# $M$-bands wavelet multiresolution analysis of assets

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**Abstract.** Given that the predictability of financial assets is indispensable to optimize the allocation of the investors' portfolio, a large literature review was dedicated to the question of predictability. Indeed, different studies have examined the relationship between the expected returns and the financial and macroeconomic variables to determine the most predictive indicators, notably the impact of the variables fluctuations on the prediction of the expected returns. Consequently, this paper consists on investigating the impact of the fluctuations in the aggregate price-earnings ratio at different timescales on the stock returns by using financial data from the USA. The data frequency is quarterly from 1952 to 2011. By aggregating the price-earnings ratio via multiresolution wavelets analysis, the results of the estimation of the Vector Autoregressive Model (VAR) demonstrated that the cycles in the price-earnings ratio presented strong predictors for the stock returns at short and intermediate horizons.

**Keywords:** wavelets, decomposition, time series, financial ratios, multiscale analysis, VAR, stock returns.

## 1. Introduction

The predictability of the financial assets is an indispensable element to optimize the allocation of the investor's portfolio. Some studies approved the predictability of the returns and other studies denied it. For example, Fama and French (1989) approved this predictability as it could not be inconsistent with the ef-

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ficiency market theory, on the other hand, other researchers as Mankiw et al. (1989) who denied it due to the same reason of the efficiency market theory.

This research will adopt the hypothesis of the predictability according to the paper of (Fama and French, 1989). In this context, the researchers have studied the algebraic relations between the expected returns, the macroeconomic variables and the financial variables to predict the future returns and to get the most predictive financial variables. Fama and French (1989) and Lettau and Ludvigson(2001) demonstrated that the expected returns fluctuate with the economic cycle. Other studies have related the fluctuations of the expected returns to the macroeconomic variables and to the financial ones and they approved the existence of a relation between the logarithm of payout ratio and the excess returns ones (Campbell and Shiller, 1988). Also, Campbell(2000,2003) confirmed that the valorization ratios have a predictive capacity on the returns at long horizons. With approximating the total wealth by a linear combination of the log labor income and log observable assets, Lettau and Ludvigson (2001) created a new ratio called consumption-wealth ratio (the cay ratio), they approved that the cay ratio has a predictive capacity at the short horizons. Also, Lettau and Nieuwerburgh (2008) have approved that the aggregation of the price earnings-ratio has notable effects on the prediction of the future returns. Thus, according to the paper of Lettau and Nieuwerburgh (2008) and Boucher and Maillet (2011) adopted the idea of aggregating the price-earnings ratio(See also Boldrin et al., 1995; Boldrin et al., 1997; Boldrin et al., 1999;Boldrin et al., 2001;Rouwenhorst, 1995; Jermann, 1998 and Jermann and Quadrini, 2007)

The problematic was how to aggregate this ratio to extract the significant information. Two approaches have been most known; the first one is the spectral analysis which is used to distinguish the different frequency components (Hamilton, 1994) and the second one is the standard filters which allows the extraction of specific features such as seasonality (periodicity) and trends. Hodrick and Prescott(1997) (see also Genay et al., 2001 and the references sited there). Nevertheless, two major drawbacks were detected in the standard methods of the time series analyzing. The first one is the restrictive hypothesis like the stationarity of the series, or this is not always the case of the financial series. The second one is the pure frequency-domain representation of the data i.e. all information from the time-domain representation is not captured.

Recently, a new method became increasingly famous in the economics and finance, called the wavelets. The wavelets analysis provides a simultaneous representation of a data series in the frequency and time domains. In fact, this technic can be used to study the financial variables over time and frequency, and it avoids the drawbacks of the spectral analysis. Boucher and Maillet (2011) decomposed the price-earnings ratio, by using the dyadic wavelets (M-bands equals 2), to identify the source of fluctuations of the assets and thus to define the most predictive variables for the future returns. They identified a timescale for that price earnings ratio to isolate the stable movements from the fluctuations.

They concluded that the first-time scale component of the price-earnings ratio explains a large part of the one-quarter stock returns.

Bianchi and Tamoni (2016) defined the expected returns as the outcome of an updating process based on medium and long-term information, showed that accounting for the low- to mid-frequency information conveyed by standard predictors, allows to identify long-lasting effects in the dynamics of high-frequency expected returns. Bandi et al. (2017) worked on the predictability over short horizons by proposing a novel modeling framework in which predictability is specified as a property of low-frequency components of both excess markets returns and economic uncertainty. This research consists on studying the predictability of the expected returns at short-term. The used variables in this analysis are the ones provided in the paper of (Boucher and Maillet, 2011) and largely discussed in different researches. Taji and Gore (2014) studied the Adaptive Texture Segmentation by using the M-bands wavelets. Also Schrrer (2015) suggested an analysis of the high frequency time series by using wavelets. Kl and Uur (2018) used the Multiresolution method to examine the SP500 time series. Here, the M-bands wavelets analysis will be used were M-bands in 3, 4 to see if the obtained results of (Boucher and Maillet, 2011) depends on the number of bands or not. Using the M-bands with M¿2 allows the analysis of higher frequency components and more numerous ones. The rest of the paper is organized as follows. Section 2 explains the methodology used in the study. Section 3 presents the results of the study and section four concludes.

## 2. Methodology and data

The methodology section consists on providing a general overview about the wavelets, the multiresolution decomposition and of the Vector Autoregressive model (VAR).

### 2.1 Wavelets and multiresolution decomposition

Wavelets have been known as an extension of Fourier analysis. The wavelets analysis was developed and extended by authors such as Grossman and Morlet in 1985, Daubechies in 1990 and 1992 and Meyer in 1992. The success of the wavelets was due to a discretization method introduced in 1986 by Stphane Mallat and then developed by Meyer and Daubechies and others as continued method. (See Al Ani, 2016; Emeric and Hammadi, 2003; Quarta,2011;Conway and Frame, 2000 for further details)

As the name indicates, a wavelet is a small wave. The term "small" mainly explains in this context that the wave increases and decreases in a limited time support (Masset, 2008). Thus, the main feature of a wavelet is its compact support, i.e. the wavelet function is limited in both the time and frequency domains. In fact, wavelets can decompose the non-stationary series without any restrictive hypothesis with localization at time and frequency. Yves MEYER defined the wavelets as following *"... unlike the Fourier series, the coefficients of a series of wavelets translate the properties of the functions in a simple, precise and reliable way, at least the properties that correspond to a discontinuity, an unplanned event."* Theoretically, a wavelet is simply a time function that follows a basic rule, known as the wavelet eligibility condition:

$$\mathbb{C}_{\psi} = \int_{0}^{\infty} \frac{|\Psi(f)|}{f} df < \infty \tag{1}$$

With

$$\Psi(f) = \int_{-\infty}^{+\infty} \psi(t) e^{-2\pi i f t} dt \tag{2}$$

Equation 2.2 presents Fourier transform. $f$ is a frequency function for $\Psi(f)$, $\Psi$ is called the mother wavelet. To ensure that $C_{\psi} < \infty$, the following conditions, related to the mother wavelet, must be imposed:

1. $\Psi(0) = 0$, or $\int_{-\infty}^{+\infty} \Psi(t) = 0$
2. $\int_{-\infty}^{+\infty} |\Psi(t)|^2 dt = 1$ (the energy unit).

Generally, wavelet transforms are classified into three categories; transformed into continuous wavelets, transformed into discrete wavelet and transformed into wavelets based on multiresolution analysis. In this analysis, the multiresolution decomposition analysis will be considered (for further details see (Boucher and Maillet, 2011) and the references stated there). Despite the problems of resolutions (time and frequency)[1] are the results of a similar phenomenon to the uncertainty's problem of Heisenberg and they exist independently from the used transformation, it is possible to analyze any signal by using an alternative approach named "multiresolution analysis". A multiresolution analysis allows a good time-resolution and a poor frequency-resolution for the high frequencies. In addition, a good frequency-resolution and a poor time-frequency for the low frequencies. This approach has a sense, especially when the real signal has components with high frequencies at the short-horizon and components with low frequencies at the long horizon. Mathematically, it is usually easier to present a development of signal on a countable group of wavelets.

This could be realized by varying the scale factor with dyadic way, by choosing $e = 2^j, j \in \mathbb{Z}$ $u = k2^j, j \in \mathbb{Z}$. In this context, a multiresolution analysis is defined as series of closed under- vectoral spaces of $(V_j)_{j \in \mathbb{Z}}$ of $L^2(\mathbb{R})$ and verifying the following proprieties:

- $\forall j \in \mathbb{Z}, V_{j+1} \subset V_j$

1. Time resolution: How far two peaks in time can be separated from one another in the domain of transformation. Frequency resolution: How far two spectral components can be separated from one another in the field of transformation.

