Event-triggered consensus control for the first-order multi-agent systems

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Abstract. In this paper, event-triggered consensus control problem for the first-order multi agent systems is studied. The control law updating considered is event-driven, which depends on the measurement error with state function norm and its neighboring agents. The proposed updating law is only triggered at its event time instants for each agent. The proposed event-based strategy doesn't require continuous monitoring the neighbors states that can lessen the rate of the control law updating, therefore, communication resources can be saved. The proposed control protocols ensure convergence to a ball focused on the average consensus, and provide a minimum interevent time to avoid Zeno behavior. Finally, a simulation example is shown to illustrate the theoretical result.
Keywords: event-triggered control, multi-agent systems (MASs), consensus.

1. Introduction

Multi-agent systems (MASs) have become an active research topic in control communities. MASs have been widely applied in practical fields, for example, intelligent robot design and traffic control. Average consensus is a basic research for multi-agent coordination that drives all agents to obtain the desired common goal relying solely on neighboring information. In [1], consensus analysis for single-integrator agents is first discussed. Nowadays, there are many excellent control strategies about consensus of MASs, for instance, continuous control [2] and periodic sampling control [3]. Consensus analysis has been widely investigated in Refs. [1-14] and many references therein.

Continuous communication among neighbor agents is commonly utilized for distributed consensus problem design. However, continuous control strategy requires that the control laws are updated uninterruptedly. This may result in large communication load and high frequencies of updating laws. Therefore, periodic sampling control strategy is introduced to reduce the rate of control updating. From the perspective of practical application, such control strategies must be implemented on a digital platform. Typically, control task is executed periodically. The measurements are carried out periodically based on a given constant sampling interval, which control law is updated in a fixed period. Although the traditional periodic sampling fashion is well-developed, it may not be suitable for resource limited systems, because with the fixed sampling period setup, communication resource can be used even though it is not necessary from the performance perspectives. Therefore, an event-triggered method is more effective first proposed in [4, 5] by considering the limited resources for embedded processors. The author in [4] gives an event-triggered PID controller, where only discrete states are monitored so the CPU usage can be reduced. In [5], event based sampling is presented instead of periodic sampling, where control laws update only if the measured value exceeds a given threshold. Therefore, event-triggered control policy can be utilized for reducing the frequency of control updating, the numbers of re-computation and transmission. At the same time, it guarantees the desired performance so that event-based controls for MASs have been developed in the above papers.

Because of the advantage of event-triggered control, event-based consensus problems of MASs are discussed in [6]. But the triggering conditions require that agent i must monitor agent j's state continuously. In [7], an original control scheme for multi-agent coordination under event-triggered broadcasting is proposed. In this work, the state independent and exponentially declining threshold is introduced in the triggering condition. Then, continuous monitoring among neighbor agents can be averted by using this method. In [8], the distributed event-triggered control scheme was given on networked systems, whose subsystems are linear time-invariant and interconnected. In [9], event-based average consensus problem for linear system is investigated, where the state independent trigger conditions are introduced for reducing communication load. In [10], an event-based control scheme for general linear systems is provided, where the state independent trigger condition is utilized for saving communication resources. However, the trigger conditions used in [8-10] emphasize some state independent triggering function. The authors in [11-13] studied event-triggered control mechanisms for MASs. Here control updating relies on triggering condition that requires the state dependent threshold. Event-based consensus problem for distributed MASs is investigated by utilizing state-dependent trigger function in [14]. But the proof of the inequality (28) in [14] is incorrect. This motivates us to search the suitable triggering condition, which guarantees consensus of MASs.

In this paper, event-triggering control scheme will be planned for MASs. The control updating of each agent is decided by definitional events that rely on the states of neighboring agents and measurement error. The primary contribution is that one sufficient consensus condition is presented by using the state-dependent trigger function, and it avoids continuous communication between neighboring agents. By using this policy, the signals transmissions number is reduced, and then the inter-communication will decrease.

The rest is organized in the following. In section 2, we introduce some concepts on algebraic graph theory and statement of the problem. In section 3, we state the main results. The simulation result is presented to illustrate our analysis result in section 4. Finally, conclusions are given in section 5.

