Supra soft b-R_0 and supra soft $b - R_1$ spaces

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Abstract. The purpose of this paper is to introduce supra soft b-R_0 and supra soft b-R_1 spaces using supra soft b-open sets. Also we study several of their properties and characterizations in details. Furthermore, we investigate the relationships between these supra soft spaces and the relationships with some other supra soft spaces.

Keywords: supra soft b-open set, supra soft topological space, supra soft b-R0 space, supra soft b-R1 space.

1. Introduction

In 1999, Russian researcher Molodtsov [22] initiated the concept of soft sets as a new general mathematical tool for dealing with uncertain objects while modelling the problems with incomplete information in engineering, physics, computer science, economics, social sciences and medical sciences. In [23], Molodtsov applied soft sets successfully in directions such as smoothness of functions, game theory, operations research, Riemann-integration, Perron integration, probability and theory of measurement. Maji et. al [20] applied soft sets in a multicriteria decision making problems. It is based on the notion of knowledge reduction of rough sets. They applied the technique of knowledge reduction to the information table induced by the soft set. In [21], they defined and studied several basic notions of soft set theory. In 2005, Pei and Miao [27] and Chen [4] improved the work of Maji et.al [20-21]. A. Kharal and B. Ahmad [19] defined and discussed the several properties of soft images and soft inverse images of soft sets. They also applied these notions to the problem of medical diagnosis in medical systems. Many researchers have contributed towards the algebraic structure of soft set theory [1-2],[3], [6], [9-19], [26], [29-30]. In 2011, Shabir and Naz [29] initiated the study of soft topological spaces. Also in 2011, S. Hussain and B. Ahmad [8] continued investigating the properties of soft open(closed), soft neighborhood and soft closure. They also defined and discussed the properties of soft interior, soft exterior and soft boundary. Shabir et. al [29] and D. N. Georgiou et. al [6], defined and studied some soft separation axioms, soft -continuity and soft connectedness in soft spaces using (ordinary) points of a topological space X. Ittanagi [9] introduced the concept of soft bitopological space and studied some types of soft separation axioms for soft bitopological spaces from his point of view. Recently, J. M. Mustafa [25], defined and studied supra soft b-compact and supra soft b-Lindelof spaces. Also, J. M. Mustafa [24], defined and studied b-R_0 and b-R_1 spaces. In this paper, we introduce the concepts of supra soft b-R_0 and supra soft b-R_1 spaces. Furthermore, we investigate the relationships between these supra sof spaces and the relatioships with some other supra soft spaces.

2. Preliminaries

In this section we will introduce necessary definitions and theorems for soft sets.

Definition 2.1 ([22])**.** Let X be an initial universe and E a set of parameters. Let $P(X)$ denote the power set of X. A pair (F,E) is called a soft set over X, where F is a mapping given by $F : E \to P(X)$. In other words, a soft set over X is a parameterized family of subsets of the universe X. For $e \in E$, $F(e)$ may be considered as the set of e-approximate elements of the soft set (F,E).

Definition 2.2 ([22])**.** The complement $\widetilde{X}-(F,E)$ of a soft set (F,E) is denoted by $(F,E)^c$ and is defined by $(F,E)^c = (F^c,E)$ where, $F^c : E \to P(X)$ is a mapping given by $F^c(e) = X - F(e)$, for all $e \in E$.

Let us call F^c to be the soft complement function of F. Clearly $(F^c)^c$ is the same as F and $((F,E)^c)^c = (F,E)$.

Definition 2.3 ([32])**.** A soft set (F,E) over X is said to be a null soft set, denoted by $\widetilde{\phi}$, if for all $e \in E$, $F(e) = \phi$. Clearly, $(\widetilde{\phi}^c)^c = \widetilde{\phi}$.

Definition 2.4 ([32])**.** A soft set (F,E) over X is said to be an absolute soft set, denoted by $\widetilde{X}$, if for all $e \in E$, $F(e) = X$. Clearly, $\widetilde{X}^c = \widetilde{\phi}$.

Definition 2.5 ([21])**.** Let (F,E), (G,E) be two soft sets over X. Then:

(1) (F,E) is a soft subset of (G,E), denoted by $(F,E)\widetilde{\subseteq}(G,E)$, if $F(e) \subseteq G(e)$, $\forall e \in E$. In this case, (F,E) is said to be a soft subset of (G,E) and (G,E) is said to be a soft superset of (F,E).

(2) Two soft sets (F,E) and (G,E) over a common universe set X are said to be equal, denoted by $(F,E) = (G,E)$, if $F(e) = G(e)$, $\forall e \in E$.

(3) The union of two soft sets (F,E) and (G,E) over the common universe X, denoted by $(F,E)\widetilde{\cup}(G,E)$, is the soft set (H,E), where$H(e) = F(e) \cup G(e)$, for all $e \in E$.