- $\forall j \in \mathbb{Z}, f(t) \in V_j \Leftrightarrow f(\frac{t}{2}) \in V_{j+1}$
- $\lim_{j\to\infty} V_j = U_{j\in\mathbb{Z}} V_j = 0$ and $\lim_{j\to-\infty} V_j = \overline{U_{j\in\mathbb{Z}} V_j} = L^2(\mathbb{R})$
- $\exists\psi_0$ such as $\psi_0(t-n), n \in \mathbb{Z}$ is an orthonormal basis of $V_0$; $\psi_0$ is the scale function.

Thus, it could be confirmed that $2^{\frac{-j}{2}}\psi_0(\frac{t}{2^j} - k), k \in \mathbb{Z}$ constructs an orthonormal basis of $V_j$. The principal of multiresolution analysis of a signal $f$ is the realizations of successive orthogonal projections of the signal in the spaces $V_j$ which conducts to bigger approximations of $f$ as $j$ increases. The difference between two consecutive approximations represents the information "detail" or "cycle" which is lost in passing from a scale to another; this information is in the under-space $W_j$ orthogonal to $V_j$ with:$V_{j-1} = V_j \oplus W_j$. The existence of a wavelet $\psi_1 \in L^(\mathbb{R})$ is demonstrated with $2^{\frac{-j}{2}}\psi_1(\frac{t}{2^j} - k), k \in \mathbb{Z}$ is an orthonormal basis of $W_j$. The orthogonal wavelet decomposition of a signal $f$ could be done in an efficient way by following the algorithm of (Mallat, 1998). Thus, the coefficients of approximations $(c_{j,0}[k])_{k\in\mathbb{Z}}$ and the details $(c_{j,1}[k])_{k\in\mathbb{Z}}$ are determined in each level of resolution and they are defined as:

$$\forall K \in \mathbb{Z}, c_{j,0}[k] = \left\langle f, \frac{1}{2^{j/2}}\psi_0(\frac{.}{2^j} - k)\right\rangle \tag{3}$$

$$\forall K \in \mathbb{Z}, c_{j,1}[k] = \left\langle f, \frac{1}{2^{j/2}}\psi_1(\frac{.}{2^j} - k)\right\rangle \tag{4}$$

$\langle .,. \rangle$ is the scalar product of $L^2(\mathbb{R})$.

## 2.2 Vector autoregressive model

The modeling part of this research is carried out in the context of Vector Autoregressive Model (VAR). This methodology has gained widespread use in empirical business cycles analysis, as it has proved to be a flexible and tractable way to analyze economic time series. Vector autoregression (VAR) models have been capable of describing the rich dynamic structure of the relationships between economic variables (Bjornland, 2000). The VAR models are usually presented through impulse responses (that measure the effects of the different shocks on the variables of study), and variance decomposition (which measures the relative importance of the different shocks to the variation in the different variables). (Sims, 1980) was the first who introduced VAR models as an alternative to the large-scale macroeconometric models. According to Sims, all variables appearing in the structural models could be argued to be endogenous. Economic theory place only weak restrictions on the reduced form coefficients and on which variables that should enter a reduced form model. Similar ideas had already been put forward by (Liu, 1960), but the proposed solution by Sims was new. Sims suggested that empirical research should use small-scale models identified via a small number of constraints (Bjornland, 2000).

Sims argued that VARs provide a more systematic approach to imposing restrictions and could lead one to capture empirical regularities which remain hidden to standard procedures. In contrast, the results from policy exercises on large-scale macroeconometric models are hard to compare and recreate and can easily be amended by their users with judgmental ex-post decisions (Bjornland, 2000). Before the VAR estimation, the unit roots using the augmented Dickey-Fuller (ADF) and KPSS tests [2] for all the variables will be tested. The VAR is often perceived as an alternative to the simultaneous equation method. It is a systems regression model in that there is more than one dependent variable. In the most basic bivariate example, where there are just two variables, then each of their current values will depend on combinations of the previous values of variables and error terms.

$$\begin{cases} y_t = \alpha_0 + \alpha_1 x_{t-1} + \beta_1 y_{t-1} + ... + \alpha_k x_{t-k} + \beta_k y_{t-k} + u_t \\ x_t = \chi_0 + \chi_1 x_{t-1} + \delta_1 y_{t-1} + ... + \chi_k x_{t-k} + \delta_k y_{t-k} + v_t \end{cases} \tag{5}$$

The number of lags included in the VAR depends on either the data (i.e. monthly data would require 12 lags) or the minimization of the Akaike or Schwarz-Bayesian criteria (maximizing in some textbooks depending on how the criteria are set up). In addition, it is assumed that the error term is not serially correlated. The system can be expanded to include any number of variables and is used extensively in the finance literature. VAR models have a number of advantages over univariate time series models, for instance, there is no need to specify which variables are exogenous and which endogenous, variables are endogenous. In addition, the issue of model identification does not occur when using a VAR. Providing there are no contemporaneous terms acting as regressors, OLS [3] can be used to estimate each equation individually, as the regressors are lagged so treated as pre-determined. In addition, VARs are often highly efficient at forecasting compared to traditional models.

A limitation of the VAR approach is that it must be estimated to low order systems and the effects of omitted variables will be in the residuals. This may lead to major distortions in the impulse responses, making them of little use for structural interpretations (Hendry, 1995), although the system may still be used for predictions (see e.g. (Hendry and Doornik, 1997), and the references stated there). Further, measurement errors or misspecifications of the model will also induce unexplained information left in the disturbance terms, making interpretations of the impulse responses even more difficult . (See also Bjornland,

2. To confirm the test results obtained from the ADF and PP tests, Kwiatkowski Phillips, Schmidt and Shins test (1992) (KPSS) is suggested to eliminate a possible low power against stationary near unit root processes which occurs in the ADF and PP tests.
3. OLS estimator,$\hat{beta} = (X'X)^{-}1X'Y$, $Y$ dependent variable and $X$ dependent variable.

2000; Hendry,1995;Lubrano,2007 for further details about VAR modelling). The following diagram presents this research framework.

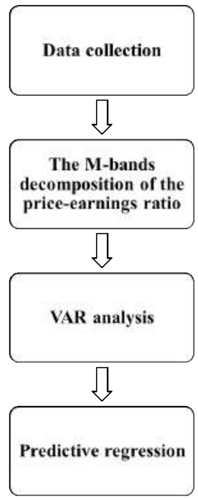


Figure 1: Research framework.

### 2.3 Dataset

The study sample is composed of financial data from the USA. The frequency is quarterly from the first quarter of the year 1952 to the earliest quarter of the year 2011. The data is extracted from different sources; the returns are from the CRSP (Center for Research in Security Prices). The Stock prices, the dividends per share and the quarterly earnings per share are available in the site of Standard and Poors and the site of Saint Louis bank. For the macro Financial and monetary indicators, the source is Federal Reflectance Database (FREDII). The input variables are described as following:

The dividend-earnings ratio or the dividend payout ratio (PER), defined as the ratio between the price of a stock and its earnings per share. Rancourt (2008) provided a full paper related to this variable.

Price-earnings ratio (PAY), calculated by dividing the dividend per share of earnings. This variable was considered in the paper of Lamont (1998) as a control variable.

The consumption-wealth ratio (CAY), defined by Lettau and Ludvigson (2001). It is the measure of short-term deviations from the long run cointegration relationship among the natural logarithm of consumption, labor income and aggregate wealth. The data related to cay variable is available at the official website of the authors.

Default spread (DEF), the default spread known also as a credit default swap spread or a credit spread. It is defined as the yield or return differential between long-term BAA corporate bonds and long-term AAA or U.S. Treasury bonds.

Elton et al. (2001) presented further details related to default spread. Different studies showed that the yield spread between BAA and AAA corporate bond spread can predict expected returns in stocks and bonds such as (Fama and Schwert, 1977; Keim and Stambaugh, 1986;Campbell,1987 and Fama and French, 1989 among others).
Term spread (TRM), defined as the difference between the 10-year Treasury bond yield and the 3-month Treasury bond yield. This variable was considered in the papers of Fama and French (1989) and Campbell (1987).
Detrended risk-free rate (RREL), is the stochastically detrended risk-free rate defined as the T-bill rate minus its last four-quarter average.
Inflation rate (INF), Campbell and Vuolteenaho(2004) demonstrated that the inflation rate is highly correlated with the price earnings ratio. In the following analysis part, the coefficients obtained from the decomposition of the price-earnings ratio by using Meyer wavelets will be used.[4]

## 3. Empirical findings

The aim of this section is to analyze the empirical results of this research; first, the M-bands decomposition of the price-earnings ratio will be provided (where the number of bands in 3,4) and then the relationship between the cycles of the price-earnings ratio and the macroeconomic variables by using VAR model will be investigated.