2. Preliminaries

2.1 Algebraic graph theory

Let $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ be an undirected graph of order N, which set of nodes $\mathcal{V} = \{v_1, v_2, \cdots, v_N\}$ represents N agents, and set of edges $\mathcal{E} \in \mathcal{V} \times \mathcal{V}$ corresponds to the communication links between agents; v_i and v_j are called adjacent, if there exists an edge (v_i, v_j) between vertices v_i and v_j. In undirected graph, one has $(v_i, v_j) \in \mathcal{E} \Leftrightarrow (v_j, v_i) \in \mathcal{E}$. The adjacency matrix $A = (a_{ij})$ is the symmetric matrix given by $a_{ij} = 1$, if $(v_i, v_j) \in \mathcal{E}$, and $a_{ij} = 0$, otherwise. $\mathcal{G}$ is called connected, if there is a path between any two nodes in undirected graph $\mathcal{G}$. Given $\mathcal{N}_i = \{v_j \in \mathcal{V} : (v_i, v_j) \in \mathcal{E}\}$, which denotes the indexes of agents that agent v_i can communicate. Let d_i be the cardinality of agent v_i's neighbor set $\mathcal{N}_i$. The degree D for an undirected graph $\mathcal{G}$ is a diagonal matrix $diag\{d_1, d_2, \cdots, d_N\}$. The Laplacian of $\mathcal{G}$ is the symmetric and positive semidefinite matrix $L = D - A$, i.e. $L = L^T > 0$. Note that the Laplacian matrix is zero average, i.e. $L\mathbf{1} = 0$ with $\mathbf{1}^T = [1, 1, \cdots, 1]$. The Laplacian matrix has a single zero eigenvalue for connected graph and corresponding eigenvector is the vector $\mathbf{1}$, which all entries equal to 1. We denote by $0 = \lambda_1(\mathcal{G}) \le \lambda_2(\mathcal{G}) \le \cdots \le \lambda_N(\mathcal{G})$ the eigenvalues of L. If $\mathcal{G}$ is connected, then $\lambda_2(\mathcal{G}) > 0$.

2.2 System modelling and problem statement

By considering the system consists of N agents, we assume that each agent's dynamics follow a single integrator model:

$$\dot{x}_i(t) = u_i(t), \quad i \in \mathcal{V}, \tag{1}$$

where, $x_i(t) \in R$ denotes the state of agent i. $u_i(t) \in R$ is the control input of the ith agent. As is known [1] that continuous distributed control law as follows:

$$u_i(t) = -\sum_{j \in \mathcal{N}_i} (x_i(t) - x_j(t)). \tag{2}$$

Let's drive each agent to asymptotically converge average of the agents' initial conditions. Then, the closed-loop system can be written as follows:

$$\dot{x}(t) = Lx(t),$$

where $x^T(t) = (x_1^T(t), \cdots, x_N^T(t))$.

This paper will consider the continuous distributed control law with an event-based implementation, which is considered as:

$$u_i(t) = -\sum_{j \in \mathcal{N}_i} (\hat{x}_i(t) - \hat{x}_j(t)). \tag{3}$$

While, in stack vector form $u(t) = -L\hat{x}(t)$, where $u^T(t) = (u_1^T(t), \cdots, u_N^T(t))$ and $\hat{x}^T(t) = (\hat{x}_1^T(t), \cdots, \hat{x}_N^T(t))$. In order to make sure that the agents' initial states average is maintained throughout the evolution of the system, and the agent i does not use its actual state $x_i(t)$ but the last broadcast value $\hat{x}_i(t)$ with $\hat{x}_i(t) = x_i(t_k^i)$, $t \in [t_k^i, t_{k+1}^i)$, where $t_0^i, t_1^i, \cdots$ denotes the sequence of event triggers of agent i. The next event-triggering instant t_k^i is decided by triggering function $f_i(t, e_i(t), x_i(t)) > 0$, and it is shown as follows:

$$t_{k+1}^i = \inf\{t : t > t_k^i, f_i(t, e_i(t), x_i(t)) > 0\}, k = 0, 1, \cdots, \infty. \tag{4}$$

For each $i \in \mathcal{V}$ and $t \geq 0$, the measurement error is defined as

$$e_i(t) = \hat{x}_i(t) - x_i(t). \tag{5}$$

It can be expressed in compact form as

$$\dot{x}(t) = -\mathcal{L}\hat{x}(t) = -\mathcal{L}(x(t) + e(t)), \tag{6}$$

where $e^T(t) = [e_1^T(t), \cdots, e_N^T(t)]$.

Denote $a(t) = \frac{1}{N}\sum_{i=1}^{N} x_i(t)$ as the average of all agents' states. Then, the derivative of $a(t)$ is

$$\dot{a}(t) = \frac{1}{N}\sum_{i=1}^{N} \dot{x}_i(t) = \frac{1}{N}\sum_{i=1}^{N} \mathbf{1}^T \dot{x}(t) = -\frac{1}{N}\sum_{i=1}^{N} \mathbf{1}^T \mathcal{L}\hat{x}(t) = 0.$$

Therefore, $a(t) = a(0) = a$ for all $t \geq 0$ and the state $x(t)$ can be decomposed as

$$x(t) = a\mathbf{1} + \delta(t), \tag{7}$$

where, $\delta(t)$ is the disagreement vector of the MASs, i.e. $\mathbf{1}^T\delta(t) = 0$. From (7) and $\delta(t) = x(t) - a\mathbf{1}$, it follows that

$$\dot{\delta}(t) = \dot{x}(t) = -\mathcal{L}(e(t) + x(t)) = -\mathcal{L}e(t) - \mathcal{L}\delta(t). \tag{8}$$

This means that

$$\delta(t) = e^{-\mathcal{L}t}\delta(0) - \int_0^t \mathcal{L}e(\tau)e^{-\mathcal{L}(t-\tau)}d\tau, \tag{9}$$

where, $\delta(0)$ means the initial value of $\delta(t)$.

