A corollary that provides seat arrangements for even numbers of seats

H. Yingtaweesittikul*
Department of Mathematics
Faculty of Science
Chiang Mai University
Chiang Mai
Thailand
hatairat.y@gmail.com

V. Longani
Department of Mathematics
Faculty of Science
Chiang Mai University
Chiang Mai
Thailand
vites.l@cmu.ac.th

Abstract. Let $1, 2, 3, \ldots, n$ be n students and $s_1, s_2, s_3, \ldots, s_n$ be n row seats. A seat is arranged for each student on each day of the following n days. It is required that each student shall have different seat every day of the n days. Also, it is required that for these n day every student shall have a chance to sit next to every other student on one of his side and shall have another chance to sit next to every other student on the other of his side. In this paper, it is shown that such arrangements are possible when the number of students is even. Also, an algorithm for such arrangements is provided.

Keywords: combinatorics, seat arrangement.

1. Introduction

Let $1, 2, 3, \ldots, n$ be n students and $s_1, s_2, s_3, \ldots, s_n$ be n row seats. A seat is arranged for each student on each day of the following n days. It is required that each student shall have different seat every day of the n days. Also, it is required that for these n day every student shall have a chance to sit next to every other student on one of his side and shall have another chance to sit next to every other student on the other of his side. For example, when $n = 4$ the following is one possible way of the arrangement for 4 days.

∗. Corresponding author

	s_1	s_2	s_3	s_4
Day 1	1	2	3	4
Day 2	2	4	1	3
Day 3	3	1	4	2
Day 4	4	3	2	1

Another example of the arrangement is when $n = 6$.

	s_1	s_2	s_3	s_4	s_5	s_6
Day 1	1	2	3	4	5	6
Day 2	2	4	6	1	3	5
Day 3	3	6	2	5	1	4
Day 4	4	1	5	2	6	3
Day 5	5	3	1	6	4	2
Day 6	6	5	4	3	2	1

From the first example, we can see that the arrangements for Day 3, and Day 4 are simply the reverses of Day 2, and Day 1 respectively. Also, from the second example, the arrangements for Day 4, Day 5, Day 6 are the reverses of Day 3, Day 2, and Day 1 respectively.

In [1], it is shown that the arrangement is possible when $n = p - 1$ for every prime number $p \geq 3$ and the algorithm (Seat Arrangement Algorithm or SAA) for the arrangement is also provided.

Theorem 1.1 ([1])**.** *The SAA is possible when the number of students* $n = p-1$ *for any given prime number* $p \geq 3$.

When n is odd numbers, it is not difficult to verify that it is not possible to arrange the seats if all above conditions are required.

The above theorem shows that we can always have the required seat arrangement if $n = p-1$ where $p \geq 3$ is prime number. We note that $n = p-1$ is even, so there is a question that whether it is still possible to have the required seat arrangements for some other even values of n. Consider the following arrangement when $n = 8$ which satisfies the conditions of seat arrangement.

s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8
1	2	3	4	5	6	7	8
2	4	1	3	6	8	5	7
3	8	4	7	2	5	1	6
4	6	2	8	1	7	3	5
8	7	6	5	4	3	2	1
7	5	8	6	3	1	4	2
6	1	5	2	7	4	8	3
5	3	7	1	8	2	6	4

We note that $n = 8$ is not of the form $n = p - 1$, since 9 is not prime. In fact, in section 2, we shall show that seat arrangements can be arranged for all even values of $n \geq 2$.

2. Seat arrangements when n are even numbers

When $n = 8$, for example, one can try to arrange the seats and shall quickly find out that, without an algorithm, it is not an obvious work to do the arrangement. We have tried to find practical algorithms to serve the purpose, and have found that the simple algorithm can follow from Theorem 2.1.