### 3.1 M-bands decomposition of the price-earnings ratio

To decompose the price-earnings ratio, the wavelet multiresolution analysis will be used. As shown in Figure 2, the x-axis presents the quarterly period 1952-2011. The y-axis presents the details and the approximation. The details show that the order of regular cycles of the price-earnings ratio is increasing with the level of decomposition. Thus, the wavelet decomposition allows the disaggregating of the variable in function of the degree of regularity. It could be explained otherwise, d1 presents the shorter term; d2 presents the short-term, d4 present the intermediate term and a4 presents the approximation. The variation of these cycles can be explained as follows:

- The cycles of the first decomposition d1 exhibit volatility during certain periods due to geopolitical, economic or financial conditions including the oil shocks of 1973, 1979 and 2008. Also, the Crash of October 1987 or "Black Monday", the attacks of September 2001, the Iraq war in 2003, the subprime crisis in 2007 and the banking and financial crisis of autumn 2008.

4. It was concluded that the results are independent from the choice of the type of wavelets. Results are not reported here.

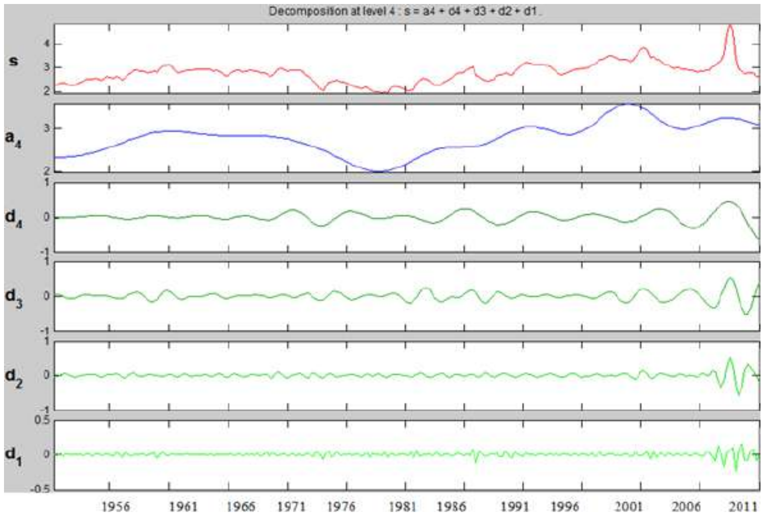


Figure 2: Decomposition of the price earning-ratio.

- The cycles of the details d2 and d3 show also similar events. They exhibit a common model regular than the d4.
- The a4 trend is very smooth with a stable phase almost of 20 years.

To approve if these cycles contain information about expected returns, an estimate of the autoregressive model VAR will be used with a lag of one quarter. The procedure will be detailed in the next section.

### 3.2 VAR analysis

Before carrying out the VAR model, the Granger causality test to examine the causality between the different variables will be provided.

#### 3.2.1 VAR model

VAR model is especially useful for describing the dynamic behavior of economic and financial time series and forecasting. Thus, to examine the relationships between the cycles of the price-earnings ratio and the macroeconomic conditions, a model VAR was estimated twice by using the lagged variables. Following the Akaike criteria, the number of lag was 1 ( which corresponds to the minimum Akaike value). As shown in (Table 2), the estimation of VAR model after the decomposition of price-earnings ratio (via 3-bands multiresolution), demonstrated that the first-decomposition (or detail d1) contains a distinguished information in short-term; the first scale decomposition has a strong ability to predict the variations in short-term. However, d1 appears related to the default spread and dividend payout ratio. The estimation of VAR model after including the decomposition of price-earnings ratio (via 4-bands) is presented in (Table 3), the estimation resutls confirmed that the first cycle of decomposition of the ratio price-earnings capture the short-term variation (over one quarter) and still related to the default spread and to the dividend payout ratio. Therefore, it could be concluded that the variables defined in the literature do not appear at short horizons i.e. they have a low capacity for forecasting at short-term except the default spread and the dividend payout ratio However, their predictive power to capture the variations of returns over the intermediate-term appears at medium-term (more than one quarter).

### 3.2.2 Granger causality

As is well known, to carry out the Granger causality test, the series should be stationary. When the causal relations between the first decomposition d1 and the other macro-financial variables was examined, no causality between d1 and those variables (Table 4) was found. These results confirm that most of the macro-financial variables used in the literature don't capture the fluctuations at short-horizon but at intermediate-horizon (from few quarters to few years) and this is confirmed too by the presence of causality between some variables such as the default spread and the dividend-earnings ratio and the second decomposition of the price-earnings ratio (d2).

### 3.2.3 Predictive regression

To approve the predictive ability of price-earnings ratio after decomposition, univariate predictive regression and multivariate predictive regression have been made at the short-horizon (one quarter) (Table 5). The regressions contain an intercept and an autoregressive term at the order 1 (AR (1)) referring one quarter. The Newey-test for the correction of the t-student statistics was used. The first line of the table shows that the predictive ability of the price-earnings ratio is significant but not so important. In fact, it predicts just 1% from the returns of the next quarter. However, the first decomposition d1 has a predictive ability of 10%. So, using d1 instead of the PER gives 9% more to predict the future returns. The third line and the fourth one from the table of regression demonstrate that the predictive ability of the PER's ratio decreases with the augmentation of the resolution's level. The predictive regressions of the other variables such as the PAY and the default spread didn't demonstrate any effect on the prediction of the future returns. The predictions related to the CAY, RREL and TRM variables demonstrated a predictive ability of those variables but which is negligent compared to d1 and d2. For the multivariate predictive regressions, without including the price-earnings ratio, no predictive ability to get the future returns has been found. However, when the cycles of the PER were included, it appears that the cycles d1 and d2 have an important predictive capacity at the short horizon.

## 4. Conclusion and perspective

The paper results showed that the decomposition of the price-earnings ratio avoids the problem of non-stationarity. Moreover, the first decomposition scale-time (d1) of the price-earnings (PER) ratio demonstrated a strong capacity to predict the variations of the assets at short-horizon (which approves the results obtained by Boucher and Maillet (2011). The details d1, d2, d3, and d4 contain information which doesn't exist in the other variables defined in the literature. It was noted that the macroeconomic and the financial variables, such as the payout dividend ratio and the default spread, have acceptable predictive capacities at short horizon, but they show more predictive capacity at intermediate horizons. To improve this work, a lot of perspectives could be mentioned as including the decomposition of the dividends ratio and check the predictive capacity of his cycles as it has been considered as important as the price-earnings ratio.

## 5. Appendices

Table 1: Estimation of VAR model (after a 3-bonds decomposition)