The following lemma and definitions are introduced, which will be utilized in the proof of ours.

Lemma 1 ([7])**.** *Suppose $\mathcal{L}$ is the Laplacian matrix of an connected and undirected graph $\mathcal{G}$. Then, for $t \geq 0$ and all vectors $\nu \in R^n$ with $\mathbf{1}^T\nu = 0$, we have*

$$\| e^{-\mathcal{L}t}\nu \| \leq e^{-\lambda_2(\mathcal{G})t} \| \nu \| . \tag{10}$$

Definition 1 ([1]). MAS is called to achieve average consensus, if all agents' states converge to average of the initial states, i.e., and $\forall i \in \mathcal{V}$,

$$\lim_{t\to\infty} x_i(t) = \frac{1}{N}\sum_{i\in\mathcal{V}} x_i(0), \tag{11}$$

where $x_i(0)$ is the initial state of agent i.

Definition 2. A hybrid system H is Zeno, if for some execution ε of H , there exists a finite constant τ_∞ such that $\lim_{i\to\infty}\tau_i = \sum_{i=0}^{\infty}(\tau_{i+1}-\tau_i) = \tau_\infty$. The execution ε is called a Zeno execution.

Remark 1. The definition of a Zeno execution results in the types of Zeno behavior. For an execution ε that is Zeno, ε is Chattering Zeno: If there exists a finite C such that $\tau_{i+1}-\tau_i > 0$ for all $i > C$.

Remark 2. The definition for the closed-loop system is following. Feedback occurs when outputs of a system are routed back as inputs as part of a chain of cause-and-effect that forms a circuit or loop. The system can then be said to feed back into itself.

3. Consensus analysis

Next, a sufficient consensus policy for MAS (1) is obtained in the following theorem, where the trigger condition only requires the discrete measurement of neighbor states.

Theorem 1. *Consider the MASs (1) with control rule (3) and event-triggering times is decided by (4). Suppose that the triggering function is devised as:*

$$f_i(t, e_i(t), x_i(t)) = \mid e_i(t)\mid - b_0\sqrt{\sum_{j\in\mathcal{N}_i} \mid \hat{x}_j(t) - \hat{x}_i(t)\mid^2}, \tag{12}$$

and $b_0 \in (0, \frac{\lambda_2(\mathcal{G})}{\sqrt{2}\left((\sqrt{\|L\|})^3+\lambda_2(\mathcal{G})\sqrt{\|L\|}\right)})$. *Then, for all initial conditions* $x(0) \in R^N$, *zeno behavior does not occur. Furthermore, the disagreement vector* $\delta(t)$ *for closed-loop system will exponential converge to a ball focus on the rendezvous with radius* Δ, *where,* $\Delta = \frac{\sqrt{2}b_0(\sqrt{\|\mathcal{L}\|})^3\sqrt{N}|a|}{\lambda_2(\mathcal{G})-\sqrt{2}b_0\sqrt{\|L\|}(\lambda_2(\mathcal{G})+\|\mathcal{L}\|)}$.

Proof. For the triggering condition (12), the upper bound of measurement error is shown as

$$\| e(t) \| = \sqrt{\sum_{i=1}^{N} \mid e_i(t)\mid^2} \le b_0\sqrt{\sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i} \mid \hat{x}_j(t) - \hat{x}_i(t)\mid^2}$$

$$
\begin{aligned}
&= b_0\sqrt{2\hat{x}^T(t)\mathcal{L}\hat{x}(t)} \leq \sqrt{2}b_0\sqrt{\parallel \mathcal{L} \parallel} \parallel \hat{x}(t) \parallel \\
&\leq \sqrt{2}b_0\sqrt{\parallel \mathcal{L} \parallel}(\parallel e(t) + x(t) \parallel) \\
&\leq \sqrt{2}b_0\sqrt{\parallel \mathcal{L} \parallel} \parallel e(t) \parallel + \sqrt{2}b_0\sqrt{\parallel \mathcal{L} \parallel} \parallel x(t) \parallel, \qquad (13)
\end{aligned}
$$

from which we can get

$$
\begin{aligned}
(1-\sqrt{2}b_0\sqrt{\parallel \mathcal{L} \parallel}) \parallel e(t) \parallel &\leq \sqrt{2}b_0\sqrt{\parallel \mathcal{L} \parallel} \parallel x(t) \parallel \\
&\leq \sqrt{2}b_0\sqrt{\parallel \mathcal{L} \parallel}(\parallel \delta(t) \parallel + \sqrt{N} \mid a \mid) \\
&= \sqrt{2}b_0\sqrt{\parallel \mathcal{L} \parallel} \parallel \delta(t) \parallel + \sqrt{2}b_0\sqrt{\parallel \mathcal{L} \parallel}\sqrt{N} \mid a \mid . \qquad (14)
\end{aligned}
$$