(4) The intersection of two soft sets (F,E) and (G,E) over the common universe X, denoted by $(F,E)\widetilde{\cap}(G,E)$, is the soft set (M,E), where $M(e) = F(e) \cap G(e)$, for all $e \in E$.

Definition 2.6 ([29])**.** Let (F,E) be a soft set over X and $x \in X$. We say that $x \in (F,E)$ read as x belongs to the soft set (F,E) whenever $x \in F(e)$ for all $e \in E$.

Note that for any $x \in X$, $x \notin (F,E)$ if $x \notin F(e)$ for some $e \in E$.

Definition 2.7 ([29])**.** Let τ be a collection of soft sets over a universe X with a fixed set of parameters E, then τ is called a soft topology on X if
(1) $\widetilde{\phi}$, $\widetilde{X} \in \tau$.
(2) the union of any number of soft sets in τ belongs to τ.
(3) the intersection of any two soft sets in τ belongs to τ.

The triplet (X, τ, E) is called a soft topological space over X.

Definition 2.8 ([29])**.** Let (X, τ, E) be a soft topological space, and (F,E) be a soft set over X. If Y is a non empty subset of X, then the sub soft set of (F,E) over Y denoted by (F_Y, E), is defined as follows

$$F_Y(e) = Y \cap F(e) \text{ for all } e \in E.$$

Definition 2.9. Let (X, τ, E) be a soft topological space and Y be a non empty subset of X. Then, $\tau_Y = \{(F_Y, E) : (F,E) \in \tau\}$ is called the soft relative topology on Y and (Y, τ_Y, E) is called a soft subspace of (X, τ, E).

Definition 2.10 ([19])**.** Let $SS(X)_E$ and $SS(Y)_K$ be families of soft sets on X and Y respectively. Let $u : X \to Y$ and $p : E \to K$ be mappings. We define a soft mapping $f_{pu} : SS(X)_E \to SS(Y)_K$ as:
(1) If $(F,E) \in SS(X)_E$, then, the image of (F,E) under f_{pu}, written as $f_{pu}(F,E) = (f_{pu}(F), p(E))$, is a soft set in $SS(Y)_K$ such that $f_{pu}(F,E)(k) = \cup\{u[F(e)] : e \in p^{-1}(k)\}$ if $p^{-1}(k) \neq \phi\phi$ if $p^{-1}(k) = \phi$, for all $k \in K$.
(2) If $(H,K) \in SS(Y)_K$, then the inverse image of (H,K) under f_{pu}, written as $f_{pu}^{-1}(H,K) = (f_{pu}^{-1}(H), E)$, is a soft set in $SS(X)_E$ such that:

$$f_{pu}^{-1}(H,K)(e) = u^{-1}[H(p(e))] \text{ for all } e \in E.$$

3. Supra soft b-open sets

Definition 3.1 ([5])**.** Let μ be a collection of soft sets over a universe X with a fixed set of parameters E. Then μ is called a supra soft topology on X with a fixed set E if:
(1) $\widetilde{\phi}$, $\widetilde{X} \in \mu$
(2) The union of any number of soft sets in μ belongs to μ.

(X, μ, E) is called supra soft topological space and the elements of μ are called supra soft open sets. The soft complement of any supra soft open set is called supra soft closed.

Definition 3.2. Let (X, μ, E) be a supra soft topological space and Y be a non empty subset of X. Then, $\mu_Y = \{(F_Y, E) : (F,E) \in \mu\}$ is called the supra soft relative topology on Y and (Y, μ_Y, E) is called a supra soft subspace of (X, μ, E).

Definition 3.3 ([5])**.** Let (X, μ, E) be a supra soft topological space and (F, E) be a soft set over X. Then the supra soft interior of (F, E) , denoted by $int^s(F, E)$ is the soft union of all supra soft open subsets of (F, E) . Clearly $int^s(F, E)$ is the largest supra soft open set over X which contained in (F, E).

Definition 3.4 ([5])**.** Let (X, μ, E) be a supra soft topological space and (F, E) be a soft set over X. Then the supra soft closure of (F, E) , denoted by $cl^s(F, E)$ is the soft intersection of all supra soft closed sets containing (F, E) . Clearly $cl^s(F, E)$ is the smallest supra soft closed set over X which contains (F, E).

Definition 3.5. A soft set (F, E) in a supra soft topological space X is called

(1) supra soft b-open (ssb-open) set if $(F, E)\widetilde{\subseteq} int^s(cl^s(F, E))\widetilde{\cup} cl^s(int^s(F, E))$.

