A comment on weakly πg-closed sets in topological spaces

J.B. Toranagatti
Department of Mathematics
Karnatak University's Karnatak College
Dharwad-580 001
India
jagadeeshbt2000@gmail.com

Abstract. In this paper, we show that the notions of πgp-closed sets and weakly πg-closed sets are equivalent.
Keywords: π-open sets, preclosed sets, πgp-closed sets, weakly πg-closed sets.

1. Introduction

Mashhour et al. [2] introduced the notion of preopen sets and obtained their properties. In 1968, Zaitsav [5] introduced the concept of π-open sets in topological spaces. Many results had been obtained by using the concept of π-open sets.

In 2004, J. H Park [3] introduced and studied the concept of πgp-closed sets. Recently, O. Ravi et al. [4] discussed and established the concept of weakly πg-closed sets in topological spaces.

In this paper, we show that the concept of πgp-closed set and a weakly πg-closed set are same.

2. Preliminaries

Definition 2.1. *Let (X, τ) be a topological space. A subset A of X is said to be preclosed* [2] *if $cl(int(A)) \subseteq A$.*

Definition 2.2 ([5])**.** *Let (X, τ) be a topological space. A subset A of X is said to be π-open if A is the finite union of regular open sets.*

Definition 2.3 ([3])**.** *Let (X, τ) be a topological space. A subset A of X is said to be πgp-closed if $pcl(A) \subseteq G$ whenever $A \subseteq U$ and U is π-open in X.*

Definition 2.4 ([4])**.** *Let (X, τ) be a topological space. A subset A of X is said to be weakly πg-closed if $cl(int(A)) \subseteq G$ whenever $A \subseteq U$ and U is π-open in X.*

Lemma 2.1 ([1])**.** *Let (X, τ) be a topological space and $A \subseteq X$. Then, $pcl(A) = A \cup cl(int(A))$.*

3. Main result

Theorem 3.1. *Let (X, τ) be a topological space and $A \subseteq X$. Then, the following are equivalent:*

(i) A is πgp-closed.

(ii) A is weakly πg-closed.

Proof. (i)⇒(ii). Let $A \subseteq X$ be a πgp-closed set and U be an π-open set containing A. By Definition 2.3, pcl(A)⊆U. Using Lemma 2.5, A∪cl(int(A))⊆U which implies cl(int(A))⊆U.

This shows that A is weakly πg-closed.

(ii)⇒(i). Let A⊆X be a weakly πg-closed set such that A⊆ V and V is π-open set in X. Then by Definition 2.4, cl(int(A))⊆V. Using Lemma 2.1, we have pcl(A)=A∪cl(int(A))⊆A∪V=V.

Therefore, pcl(A)⊆V. Hence, A is πgp-closed. □

4. Illustrative examples

In this section, we give some examples to confirm our claim.

Example 4.1. Let X = {a,b,c} and τ = {X, ϕ, {a}, {b}, {a,b}, {a,c}}, then

$$\begin{aligned} O(X) &= \{X, \phi, \{a\}, \{b\}, \{a,b\}, \{a,c\}\}, \\ C(X) &= \{X, \phi, \{b\}, \{c\}, \{b,c\}, \{a,c\}\}, \\ \pi O(X) &= \{X, \phi, \{b\}, \{a,c\}\}, \\ \pi GPC(X) &= P(X) = W\pi GC(X). \end{aligned}$$

Therefore, πGPC(X) = WπGC(X).

Example 4.2. Let X = {a,b,c} and τ = {X, ϕ, {a}, {b,c}}, then

$$\begin{aligned} O(X) &= \{X, \phi, \{a\}, \{b,c\}\}, \\ C(X) &= \{X, \phi, \{a\}, \{b,c\}\}, \\ \pi O(X) &= \{X, \phi, \{a\}, \{b,c\}\}, \pi GPC(X) = P(X) = W\pi GC(X). \end{aligned}$$

Therefore, πGPC(X) = WπGC(X).

Example 4.3. Let X = {a,b,c,d} and τ = {X, ϕ, {a}, {b}, {a,b}, {a,b,c}}, then O(X) = {X, ϕ, {a}, {b}, {a,b}, {a,b,c}},
C(X) = {X, ϕ, {d}, {c,d}, {a,c,d}, {b,c,d}},
πO(X) = {X, ϕ, {a}, {b}, {a,b}},
πGPC(X) = {X, ϕ, {c}, {d}, {a,c}, {a,d}, {b,c}, {b,d}, {c,d}, {a,b,c}, {a,b,d}, {a,c,d}, {b,c,d}},
WπGC(X) = {X, ϕ, {c}, {d}, {a,c}, {a,d}, {b,c}, {b,d}, {c,d}, {a,b,c}, {a,b,d}, {a,c,d}, {b,c,d}}.

Therefore, πGPC(X) = WπGC(X).

Example 4.4. Let X = {a,b,c,d} and τ = {X, ϕ, {a,b}, {c,d}}, then

$$\begin{aligned} O(X) &= \{X, \phi, \{a,b\}, \{c,d\}\}, \\ C(X) &= \{X, \phi, \{a,b\}, \{c,d\}\}, \\ \pi O(X) &= \{X, \phi, \{a,b\}, \{c,d\}\}, \\ \pi GPC(X) &= P(X) = W\pi GC(X). \end{aligned}$$

Therefore, πGPC(X) = WπGC(X).

Example 4.5. Let X = {a,b,c,d,e} and τ = {X, ϕ, {a}, {b}, {c}, {a,b}, {a,c}, {b,c}, {a,b,c}}, then O(X) = {X, ϕ, {a}, {b}, {c}, {a,b}, {a,c}, {b,c}, {a,b,c}},

C(X) = {X, ϕ, {d,e}, {a,d,e}, {b,d,e}, {c,d,e}, {a,b,d,e}, {a,c,d,e}, {b,c,d,e}},

πO(X) = {X, ϕ, {a}, {b}, {c}, {a,b}, {a,c}, {b,c}, {a,b,c}},

πGPC(X) = {X, ϕ, {d}, {e}, {a,d}, {a,e}, {b,d}, {b,e}, {c,d}, {c,e}, {d,e}, {a,b,e}, {a,b,d}, {a,c,d}, {a,c,e}, {a,d,e}, {b,c,d}, {b,c,e}, {b,d,e}, {c,d,e}, {a,b,c,d}, {a,c,d,e}, {b,c,d,e}, {a,b,d,e}, {a,b,c,e}},

WπGC(X) = {X, ϕ, {d}, {e}, {a,d}, {a,e}, {b,d}, {b,e}, {c,d}, {c,e}, {d,e}, {a,b,e}, {a,b,d}, {a,c,d}, {a,c,e}, {a,d,e}, {b,c,d}, {b,c,e}, {b,d,e}, {c,d,e}, {a,b,c,d}, {a,c,d,e}, {b,c,d,e}, {a,b,d,e}, {a,b,c,e}}.

Therefore, πGPC(X) = WπGC(X).

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