It follows that

$$
\parallel e(t) \parallel \leq \frac{\sqrt{2}b_0\sqrt{\parallel \mathcal{L} \parallel}}{1-\sqrt{2}b_0\sqrt{\parallel \mathcal{L} \parallel}}(\parallel \delta(t) \parallel + \sqrt{N} \mid a \mid), \qquad (15)
$$

where, $0 < b_0 < \frac{\lambda_2(\mathcal{G})}{\sqrt{2}\left((\sqrt{\|L\|})^3+\lambda_2(\mathcal{G})\sqrt{\|L\|}\right)} < \frac{1}{\sqrt{2}\sqrt{\|L\|}}$.

Let $b_1 = \frac{\sqrt{2}b_0\sqrt{\|\mathcal{L}\|}}{1-\sqrt{2}b_0\sqrt{\|\mathcal{L}\|}}, b_2 = \frac{\sqrt{2}b_0\sqrt{\|\mathcal{L}\|}}{1-\sqrt{2}b_0\sqrt{\|\mathcal{L}\|}}\sqrt{N} \mid a \mid$. Rewrite the upper bound of measurement errors as:

$$
\parallel e(t) \parallel \leq b_1 \parallel \delta(t) \parallel + b_2. \qquad (16)
$$

From Eq. (9), we can get that the disagreement vector is bounded by

$$
\begin{aligned}
\parallel \delta(t) \parallel &= \parallel e^{-\mathcal{L}t}\delta(0) \parallel + \int_0^t e^{-\mathcal{L}(t-\tau)} \parallel \mathcal{L}e(\tau) \parallel d\tau \\
&\leq e^{-\lambda_2(\mathcal{G})t} \parallel \delta(0) \parallel + \int_0^t e^{-\lambda_2(\mathcal{G})(t-\tau)} \parallel \mathcal{L}e(\tau) \parallel d\tau \\
&\leq e^{-\lambda_2(\mathcal{G})t} \parallel \delta(0) \parallel + \parallel \mathcal{L} \parallel \int_0^t e^{-\lambda_2(\mathcal{G})(t-\tau)} \parallel e(\tau) \parallel d\tau \\
&= e^{-\lambda_2(\mathcal{G})t} \parallel \delta(0) \parallel + b_2 \parallel \mathcal{L} \parallel \int_0^t e^{-\lambda_2(\mathcal{G})(t-\tau)} d\tau \\
&\quad + b_1 \parallel \mathcal{L} \parallel \int_0^t \parallel \delta(\tau) \parallel e^{-\lambda_2(\mathcal{G})(t-\tau)} d\tau. \qquad (17)
\end{aligned}
$$

Since $b_0 \in (0, \frac{\lambda_2(\mathcal{G})}{\sqrt{2}((\sqrt{\|L\|})^3+\lambda_2(\mathcal{G})\sqrt{\|L\|})}) \subset (0, \frac{1}{\sqrt{2}\sqrt{\|\mathcal{L}\|}})$, we have $\frac{b_1\|\mathcal{L}\|}{\lambda_2(\mathcal{G})} < 1$. Therefore, there is a positive constant $\rho \in (0, \lambda_2(\mathcal{G}))$ such that $\frac{b_1\|\mathcal{L}\|}{\lambda_2(\mathcal{G})-\rho} < 1$. We can claim that

$$
\parallel \delta(t) \parallel \leq \parallel \delta(0) \parallel e^{-\rho t} + \Delta, \qquad (18)
$$

where, $\Delta = \frac{\sqrt{2}b_0(\sqrt{\|\mathcal{L}\|})^3\sqrt{N}|a|}{\lambda_2(\mathcal{G})-\sqrt{2}b_0\sqrt{\|L\|}(\lambda_2(\mathcal{G})+\|\mathcal{L}\|)}$.

In order to prove (18), for any $\eta > 1$ and $t > 0$, we first show that the following inequality is true,

$$\| \delta(t) \| \leq \eta \| \delta(0) \| e^{-\rho t} + \Delta = v(t). \tag{19}$$