| | | D1_1 | D1_2 | A1 | D2_1 | D2_2 | A2 | PAY | CAY | DEF | RREL | INF |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| D1_1(-1) | coef. | 0.451522 | 0.227506 | -0.712132 | -0.026192 | 0.051286 | -0.806962 | -0.389208 | -0.004373 | -0.396572 | 0.359558 | 0.007037 |
| | Std-error | (0.06160) | (0.26016) | (0.14312) | (0.01826) | (0.06784) | (0.41142) | (0.04112) | (0.00277) | (0.07104) | (0.26894) | (0.00303) |
| | t-statistic | [ 7.33017] | [ 0.87449] | [-4.97592] | [-1.43417] | [ 0.75597] | [-1.96142] | [-9.46498] | [-1.57606] | [-5.58204] | [ 1.33695] | [ 2.32549] |
| | p-value | 0.0000 | | 0.0000 | | | 0.0499 | 0.0000 | | 0.0000 | | 0.0237 |
| D1_2(-1) | coef. | -0.001815 | 0.819551 | -0.046118 | -0.008993 | 0.001517 | -0.136548 | -0.008080 | -2.47E-05 | 0.002431 | 0.034922 | 0.000643 |
| | Std-error | (0.01001) | (0.04229) | (0.02327) | (0.00297) | (0.01103) | (0.06688) | (0.00668) | (0.00045) | (0.01155) | (0.04372) | (0.00049) |
| | t-statistic | [-0.18123] | [ 19.3784] | [-1.98226] | [-3.02908] | [ 0.13751] | [-2.04167] | [-1.20873] | [-0.05479] | [ 0.21050] | [ 0.79877] | [ 1.30786] |
| | p-value | | 0.0000 | 0.0476 | 0.0025 | | 0.0413 | | | | | |
| A1(-1) | coef. | -0.015004 | -0.060076 | 0.858930 | -0.013068 | -0.046500 | 0.484274 | -0.019228 | 0.000469 | 0.010832 | 0.028079 | -0.000875 |
| | Std-error | (0.01871) | (0.07904) | (0.04348) | (0.00555) | (0.02061) | (0.12499) | (0.01249) | (0.00084) | (0.02158) | (0.08171) | (0.00092) |
| | t-statistic | [-0.80176] | [-0.76010] | [ 19.7550] | [-2.35528] | [-2.25613] | [ 3.87450] | [-1.53915] | [ 0.55601] | [ 0.50186] | [ 0.34367] | [-0.95162] |
| | p-value | | | 0.0000 | 0.0186 | 0.0242 | 0.0001 | | | | | |
| D2_1(-1) | coef. | -0.133019 | -1.092295 | -0.855850 | 0.674840 | 0.066683 | -1.089277 | 0.052499 | 0.018918 | -0.049720 | 1.175235 | -0.006197 |
| | Std-error | (0.16948) | (0.71580) | (0.39376) | (0.05025) | (0.18666) | (1.13196) | (0.11314) | (0.00763) | (0.19547) | (0.73995) | (0.00833) |
| | t-statistic | [-0.78488] | [-1.52599] | [-2.17351] | [ 13.4302] | [ 0.35725] | [-0.96229] | [ 0.46403] | [ 2.47788] | [-0.25436] | [ 1.58826] | [-0.74422] |
| | p-value | | | 0.0298 | 0.0000 | | | | 0.0118 | | | |
| D2_2(-1) | coef. | 0.004768 | -0.115373 | -0.011727 | 0.003038 | 0.966746 | -0.087157 | -0.016796 | -0.001772 | -0.043976 | -0.024513 | -0.001547 |
| | Std-error | (0.01817) | (0.07674) | (0.04221) | (0.00539) | (0.02001) | (0.12135) | (0.01213) | (0.00082) | (0.02096) | (0.07933) | (0.00089) |
| | t-statistic | [ 0.26245] | [-1.50350] | [-0.27780] | [ 0.56401] | [ 48.3121] | [-0.71822] | [-1.38481] | [-2.16564] | [-2.09859] | [-0.30901] | [-1.73273] |
| | p-value | | | | | 0.0000 | | | 0.0300 | 0.0350 | | |
| A2(-1) | coef. | 0.009684 | 0.016784 | 0.029960 | 0.002104 | 0.015080 | 0.807061 | 0.005730 | -0.000218 | 0.003217 | -0.010606 | -0.000200 |
| | Std-error | (0.00588) | (0.02483) | (0.01366) | (0.00174) | (0.00647) | (0.03927) | (0.00392) | (0.00026) | (0.00678) | (0.02567) | (0.00029) |

Vector Autoregression Estimates

Sample (adjusted): 1952Q2 2010Q2

Included observations: 233 after adjustments

Standard errors in () & t-statistics in []

| | | D1_1 | D1_2 | A1 | D2_1 | D2_2 | A2 | PAY | CAY | DEF | RREL | INF |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | **Std-error** | (0.00588) | (0.02483) | (0.01366) | (0.00174) | (0.00647) | (0.03927) | (0.00392) | (0.00026) | (0.00678) | (0.02567) | (0.00029) |
| | **t-statistic** | [ 1.64727] | [ 0.67596] | [ 2.19341] | [ 1.20693] | [ 2.32898] | [ 20.5538] | [ 1.45996] | [-0.82340] | [ 0.47445] | [-0.41319] | [-0.69233] |
| | **p-value** | | | 0.0284 | | 0.0199 | 0.0000 | | | | | |
| **PAY (-1)** | **coef.** | 0.151513 | 0.569953 | 0.473201 | 0.019467 | 0.004449 | 0.462579 | 0.979924 | 0.000424 | 0.033812 | -0.187222 | -0.003762 |
| | **Std-error** | (0.04902) | (0.20703) | (0.11389) | (0.01453) | (0.05399) | (0.32739) | (0.03272) | (0.00221) | (0.05654) | (0.21401) | (0.00241) |
| | **t-statistic** | [ 3.09098] | [ 2.75303] | [ 4.15498] | [ 1.33950] | [ 0.08242] | [ 1.41291] | [ 29.9462] | [ 0.19185] | [ 0.59807] | [-0.87481] | [-1.56224] |
| | **p-value** | 0.0020 | 0.0059 | 0.000 | | | | 0.0000 | | | | |
| **CAY (-1)** | **coef.** | 0.181601 | -5.450554 | 0.279563 | -0.117367 | 0.867025 | -3.691627 | 0.231753 | 0.896889 | -0.204919 | 0.284806 | -0.071730 |
| | **Std-error** | (0.65938) | (2.78492) | (1.53201) | (0.19550) | (0.72622) | (4.40408) | (0.44018) | (0.02970) | (0.76051) | (2.87891) | (0.03239) |
| | **t-statistic** | [ 0.27541] | [-1.95717] | [ 0.18248] | [-0.60035] | [ 1.19389] | [-0.83823] | [ 0.52649] | [ 30.1947] | [-0.26945] | [ 0.09893] | [-2.21427] |
| | **p-value** | | | | | | | | 0.0000 | | | 0.0272 |
| **DEF (-1)** | **coef.** | 0.072143 | -0.022269 | 0.213889 | -0.001579 | -0.012180 | -0.011899 | -0.000838 | -0.003021 | 0.831864 | -0.221713 | -0.005297 |
| | **Std-error** | (0.03045) | (0.12863) | (0.07076) | (0.00903) | (0.03354) | (0.20341) | (0.02033) | (0.00137) | (0.03512) | (0.13297) | (0.00150) |
| | **t-statistic** | [ 2.36889] | [-0.17313] | [ 3.02284] | [-0.17488] | [-0.36314] | [-0.05850] | [-0.04122] | [-2.20225] | [ 23.6830] | [-1.66744] | [-3.54052] |
| | **p-value** | 0.0179 | | 0.0025 | | | | | 0.0276 | 0.0000 | | 0.0003 |
| **RREL (-1)** | **coef.** | -0.021381 | 0.024832 | -0.063071 | -0.006890 | -0.012158 | -0.055268 | -0.003080 | 0.000204 | 0.032118 | 0.018134 | 0.003915 |
| | **Std-error** | (0.01552) | (0.06557) | (0.03607) | (0.00460) | (0.01710) | (0.10369) | (0.01036) | (0.00070) | (0.01791) | (0.06778) | (0.00076) |
| | **t-statistic** | [-1.37724] | [ 0.37872] | [-1.74856] | [-1.49688] | [-0.71107] | [-0.53301] | [-0.29723] | [ 0.29141] | [ 1.79374] | [ 0.26754] | [ 5.13300] |
| | **p-value** | | | | | | | | | | | 0.0000 |
| **INF (-1)** | **coef.** | -0.237042 | 0.285119 | -2.844377 | 0.006661 | -0.089562 | -2.709856 | 0.006615 | 0.015018 | 2.901643 | 1.676442 | 0.938542 |
| | **Std-error** | (0.57999) | (2.44961) | (1.34755) | (0.17196) | (0.63878) | (3.87381) | (0.38718) | (0.02613) | (0.66894) | (2.53228) | (0.02849) |
| | **t-statistic** | [-0.40870] | [ 0.11639] | [-2.11078] | [ 0.03874] | [-0.14021] | [-0.69953] | [ 0.01708] | [ 0.57482] | [ 4.33769] | [ 0.66203] | [ 32.9383] |

Vector Autoregression Estimates

Sample (adjusted): 1952Q2 2010Q2

Included observations: 233 after adjustments

Standard errors in () & t-statistics in []

