# Magnetic effect of non-commutativity

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**Abstract.** In this paper we argue that 2D crystals emit electromagnetic radiation and exhibit extra electromagnetic effects, by virtue of the noncommutative nature of the 2D crystal space.
**Keywords:** electromagnetism, noncommutativity, extra effects.

## 1. Introduction

In an earlier communication [1] we had shown the effects of non–commutativity on Maxwell's equations starting from the usual relation

$$E^2 = p^2c^2 + m^2c^4. \tag{1}$$

Now, if we take natural units i.e.$\hbar = c = 1$ we can rewrite this equation as

$$E^2 = p^2 + m^2. \tag{2}$$

## 2. Non-commutative Maxwell's equation

Maxwell's equation in covariant format can be written as

$$\partial^\mu F_{\mu\nu} = \frac{4\pi}{c} j_\nu. \tag{3}$$

This particular equation is an abridge form with $\mu = 1, 2, 3, 4$ and $\nu = 1, 2, 3, 4$. More fully

$$\partial_\mu(\partial_\mu A_\nu - \partial_\nu A_\mu) = \frac{4\pi}{c} j_\nu, \tag{4}$$

$$\partial_\mu \partial_\mu A_\nu - \partial_\mu \partial_\nu A_\mu = \frac{4\pi}{c} j_\nu. \tag{5}$$

Let us consider a transformation [4, 6] in which

$$p_\mu p^\mu = D_{\mu\mu} - \Gamma^\mu_{\lambda\lambda} \partial_\mu, \tag{6}$$

$$p_\mu p^\nu = D_{\mu\nu} - \Gamma^\mu_{\lambda\lambda} \partial_\nu, \tag{7}$$

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where $p_\mu \equiv \partial_\mu$ and which is occurring due to the fact that space-time is non-commutative [4]. As we can see, the last term on the right side is zero then we return to continuous space-time. As shown in detail elsewhere the last term occurs below Compton wavelength when negative energies begins to dominate [4]. Now, we substitute (6) and (7) in equation (5) and try to analyse the physical behavior

$$(D_{\mu\mu} - \Gamma^{\mu}_{\lambda\lambda}\partial_\mu)A_\nu - (D_{\mu\nu} - \Gamma^{\mu}_{\lambda\lambda}\partial_\nu)A_\mu = \frac{4\pi}{c}j_\nu, \tag{8}$$

$$(D_{\mu\mu}A_\nu - D_{\mu\nu}A_\mu) = \frac{4\pi}{c}j_\nu + (\Gamma^{\mu}_{\lambda\lambda}\partial_\mu A_\nu - \Gamma^{\mu}_{\lambda\lambda}\partial_\nu A_\mu), \tag{9}$$

$$(D_{\mu\mu}A_\nu - D_{\mu\nu}A_\mu) = \frac{4\pi}{c}j_\nu + \Gamma^{\mu}_{\lambda\lambda}(\partial_\mu A_\nu - \partial_\nu A_\mu), \tag{10}$$

$$\partial^\mu F_{\mu\nu} = \frac{4\pi}{c}j_\nu + \Gamma^{\mu}_{\lambda\lambda}F_{\mu\nu}, \tag{11}$$

or this can be written as

$$\partial^\mu F_{\mu\nu} = \frac{4\pi}{c}j_\nu + \partial_\lambda \epsilon F_{\lambda\nu}, \tag{12}$$

where $\varepsilon$ in (12) is a dimensionless number which is equal to one for our non-commutative case (6) and equals zero for the usual commutative case and has been introduced for clarity. In the usual commutative space and time $\varepsilon = 0$ and we get back the usual covariant Maxwell's equations given in (3). So now there is an additional term in the Maxwell equations. Now let us try to derive these equations for two dimensions. We will see that this is a non-trivial case. We get

$$\partial^1 F_{12} = \frac{4\pi}{c}j_2 + \partial_4 \varepsilon F_{42}, \tag{13}$$

$$\partial^2 F_{21} = \frac{4\pi}{c}j_1 + \partial_4 \varepsilon F_{41}, \tag{14}$$

$$\partial^4 F_{42} = \frac{4\pi}{c}j_2 + \partial_1 \varepsilon F_{12}, \tag{15}$$

$$\partial^4 F_{41} = \frac{4\pi}{c}j_1 + \partial_2 \varepsilon F_{21} \tag{16}$$

$$\partial^2 F_{24} = \frac{4\pi}{c}j_4 + \partial_1 \varepsilon F_{14}, \tag{17}$$

$$\partial^1 F_{14} = \frac{4\pi}{c}j_4 + \partial_2 \varepsilon F_{24}. \tag{18}$$

