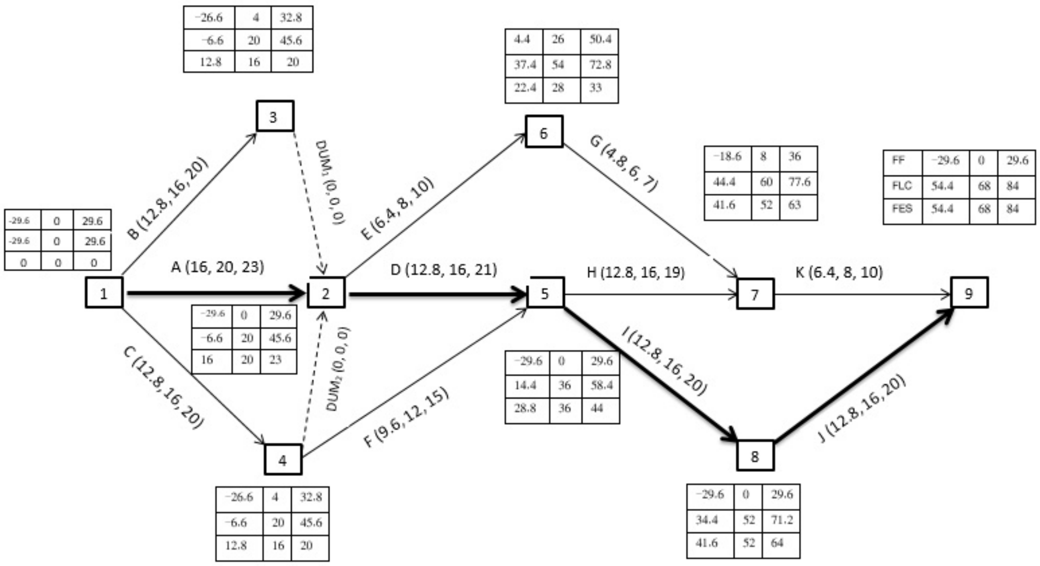

Figure 3: Network Project Includes (FES, FLC, FF) and FCP

Note that the median value for critical activities is (M=0) (FF1, FF2, FF5, FF8, FF9) and the fuzzy critical path is FCP=(A, D, I, J).

8. Calculation of relationship and $\alpha - Cut$:

$$F\tilde{A}T_{12} = (16, 20, 23),\ \mu_{D12}(X) = \begin{cases} \frac{X-16}{4}, & 16 \leq X \leq 20, \\ 1, & X = 20, \\ \frac{23-X}{3}, & 20 \leq X \leq 23. \end{cases}$$

The duration of the activity is (A = 20) which is the confirmed period and everywhere we further for it, it less the relationship, So the clear value determined by the $\alpha - Cut$ for activity (A), $\alpha - Cutof12 = [4\alpha + 16, 23 - 3\alpha]$ By fixing the value (α) of the information available to the decision maker $F\tilde{A}T_{25} =$

$$(12.8, 16, 21),\ \mu_{D25}(X) = \begin{cases} \frac{X-12.8}{3.2}, & 12.8 \leq X \leq 16, \\ 1, & X = 16, \\ \frac{21-X}{5}, & 16 \leq X \leq 21. \end{cases}$$

The duration of the activity is (D = 16) which is the confirmed period and everywhere we further for it, it less the relationship, So the clear value determined by the $\alpha - Cut$ for activity (D):

$\alpha - Cutof25 = [3.2\alpha + 12.8, 21 - 5\alpha]$. By fixing the value ($\alpha$) of the information available to the decision maker

$$F\tilde{A}T_{58} = (12.8, 16, 20),\ \mu_{D58}(X) = \begin{cases} \frac{X-12.8}{3.2}, & 12.8 \leq X \leq 16, \\ 1, & X = 16, \\ \frac{20-X}{4}, & 16 \leq X \leq 20. \end{cases}$$

The duration of the activity is (I = 16) which is the confirmed period and everywhere we further for it, it less the relationship, So the clear value determined by the $\alpha - Cut$ for activity (I):

$\alpha - Cutof58 = [3.2\alpha + 12.8, 20 - 4\alpha]$

By fixing the value (α) of the information available to the decision maker.

$$F\tilde{A}T_{89} = (12.8, 16, 20)$$

$$\mu_{D89}(X) = \begin{cases} \frac{X-12.8}{3.2}, & 12.8 \leq X \leq 16, \\ 1, & X = 16, \\ \frac{20-X}{4}, & 16 \leq X \leq 20. \end{cases}$$

The duration of the activity is (J = 16) which is the confirmed period and everywhere we further for it, it less the relationship, So the clear value determined by the $\alpha - Cut$ for activity (J):

$\alpha - Cutof58 = [3.2\alpha + 12.8, 20 - 4\alpha]$

By fixing the value (α) of the information available to the decision maker.

Project end time:

$$\mu_{D19}(X) = \begin{cases} \frac{X-54.4}{13.6} & 54.4 \leq X \leq 68, \\ 1 & X = 68, \\ \frac{84-X}{16} & 68 \leq X \leq 84 \end{cases}$$

$\alpha - CutofD_{19} = [13.6\alpha + 68, 84 - 16\alpha]$.

By fixing the value (α) of the information available to the decision maker.

9. Conclusions

1- It is very oftentimes that information available for making the decision is uncertain and vague, in analyzing the evaluation of the project. Thus, it is difficult to get the exact data of the activity assessment. Accordingly, the traditional accuracy-based/normal oriented analysis of the project seems to be of less effectivity in transmitting the information available in such an inaccurate and fuzzy decision society or environment.

2- Therefore, using the fuzzy critical path method is more realistic than jealousy in fuzzy project network.

3- The fuzzy critical path of the project is (A, D, I, j) and these activities should be observed and followed up to include the project is not delayed.

4- The total expected time to complete the project will be between (54.4, 68, 84) weeks, which was calculated by using the fuzzy critical path method, which is identical to the length of the first fuzzy critical path when calculating the lengths of all paths, and this estimate of the expected time with this method It enables the decision-maker to follow two policies in the completion of the project that accommodate all the conditions that can be encountered without making mistakes in incomplete accounts, and it also allows him to obtain a clear value in the amount determined by the level of the cut (α).

5- By using the network analysis and fuzzy set we show that the compilation time to complete the project under study is equal to (68) weeks the same as the company determinant.

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