ITALIAN JOURNAL OF PURE AND APPLIED MATHEMATICS – N. 45–2021 (894–903)

Bounds on minimum distance for linear codes over $GF(q)$

Fardos N. Abdullah
Department of Mathematics
College of Computer Sciences and Mathematics
University of Mosul
Mosul
Iraq
math3a9c@yahoo.com

Nada Yassen Kasm Yahya*
Department of Mathematics
College of Education for Pure Sciences
University of Mosul
Mosul
Iraq
drnadaqasim1@gmail.com

Abstract. Let $[n, k, d; q]$-codes be linear codes of length n, dimension k and minimum hamming distance d over $GF(q)$. Let $m_{47}(n, k)$ be the maximum possible minimum Hamming distance of a linear $[n, k, d; 47]$ - codes for given values of n and k. In this paper 21new linear codes over GF(47) are constructed, and a table of $m_{47}(n, k) k \leq 47$, $n \leq 6267$ is presented. First: we construct three Griesmer $[n, 3, d]_{47}-$ codes. Second: Also, a (k, r) - arc K corresponds to a projective $[k, n, d]_q$-code of length k, dimension n, and minimum distance $d = k - n$.
Keywords: bounds on $[n, k, d]q$-code, Griesmer codes, projective geometries, MDS-code, NMDS-code

1. Introduction

Let GF(q) denote the Galois field of q elements, and let $V(n, k)$ denote the vector space of all ordered n-tuples over $GF(q)$. A linear codes C of length n and dimension k over $GF(q)$ is a k-dimensional subspace of $V(n, q)$. Such a code is called $[n, k, d; q]$-code if its minimum Hamming distance is d.

A central problem in coding theory is that of optimizing one of the parameters n,k and d for given values of the other two. Two versions are:

Problem 1: Find $m_q(n, k)$, the largest value of d for which there exists an $[n, k, d; q]-$code.

Problem 2: Find $n_q(k, d)$, the smallest value of n for which there exists an $[n, k, d; q]$-code.

∗. Corresponding author

A code which achieves one of these two values is called optimal.

For the case of linear codes over GF(47), problem 2 has been solved for $k \leq 3$ (see [11]). In this paper consider the next four dimensions. 21new linear codes over GF(47) are constructed, the nonexistence of sixteen linear codes is proved and a table of $m_q(n,k)k \leq 47, n \leq 6267$ is proved.

A number of researcher have studied the nonexistence of some (k,r)-arcs in $PG(2,q)$ have been proved by (Daskalov, 2004, 2008, 2013) in and (Najem, 2010), (Hirschfield, 2007), (Salam, 2018), (Khalied, 2013), when $q = 7, 17, 19, 23, 29, 31, 37, 41, 43$. Complete arcs have important connections with a number of other objects, see [2,3,4,5,6,7,8,9,10,12,13,14,15,16,19] and the references therein.

Definition 1.1. *A(k,r)-arc in PG(2,q) is a set K of k points, no r_+1 of which are collinear, but with at least one set of r points collinear. When $r = 2, a(k,2)$-arc is called a k-arc ([17]). The maximum size of a $(k,r)_-arc$ in $PG(2,q)$ is denoted by $m_r(2,q)$.*

Definition 1.2. *An $\{l,n\}-$blocking set S in $PG(2,q)$ is a set of I points such that every line of $PG(2,q)$ intersects S in at least n points, and there is a line intersecting S in exactly n points.*

Definition 1.3. *Let M be a set of point in any plane. An i-secant is a line meeting M in exactly I points. Define t_i as the number of i-secants to a set M.*

The t_i satisfy the next three Diophantine equations in any projective plane, which are known as the standard equations [5].

Definition 1.4 (Griesmer [18])**.** *The parameters of a linear $[n,k,d]_q$ code satisfy Griesmer bound $n \geq \sum_{i=0}^{k-1} \lceil d/q^i \rceil$.*

Lemma 1.1. *For any set of k points in $PG(2,q)$ the following hold:*
1. $\sum_{i=0}^{q+1} t_i = q^2 + q + 1$,
2. $\sum_{i=1}^{q+1} it_i = k(q+1)$,
3. $\sum_{i=2}^{q+1} i(i-1)t_i = b(b-1)$.