(2) supra soft b-closed (ssb-closed) set if

$$(F, E)\widetilde{\supseteq} nt^s(cl^s(F, E))\widetilde{\cap} cl^s(int^s(F, E)).$$

We denote the family of all ssb-open (resp. ssb-closed) sets in a supra soft topological space X by SSBO(X) (resp. SSBC(X)).

Definition 3.6. Let (X, μ, E) be a supra soft topological space. Then a soft set (F, E) is called a supra soft b-neighborhood of x if there exists a supra soft b-open set (G, E) such that $x \in (G, E)\widetilde{\subseteq}(F, E)$

Definition 3.7. Let (X, μ, E) be a supra soft topological space and (F, E) be a soft set over X. Then the supra soft b-interior of (F, E) , denoted by $int_b^s(F, E)$ is the soft union of all supra soft b-open subsets of (F, E) . Clearly $int_b^s(F, E)$ is the largest supra soft b-open set over X which contained in (F, E).

Definition 3.8. Let (X, μ, E) be a supra soft topological space and (F, E) be a soft set over X. Then the supra soft b-closure of (F, E) , denoted by $cl_b^s(F, E)$ is the soft intersection of all supra soft b-closed sets containing (F, E) . Clearly $cl_b^s(F, E)$ is the smallest supra soft b-closed set over X which contains (F, E).

Definition 3.9. A soft mapping $f_{pu} : (X, \mu, E) \to (Y, \sigma, K)$ is said to be

(1) supra soft b-continuous (briefly ssb-continuous) if the inverse image of each supra soft open set of (Y, σ, K) is a supra soft b-open set in (X, μ, E).

(2) supra soft b-open if the image of each supra soft open set of (X, μ, E) is supra soft b -open set in (Y, σ, K).

(3) supra soft b-irresolute if the inverse image of each supra soft b-open set of (Y, σ, K) is a supra soft b-open set in (X, μ, E).

(4) supra soft b*-open if the image of each supra soft b-open set of (X, μ, E) is supra soft b-open set in (Y, σ, K).

(5) supra soft b*-closed if the image of each supra soft b-closed set of (X, μ, E) is supra soft b-closed set in (Y, σ, K).

4. Supra soft b-R_0 and supra soft b-R_1 spaces

Definition 4.1. Let (F,E) be a supra soft set over X. The supra soft b-kernel of (F,E) denoted by $Ker^s_b(F,E)$ is defined to be the set $Ker^s_b(F,E) = \widetilde{\cap}\{(G,E) \in SSBO(X) : (F,E)\widetilde{\subseteq}(G,E)\}$.

Definition 4.2. Let x be a point of a supra soft topological space (X,μ,E). The supra soft b-kernel of x, denoted by $Ker^s_b(\{x\})$ is defined to be the set $Ker^s_b(\{x\}) = \widetilde{\cap}\{(G,E) \in SSBO(X) : x \in (G,E)\}$.

Theorem 4.3. *Let (X,μ,E) be a supra soft topological space and $x \in X$. Then $Ker^s_b(F,E) = \{x \in X : cl^s_b(\{x\})\widetilde{\cap}(F,E) \neq \widetilde{\phi}\}$.*

Proof. To show that $Ker^s_b(F,E)\widetilde{\subseteq}\{x \in X : cl^s_b(\{x\})\widetilde{\cap}(F,E) \neq \widetilde{\phi}\}$ let $x \in Ker^s_b(F,E)$ and $cl^s_b(\{x\})\widetilde{\cap}(F,E) = \widetilde{\phi}$. Now $X - cl^s_b(\{x\})$ is a supra soft b-open set containing (F,E), but $x \notin \widetilde{X} - cl^s_b(\{x\})$ which is impossible since $x \in Ker^s_b(F,E)$. Consequently, $cl^s_b(\{x\})\widetilde{\cap}(F,E) \neq \widetilde{\phi}$. Now, let $cl^s_b(\{x\})\widetilde{\cap}(F,E) \neq \widetilde{\phi}$ and $x \notin Ker^s_b(F,E)$. Then, there exists a supra soft b-open set (G,E) containing (F,E) and $x \notin (G,E)$. Let $y \in cl^s_b(\{x\})$. Hence, (G,E) is a supra soft b-neighborhood of y with $x \notin (G,E)$. By this contradiction, $x \in Ker^s_b(F,E)$ and the claim. □

Definition 4.4. A supra soft topological space (X,μ,E) is said to be supra soft $b-R_0$ space if every supra soft b-open set contains the supra soft b-closure of each of its singletons.