Otherwise, there is a $t^* > 0$ such that $\| \delta(t^*) \| = v(t^*)$ and $\| \delta(t) \| < v(t)$ as well as $t \in (0, t^*)$. From (17), we have:

$$\begin{aligned}
v(t^*) = \| \delta(t^*) \| &\leq e^{-\lambda_2(\mathcal{G})t^*} \| \delta(0) \| + b_2 \| \mathcal{L} \| \int_0^{t^*} e^{-\lambda_2(\mathcal{G})(t^*-\tau)} d\tau \\
&\quad + b_1 \| \mathcal{L} \| \int_0^{t^*} \| \delta(\tau) \| e^{-\lambda_2(\mathcal{G})(t^*-\tau)} d\tau \\
&= e^{-\lambda_2(\mathcal{G})t^*} \| \delta(0) \| + \frac{b_1 \eta \| \mathcal{L} \| \| \delta(0) \|}{\lambda_2(\mathcal{G}) - \rho} (e^{-\rho t^*} - e^{-\lambda_2(\mathcal{G})t^*}) \\
&\quad + \Big(\frac{b_2 \| \mathcal{L} \|}{\lambda_2(\mathcal{G})} + \frac{b_1 \| \mathcal{L} \| \Delta}{\lambda_2(\mathcal{G})} \Big) (1 - e^{-\lambda_2(\mathcal{G})t^*}) \\
&< \eta \| \delta(0) \| \Big[e^{-\lambda_2(\mathcal{G})t^*} + \frac{b_1 \| \mathcal{L} \|}{\lambda_2(\mathcal{G}) - \rho} (e^{-\rho t^*} - e^{-\lambda_2(\mathcal{G})t^*}) \Big] \\
&\quad + \Big(\frac{b_2 \| \mathcal{L} \|}{\lambda_2(\mathcal{G})} + \frac{b_1 \| \mathcal{L} \| \Delta}{\lambda_2(\mathcal{G})} \Big) (1 - e^{-\lambda_2(\mathcal{G})t^*}) \\
&< \eta \| \delta(0) \| e^{-\rho t^*} + \Big(\frac{b_2 \| \mathcal{L} \|}{\lambda_2(\mathcal{G})} + \frac{b_1 \| \mathcal{L} \| \Delta}{\lambda_2(\mathcal{G})} \Big) (1 - e^{-\lambda_2(\mathcal{G})t^*}) \\
&< \eta \| \delta(0) \| e^{-\rho t^*} + \frac{b_2 \| \mathcal{L} \| + b_1 \| \mathcal{L} \| \Delta}{\lambda_2(\mathcal{G})} \\
&= \eta \| \delta(0) \| e^{-\rho t^*} + \Delta = v(t^*). \qquad (20)
\end{aligned}$$

Obviously, this is a contradiction which implies the inequality (19) holds for any $\eta > 1$. Then, by letting $\eta \to 1$, the inequality (18) holds. Since $b_0 \in (0, \frac{\lambda_2(\mathcal{G})}{\sqrt{2}\big((\sqrt{\|L\|})^3 + \lambda_2(\mathcal{G})\sqrt{\|L\|}\big)})$, we have

$$\lambda_2(\mathcal{G}) - \sqrt{2} b_0 \sqrt{\| L \|} (\lambda_2(\mathcal{G}) + \| \mathcal{L} \|) > 0.$$

This means that $\Delta > 0$. Consequently, $\| \delta(t) \|$ is exponentially converged to a ball with radius Δ as $t \to \infty$. Then, the state of MASs converge exponentially to a ball focuses on the average consensus.

To avoid the Zeno-behavior, we have to indicate that there exists a minimum interevent time τ, for $t \geq t_0$, $\hat{x}_i(t)$ and $\hat{x}_j(t)$ remain constant. Since $\dot{e}_i(t) = -\dot{x}_i(t)$, the evolution of the error is simply written as

$$e_i(t) = -(t - t_0) \sum_{j \in N_i} (\hat{x}_j(t) - \hat{x}_i(t)). \tag{21}$$

Then, when $f_i(t, e_i(t), x_i(t)) = 0$ occurs, we will find the time t^* at which a broadcast of agent i's state is triggered. If $\sum_{j \in N_i} (\hat{x}_j(t) - \hat{x}_i(t)) = 0$, which implies $e_i(t) = 0$ for all $t \geq t_0$, no broadcast will ever happen ($t^* = \infty$). Hence, if $\sum_{j\in N_i}(\hat{x}_j(t) - \hat{x}_i(t)) \neq 0$, we can derive $(\sum_{j\in N_i}(\hat{x}_j(t) - \hat{x}_i(t)))^2 \neq 0$. By using (21), the trigger condition (12) prescribes a broadcast at the time $t^* \geq t_0$, which can satisfy that:

$$(t^* - t_0)^2\Big(\sum_{j\in N_i}(\hat{x}_j(t) - \hat{x}_i(t))\Big)^2 - b_0^2\sum_{j\in N_i}(\hat{x}_j(t) - \hat{x}_i(t))^2 = 0, \tag{22}$$

or, equivalently:

$$(t^* - t_0)^2 = \frac{b_0^2\sum_{j\in N_i}(\hat{x}_j(t) - \hat{x}_i(t))^2}{\Big(\sum_{j\in N_i}(\hat{x}_j(t) - \hat{x}_i(t))\Big)^2}. \tag{23}$$

For any $y_1, y_2, \cdots, y_p \in R$ and $p \in Z$, using the Cauchy-Schwarz inequility $(\sum_{k=1}^p y_k)^2 \leq p\sum_{k=1}^p y_k^2$, we get

$$\Big(\sum_{j\in N_i}(\hat{x}_i(t) - \hat{x}_j(t))\Big)^2 \leq \mid N_i \mid \sum_{j\in N_i}(\hat{x}_i(t) - \hat{x}_j(t))^2. \tag{24}$$

Therefore, we substitute Eq.(24) into Eq.(23) so that we can get the minimum interevent time by

$$\tau = t^* - t_0 \geq \frac{b_0}{\sqrt{\mid N_i \mid}} > 0.$$

From the above discussions, it can be easy to find that there exists a minimum inter-event time τ, which implies that the Zeno-behavior is avoided in agent i. This completes the proof.