Theorem 1.1. *Let K be a (k,r)- arc in $PG(2,q)$ where q is prime:*

1. *If $r \leq (q+1)/2$ then $m_r(2,q) \leq (r-1)q+1$.*
2. *If $r \leq (q+3)/2$ then $m_r(2,q) \leq (r-1)q + r - (q+1)/2$.*

For $r = (q+3)/2$ the bound is achieved by the Barlotti construction [3]*. In this paper we shall prove that if $r > (q+3)/2$ then $m_r(2,q) \leq (r-1)q + r - (q+3)/2$ for $q = 47$.*

Theorem 1.2. *Let K be a (k,r)- arc in PG(2,q) with $r > (q+3)/2$ and q=47. Than $m_r(2,q) \leq (r-1)q + r - -(q+3)/2$.*

Proof. Finding a maximum $((r-1)q+r-(q+1)/2, r)\_arc$ is equivalent to finding a $q^2+q+1-(r-1)q-r+(q_+1)/2, q+1-r$ -blocking set B. Since:

$$q^2+q+1-(r-1)q-r+(q+1)/2=(q+1)(2q+3\_2r)/2.$$

Then B is a $(q+1)(2q+3\_2r)/2, q+1\_r-$ blocking set. □

Theorem 1.3. *Let B be an $\{l,n\}-$ blocking set in $PG(2,q)(q')$. Where $n = q+1-rl = n(q+1)+(q+1)/2$, then each point of B has exactly $(q+3)/2$ lines If through.*

Let $T=(q^2_+1)(2q+3\_2r)/4(q+1-r)$ is a total number of $(q+1-r)-$secants.

Let t be the length of the longest secant for the blocking set B. If $t=q+1$ than B contains a line.

Theorem 1.4. *Let B be an $\{l,n\}-$-blocking set in $PG(2.q)$ that contains a line. If $(n-1,q)=1$, then: $B=1\geq q(n+1)$.*

It follows from this theorem that if $t=q+1$ then $B\geq q(q+2-r)$, which cannot occur since $r>(q+3)/2 \leftrightarrow 0>1-r+(q+1)/2 \rightarrow L=q^2+2q-rq> q^2+2q+1-rq-r+(q+1)/2=(q+1)(2q+3-2r)/2=B$.

If $3(q+1)/2-r<t\leq$ q then considering lines through a point on the longest secant but not in B. B must have at least $q(q+1-r)+t$ point. Now, we have a contradiction, since: $t>3(q+1)/2-r \rightarrow q(q+1--r)+t> q(q+1)-qr+3(q+1)/2-r=(q+1)(2q+3-2r)/2$.

Let $t_i=3\frac{48}{2}-r-i, i=0,1\ldots,\frac{q-1}{2}$.

Now, consider $(q+1-r)$-secants with the longest secant.

Let $t=t_i$. Suppose that p is a point on a t_i -secant L and P is not in B. Of the q other secants that P is on, let s of them be $(48-r)$- secants and $q-s$ be longer secants. Then:

$$B\geq 72+s(q+1-r)+(q--s)(q+2--r)=t_i+q(q+2-r)-s.$$

So, that $B-t_i=q(q+2-r)-q+i\geq q(q+2-r)-s$ making $s\geq q-i$, $T_q+1-r\geq t_i\frac{q_1}{2}+(q+1-ti)(q-i)=M_i$.

The inequality

$$M_i>T \tag{1}$$

is equivalent to $f(r)=2(q-2i+1)r^2-(3q^2-8iq+6q+4i^2\_8i+3)r+q^3+4q^2+3q-4iq^2+4i^2q-8iq+4i2-4i<0$.

It is clear that if $f((47+5)/2)<0$ and $f(47-1)<0$ for some values of I, then for these values condition (1) will be fulfilled for $r=(47+5)/2,\ldots,q-1$ and we shall have a contradiction

$$f(q+1/2)<0 \leftrightarrow 2(q-3)i^2--(q-3)^2i-q^2+4q+5<0 \tag{2}$$

$$f(q-1)<0 \leftrightarrow 8i^2-16i--q2+4q+5<0. \tag{3}$$

The solutions of inequalities (2) and (3) are as follows:
$i \in (\frac{(q-3)^2-\sqrt{q^4-4q^3\ -2q^2\ -52q+201}}{4\ (q-3)}. \frac{(q-3)^2+\sqrt{q^4-4q^3-2-52q+201}}{4(q-3)})$.
$i \in (1-\frac{1}{4}\sqrt{2q^2-8q+6}, 1+\frac{1}{4}\sqrt{2q^2-8q+6})$.
It is easy show that $1-\frac{1}{4}\sqrt{2q^2-8q+6}\ < 0$ and