| | | D1_1 | D1_2 | A1 | D2_1 | D2_2 | A2 | PAY | CAY | DEF | RREL | INF |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | **p-value** | | | 0.0349 | | | | | | 0.0000 | | 0.0000 |
| **C** | **coef.** | -0.069966 | 0.547800 | 0.693010 | 0.071887 | 0.016901 | 1.285328 | 0.000824 | 0.004159 | -0.100015 | 0.047553 | 0.017183 |
| | **Std-error** | (0.14220) | (0.60059) | (0.33039) | (0.04216) | (0.15661) | (0.94977) | (0.09493) | (0.00641) | (0.16401) | (0.62086) | (0.00699) |
| | | | | | | | | | | | | |
| | **t-statistic** | [-0.49203] | [ 0.91211] | [ 2.09756] | [ 1.70508] | [ 0.10791] | [ 1.35330] | [ 0.00868] | [ 0.64933] | [-0.60981] | [ 0.07659] | [ 2.45963] |
| | **p-value** | | | 0.0360 | | | | | | | | 0.0147 |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **R-squared** | 0.382508 | 0.796053 | 0.933770 | 0.636466 | 0.947024 | 0.928666 | 0.884338 | 0.848571 | 0.868907 | 0.040117 | 0.929710 |
| **Adj. R-squared** | 0.351773 | 0.785902 | 0.930473 | 0.618372 | 0.944387 | 0.925115 | 0.878581 | 0.841033 | 0.862382 | -0.007660 | 0.926212 |
| **Sum sq. resids** | 4.709703 | 84.01260 | 25.42369 | 0.414003 | 5.712838 | 210.1008 | 2.098884 | 0.009557 | 6.265022 | 89.77891 | 0.011367 |
| **S.E. equation** | 0.145982 | 0.616561 | 0.339174 | 0.043282 | 0.160779 | 0.975029 | 0.097454 | 0.006576 | 0.168370 | 0.637369 | 0.007172 |
| **F-statistic** | 12.44540 | 78.41966 | 283.2590 | 35.17467 | 359.1536 | 261.5538 | 153.6122 | 112.5842 | 133.1657 | 0.839663 | 265.7399 |
| **Log likelihood** | 123.9020 | -211.7743 | -72.52356 | 407.1725 | 101.4069 | -318.5606 | 218.0595 | 846.2108 | 90.65789 | -219.5080 | 826.0059 |
| **Akaike AIC** | -0.960532 | 1.920810 | 0.725524 | -3.392039 | -0.767441 | 2.837430 | -1.768751 | -7.160608 | -0.675175 | 1.987193 | -6.987176 |
| **Schwarz SC** | -0.782796 | 2.098546 | 0.903260 | -3.214303 | -0.589705 | 3.015166 | -1.591015 | -6.982872 | -0.497439 | 2.164929 | -6.809440 |

Table 2: Estimation of VAR model (after a 4-bonds decomposition)

| | | D1 | D2 | D3 | A1 | PAY | CAY | RREL | TRM | DEF | INF |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **D1**(-1) | **coef.** | 0.622264 | 0.005388 | 0.020981 | -0.435114 | -0.181114 | -0.002241 | 0.093635 | -0.058095 | -0.197808 | 0.002905 |
| | **Std-error** | (0.05006) | (0.01099) | (0.00807) | (0.09392) | (0.01836) | (0.00126) | (0.11847) | (0.15166) | (0.03128) | (0.00136) |
| | **t-statistic** | [ 12.4316] | [ 0.49042] | [ 2.59836] | [-4.63298] | [-9.86698] | [-1.77804] | [ 0.79038] | [-0.38307] | [-6.32338] | [ 2.14326] |
| | **p-value** | 0.0000 | | 0.0094 | 0.0000 | 0.0000 | | | | 0.0000 | 0.0322 |
| **D2**(-1) | **coef.** | 0.096829 | 0.689666 | 0.053358 | -0.257143 | 0.091788 | 0.002897 | 0.404787 | -0.042290 | 0.265617 | -0.004075 |
| | **Std-error** | (0.20949) | (0.04598) | (0.03379) | (0.39306) | (0.07682) | (0.00527) | (0.49582) | (0.63473) | (0.13092) | (0.00567) |
| | **t-statistic** | [ 0.46221] | [ 15.0001] | [ 1.57891] | [-0.65421] | [ 1.19481] | [ 0.54934] | [ 0.81640] | [-0.06663] | [ 2.02881] | [-0.71852] |
| | **p-value** | | 0.0000 | | | | | | | 0.0426 | |
| **D3**(-1) | **coef.** | -0.024764 | -0.157602 | 0.724141 | 1.054133 | 0.151005 | 0.004616 | -0.770168 | 1.285105 | 0.145387 | -0.007444 |
| | **Std-error** | (0.27932) | (0.06130) | (0.04506) | (0.52408) | (0.10243) | (0.00703) | (0.66109) | (0.84630) | (0.17456) | (0.00756) |
| | **t-statistic** | [-0.08866] | [-2.57088] | [ 16.0711] | [ 2.01140] | [ 1.47424] | [ 0.65641] | [-1.16500] | [ 1.51850] | [ 0.83286] | [-0.98430] |
| | **p-value** | | 0.0102 | 0.0000 | 0.0444 | | | | | | |
| **A1**(-1) | **coef.** | -0.002084 | 0.004345 | 0.000909 | 0.930373 | -0.013678 | -0.000206 | -0.009252 | -0.008118 | 0.004669 | 0.000242 |
| | **Std-error** | (0.01540) | (0.00338) | (0.00248) | (0.02889) | (0.00565) | (0.00039) | (0.03644) | (0.04665) | (0.00962) | (0.00042) |
| | **t-statistic** | [-0.13538] | [ 1.28595] | [ 0.36581] | [ 32.2061] | [-2.42255] | [-0.53198] | [-0.25390] | [-0.17402] | [ 0.48528] | [ 0.58033] |
| | **p-value** | | | | 0.0000 | 0.0155 | | | | | |
| **PAY (-1)** | **coef.** | 0.213956 | 0.012174 | -0.056182 | 0.716768 | 0.934418 | 0.000354 | -0.156664 | 0.087650 | -0.025471 | -0.003339 |
| | **Std-error** | (0.07701) | (0.01690) | (0.01242) | (0.14449) | (0.02824) | (0.00194) | (0.18226) | (0.23332) | (0.04813) | (0.00208) |
| | **t-statistic** | [ 2.77838] | [ 0.72033] | [-4.52261] | [ 4.96082] | [ 33.0895] | [ 0.18274] | [-0.85957] | [ 0.37566] | [-0.52925] | [-1.60144] |
| | **p-value** | 0.0055 | | 0.0000 | 0.0000 | 0.0000 | | | | | |
| **CAY (-1)** | **coef.** | 0.289964 | 0.140723 | -0.058030 | 3.139056 | 0.858319 | 0.915426 | -1.010917 | 6.763216 | 0.406904 | -0.076494 |
| | **Std-error** | (1.10777) | (0.24312) | (0.17870) | (2.07846) | (0.40623) | (0.02789) | (2.62183) | (3.35636) | (0.69230) | (0.02999) |
| | **t-statistic** | [ 0.26175] | [ 0.57881] | [-0.32474] | [ 1.51028] | [ 2.11291] | [ 32.8221] | [-0.38558] | [ 2.01504] | [ 0.58775] | [-2.55051] |
| | **p-value** | | | | | 0.0347 | 0.0000 | | 0.0440 | | 0.0108 |
| **RREL (-1)** | **coef.** | 0.002557 | -0.006993 | -0.001043 | -0.151754 | -0.002472 | 0.000304 | 0.028421 | 0.366435 | 0.024616 | 0.004355 |
| | **Std-error** | (0.03134) | (0.00688) | (0.00506) | (0.05881) | (0.01149) | (0.00079) | (0.07418) | (0.09496) | (0.01959) | (0.00085) |
| | **t-statistic** | [ 0.08158] | [-1.01661] | [-0.20624] | [-2.58039] | [-0.21509] | [ 0.38501] | [ 0.38314] | [ 3.85879] | [ 1.25676] | [ 5.13192] |
| | **p-value** | | | | 0.0099 | | | | 0.0001 | | 0.0000 |
| **TRM (-1)** | **coef.** | -0.000384 | -0.009627 | -0.001929 | -0.031492 | -0.007214 | 4.94E-05 | 0.032578 | 0.787435 | -0.016222 | 0.000349 |