Theorem 2.1 ([2]). *The graph K_{2n+1} is the sum of n spanning cycles.*

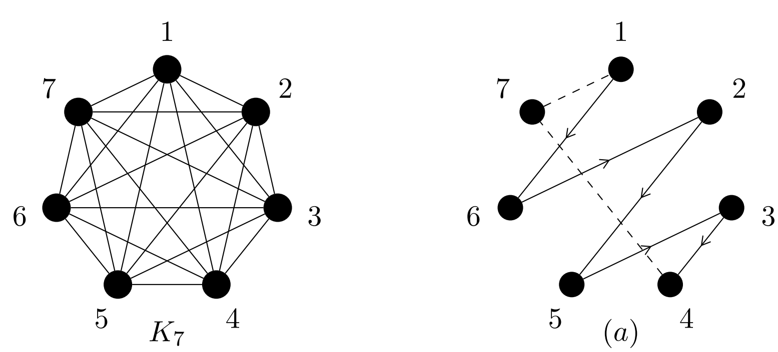
From, Theorem 2.1, we can have Corollary 2.1 which guarantees the possibility of seat arrangements when n are even numbers.

Corollary 2.1. *The seat arrangements are possible for all numbers of students that are positive even numbers.*

Proof. From the proof of Theorem 2.1, there are $2n + 1$ numbers of points : $v_1, v_2, v_3, \ldots, v_{2n+1}$. The proof shows the constructions of n paths: $P_1, P_2, \ldots, P_n$ from $2n$ points $v_1, v_2, v_3, \ldots, v_{2n}$. Each path consists of $2n - 1$ lines, and so all paths together have $n(2n-1) = \binom{2n}{2}$ lines, i.e. all lines that join between $2n$ points. All these lines are different, and, according to the proof of Theorem 2.1, the j-th points of P_i are different. We can see that $P_1, P_2, \ldots, P_n$ can represent seat arrangement for Day 1, Day 2, ... , Day n respectively. The reverse paths of $P_1, P_2, , P_n$ can represent the arrangements for Day $n+1$, Day $n+2$, ... , Day $2n$. □

For example, when $2n = 6$, we use Theorem 2.1 to construct 3 paths of K_7, see Fig. 2.1 which is from [2].

1	6	2	5	3	4
2	1	3	6	4	5
3	2	4	1	5	6



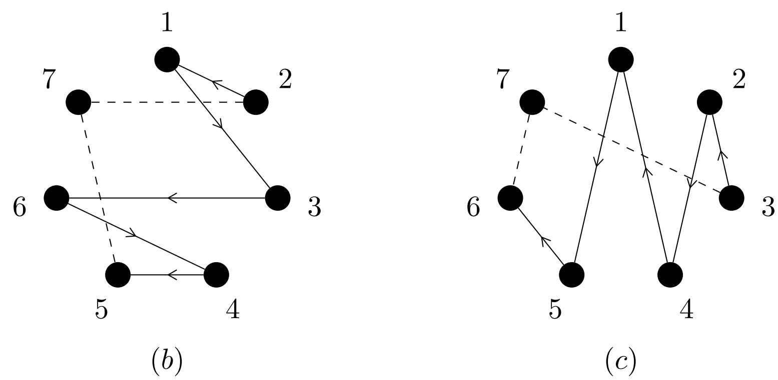


Fig. 2.1

These 3 paths provide seat arrangements for Day 1, Day 2, Day 3. Together with the reverses of the three paths, we have the following seat arrangements for 6 days:

	s_1	s_2	s_3	s_4	s_5	s_6
Day 1	1	6	2	5	3	4
Day 2	2	1	3	6	4	5
Day 3	3	2	4	1	5	6
Day 4	4	3	5	2	6	1
Day 5	5	4	6	3	1	2
Day 6	6	5	1	4	2	3

3. Algorithm for seat arrangements when number of students is even

We shall simplify the path construction in the proof of Theorem 2.1 in obtaining the seat arrangement algorithm. We only need to construct Day 1 arrangement, the other day arrangements shall readily follow.

For example, consider the above seat arrangement when $2n = 6$. First, consider the arrangements for s_1, s_3, and s_5.

s_1	s_3	s_5
1	2	3
2	3	4
3	4	5
4	5	6
5	6	1
6	1	2

and consider the arrangements for s_2, s_4, and s_6.

s_2	s_4	s_6
6	5	4
1	6	5
2	1	6
3	2	1
4	3	2
5	4	3

Each column of the arrangement is simply related to clockwise circular arrangement of 1, 2, 3, 4, 5, and 6, see Fig. 3.1.