$$\frac{(q-3)^2\ -\sqrt{q^4-4q^3-2q^2-52q+201}}{4(q-3)} < 0,$$

for $q \geq 7$.
Let

$$i_1 = 1+\frac{1}{4}\sqrt{2q^2-8q+6}, i_2 = \frac{(q-3)^2+\sqrt{q^4-4q^3-2q^2-52q+201}}{4(q-3)}v.$$

When $i_1 < i_2$, $q^4-4q^3-6q^2+4q+5 = (q-1)(q-5)(q+1)^2 \geq 0$ inequalities (2) and (3) are true when $0 \leq i < 1+\frac{1}{4}\sqrt{2q^2-8q+6}$.

Furthermore:

$$1+\frac{1}{4}\sqrt{2q^2-8q+6} \leq \frac{q-5}{2} \leftrightarrow (q-5)(q-19) \geq 0.$$

Hence $i_1 = 1+\frac{1}{4}\sqrt{2q^2-8q+6} \leq \frac{q-5}{2}$ for q ≥ 19.

Let $i = (q-3)/2$, in this case we shall use the standard equations for the set B. Let $x = t_{q-r+1}, y = t_{q-r+2}, z = t_{q-r+3}$, $a_1 = q-r+1, b1 = q-r+2, c_1 = q-r+3, a_2 = (q-r+1)(q-r), b_2 = (q-r+2)(q-r+1)$ and $c_2 = (q-r+3)(q-r+2)$.

Then the standard equation are $x+y+z = q^2+q+1, a_1x+b1y+c1z = (q+1)^2(q-r+3/2)$, $a_2x+b_2y+c2z = (q+1)(q-r+3/2)((q+1)(q-r+3/2)-1)$.

The unique solution of this system is $X = [q^2(4r-7)-4q(r^2-4r+3)+3]/8$, $Y = [4qr^2-4q(q+3)r+9q^2+12q+3]/4$, $Z = [q^2(4r-3)-4q(r-1)^2-1]/8$.

Let us consider the function $G(r) = 4qr^2-4q(q+3)r+9q^2+12q+3$.

This function increases when $(q+3)/2 < r < q$. therefore $q(r) \leq g(q-1) = -7q^2+28q+3 < 0$ if $q \geq 5$. So, $Y = t_{q-r+2} < 0$, a contradiction.

Let $i = (q-1)/2$. We shall use again the standard equation for the set B let $x = t_{q-r+1}, y = t_{q-r+2}, a = q-r+1, b = q-r+2, c = (q-r+1)$ and $d = (q-r+2)(q-r+1)$. Then the standard equation $Ax+by = (q+1)^2(q-r+3/2)$, $Cx+dy = (q+1)(q-r+3/2)((q+1)(q-r+3/2)-1)$. From the first two equations we obtain $y = -\frac{q+2q(1-r)-1}{2}, y = \frac{3q+2q(2-r)+1}{2}$. The difference between the left-hand side and right-hand side of the third equation now is $D(r) = qr^2-q(q+2)r+3q^2/4+q+1/4$.

The function D(r) is an increasing function when $(q+3)/2 < r < q$.

Hence, $D(q-1) = (-9q^2+16+1)/4 < 0$ when $q > (8+\sqrt{73})/9 \approx 1$ and so the system of standard equations no solutions.

On the basis of our observations up to now we can conclude that: $D_r(2,q) \leq (r-1)q+r-(q+3)/2$, $r > (q+3)/2$ and $7 \leq 17$.

If $q \geq 19$ then $i_1 \leq (q-5)/2$ (see(5)) and we have to check directly only the cases when $i_1 \leq i \leq \frac{q-5}{2}$. For $q = 47$ the results are as follow. Then $i_1 = 1+\sqrt{253} \approx 16.$ 96,(q-5) /2=21 and we check directly that inequality (1) is fulfilled for $i = 17, 18, 19, 20, 21$ and $r = 26, \ldots, 43$.