Example 4.5. Let $X = \{h_1,h_2\}$ and $E = \{e_1,e_2\}$. Then the soft subsets over X are $(F_1,E) = \widetilde{\phi}$, $(F_2,E) = \widetilde{X}$, $(F_3,E) = \{(e_1,\phi),(e_2,X)\}$, $(F_4,E) = \{(e_1,\phi),(e_2,\{h_1\})\}$, $(F_5,E) = \{(e_1,\phi),(e_2,\{h_2\})\}$, $(F_6,E) = \{(e_1,X),(e_2,\phi)\}$, $(F_7,E) = \{(e_1,X),(e_2,\{h_1\})\}$, $(F_8,E) = \{(e_1,X),(e_2,\{h_2\})\}$, $(F_9,E) = \{(e_1, \{h_1\}),(e_2,\phi)\}$, $(F_{10},E)=\{(e_1,\{h_1\}),(e_2,X)\}$, $(F_{11},E) = \{(e_1,\{h_1\}),(e_2,\{h_1\})\}$, $(F_{12},E) = \{(e_1,\{h_1\}),(e_2,\{h_2\})\}$, $(F_{13},E) = \{(e_1,\{h_2\}),(e_2,\phi)\}$, $(F_{14},E) = \{(e_1,\{h_2\}),(e_2,X)\}$, $(F_{15},E) = \{(e_1,\{h_2\}),(e_2,\{h_1\})\}$ and $(F_{16},E)=\{(e_1,\{h_2\}), (e_2,\{h_2\})\}$.

Let $\mu = \{\widetilde{\phi},\widetilde{X},(F_{11},E),(F_{16},E)\}$. Then (X,μ,E) is a supra soft $b-R_0$ space.

Example 4.6. Let $X = \{h_1,h_2,h_3\}$ and $E = \{e\}$. Then the soft subsets over X are $(F_1,E) = \widetilde{\phi}$, $(F_2,E) = \widetilde{X}$, $(F_3,E) = \{(e,\{h_1\})\}$, $(F_4,E) = \{(e,\{h_2\})\}$, $(F_5,E) = \{(e,\{h_3\})\}$, $(F_6,E) = \{(e,\{h_1,h_2\})\}$, $(F_7,E) = \{(e,\{h_1,h_3\})\}$, $(F_8,E) = \{(e,\{h_2,h_3\})\}$.

Let $\mu = \{\widetilde{\phi},\widetilde{X},(F_3,E),(F_6,E)\}$. Then (X,μ,E) is not supra soft $b-R_0$ since (F_3,E) is a supra soft b-open set and $h_1 \in (F_3,E)$, but $cl^s_b(\{h_1\}) = \widetilde{X}\widetilde{\nsubseteq}(F_3,E)$.

Theorem 4.7. *Let (X,μ,E) be a supra soft topological space and $x \in X$. Then $y \in Ker^s_b(\{x\})$ if and only if $x \in cl^s_b(\{y\})$.*

Proof. Assume that $x \notin cl_b^s(\{y\})$ then $X - cl_b^s(\{y\})$ is a supra soft b-open set containing x but not y. Therefore we have $y \notin Ker_b^s(\{x\})$. The prove of the converse case can be done similarly. □

Theorem 4.8. *Let (X, μ, E) be a supra soft topological space and $x, y \in X$, then the following are equivalent:*
(1) $Ker_b^s(\{x\}) \neq Ker_b^s(\{y\})$;
(2) $cl_b^s(\{x\}) \neq cl_b^s(\{y\})$.

Proof. (1) $\rightarrow$ (2). Assume that $Ker_b^s(\{x\}) \neq Ker_b^s(\{y\})$, then there exists a point z in X such that $z \in Ker_b^s(\{x\})$ and $z \notin Ker_b^s(\{y\})$ (or $z \in Ker_b^s(\{y\})$ and $z \notin Ker_b^s(\{x\})$). It follows from $z \in Ker_b^s(\{x\})$ that $x \in cl_b^s(\{z\})$. Since $z \notin Ker_b^s(\{y\})$, we have $y \notin cl_b^s(\{z\})$. Also, since $x \in cl_b^s(\{z\})$, $cl_b^s(\{x\}) \widetilde{\subseteq} cl_b^s(\{z\})$ and $\{y\} \widetilde{\cap} cl_b^s(\{x\}) = \widetilde{\phi}$. Therefore, $cl_b^s(\{x\}) \neq cl_b^s(\{y\})$. Now $Ker_b^s(\{x\}) \neq Ker_b^s(\{y\})$ implies $cl_b^s(\{x\}) \neq cl_b^s(\{y\})$.