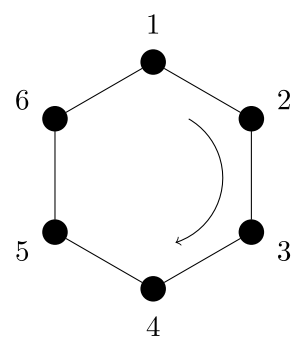


Fig. 3.1

We can generalize the idea to arrange for $2n$ students $1, 2, 3, \ldots, 2n$. Note that we only need to construct the seat arrangement for Day 1 and all the arrangements for other days shall follow by using the clockwise circular arrangements for each column, see Fig. 3.2, as mentioned in the case $2n = 6$ above.

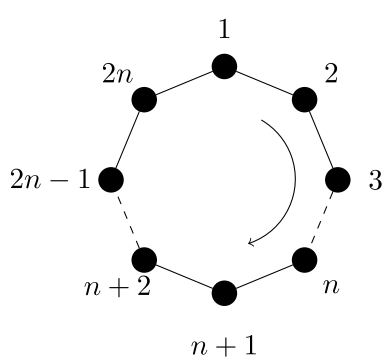


Fig. 3.2

The algorithm for general cases is as follow. For Day 1 arrangement, let $1, 2, 3, \ldots, n$ sit on seats $s_1, s_3, s_5, \ldots, s_{2n-1}$ respectively, and let $2n, 2n-1, 2n-$

$2, \ldots, n+1$ sit on seat $s_2, s_4, s_6, \ldots, s_{2n}$ respectively. From this, the arrangements for Day 2, Day 3, …, Day $2n$ shall follow.

s_1	s_3	s_5	$\ldots$	s_{2n-3}	s_{2n-1}
1	2	3	$\ldots$	$n-1$	n
2	3	4	$\ldots$	n	$n+1$
3	4	5	$\ldots$	$n+1$	$n+2$
$\vdots$	$\vdots$	$\vdots$		$\vdots$	$\vdots$
$2n-1$	$2n$	1	$\ldots$	$n-3$	$n-2$
$2n$	1	2	$\ldots$	$n-2$	$n-1$

s_2	s_4	s_6	$\ldots$	s_{2n-2}	s_{2n}
$2n$	$2n-1$	$2n-2$	$\ldots$	$n+2$	$n+1$
1	$2n$	$2n-1$	$\ldots$	$n+3$	$n+2$
2	1	$2n$	$\ldots$	$n+4$	$n+3$
$\vdots$	$\vdots$	$\vdots$		$\vdots$	$\vdots$
$2n-2$	$2n-3$	$2n-4$	$\ldots$	n	$n-1$
$2n-1$	$2n-2$	$2n-3$	$\ldots$	$n+1$	n

For example, consider K_9 or $K_{2(4)+1,}$ we can construct 4 paths (and another 4 reverse paths) by using the above algorithm.

For Day 1 arrangement. let 1,2,3,4 sit on seats s_1, s_3, s_5, s_7 respectively, and let 8, 7, 6, 5 sit on seats s_2, s_4, s_6, s_8 respectively. After that, we can arrange seats, using the above algorithm, as follow.

s_1	s_3	s_5	s_7
1	2	3	4
2	3	4	5
3	4	5	6
4	5	6	7
5	6	7	8
6	7	8	1
7	8	1	2
8	1	2	3

s_2	s_4	s_6	s_8
8	7	6	5
1	8	7	6
2	1	8	7
3	2	1	8
4	3	2	1
5	4	3	2
6	5	4	3
7	6	5	4

Combining the above arrangements we obtain seat arrangement for $2n = 8$ students.

s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8
1	8	2	7	3	6	4	5
2	1	3	8	4	7	5	6
3	2	4	1	5	8	6	7
4	3	5	2	6	1	7	8
5	4	6	3	7	2	8	1
6	5	7	4	8	3	1	2
7	6	8	5	1	4	2	3
8	7	1	6	2	5	3	4

References

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