If $r = 44$ then $M21 = 1227$, $T = 1242$ and inequality (1) is not satisfied for $i = 21$, then $t_2 1 = 7$ and the respective system of standard equations in unknowns t_4, t_4, t_6 and t_7 has no solutions in nonnegative integers. More precisely, the standard equations are $t_4 + t_5 + t_6 + t_7 = 216$, $4t_4 + 5t_5 + 6t_6 + 7t_7 = 10368$, $12t_4 + 20t_5 + 30t_6 + 42t_7 = 46440$.

The solution of this system in term of t$_7$, $t_4 = -25380 - t_7, t_5 = 4168 + 3t_7, t_6 = -16092 - 3t_7$.

It follows now that $t_5 + t_6 = -11924$ and we have a contradiction.

If $r = 45$, then $M_{20} = 1268$, $M_{21} = 1230$, $T = 1288$ and inequality (1) is not satisfied for $i = 17, 18, 19, 20, 21$. If $i = 20$ then $t_{20} = 7$ and the respective system of standard equations in unknowns t_3, t_4, t_5, t_6 and t_7 has no solutions in nonnegative integers. More precisely, the standard equations are $t_3+t_4+t_5+t_6+t_7 = 168$, $3t_3+4t_4+5t_5+6t_6+7t_7 = 8064$, $6t_3+12t_4+20t_5+30t_6+42t_7 = 28056$.

The solution of this system in terms of t_6 and t_7 is $t_3 = -16548-t_6-3t_7, t_4 = 25872 + 3t_6 + 8t_7, t_5 = -9156 - 3t_6 - 6t7$, $t_4 + t_5 = -1192$ and we have a contradiction.

If $i = 21$ then $t_{21} = 6$ and the respective system of standard equations in unknowns t_3, t_4, t_5 and t_6, $t_3 + t_4 + t_5 + t_6 = 168$, $3t_3 + 4t_4 + 5t_5 + 6t_6 = 8064$, $6t_3 + 12t_4 + 20t_5 + 30t_6 = 28056$.

The solution of this system in terms of t_6, $t_3 = -16548 - t_6, t_4 = 2587 + 3t_6, t_5 = -9156 - 3t_6$ it follows now that $t_4 + t_5 = -6569$ and we have a contradiction.

If $r = 46$ then $M_{17} = 1377, M_{18} = 1344, M_{19} = 1309, M_{20} = 1272, M_{21} = 1233, T = 1380$ and inequality (1) is not satisfied for $i = 17, 18, 19, 20, 21$.

If i= 17 then t$_{17}$= 9 and the respective system of standard equations in unknowns t_2,t_3,t_4,t_5,t_6,t_7,t_8 and t_9 has no solutions in nonnegative integers. More precisely, the standard equations are $t_2 + t_3 + t_4 + t_5 + t_6 + t_7 + t_8 + t_9 = 120$, $2t_2 + 3t_3 + 4t_4 + 5t_5 + 6t_6 + 7t_7 + 8t_8 + 9t_9 = 5760$, $2t_2 + 6t_3 + 12t_4 + 20t_5 + 30t_6 + 42t_7 + 56t_8 + 72t_9 = 14280$

The solution of this system in terms of t_5, t_6, t_7, t_8, t_9 is $t_2 = -9420 - 5t_5 - 3t_6 - 6t_7 - 10t_8 - 15t_9, t_3 = 13560 + 3t_5 + 8t_6 + 15t_7 + 24t_8 + 35t_9, t_4 = -4020 - 3t_5 - 6t_6 - 10t_7 - 15t_8 - 12t_9$ it follows now that $t_3 + t_4 = -414$ and we have a contradiction.

If $i = 18$ then $t_{18} = 8$ and the respective system of standard equations in unknowns $t_2, t_3, t_4, t_5, t_6, t_7$ and t_8 has no solutions in nonnegative integers. More precisely, the standard equations are $t_2 + t_3 + t_4 + t_5 + t_6 + t_7 + t_8 = 120$, $2t_2 + 3t_3 + 4t_4 + 5t_5 + 6t_6 + 7t_7 + 8t_8 = 5760$, $2t_2 + 6t_3 + 12t_4 + 20t_5 + 30t_6 + 42t_7 + 56t_8 = 14280$.