| | | D1 | D2 | D3 | A1 | PAY | CAY | RREL | TRM | DEF | INF |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | **Std-error** | (0.01679) | (0.00369) | (0.00271) | (0.03151) | (0.00616) | (0.00042) | (0.03974) | (0.05088) | (0.01049) | (0.00045) |
| | **t-statistic** | [-0.02285] | [-2.61240] | [-0.71208] | [-0.99957] | [-1.17147] | [ 0.11685] | [ 0.81972] | [ 15.4775] | [-1.54582] | [ 0.76850] |
| | **p-value** | | 0.0090 | | | | | | 0.0000 | | |
| **DEF (-1)** | **coef.** | 0.102748 | 0.002173 | -0.002058 | 0.152938 | -0.000677 | -0.001787 | -0.199414 | 0.359959 | 0.860669 | -0.003848 |
| | **Std-error** | (0.04738) | (0.01040) | (0.00764) | (0.08890) | (0.01738) | (0.00119) | (0.11215) | (0.14356) | (0.02961) | (0.00128) |
| | **t-statistic** | [ 2.16844] | [ 0.20900] | [-0.26921] | [ 1.72027] | [-0.03894] | [-1.49779] | [-1.77817] | [ 2.50730] | [ 29.0645] | [-2.99922] |
| | **p-value** | 0.0302 | | | | | | | 0.0122 | 0.0000 | 0.0027 |
| **INF (-1)** | **coef.** | -0.270133 | -0.009270 | -0.057422 | -1.504212 | -0.164514 | 0.016307 | 1.460967 | -3.058649 | 2.463667 | 0.974772 |
| | **Std-error** | (1.06623) | (0.23401) | (0.17200) | (2.00053) | (0.39099) | (0.02684) | (2.52353) | (3.23052) | (0.66634) | (0.02887) |
| | **t-statistic** | [-0.25335] | [-0.03961] | [-0.33385] | [-0.75191] | [-0.42076] | [ 0.60747] | [ 0.57894] | [-0.94680] | [ 3.69728] | [ 33.7676] |
| | **p-value** | | | | | | | | | 0.0002 | 0.0000 |
| **C** | **coef.** | 0.081573 | -0.017218 | -0.046160 | 1.254865 | 0.113684 | 0.003438 | 0.073631 | 0.187818 | -0.023335 | -0.000617 |
| | **Std-error** | (0.20432) | (0.04484) | (0.03296) | (0.38336) | (0.07493) | (0.00514) | (0.48358) | (0.61906) | (0.12769) | (0.00553) |
| | **t-statistic** | [ 0.39923] | [-0.38395] | [-1.40047] | [ 3.27331] | [ 1.51727] | [ 0.66825] | [ 0.15226] | [ 0.30339] | [-0.18274] | [-0.11162] |
| | **p-value** | | | | 0.0011 | | | | | | |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| **R-squared** | 0.455682 | 0.627655 | 0.673736 | 0.915145 | 0.889930 | 0.851315 | 0.035878 | 0.600747 | 0.868995 | 0.927653 |
| **Adj. R-squared** | 0.431808 | 0.611324 | 0.659426 | 0.911424 | 0.885102 | 0.844794 | -0.006408 | 0.583236 | 0.863249 | 0.924479 |
| **Sum sq. resids** | 16.09978 | 0.775489 | 0.418955 | 56.67694 | 2.164999 | 0.010206 | 90.18453 | 147.7948 | 6.288014 | 0.011801 |
| **S.E. equation** | 0.265731 | 0.058320 | 0.042866 | 0.498581 | 0.097445 | 0.006690 | 0.628925 | 0.805123 | 0.166069 | 0.007194 |
| **F-statistic** | 19.08724 | 38.43350 | 47.08207 | 245.8952 | 184.3402 | 130.5446 | 0.848457 | 34.30670 | 151.2386 | 292.3460 |
| **Log likelihood** | -16.75620 | 345.6953 | 419.2751 | -167.1543 | 223.0069 | 863.1979 | -222.6609 | -281.6900 | 95.59296 | 845.8395 |
| **Akaike AIC** | 0.232269 | -2.800797 | -3.416528 | 1.490831 | -1.774117 | -7.131363 | 1.955322 | 2.449289 | -0.707891 | -6.986105 |
| **Schwarz SC** | 0.392274 | -2.640793 | -3.256524 | 1.650836 | -1.614112 | -6.971359 | 2.115326 | 2.609293 | -0.547886 | -6.826100 |

Table 3: Granger causality

Tests of Granger causality by pairs
Sample: 1952Q1 2011Q4
Delay: 1

| Null Hypothesis: | Obs. | F-Statistic | Prob. | Null Hypothesis: | Obs. | F-Statistic | Prob. |
|---|---|---|---|---|---|---|---|
| CAY does not Granger Cause A4 | 239 | 13.4976 | 0.0003 | D4 does not Granger Cause D3 | 239 | 3.95479 | 0.0479 |
| A4 does not Granger Cause CAY | | 0.37348 | 0.5417 | D3 does not Granger Cause D4 | | 1.44909 | 0.2299 |
| D1 does not Granger Cause A4 | 239 | 0.00076 | 0.9781 | DEF does not Granger Cause D3 | 239 | 7.49352 | 0.0067 |
| A4 does not Granger Cause D1 | | 0.00306 | 0.9559 | D3 does not Granger Cause DEF | | 8.41002 | 0.0041 |
| D2 does not Granger Cause A4 | 239 | 0.03759 | 0.8464 | INF does not Granger Cause D3 | 239 | 0.33997 | 0.5604 |
| A4 does not Granger Cause D2 | | 0.00867 | 0.9259 | D3 does not Granger Cause INF | | 1.26270 | 0.2623 |
| D3 does not Granger Cause A4 | 239 | 0.09956 | 0.7526 | PAY does not Granger Cause D3 | 239 | 10.0315 | 0.0017 |
| A4 does not Granger Cause D3 | | 0.08271 | 0.7739 | D3 does not Granger Cause PAY | | 30.1070 | 1.E-07 |
| D4 does not Granger Cause A4 | 239 | 8.73344 | 0.0034 | PER does not Granger Cause D3 | 239 | 0.04116 | 0.8394 |
| A4 does not Granger Cause D4 | | 0.18646 | 0.6663 | D3 does not Granger Cause PER | | 5.10839 | 0.0247 |
| DEF does not Granger Cause A4 | 239 | 0.01338 | 0.9080 | RREL does not Granger Cause D3 | 239 | 1.94303 | 0.1647 |
| A4 does not Granger Cause DEF | | 0.85459 | 0.3562 | D3 does not Granger Cause RREL | | 0.00295 | 0.9567 |
| INF does not Granger Cause A4 | 239 | 1.47743 | 0.2254 | TRM does not Granger Cause D3 | 239 | 4.66240 | 0.0318 |
| A4 does not Granger Cause INF | | 4.39646 | 0.0371 | D3 does not Granger Cause TRM | | 1.55106 | 0.2142 |
| PAY does not Granger Cause A4 | 239 | 0.29295 | 0.5888 | DEF does not Granger Cause D4 | 239 | 4.73804 | 0.0305 |

Tests of Granger causality by pairs
Sample: 1952Q1 2011Q4
Delay: 1

| Null Hypothesis: | Obs. | F-Statistic | Prob. | Null Hypothesis: | Obs. | F-Statistic | Prob. |
|---|---|---|---|---|---|---|---|
| A4 does not Granger Cause PAY | | 8.3E-05 | 0.9927 | D4 does not Granger Cause DEF | | 3.15058 | 0.0772 |
| | | | | | | | |
| PER does not Granger Cause A4 | 239 | 3.09404 | 0.0799 | INF does not Granger Cause D4 | 239 | 2.08720 | 0.1499 |
| A4 does not Granger Cause PER | | 16.4743 | 7.E-05 | D4 does not Granger Cause INF | | 0.21535 | 0.6430 |
| | | | | | | | |
| RREL does not Granger Cause A4 | 239 | 0.09676 | 0.7560 | PAY does not Granger Cause D4 | 239 | 20.8948 | 8.E-06 |
| A4 does not Granger Cause RREL | | 0.32156 | 0.5712 | D4 does not Granger Cause PAY | | 1.64974 | 0.2003 |
| | | | | | | | |
| TRM does not Granger Cause A4 | 239 | 21.1850 | 7.E-06 | PER does not Granger Cause D4 | 239 | 0.02341 | 0.8785 |
| A4 does not Granger Cause TRM | | 0.05160 | 0.8205 | D4 does not Granger Cause PER | | 0.04542 | 0.8314 |
| | | | | | | | |
| D1 does not Granger Cause CAY | 239 | 8.83628 | 0.0033 | RREL does not Granger Cause D4 | 239 | 0.00044 | 0.9834 |
| CAY does not Granger Cause D1 | | 0.24510 | 0.6210 | D4 does not Granger Cause RREL | | 1.36674 | 0.2436 |
| | | | | | | | |
| D2 does not Granger Cause CAY | 239 | 0.03556 | 0.8506 | TRM does not Granger Cause D4 | 239 | 0.54263 | 0.4621 |
| CAY does not Granger Cause D2 | | 0.53357 | 0.4658 | D4 does not Granger Cause TRM | | 1.03277 | 0.3105 |
| | | | | | | | |
| D3 does not Granger Cause CAY | 239 | 1.25861 | 0.2631 | INF does not Granger Cause DEF | 239 | 1.20379 | 0.2737 |
| CAY does not Granger Cause D3 | | 0.45915 | 0.4987 | DEF does not Granger Cause INF | | 1.02397 | 0.3126 |
| | | | | | | | |
| D4 does not Granger Cause CAY | 239 | 3.51043 | 0.0622 | PAY does not Granger Cause DEF | 239 | 5.79461 | 0.0168 |
| CAY does not Granger Cause D4 | | 15.5673 | 0.0001 | DEF does not Granger Cause PAY | | 0.11648 | 0.7332 |
| | | | | | | | |
| DEF does not Granger Cause CAY | 239 | 1.67459 | 0.1969 | PER does not Granger Cause DEF | 239 | 11.0063 | 0.0011 |
| CAY does not Granger Cause DEF | | 0.00886 | 0.9251 | DEF does not Granger Cause PER | | 11.1929 | 0.0010 |

