Two simple confidence intervals for the population coefficient of variation under the non-normal and skewed distributions

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Abstract. This paper presents two simple confidence intervals (CIs) for the population coefficient of variation (CV) in the case of non-normal distributions. The first is based on the Bonett [6] formula for calculating an approximate CI for the variance (σ^2) of the non-normal distributions and the other is based on the Niwitpong and Kirdwichai [29] formula for calculating an adjusted Bonett [6] confidence interval for the variance of the non-normal distributions. An extensive Monte-Carlo simulation study was conducted to compare the performance of the proposed CIs with the other existing CIs available in the literature. The simulation results showed that the proposed two simple confidence intervals perform well in terms of coverage probability and expected average width. The proposed two methods are illustrated using two real life data which reinforced the findings of the simulation study to some extent.

Keywords: confidence interval, coefficient of variation, non-normal distribution, coverage probability, expected average width.

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1. Introduction

The coefficient of variation (hereinafter termed CV) is a quantity describing the variation between results in different populations. It is a relative measure of dispersion based on mean and standard deviation [12]. The concept of CV was introduced by Pearson [16]and since then it has been employed in a wide variety of fields such as medicine, engineering, economic, and science (See for example [24, 25]). The population CV is defined as a ratio of the population standard deviation (σ) to the population mean (μ):

$$CV = \frac{\sigma}{\mu}, \quad \mu \neq 0. \tag{1}$$

In practice, the population CV is unknown and needs to be estimated. The typical sample estimate of the population CV is given as:

$$CV = \frac{S}{\bar{X}} \tag{2}$$

where S and $\bar{X}$ are the sample standard deviation and the sample mean, respectively.

Developing a CI for the population CV is a key problem; see, for example, Koopmans et al. [20], Miller [9], Sharma and Krishna [18], Vangel [22], Tian [19], Wong and Wu [2], Mahmoudvand and Hassani [26], Curto and Pinto [15], Banik and Kibria [28], Niwitpong [27], Albatineh et al. [3] etc. The classical CIs for CV are not robust to violations of normality [7]. In this study, we propose two simple approximate confidence intervals for the population CV in non-normal distributions. We compare the performance of the proposed confidence intervals with the existing confidence intervals in terms of coverage probability and expected average width. An extensive Monte-Carlo simulation study is conducted to compare the performance of the interval estimators. Since in real-life the data could be non-normal or skewed, we also conducted simulations under skewed distributions. Because of this study is related to the performance comparison of the available and proposed methods for population coefficient of variation (CV), we want to express that the proposed two methods performs well and are good competitors compared with the methods in the literature.

The organization of this paper is as follows: Section 2 details the literature review of the study. In Sections 3 and 4 we present the existing and proposed confidence intervals. Section 5 includes simulation technique and results that represent simulations for normal and skewed distributions. Two scenarios using real-life data illustrate the implementation of the methods in Section 6. Finally, some concluding remarks are presented in Section 7.

2. Literature review

There are several approaches and various techniques considered by many researchers available for constructing the CI for the population CV. McKay [4] proposed a confidence interval (hereinafter termed CI) for the population CV based on the chi-square distribution; this confidence interval works well when $CV \leq 0.33$ [11]. Hendricks and Robey [33] studied the distribution of the sample CV when the random sample is drawn from a normal distribution. Later, Koopmans et al. [20], Iglewicz [5], and others described confidence interval (CI) for population CV for normal and lognormal distributions. Lehmann [10] proposed an exact method for the confidence interval of population CV. Vangel [22] proposed a new confidence interval for population CV also called a modified McKay's confidence interval.

Miller [9] discussed the approximate distribution of population CV and proposed the approximate CI for population CV in the case of a normal distribution. Sharma and Krishna[18] introduced the asymptotic distribution and CI of the reciprocal of the population CV, which does not require any assumptions to be made about the population distribution. Wong and Wu [2] obtained approximate CI for the population CV of normal and non-normal models. Verrill [30] reviewed the exact approach appropriate for normally distributed data.

The performance of CIs for population CV obtained by McKay's, Miller's, and Sharma-Krishna's methods was compared under the same simulation conditions by Ng [17]. Panichkitkosolkul [31] modified McKay's CI by replacing the sample CV with the maximum likelihood estimator for a normal distribution. Panichkitkosolkul [32] presented three CIs for the CV in a normal distribution with a known population mean where all three proposed confidence intervals performed well in terms of coverage probability and expected length.

Mahmoudvand and Hassani [26] proposed an approximately unbiased estimator for population CV in a normal distribution and used this estimator for constructing two approximate CIs for the population CV. Curto and Pinto[15] constructed the CI for the population CV when random variables are not independently and identically distributed. Banik and Kibria [28] reviewed several interval estimators for estimating the population CV and compared them to bootstrap interval estimators for various distributions.