The solution of this system in terms of t_5, t_6, t_7, t_8 is $t_2 = -9420-5t_5-3t_6-6t_7-10t_8$, $t_3 = 13560+3t_5+8t_6+15t_7+24t_8$, $t_4 = -4020-3t_5-6t_6-10t_7-15t_8$.

It follows now that $t_3 + t_4 = -1060$ and we have a contradiction.

If $i = 19$ then $t_{19} = 7$ and the respective system of standard equations in unknowns t_2, t_3, t_4, t_5, t_6 and t_7 has no solutions in nonnegative integers. More precisely, the standard equations are $t_2 + t_3 + t_4 + t_5 + t_6 + t_7 = 120$, $2t_2 + 3t_3 + 4t_4 + 5t_5 + 6t_6 + 7t_7 = 5760$, $2t_2 + 6t_3 + 12t_4 + 20t_5 + 30t_6 + 42t_7 = 14280$.

The solution of this system in terms of t_5, t_6, t_7 is $t_2 = -9420 - 5t_5 - 3t_6 - 6t_7$, $t_3 = 13560 + 3t_5 + 8t_6 + 15t_7+$, $t_4 = -4020 - 3t_5 - 6t_6 - 10t_7$. It follow now that $t_3 + t_4 = -904$ and we have a contradictions.

If $i = 20$ then $t_{20} = 6$ and the respective system of standard equations in unknowns t_2, t_3, t_4, t_6 and $t6$ has no solutions in nonnegative integers More precisely, the standard equations are $t_2 + t_3 + t_4 + t_5 + t_6 = 120$, $2t_2 + 3t_3 + 4t_4 + 5t_5 + 6t_6 = 5760$, $2t_2 + 6t_3 + 12t_4 + 20t_5 + 30t_6 = 14280$.

The solution of this system in terms of t_5, t_6 is $t2 = -9420 - 5t_5 - 3t_6, t_3 = 13560 + 3t_5 + 8t_6, t_4 = -4020 - 3t_5 - 6t_6$.

It follow now that $t_3 + t_4 = -4770$ and we have a contradictions.

If $i = 21$ then $t_{21} = 5$ and the respective system of standard equations in unknowns t_2, t_3, t_4 and t_6 has no solutions in nonnegative integers More precisely, the standard equations are $t_2 + t_3 + t_4 + t_5 = 120$, $2t_2 + 3t_3 + 4t_4 + 5t_5 = 5760$, $2t_2 + 6t_3 + 12t_4 + 20t_5 = 14280$.

The solution of this system in terms of t_5 is $t2 = -9420 - 5t_5, t_3 = 1356 + 3t_5, t_4 = -4020 - 3t_5$. It follow now that $t_3 + t_4 = -2664$ and we have contradiction.

2. The related linear cods

Let $GF(q)$ denote the Galois field of q elements, end let $V(n, q)$ denote the vector space of all ordered n-tuples over $GF(q)$. A linear code C over $GF(q)$ of length n and dimension k is a k-dimensional subspace of $V(n, q)$. The vectors of C are called codewords. The Hamming distance between two codewords is defined to be number of coordinate places in which they differ. The minimum distance of a code is the smallest of the distances between distinct codewords. Such a code is called an $[n, k, d]_q$-code if its minimum Hamming distance is d. The code C is called projective if no two coordinate function on C are scalar multiples of each other.

Theorem 2.1. *There exists a projective* $[n, 3, d]_q$ *code if and only if there exists an* $(n, n-d)-$ *arc in PG(2,q).*

We know also that every Griesmer code with $d \leq q^{k-1}$ is projective [4]. Let $d = (r - i)q - (q + 1)/2$ and $(q + 3)/2 < r < q$. Since $[d] + [d/q^2] = (r - 1)q + r - (q + 1)/2$, using Theorems 2.2 and Theorem 2.1 we can formulate the following:

Corollary 2.1. *There do not exist Griesmer codes with parameters* $[(r-1)q + r - (q+1)/2, 3, (r-1)q - (q+1)/2]_q$, *where* $(q + 3)/2 < r < q$ *and* $q = 47$.

A central problem in coding theory is that of optimizing one of the parameters n, k and d for given values of the other two and q fixed.