Remark 1. The consensus analysis of a class of MASs with single-integrator is investigated in [14]. However, the proof of the inequality (28) in [14] is incorrect. By Eq. (27), the author in [14] gets:

$$\begin{aligned}&\frac{\lambda_2(\mathcal{G}) - \parallel \mathcal{L} \parallel (b_1 + 1)}{\lambda_2(\mathcal{G})}\int_0^t e^{-\lambda_2(\mathcal{G})(t-\tau)} \parallel \delta(\tau) \parallel d\tau \\ &\leq \frac{1}{\lambda_2(\mathcal{G})} \parallel \delta(t) \parallel - \Big(\frac{1}{\lambda_2(\mathcal{G})} \parallel \delta(0) \parallel + \frac{\parallel \mathcal{L} \parallel b_2}{\lambda_2^2(\mathcal{G})}\Big)e^{-\lambda_2(\mathcal{G})t} + \frac{\parallel \mathcal{L} \parallel b_2}{\lambda_2^2(\mathcal{G})}.\end{aligned}$$

Obviously, by the condition in [21],

$$\lambda_2(\mathcal{G}) - \parallel \mathcal{L} \parallel (b_1 + 1) = \frac{(1 - b_0\sqrt{\parallel \mathcal{L} \parallel})\lambda_2(\mathcal{G}) - \parallel \mathcal{L} \parallel}{(1 - b_0\sqrt{\parallel \mathcal{L} \parallel})\lambda_2(\mathcal{G})}.$$

Then, since $b_0 \in (0, \frac{1}{\sqrt{\|\mathcal{L}\|}})$ and $\lambda_2(\mathcal{G}) \leq \| \mathcal{L} \|$, it can be easy to get $\lambda_2(\mathcal{G}) - \| \mathcal{L} \| (b_1 + 1) < 0$. That implies

$$\int_0^t e^{-\lambda_2(\mathcal{G})(t-\tau)} \| \delta(\tau) \| d\tau \geq \frac{\| \delta(t) \|}{\lambda_2(\mathcal{G}) - \| \mathcal{L} \| (b_1+1)} - \frac{\| \delta(0) \| \lambda_2(\mathcal{G}) + \| \mathcal{L} \| b_2}{\lambda_2(\mathcal{G})\big(\lambda_2(\mathcal{G}) - \| \mathcal{L} \| (b_1+1)\big)} e^{-\lambda_2(\mathcal{G})t} + \frac{\| \mathcal{L} \| b_2}{\lambda_2(\mathcal{G})\big(\lambda_2(\mathcal{G}) - \| \mathcal{L} \| (b_1+1)\big)}.$$

Then, the inequality (28) is incorrect, which results that the following conclusions are wrong. In our paper, we correct this error and obtain the ideal result to guarantee the consensus of MASs. Our design is based on the evolution of the network divergence, and triggers are designed by using locally available information, which allows each agent to resolve when to broadcast its current state to neighbors.

Remark 2. The graph G is considered to be undirected in this paper. However, it can be easily found that Theorem 1 extends to strongly connected and balanced directed graphs.

4. Illustrative example

Now, an illustrative example is provided to show the validity of our approach. A MAS which consists of five agents is considered, and the system can be expressed as the form of (1).

Example. By considering a system of five agents, the Laplacian matrix is shown as:

$$\mathcal{L} = \begin{bmatrix} 2 & -1 & 0 & -1 & 0 \\ -1 & 3 & -1 & -1 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ -1 & -1 & 0 & 3 & -1 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix}. \tag{25}$$

By using the control policy proposed in Theorem 1, Fig.1 depicts the communication topology between agents. Given $b_0 = 0.04 \in (0, 0.0475)$, by using the controller (3) and the triggering mechanism (12), Fig.2 shows the response of states of all agents, from which it can easy to find that five agents will achieve consensus.

Fig. 3-Fig. 7 show the evolution of error norm of each agent that embodies the distributed event-triggered strategy. When the error norm of each agent reach threshold $b_0\sqrt{\sum_{j\in\mathcal{N}_i} | \hat{x}_j(t) - \hat{x}_i(t) |^2}$, the error norm of each agent is reset to 0.