Now, from Covariant electrodynamics we know that $F_{\mu\nu}$ is the electromagnetic tensor and is given by [7] $\nu \rightarrow$

$$F_{\mu\nu} = \mu \downarrow \begin{pmatrix} 0 & -cB_z & cB_y & -E_x \\ cB_z & 0 & -cB_x & -E_y \\ -cB_y & cB_x & 0 & -E_z \\ E_x & E_y & E_z & 0 \end{pmatrix}. \tag{19}$$

So, this electromagnetic tensor will give Maxwell's equations. We can see that the electromagnetic tensor is asymmetric. Now from the different components from (18)-(16) given by using equation (19) Taking $c = 1$ we get

$$-\frac{\partial B_z}{\partial x} = 4\pi j_y + \epsilon \frac{\partial E_y}{\partial t}, \tag{20}$$

$$\frac{\partial B_z}{\partial y} = 4\pi j_x + \epsilon \frac{\partial E_x}{\partial t}, \tag{21}$$

$$\frac{\partial E_y}{\partial t} = 4\pi j_y - \epsilon \frac{\partial B_z}{\partial x}, \tag{22}$$

$$\frac{\partial E_x}{\partial t} = 4\pi j_x + \epsilon \frac{\partial B_z}{\partial y}, \tag{23}$$

$$-\frac{\partial E_y}{\partial y} = 4\pi \frac{\partial \rho}{\partial t} - \epsilon \frac{\partial E_x}{\partial x}, \tag{24}$$

$$-\frac{\partial E_x}{\partial x} = 4\pi \frac{\partial \rho}{\partial t} - \epsilon \frac{\partial E_y}{\partial y}, \tag{25}$$

where $E$ is for the extra term.

## 3. Results and discussions

So, from equations (20) to (25) we can see that there is an extra term in the Maxwell equations and if $\varepsilon = 0$ then we get the usual Maxwell's equations.

Thus, we can see that introduction of non-commutative space-time leads to extra effects in Maxwell's equations (20) to (25) which indicate an extra electromagnetic field.

In commutative space-time $\varepsilon = 0$ and the extra magnetic effects will disappear. To get more insight into the extra magnetic effect we observe an alternate way of expressing the non-commutativity above. This is with a minimum space-time extension which also leads to non-commutative geometry.

We have now a relation such that

$$[x, y] = l^2\Theta, \tag{26}$$

where $x$ and $y$ are coordinates and $l$ is the minimum extension and $\Theta$ are matrices. Because of non-commutative geometry (26) it is known that there is the generation of a magnetic field which was shown independently by the author and Saito [8, 9], with the relation

$$Bl^2 = hc/e. \tag{27}$$

In other words, the non-commutativity of space-time generates the extra electromagnetic effects.

We specialize to the case of 2D crystals where also we have this situation. The result is that we can expect some additional radiation for example. This effect would be stronger than in the 3D case. We can crossover from 3D to 2D by using the transformation [10, 11] which essentially replaces the velocity of light $c$ with the Fermi velocity in crystals ([10]). (The remarkable thing about this theory is we are able to deduce theoretically the entire infinite sequence of Von Klitzing levels.)

So, we have the terms like the last transformation in the RHs of equations (20)-(25). These point to effects like extra radiation and the like.

## 4. Two dimensional graphene

The best example of the foregoing is the recent understanding of space-time in Graphene or any crystal [12] which approximates the above 2D scenario. The authors concluded that electron spin is attributed to the discreteness of space-time which resembles a chess board and this type of a discrete partition of space-time will lead to the generation of a magnetic field which may even affect the intrinsic behavior of the electron.

Here space-time is discrete and during the hopping of electrons there is change in spin direction which shows that the space is not smooth and this give rise to some sort of a magnetic field which can cause the change of spin in the electron during hopping. So we can see that above behavior of electron spin can be analogous to the above with appearance of a magnetic field in non-commutative space-time. The extra terms in the equations (20)-(25) correspond to additional electromagnetic effects including radiation.

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