(2) $\rightarrow$ (1): Suppose that $cl_b^s(\{x\}) \neq cl_b^s(\{y\})$. Then there exists a point z in X such that $z \in cl_b^s(\{x\})$ and $z \notin cl_b^s(\{y\})$. Then, there exists a supra soft b-open set containing z and therefore x but not y, i.e., $y \notin Ker_b^s(\{x\})$. Hence $Ker_b^s(\{x\}) \neq Ker_b^s(\{y\})$. □

Theorem 4.9. *A supra soft topological space (X, μ, E) is supra soft $b{-}R_0$ if and only if for any x and y in X, $cl_b^s(\{x\}) \neq cl_b^s(\{y\})$ implies $cl_b^s(\{x\}) \widetilde{\cap} cl_b^s(\{y\}) = \widetilde{\phi}$.*

Proof. $\Rightarrow$) Suppose that (X, μ, E) *is supra soft* $b - R_0$ and $x, y \in X$ such that $cl_b^s(\{x\}) \neq cl_b^s(\{y\})$. Then, there exists $z \in cl_b^s(\{x\})$ such that $z \notin cl_b^s(\{y\})$ (or $z \in cl_b^s(\{y\})$ such that $z \notin cl_b^s(\{x\})$). There exists $(G, E) \in SSBO(X)$ such that $y \notin (G, E)$ and $z \in (G, E)$; hence $x \in (G, E)$. Therefore, we have $x \notin cl_b^s(\{y\})$. Thus $x \in \widetilde{X} - cl_b^s(\{y\}) \in SSBO(X)$, which implies $cl_b^s(\{x\}) \widetilde{\subseteq} \widetilde{X} - cl_b^s(\{y\})$ and $cl_b^s(\{x\}) \widetilde{\cap} cl_b^s(\{y\}) = \widetilde{\phi}$. The prove for otherwise is similar.

$\Leftarrow$) Let $(G, E) \in SSBO(X)$ and let $x \in (G, E)$. We will show that $cl_b^s(\{x\}) \widetilde{\subseteq} (G, E)$. Let $y \notin (G, E)$, i.e., $y \in \widetilde{X} - (G, E)$. Then $x \neq y$ and $x \notin cl_b^s(\{y\})$. This shows that $cl_b^s(\{x\}) \neq cl_b^s(\{y\})$. By assumption, $cl_b^s(\{x\}) \widetilde{\cap} cl_b^s(\{y\}) = \widetilde{\phi}$. Hence $y \notin cl_b^s(\{x\})$ and therefore $cl_b^s(\{y\}) \widetilde{\subseteq} (G, E)$. □

Theorem 4.10. *A supra soft topological space (X, μ, E) is supra soft $b - R_0$ if and only if for any points x and y in X, $Ker_b^s(\{x\}) \neq Ker_b^s(\{y\})$ implies $Ker_b^s(\{x\}) \widetilde{\cap} Ker_b^s(\{y\}) = \widetilde{\phi}$.*

Proof. $\Rightarrow$) Suppose that (X, μ, E) *is a supra soft* $b - R_0$ space. Thus by Theorem 4.8, for any points x and y in X if $Ker_b^s(\{x\}) \neq Ker_b^s(\{y\})$ then $cl_b^s(\{x\}) \neq cl_b^s(\{y\})$. Now we prove that $Ker_b^s(\{x\}) \widetilde{\cap} Ker_b^s(\{y\}) = \widetilde{\phi}$. Assume that $z \in Ker_b^s(\{x\}) \widetilde{\cap} Ker_b^s(\{y\})$. By $z \in Ker_b^s(\{x\})$ and Theorem 4.7, it follows that $x \in cl_b^s(\{z\})$. Since $x \in cl_b^s(\{x\})$, by Theorem 4.9, $cl_b^s(\{x\}) = cl_b^s(\{z\})$. Similarly, we have $cl_b^s(\{y\}) = cl_b^s(\{z\}) = cl_b^s(\{x\})$. This is a contradiction. Therefore, we have $Ker_b^s(\{x\}) \widetilde{\cap} Ker_b^s(\{y\}) = \widetilde{\phi}$.

$\Leftarrow$) Let (X,μ,E) *be a supra soft* space such that for any points x and y in X, $Ker_b^s(\{x\}) \neq Ker_b^s(\{y\})$ *implies* $Ker_b^s(\{x\})\widetilde{\cap}Ker_b^s(\{y\}) = \widetilde{\phi}$. If $cl_b^s(\{x\}) \neq cl_b^s(\{y\})$, then by Theorem 4.8, $Ker_b^s(\{x\}) \neq Ker_b^s(\{y\})$. Hence $Ker_b^s(\{x\}) \widetilde{\cap}Ker_b^s(\{y\}) = \widetilde{\phi}$ which implies $cl_b^s(\{x\})\widetilde{\cap}cl_b^s(\{y\}) = \widetilde{\phi}$. Because $z \in cl_b^s(\{x\})$ implies that $x \in Ker_b^s(\{z\})$. Therefore $Ker_b^s(\{x\})\widetilde{\cap}Ker_b^s(\{z\}) \neq \widetilde{\phi}$. By hypothesis, we have $Ker_b^s(\{x\}) = Ker_b^s(\{z\})$. Then $z \in cl_b^s(\{x\})\widetilde{\cap}cl_b^s(\{y\})$ implies that $Ker_b^s(\{x\}) = Ker_b^s(\{z\}) = Ker_b^s(\{y\})$. This is a contradiction. Hence $cl_b^s(\{x\})\widetilde{\cap}cl_b^s(\{y\}) = \widetilde{\phi}$. By Theorem 4.9, (X,μ,E) *is a supra soft* $b-R_0$ space. □