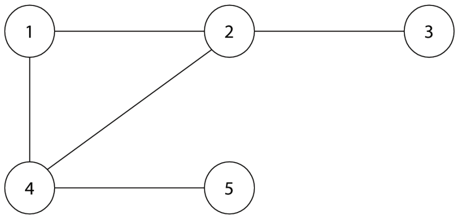

Figure 1: Communication graph $\mathcal{G}$ of the MASs

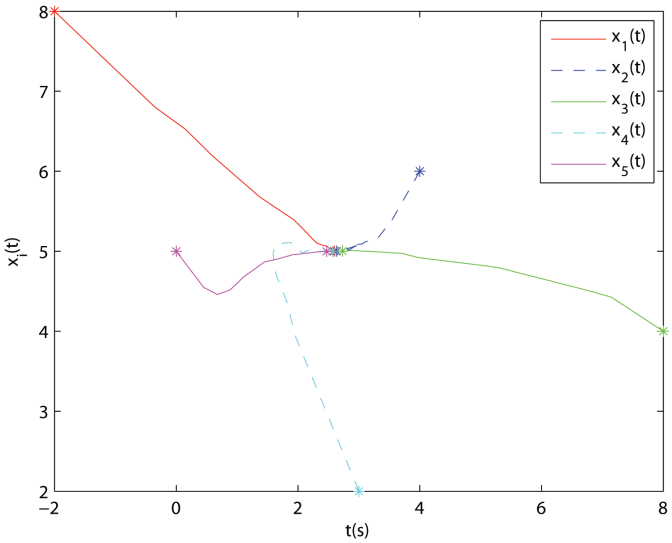

Figure 2: State trajectories of five agents with event-triggered control.