Two versions are?

Problem 1: find $d_q(n, k)$, the largest value of d for which there exists $[n, k, d]_q$-code.

Problem 2: find $n_q(k, d)$, the smallest value of n for which there exists $[n, k, d]_q$-code.

The Griesmer bound states that $n_q(k, d) \geq g_q(k, d) = \sum_{j=0}^{k-1} [\frac{d}{qj}]$. Codes with parameters $[g_q(k, d), k, d]_q$, are called Griesmer codes. There is a close relationship between $(n, r)-$ arcs in $PG(2.q)$ and $[n, 3, d]_q-$codes, given by the next theorem [4].

25<r<47	$[n,3,d]_q$	$M_{q\_}(n,k) \leq d$
26	$[1177,3,1151]_{47}$	$M_{47}(1177,3) \leq 1176$
27	$[1225,3,1198]_{47}$	$M_{47}(1225,3) \leq 1224$
28	$[1273,3,1245]_{47}$	$M_{47}(1273) \leq 1271$
29	$[1321,3,1295]_{47}$	$M_{47}(1321,3) \leq 1320$
30	$[1369,3,1339]_{47}$	$M_{47}(1369,3) \leq 1368$
31	$[1417,3,1386]_{47}$	$M_{47}(1417,3) \leq 1416$
32	$[1465,3,1433]_{47}$	$M_{47}(1465,3) \leq 1464$
33	$[1513,3,1480]_{47}$	$M_{47}(1513,3) \leq 1512$
34	$[1561,3,1527]_{47}$	$M_{47}(1561,3) \leq 1560$
35	$[1609,3,1575]_{47}$	$M_{47}(1609,3) \leq 1608$
36	$[1657,3,1621]_{47}$	$M_{47}(1657,3) \leq 1656$
37	$[1705,3,1668]_{47}$	$M_{47}(1705,3) \leq 1704$
38	$[1753,3,1715]_{47}$	$M_{47}(1753,3) \leq 1752$
39	$[1801,3,1762]_{47}$	$M_{47}(1801,3) \leq 1800$
40	$[1849,3,1809]_{47}$	$M_{47}(1849,3) \leq 1848$
41	$[1897,3,1856]_{47}$	$M_{47}(1897,3) \leq 1896$
42	$[1945,3,1903]_{47}$	$M_{47}(1945,3) \leq 1944$
43	$[1993,3,1950]_{47}$	$M_{47}(1993,3) \leq 1992$
44	$[2041,3,1997]_{47}$	$M_{47}(2041,3) \leq 2040$
45	$[2089,3,2044]_{47}$	$M_{47}(2089,3) \leq 2088$
46	$[2137,3,2091]_{47}$	$M_{47}(2137,3) \leq 2136$

3. Conclusions

1. We finding new largest Bounded:

   $M_r(2, 47) \leq 2040$

   $M_r(2, 47) \leq 2088$

   $M_r(2, 47) \leq 2136$

2. Also we finding bounded of linear $[n, 3, d]_q$ codes not exists

[2041,3,1997]47

[2089,3,2044]47

[2137,3,2091]47

References

[1] J. W. P. Hirschfeld, *Projective geometries over finite fields*, Oxford Clarendon Press, New York, Oxford University Press, 1979.

[2] H. Sh. Ibrahim, N.Y. Kasm, *The possibility of applying rumen research at the projective plane* $PG(2, 17)$, Modern Applied Science, 13 (2019).

[3] H.Sh. Ibrahim, N.Y. Kasm, *The optimal size of* b, t*-blocking set when* $t = 3, 4$ *by intersection the tangents in* $PG(2, q)$*,* Modern Applied Science, 13 (2019).

[4] N.Y. Kasm, A.Z. Hamad, *Applications of algebraic geometry in cryptography*, Modern Applied Science, 13 (2019).

[5] N.Y.K. Yahya, *A geometric construction of complete* (k, r)*-arc in* $PG(2, 7)$ *and the related projective [n,3, d]7 codes*, AL-Rafidain Journal of Computer Sciences and Mathematics, 12 (2018), 24-40.

[6] N.Y.K. Yahya, *A non* $PGL(3, q)k$*-arcs in the projective plane of order 37*, Tikrit Journal of Pure Science, 19 (2014), 135-145.