Theorem 4.11. *For a supra soft topological space (X,μ,E), the following properties are equivalent:*

(1) (X,μ,E) *is a supra soft* $b-R_0$ *space;*

(2) *For any* $(F,E) \neq \widetilde{\phi}$ *and* $(G,E) \in SSBO(X)$ *such that* $(F,E)\widetilde{\cap}(G,E) \neq \widetilde{\phi}$, *there exists* $(H,E) \in SSBC(X)$ *such that* $(F,E)\widetilde{\cap}(H,E) \neq \widetilde{\phi}$ *and* $(H,E)\widetilde{\subseteq}(G,E)$*;*

(3) *If* $(G,E) \in SSBO(X)$, *then* $(G,E) = \widetilde{\cup}\{(F,E) \in SSBC(X) : (F,E)\widetilde{\subseteq}(G,E)\}$*;*

(4) *If* $(F,E) \in SSBC(X)$, *then* $(F,E) = \widetilde{\cap}\{(G,E) \in SSBO(X) : (F,E)\widetilde{\subseteq}(G,E)\}$*;*

(5) *For any* $x \in X, cl_b^s(\{x\})\widetilde{\subseteq}Ker_b^s(\{x\})$.

Proof. (1) $\to$ (2). Let $(F,E) \neq \widetilde{\phi}$ and $(G,E) \in SSBO(X)$ such that $(F,E) \widetilde{\cap}(G,E) \neq \widetilde{\phi}$. Then there exists $x \in (F,E)\widetilde{\cap}(G,E)$. Since $x \in (G,E) \in SSBO(X)$, $cl_b^s(\{x\})\widetilde{\subseteq}(G,E)$. Let $(H,E) = cl_b^s(\{x\})$, then $(H,E) \in SSBC(X)$, $(H,E)\widetilde{\subseteq}(G,E)$ and $(F,E)\widetilde{\cap}(H,E) \neq \widetilde{\phi}$

(2) $\to$ (3). Let $(G,E) \in SSBO(X)$, then $\widetilde{\cup}\{(F,E) \in SSBC(X) : (F,E)\widetilde{\subseteq}(G,E)\}\widetilde{\subseteq} (G,E)$. Let x be any point of (G,E). There exists $(F,E) \in SSBC(X)$ such that $x \in (F,E)$ and $(F,E)\widetilde{\subseteq}(G,E)$. Therefore, we have $x \in (F,E)\widetilde{\subseteq}\widetilde{\cup}\{(F,E) \in SSBC(X) : (F,E)\widetilde{\subseteq}(G,E)\}$ and hence $(G,E) = \widetilde{\cup}\{(F,E) \in SSBC(X) : (F,E)\widetilde{\subseteq}(G,E)\}$.

(3) $\to$ (4). This is obvious.

(4) $\to$ (5). Let x be any point of X and $y \notin Ker_b^s(\{x\})$. There exists $(H,E) \in SSBO(X)$ such that $x \in (H,E)$ and $y \notin (H,E)$; hence $cl_b^s(\{y\})\widetilde{\cap}(H,E) = \widetilde{\phi}$. By (4)$(\widetilde{\cap}\{(G,E) \in SSBO(X) : cl_b^s(\{y\})\widetilde{\subseteq}(G,E)\})\widetilde{\cap}(H,E) = \widetilde{\phi}$. There exists $(G,E) \in SSBO(X)$ such that $x \notin (G,E)$ and $cl_b^s(\{y\})\widetilde{\subseteq}(G,E)$. Therefore, $cl_b^s(\{x\})\widetilde{\cap}(G,E) = \widetilde{\phi}$ and $y \notin cl_b^s(\{x\})$.

Consequently, we obtain $cl_b^s(\{x\})\widetilde{\subseteq}Ker_b^s(\{x\})$.