Tests of Granger causality by pairs
Sample: 1952Q1 2011Q4
Delay: 1

| Null Hypothesis: | Obs. | F-Statistic | Prob. | Null Hypothesis: | Obs. | F-Statistic | Prob. |
|---|---|---|---|---|---|---|---|
| INF does not Granger Cause CAY | 239 | 0.00089 | 0.9763 | RREL does not Granger Cause DEF | 239 | 4.77307 | 0.0299 |
| CAY does not Granger Cause INF | | 0.25709 | 0.6126 | DEF does not Granger Cause RREL | | 3.13008 | 0.0782 |
| PAY does not Granger Cause CAY | 239 | 1.03334 | 0.3104 | TRM does not Granger Cause DEF | 239 | 11.9161 | 0.0007 |
| CAY does not Granger Cause PAY | | 4.62580 | 0.0325 | DEF does not Granger Cause TRM | | 1.99093 | 0.1596 |
| PER does not Granger Cause CAY | 239 | 1.52341 | 0.2183 | PAY does not Granger Cause INF | 239 | 2.19933 | 0.1394 |
| CAY does not Granger Cause PER | | 9.90458 | 0.0019 | INF does not Granger Cause PAY | | 0.89006 | 0.3464 |
| RREL does not Granger Cause CAY | 239 | 0.30500 | 0.5813 | PER does not Granger Cause INF | 239 | 5.55911 | 0.0192 |
| CAY does not Granger Cause RREL | | 0.00249 | 0.9603 | INF does not Granger Cause PER | | 0.89137 | 0.3461 |
| TRM does not Granger Cause CAY | 239 | 0.38469 | 0.5357 | RREL does not Granger Cause INF | 239 | 5.52006 | 0.0196 |
| CAY does not Granger Cause TRM | | 4.52834 | 0.0344 | INF does not Granger Cause RREL | | 0.46731 | 0.4949 |
| D2 does not Granger Cause D1 | 239 | 0.00224 | 0.9623 | TRM does not Granger Cause INF | 239 | 4.21537 | 0.0412 |
| D1 does not Granger Cause D2 | | 0.57651 | 0.4484 | INF does not Granger Cause TRM | | 0.35736 | 0.5505 |
| D3 does not Granger Cause D1 | 239 | 0.04136 | 0.8390 | PER does not Granger Cause PAY | 239 | 11.9701 | 0.0006 |
| D1 does not Granger Cause D3 | | 0.00283 | 0.9576 | PAY does not Granger Cause PER | | 13.9299 | 0.0002 |
| D4 does not Granger Cause D1 | 239 | 0.10572 | 0.7454 | RREL does not Granger Cause PAY | 239 | 0.04121 | 0.8393 |
| D1 does not Granger Cause D4 | | 1.1E-05 | 0.9973 | PAY does not Granger Cause RREL | | 1.38063 | 0.2412 |

Tests of Granger causality by pairs
Sample: 1952Q1 2011Q4
Delay: 1

| Null Hypothesis: | Obs. | F-Statistic | Prob. | Null Hypothesis: | Obs. | F-Statistic | Prob. |
|---|---|---|---|---|---|---|---|
| | | | | | | | |
| DEF does not Granger Cause D1 | 239 | 0.75984 | 0.3843 | TRM does not Granger Cause PAY | 239 | 1.31066 | 0.2534 |
| D1 does not Granger Cause DEF | | 0.00625 | 0.9370 | PAY does not Granger Cause TRM | | 0.77129 | 0.3807 |
| | | | | | | | |
| INF does not Granger Cause D1 | 239 | 0.00229 | 0.9618 | RREL does not Granger Cause PER | 239 | 6.20445 | 0.0134 |
| D1 does not Granger Cause INF | | 1.75930 | 0.1860 | PER does not Granger Cause RREL | | 0.34979 | 0.5548 |
| | | | | | | | |
| PAY does not Granger Cause D1 | 239 | 0.76493 | 0.3827 | TRM does not Granger Cause PER | 239 | 0.26717 | 0.6057 |
| D1 does not Granger Cause PAY | | 1.16587 | 0.2814 | PER does not Granger Cause TRM | | 0.00108 | 0.9739 |
| | | | | | | | |
| PER does not Granger Cause D1 | 239 | 0.00013 | 0.9910 | TRM does not Granger Cause RREL | 239 | 0.29065 | 0.5903 |
| D1 does not Granger Cause PER | | 30.7299 | 8.E-08 | RREL does not Granger Cause TRM | | 13.1531 | 0.0004 |
| | | | | | | | |
| RREL does not Granger Cause D1 | 239 | 0.47075 | 0.4933 | INF does not Granger Cause D2 | 239 | 0.02597 | 0.8721 |
| D1 does not Granger Cause RREL | | 0.22222 | 0.6378 | D2 does not Granger Cause INF | | 0.00917 | 0.9238 |
| | | | | | | | |
| TRM does not Granger Cause D1 | 239 | 0.00735 | 0.9317 | PAY does not Granger Cause D2 | 239 | 8.29409 | 0.0043 |
| D1 does not Granger Cause TRM | | 0.25886 | 0.6114 | D2 does not Granger Cause PAY | | 67.8245 | 1.E-14 |
| | | | | | | | |
| D3 does not Granger Cause D2 | 239 | 4.04871 | 0.0453 | PER does not Granger Cause D2 | 239 | 0.19506 | 0.6591 |
| D2 does not Granger Cause D3 | | 9.17764 | 0.0027 | D2 does not Granger Cause PER | | 9.68642 | 0.0021 |
| | | | | | | | |
| D4 does not Granger Cause D2 | 239 | 0.23363 | 0.6293 | RREL does not Granger Cause D2 | 239 | 6.25818 | 0.0130 |
| D2 does not Granger Cause D4 | | 0.33883 | 0.5611 | D2 does not Granger Cause RREL | | 2.77613 | 0.0970 |

Tests of Granger causality by pairs
Sample: 1952Q1 2011Q4
Delay: 1

| Null Hypothesis: | Obs. | F-Statistic | Prob. | Null Hypothesis: | Obs. | F-Statistic | Prob. |
|---|---|---|---|---|---|---|---|
| | | | | | | | |
| DEF does not Granger Cause D2 | 239 | 7.67889 | 0.0060 | TRM does not Granger Cause D2 | 239 | 0.74449 | 0.3891 |
| D2 does not Granger Cause DEF | | 38.0151 | 3.E-09 | D2 does not Granger Cause TRM | | 0.76242 | 0.3835 |

Table 4: Predictive regression of the returns

**Univariate Predictive Regression**

| # | | PER | D1 | D2 | D3 | D4 | A4 | PAY | CAY | RREL | TRM | DEF | Adjusted R2 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | coef. | 0.149969 | | | | | | | | | | | 0.01 |
| | t-statistic | 2.249588 | | | | | | | | | | | |
| | p-value | 0.0254 | | | | | | | | | | | |
| 2 | coef. | | 0.191427 | | | | | | | | | | 0.1 |
| | t-statistic | | 3.213660 | | | | | | | | | | |
| | p-value | | 0.0015 | | | | | | | | | | |
| 3 | coef. | | | 0.137164 | | | | | | | | | 0.04 |
| | t-statistic | | | 2.082461 | | | | | | | | | |
| | p-value | | | 0.0384 | | | | | | | | | |
| 4 | coef. | | | | 0.145258 | | | | | | | | 0.01 |
| | t-statistic | | | | 2.184514 | | | | | | | | |
| | p-value | | | | 0.0299 | | | | | | | | |
| 5 | coef. | | | | | -0.000835 | | | | | | | 0.01 |
| | t-statistic | | | | | -0.051055 | | | | | | | |
| | p-value | | | | | 0.9593 | | | | | | | |
| 6 | coef. | | | | | | -0.004774 | | | | | | 0.01 |
| | t-statistic | | | | | | -0.904432 | | | | | | |
| | p-value | | | | | | 0.3667 | | | | | | |
| 7 | coef. | | | | | | | -0.007058 | | | | | 0.02 |
| | t-statistic | | | | | | | -0.709074 | | | | | |
| | p-value | | | | | | | 0.4790 | | | | | |
| 8 | coef. | | | | | | | | 0.148919 | | | | 0.013 |
| | t-statistic | | | | | | | | 2.242868 | | | | |
| | p-value | | | | | | | | 0.0258 | | | | |
| 9 | coef. | | | | | | | | | -0.005444 | | | 0.01 |
| | t-statistic | | | | | | | | | -2.059860 | | | |
| | p-value | | | | | | | | | 0.0405 | | | |
| 10 | coef. | | | | | | | | | | 0.003637 | | 0.03 |
| | t-statistic | | | | | | | | | | 2.298442 | | |
| | p-value | | | | | | | | | | 0.0224 | | |
| 11 | coef. | | | | | | | | | | | 0.001288 | 0.04 |
| | t-statistic | | | | | | | | | | | 0.173709 | |
| | p-value | | | | | | | | | | | 0.8622 | |