[7] N.Y.K. Yahya, *Existence of minimal blocking sets of size 31 in the projective plane* $PG(2, 17)$, Journal of University of Babylon/Pure and Applied Sciences, 20 (2012), 1138-1146,

[8] N.Y.K. Yahya, *The use of 7- blocking sets in Galois geometries*, Journal of University of Babylon/Pure and Applied Sciences, 22 (2014), 1229-1235.

[9] N.Y.K. Yahya, M.N. Salim, *The geometric approach to existences linear [n, k, d]13 code*, International Journal of Enhanced Research in science, Technology and Engineering, 2018.

[10] N.Y.K. Yahya, M.N. Salim, *New geometric methods for prove existence three- dimensional linear [97,3,87]11 and [143,3,131]13 codes*, Journal of Education and Science, 28 (2019), 312-333.

[11] N.Y.K. Yahya, M.N. Salim, *New existences linear [n, 3, d]19 codes by geometric structure method in PG (2, 19)*, AL-Rafidain Journal of Computer Sciences and Mathematics, 13 (2019).

[12] N.Y.K. Yahya, A.Z. Hamad, *A geometric construction of a* $(56, 2)$*-blocking set in* $PG(2, 19)$ *and on three dimensional linear [325,3,307]19 Griesmer*

code, AL-Rafidain Journal of Computer Sciences and Mathematics, University of Mosul, Mosul-Iraq. AL-Rafidain Journal of Computer Sciences and Mathematics, 13 (2019).

[13] A.E.M. Sulaimaan, N.Y. Kasm, *Linear code originates from complete arc by engineering building methods*, Applied Mathematical Science, 13 (2019), 1013-1020.

[14] A.E.M. Sulaimaan, N.Y. Kasm, *Linear code related ?-blocking sets*, Applied Mathematical Science, 13 (2019), 1033-1040.

[15] A.E.M. Sulaimaan, N.Y. Kasm, *New structural engineering construction methods by remove q points in PG(2,q)*, Applied Mathematical Science, 13 (2019), 1048-1041.

[16] M.G. Faraj, N.Y. Kasm, *Reverse building of complete* (k, r)*-arcs in* $PG(2, q)$, Open Access Library Journal, 6 (2019).

[17] D.G. Glynn, *Theorems of points and planes in three-dimensional projective space*, Journal of the Australian Mathematical Society 88 (2010), 75-92.

[18] J.W.P. Hirschfeld, *Projective geometries over finite fields*, Second edition, Oxford Clarendon Press, New York, Oxford University Press, 1998.

[19] J.H. Griesmer, *A bound for error-correcting codes*, IBM J. Res. Dev., 4 (1960), 532-542.

[20] A.E.M. Sulaimaan, N.Y. Kasm Yahya, *The reverse construction of complete (k,n)- arcs in three dimensional projective space PG(3,4)*, Journal of Physics: Conference Series, 1591 (2020), 012078.

[21] A.E.M. Sulaimaan, N.Y. Kasm Yahya, *The reverse construction of complete* (k, n)*- arcs in* $PG(2, q)$ *where* $q = 2, 4, 8$ *related with linear codes*, Journal of Physics: Conference Series, 1591 (2020), 012079 IOP Publishing.

[22] Ali Ahmed A. Abdulla, N.Y. Kasm Yahya, *Complete arcs and surfaces in three dimensional projective space PG (3,7)*, Open Access Library Journal, 7 (2020).

[23] Ali Ahmed A. Abdulla, N.Y. Kasm Yahya, *Application of algebraic geometry in three dimensional projective space* $PG(3, 7)$, Journal of Physics: Conference Series, 1591 (2020), 012077.

[24] A. Abdulla, Ali Ahmed, N.Y. Kasm Yahya, *A geometric construction of surface complete (k, r)-cap in PG(3,7),* in Ibn Al-Haitham 2nd Conference for Pure and Applied Science (IHICPAS)-2020, for publication in Journal of Physics: Conference Series, in press, 2021.

[25] N.Y. Kasm Yahya *Applications Geometry of Space in PG(3,P),* in Journal of Interdisciplinary Mathematics, for publication in Journal of Interdisciplinary Mathematics, in press, 2021.

Accepted: 11.09.2019