(5) $\to$ (1). Let $(G,E) \in SSBO(X)$ and $x \in (G,E)$. Suppose $y \in Ker_b^s(\{x\})$, then $x \in cl_b^s(\{y\})$ and $y \in (G,E)$. This implies that $cl_b^s(\{x\})\widetilde{\subseteq}Ker_b^s(\{x\})\widetilde{\subseteq}(G,E)$. Therefore, (X,μ,E) *is a supra soft* $b-R_0$ *space.* □

Corollary 4.12. *For a supra soft topological space (X,μ,E), the following are equivalent:*

(1) (X,μ,E) *is a supra soft* $b-R_0$ *space;*
(2) $cl_b^s(\{x\})=Ker_b^s(\{x\})$ *for all* $x\in X$.

Proof. (1) $\to$ (2). Suppose that (X,μ,E) *is a supra soft* $b-R_0$ *space*. By Theorem 4.11, $cl_b^s(\{x\})\widetilde{\subseteq}Ker_b^s(\{x\})$ for each $x\in X$. Let $y\in Ker_b^s(\{x\})$, then $x\in cl_b^s(\{y\})$ and so $cl_b^s(\{x\})=cl_b^s(\{y\})$. Therefore, $y\in cl_b^s(\{x\})$ and hence $Ker_b^s(\{x\})\widetilde{\subseteq}cl_b^s(\{x\})$. This shows that $cl_b^s(\{x\})=Ker_b^s(\{x\})$.

(2) $\to$ (1). This is obvious by Theorem 4.10. □

Theorem 4.13. *For a supra soft topological space* (X,μ,E)*, the following properties are equivalents :*
(1) (X,μ,E) *is a supra soft* $b-R_0$ *space;*
(2) $x\in cl_b^s(\{y\})$ *if and only if* $y\in cl_b^s(\{x\})$*, for any points* x *and* y *in* X.

Proof. (1) $\to$ (2). Assume that (X,μ,E) *is a supra soft* $b-R_0$ space. Let $x\in cl_b^s(\{y\})$ and (G,E) be any supra soft b-open set such that $y\in(G,E)$. Now by hypothesis, $x\in(G,E)$. Therefore, every supra soft b-open set containing y contains x. Hence $y\in cl_b^s(\{x\})$.

(2) $\to$ (1). Let (H,E) be a supra soft b-open set and $x\in(H,E)$. If $y\notin(H,E)$, then $x\notin cl_b^s(\{y\})$ and hence $y\notin cl_b^s(\{x\})$. This implies that $cl_b^s(\{x\})\widetilde{\subseteq}(H,E)$. Hence (X,μ,E) *is a supra soft* $b-R_0$ space. □

Theorem 4.14. *For a supra soft topological space* (X,μ,E)*, the following properties are equivalent :*
(1) (X,μ,E) *is a supra soft* $b-R_0$ *space;*
(2) *If* (F,E) *is supra soft b-closed, then* $(F,E)=Ker_b^s(F,E)$*;*
(3) *If* (F,E) *is supra soft b-closed and* $x\in(F,E)$*, then* $Ker_b^s(\{x\})\widetilde{\subseteq}(F,E)$*;*
(4) *If* $x\in X$*, then* $Ker_b^s(\{x\})\widetilde{\subseteq}cl_b^s(\{x\})$.

Proof. (1) $\to$ (2). Let (F,E) be supra soft b-closed and $x\notin(F,E)$. Thus $\widetilde{X}-(F,E)$ is supra soft b-open and contains x. Since (X,μ,E) *is supra soft* $b-R_0$, $cl_b^s(\{x\})\widetilde{\subseteq}\widetilde{X}-(F,E)$. Thus $cl_b^s(\{x\})\widetilde{\cap}(F,E)=\widetilde{\phi}$ and by Theorem 4.3, $x\notin Ker_b^s((F,E))$. Therefore $Ker_b^s(F,E)=(F,E)$.

(2) $\to$ (3). In general, $(A,E)\widetilde{\subseteq}(B,E)$ implies $Ker_b^s(A,E)\widetilde{\subseteq}Ker_b^s(B,E)$. Therefore, it follows from (2) that $Ker_b^s(\{x\})\widetilde{\subseteq}Ker_b^s(F,E)=(F,E)$.

(3) $\to$ (4). Since $x\in cl_b^s(\{x\})$ and $cl_b^s(\{x\})$ is supra soft b-closed, by (3) $Ker_b^s(\{x\})\widetilde{\subseteq}cl_b^s(\{x\})$.