**Multivariate Predictive Regression**

| # | | PER | D1 | D2 | D3 | D4 | A4 | PAY | CAY | RREL | TRM | DEF | Adjusted R2 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 12 | coef. | 0.004157 | | | | | | -0.012689 | -0.031856 | -0.003082 | 0.003162 | 0.002280 | 0.03 |
| | t-statistic | 0.682534 | | | | | | -1.328249 | -0.280369 | -1.095661 | 1.752707 | 0.403037 | |
| | p-value | 0.4956 | | | | | | 0.1854 | 0.7794 | 0.2744 | 0.0810 | 0.6873 | |
| 13 | coef. | | 0.185977 | 0.073399 | 0.028922 | 0.021341 | 0.028922 | -0.027233 | 0.0922 | -0,0051 | 0.00198 | 0.0019 | 0.16 |
| | t-statistic | | 2.420076 | 3.893068 | 1.923321 | 1.191262 | 1.191262 | -2.518882 | 0.9341 | -2,0309 | 1.07503 | 0.3809 | |
| | p-value | | 0.0163 | 0.0001 | 0.0557 | 0.2348 | 0.4908 | 0.0125 | 0.3512 | 0.0434 | 0.2835 | 0.7036 | |

## References


[1] T. Al Ani, *Introduction aux ondelettes*, ESIEE-Paris, 2016.

[2] F. Bandi, B. Perron, A. Tamoni, C. Tebaldi, *The scale of predictability*, Journal of Econometrics, A (2017), 120-140.

[3] D. Bianchi, A. Tamoni, *The dynamics of expected returns: evidence from multi-scale time series modeling*, SSRN, 2016.

[4] H. C. Bjornland, *VAR models in macroeconomic research*, Statistics Norway Research Department, 2000.

[5] M. Boldrin, L. J. Christiano, J. D. M Fisher, *Asset pricing lessons for modelling business cycles*, Working Papers from Federal Reserve Bank of Minneapolis, 560 (1995).

[6] M. Boldrin, L. J. Christiano, J. D. M Fisher, *Habit persistence and asset returns in an exchange economy*, Macroeconomic Dynamics, 1 (1997), 312-332.

[7] M. Boldrin, L. J. Christiano, J. D. M Fisher, *Habit persistence, asset returns, and the business cycle*, American Economic Review, 1 (2001), 149-166.

[8] C. Boucher, B. Maillet, *Une analyse temps-fréquence des cycles financiers*, CES Working Papers, 62 (2011), 441-450.

[9] J. Campbell, *Stock returns and the term structure*, Journal of Financial Economics, 18 (1987).

[10] J. Campbell, R. Shiller,*Stock prices, earnings and expected dividends*, Journal of Financial Economics, 43 (1988), 661-676.

[11] J. Campbell, *Asset pricing at the millennium*, Journal of Finance, 7589 (2000), 1515-1567.

[12] J. Campbell, *Consumption-based asset pricing*, Handbook of the Economics of Finance, (2003), 803-887.

[13] J. Campbell, T. Vuolteenaho, *Inflation illusion and stock prices*, American Economic Review Papers and Proceedings, 94 (2004), 19-23.

[14] J. Campbell, M. Yogo, *Efficient tests of stock return predictability*, Journal of Financial Economics, (2006), 27-60.

[15] P. Conway, D. Frame, *A spectral analysis of New Zealand output gaps using Fourier and wavelet techniques*, Reserve Bank of New Zealand Discussion Papers, 2000.

[16] E . J. Elton, M. J. Gruber, D. Agrawal, C. Mann, *Explaining the rate spread on corporate bonds*, Journal of Finance, 56 (2001), 247-277.

[17] E. Fama, G. W. Schwert, *Asset returns and inflation*, Journal of Financial Economics, 1977, 115-146.

[18] E. Fama, G. K. French, *Business conditions and expected returns on stocks and bonds*, Journal of Financial Economics, 25 (2001), 23-49.

[19] FREDII, *Federal reserve economic data*, 2001.

[20] R. Gençay, F. Selçuk, B. Whitcher *An introduction to wavelets and other filtering methods in finance and economics*, Elsevier Science, 2001.

[21] E. Golfier, B. A. J. Hammadi, *La théorie des ondelettes dans la multi-résolution des objets 3D*, Faculté des Sciences de Luminy, Université de la Méditerranée, 2016.

[22] A. Grossman, J. Morlet, *Decomposition of Hardy functions into square integrable wavelets of constant shape*, SIAM Journal of Mathematical Analysis, 15 (1984), 723-736.

[23] J. Hamilton, *Time series analysis*, Princeton University Press, 1984.

[24] D. Hendry, *Econometrics and business cycle empirics*, The Economic Journal, 105 (1995), 1622-1636.

[25] D. Hendry, *The implications for econometric modelling of forecast failure*, Scottish Journal of Political Economy, 4 (1997), 437-461.

[26] R. J. Hodrick, *Dividend yields and expected stock returns: alternative procedures for interference and measurement*, The Review of Financial Studies, 3 (1991), 357-386.

[27] R. J. Hodrick, E. Prescott, *Post-war U.S. business cycles: an empirical investigation*, Discussion Papers, Northwestern University, Center for Mathematical Studies in Economics and Management Science, 451 (1997).

[28] U. Jermann, V. Quadrini, *Stock market boom and the productivity gains of the 1990s*, Journal of Monetary Economics, 2 (2007),413-432.

[29] U. Jermann, *Asset pricing in production economies*, Journal of Monetary Economics, 2 (2007), 257-275.

[30] D. Kl, . Uur, *Multiresolution analysis of SP500 time series*, Annals of Operations Research, Springer, 1 (2018), 197-216.

[31] D. B. Keim, F. S. Robert, *Predicting returns in the stock and bond markets*, Journal of Financial Economics, 1 (2018), 357-390.

[32] O. Lamont, *Earnings and expected returns*, Journal of Finance, 5 (1998), 1563-1587.

[33] M. Lettau, S. Ludvigson, *Consumption, aggregate wealth, and expected stock returns*, Journal of Finance, 4 (2001), 815-849.

[34] M. Lettau, V. Nieuwerburgh, *Reconciling the return predictability evidence*, Review of Financial Studies, 4 (2008), 1607-1652.

[35] M. Lettau, S. Ludvigson, J. Wachter, *The Declining Equity Premium: What role does macroeconomic risk play?*, Review of Financial Studies, 4 (2008), 1653-1687.

[36] M. Lubrano, *Modèles VAR, modèles VAR structurels et modèles à équations simultanées*, 2007.

[37] S. Mallat, *A wavelet tour of signal processing*, Academic Press, San Diego, USA, 1998.

[38] N. G. Mankiw, D. Romer, M. D. Shapiro,*Stock market forecast ability and volatility: a statistical appraisal*, NBER Working Papers, (1989), 89-21.

[39] P. Masset,*Analysis of Financial Time-Series using Fourier and Wavelet Methods*, SSRN Electronic Journal, 2008.

[40] L. Quarta,*Une introduction (élémentaire) à la théorie des ondelettes*, Cahier pédagodique, 2011.

[41] P. Rancourt,*Le ratio cours/bénéfices*, L'entrepreneur boursier, 2008.

[42] G. Rouwenhorst, *Asset Pricing Implications of Equilibrium Business Cycles Models*, Elsevier Science B.V, (1995), 294–330.

[43] SP 500, *SP 500 Earnings and Estimate Report*, http://pages.stern.nyu.edu.

[44] SLB, *Saint Louis Bank*, https://www.stlouisbank.com/.

[45] J. Schrrer, *High frequency time series analysis using wavelets*, POSTER, 2015.

[46] Sims, *Macroeconomics and reality*, Econometrica, 40 (1980), 1-48.

[47] T. Taji, D. Gore, *Adaptive texture segmentation using M band wavelet transform and wavelet packet*, International Journal of Computer Science and Information Technologies, 5 (2014), 7305-7308.

[48] T. Vuorenmaa, *A Multiresolution Analysis of Stock Market Volatility Using Wavelet Methodology*, Licentiate thesis, Helsinki: University of Helsinki, 2005.

[49] T. Vuorenmaa, *A Wavelet analysis of scaling laws and long-memory in stock market volatility*, SSRN Electronic Journal, 2006.

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