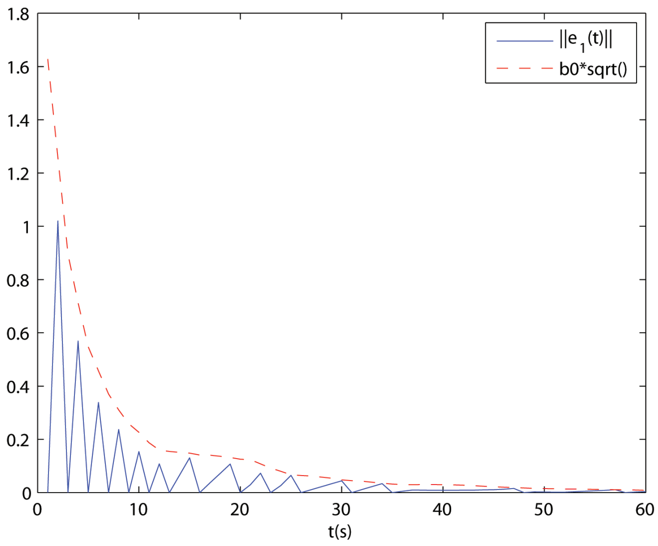

Figure 3: The evolution of $\| e_1(t) \|$ for agent 1

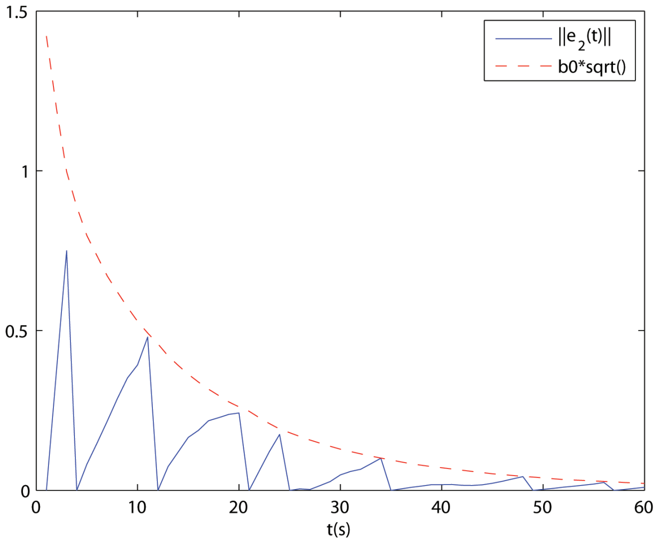

Figure 4: The evolution of $\| e_2(t) \|$ for agent 2

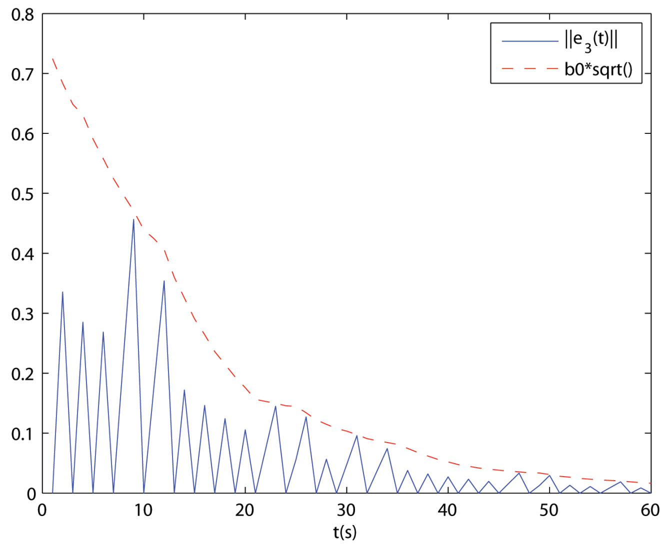

Figure 5: The evolution of $\| e_3(t) \|$ for agent 3

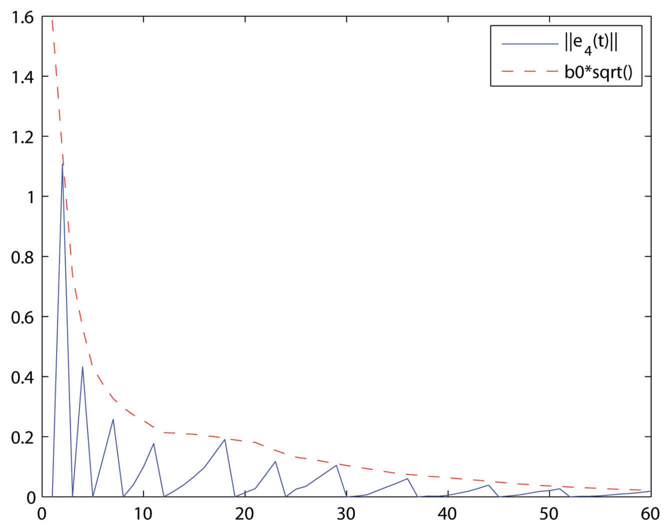

Figure 6: The evolution of $\| e_4(t) \|$ for agent 4

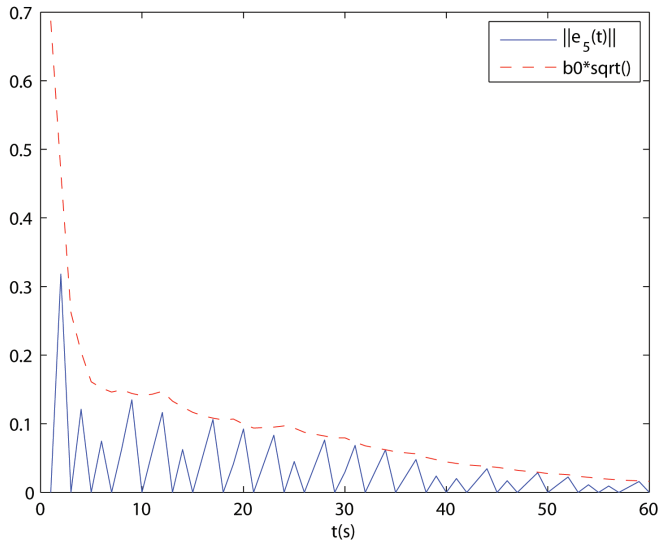

Figure 7: The evolution of $\| e_5(t) \|$ for agent 5

Fig.8 shows that event-triggered instants for each agent during $[0, 4.5]$. The distributed event-triggered scheme reduces significantly the frequency of signal transmission and control updating.

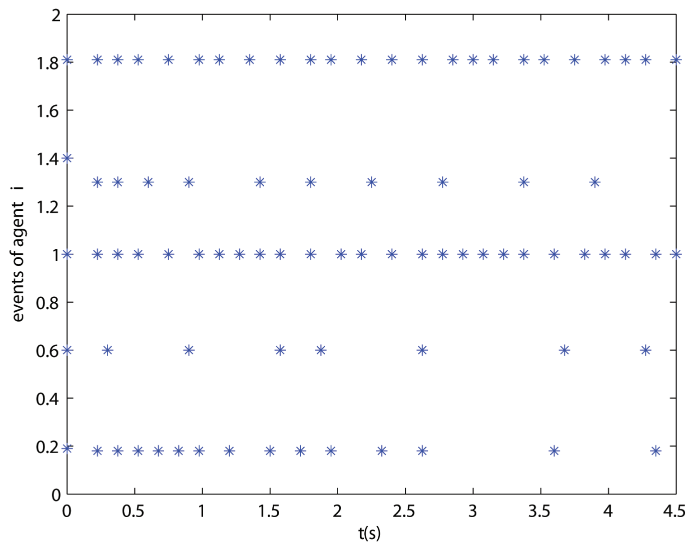

Figure 8: Event time of five agents with $b_0 = 0.04$.

5. Conclusion

This paper studied event-based consensus problem of distributed MASs by using the algebraic graph theory. One sufficient condition is presented to guarantee consensus on the trigger functions. The trigger condition avoids monitoring the states of each agents neighbors continuously. As the suggested event-triggered control policy does not require continuous local information, so it is more suitable for practical implementation. Finally, the simulation result was indicated to show the effectiveness of the proposed approach.

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