(4) $\to$ (1). We show the implication by using Theorem 4.11. Let $x\in cl_b^s(\{y\})$. Then by Theorem 4.7, $y\in Ker_b^s(\{x\})$. Since $x\in cl_b^s(\{x\})$ and $cl_b^s(\{x\})$ is supra soft b-closed, by (4) we obtain $y\in Ker_b^s(\{x\})\widetilde{\subseteq}cl_b^s(\{x\})$. Therefore, $x\in cl_b^s(\{y\})$ implies $y\in cl_b^s(\{x\})$. The converse is obvious and (X,μ,E) *is supra soft* $b-R_0$. □

Definition 4.15. *A* supra soft topological space (X,μ,E) *is said to be supra soft* $b-R_1$ *if for* x, y *in* X *with* $cl_b^s(\{x\})\neq cl_b^s(\{y\})$*, there exist disjoint supra soft b-open sets* (G,E) *and* (H,E) *such that* $cl_b^s(\{x\})\widetilde{\subseteq}(G,E)$ *and* $cl_b^s(\{y\})\widetilde{\subseteq}(H,E)$.

Theorem 4.16. *Every supra soft $b-R_1$ space is supra soft $b-R_0$.*

Proof. Let (X,μ,E) *be a supra soft* $b-R_1$ space and (G,E) be a supra soft b-open set such that $x \in (G,E)$. If $y \notin (G,E)$, then since $x \notin cl_b^s(\{y\})$, $cl_b^s(\{x\}) \neq cl_b^s(\{y\})$. Hence, there exists a supra soft b-open set (H,E) such that $cl_b^s(\{y\}) \widetilde{\subseteq} (H,E)$ and $x \notin (H,E)$, which implies $y \notin cl_b^s(\{x\})$. Thus $cl_b^s(\{x\}) \widetilde{\subseteq} (G,E)$. Therefore (X,μ,E) *is supra soft* $b-R_0$. □

Notice that the converse of the above theorem need not be true:

Example 4.17. Let $X=\{h_1,h_2,h_3\}$ and $E=\{e\}$. Then the soft subsets over X are $(F_1,E)=\widetilde{\phi}$, $(F_2,E)=\widetilde{X}$, $(F_3,E)=\{(e,\{h_1\})\}$, $(F_4,E)=\{(e,\{h_2\})\}$, $(F_5,E)=\{(e,\{h_3\})\}$, $(F_6,E)=\{(e,\{h_1,h_2\})\}$, $(F_7,E)=\{(e,\{h_1,h_3\})\}$, (F_8,E) $=\{(e,\{h_2,h_3\})\}$.

Let $\mu=\{\widetilde{\phi},\widetilde{X},(F_6,E),(F_7,E),(F_8,E\}\}$. Then (X,μ,E) is a supra soft $b-R_0$ space but not supra soft $b-R_1$.

Theorem 4.18. *A supra soft topological space (X,μ,E) is supra soft $b-R_1$ if and only if for $x,y \in X$, with $Ker_b^s(\{x\}) \neq Ker_b^s(\{y\})$, there exists disjoint supra soft b-open sets (G,E) and (H,E) such that $cl_b^s(\{x\}) \widetilde{\subseteq} (G,E)$ and $cl_b^s(\{y\}) \widetilde{\subseteq} (H,E)$.*

Proof. It follows from Theorem 4.7. □

Theorem 4.19. *In a supra soft topological space (X,μ,E), $\widetilde{\cap}\{cl_b^s(\{x\}) : x \in X\} = \widetilde{\phi}$ if and only if $Ker_b^s(\{x\}) \neq \widetilde{X}$ for every $x \in X$.*

Proof. $\Rightarrow$) Suppose that $\widetilde{\cap}\{cl_b^s(\{x\}) : x \in X\} = \widetilde{\phi}$. Assume that there is a point y in X such that $Ker_b^s(\{y\}) = \widetilde{X}$. Then $y \notin (G,E)$, where (G,E) is some proper supra soft b-open subset of (X,μ,E). This implies that $y \in \widetilde{\cap}\{cl_b^s(\{x\}) : x \in X\}$. But this is a contradiction.

$\Leftarrow$) Now assume that $Ker_b^s(\{x\}) \neq \widetilde{X}$ for every $x \in X$. If there exists a point $y \in X$ such that $y \in \widetilde{\cap}\{cl_b^s(\{x\}) : x \in X\}$, then every supra soft b-open set containing y must contain every point of X. This implies that the space $\widetilde{X}$ is the unique supra soft b-open set containing y. Hence $Ker_b^s(\{x\}) = \widetilde{X}$ which is a contradiction. Therefore, $\widetilde{\cap}\{cl_b^s(\{x\}) : x \in X\} = \widetilde{\phi}$. □

Theorem 4.20. *If $f_{pu} : (X,\mu,E) \to (Y,\sigma,K)$ is an injective supra soft b^*-closed function and $\widetilde{\cap}\{cl_b^s(\{x\}) : x \in X\} = \widetilde{\phi}$ then $\widetilde{\cap}\{cl_b^s(\{y\}) : y \in Y\} = \widetilde{\phi}$.*

Proof. Straightforward.

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