Buntao and Niwitpong [23] also introduced an interval estimating the difference of the CV for lognormal and delta-lognormal distributions. Gulhar et al. [21] compared CIs for estimating the population CV based on parametric, nonparametric, and modified methods. Niwitpong [27] presented CIs for the population CV of log-normal distribution with restricted parameter space. Finally, Sangnawakij and Niwitpong [25] proposed CIs for the single CV and the difference between coefficients of variation in two-parameter exponential distributions.

3. The Confidence intervals for the population coefficient of variation

Let X_1, X_2, X_3, ..., X_n be an independently and identically distributed (iid) random sample of size n from a distribution with finite mean (μ) and variance

(σ^2). In the following, we shall describe some interval estimation procedures for the population CV. Since we will deal with and evaluate a large number of methods for constructing CIs for population CV, we now catalog all of them and their acronyms for easy reference as follows:

HR: Hendricks and Robey.
McK: McKay.
MMcK: Modified McKay.
Panich: Panichkitkosolkul.
Mill: Miller.
S&K: Sharma and Krishna.
C&P: Curto and Pinto.
GKA&A: Gulhar, Kibria, Albatineh and Ahmed.
MOV: MOVER.
M&H(I) and M&H(II): Two versions of Mahmoudvand & Hassani.
AK&A(I) and AK&A(II): Two versions of Abu-Shawiesh, Khurshid & Akyüz.

3.1 Hendricks and Robey Confidence Interval

Hendricks and Robey [33] developed following CI for the population CV based on a random sample from a normal population:

$$CI_{HR} = \left(\hat{CV} - t_{(n-1\,,\,\alpha/2)}\; S_{\hat{CV}}\;\;,\;\; \hat{CV} + t_{(n-1\,,\,\alpha/2)}\; S_{\hat{CV}}\right), \tag{3}$$

where $t_{(\alpha/2\,,\,n-1)}$ is the $100(1-\alpha/2)$ th percentile of the Student t-distribution with $(n-1)$ degrees of freedom, $\hat{CV} = s/\bar{X}$ is the sample CV and$S_{\hat{CV}} = (\hat{CV}\Big/\sqrt{2n})$ is the standard error of $\hat{CV}$.

3.2 McKay's Confidence Interval

McKay [4] suggested the following CI for population CV:

$$CI_{McK} = \left(\left(\frac{s}{\bar{X}}\right)\sqrt{\left(\frac{\chi^2_{v\,,\,1-\alpha/2}}{v+1}-1\right)\left(\frac{s}{\bar{X}}\right)^2+\frac{\chi^2_{v\,,\,1-\alpha/2}}{v}},\right.$$
$$\left.\left(\frac{s}{\bar{X}}\right)\sqrt{\left(\frac{\chi^2_{v\,,\,\alpha/2}}{v+1}-1\right)\left(\frac{s}{\bar{X}}\right)^2+\frac{\chi^2_{v\,,\,\alpha/2}}{v}}\right) \tag{4}$$

where $\chi^2_{v\,,\,1-\alpha/2}$ and $\chi^2_{v\,,\,\alpha/2}$are $100(1-\alpha/2)$ th and $100(\alpha/2)$ th percentile of the chi-square distribution with $v = (n-1)$ degrees of freedom respectively.

3.3 Modified McKay's Confidence Interval

Vangel [22] modified the McKay's CI as:

$$CI_{MMcK} = \left(\left(\frac{s}{\bar{X}}\right) \sqrt{\left(\frac{\chi^2_{v\,,\,1-\alpha/2}+2}{v+1}-1\right)\left(\frac{s}{\bar{X}}\right)^2+\frac{\chi^2_{v\,,\,1-\alpha/2}}{v}}, \right.$$

$$\left. \left(\frac{s}{\bar{X}}\right) \sqrt{\left(\frac{\chi^2_{v\,,\,\alpha/2}+2}{v+1}-1\right)\left(\frac{s}{\bar{X}}\right)^2+\frac{\chi^2_{v\,,\,\alpha/2}}{v}} \right) \tag{5}$$

where $\chi^2_{v\,,\,1-\alpha/2}$ and $\chi^2_{v\,,\,\alpha/2}$are respectively the $100(1-\alpha/2)$ th and $100(\alpha/2)$ th percentile of the chi-square distribution with $v = (n-1)$ degrees of freedom. When data are normally distributed, CI_{McK} and CI_{MMcK} can be used very well in terms of coverage probability and average width. However, these confidence intervals cannot be used for the non-normal distributions.

3.4 Panichkitkosolkul's Confidence Interval

Panichkitkosolkul [31] improved the CI_{MMcK} obtained by Vangel [22]. Panichkitkosolkul's confidence interval is:

$$CI_{Panich} = \left(\tilde{k} \sqrt{\left(\frac{\chi^2_{v\,,\,1-\alpha/2}+2}{v+1}-1\right)\left(\tilde{k}\right)^2+\frac{\chi^2_{v\,,\,1-\alpha/2}}{v}}, \right.$$

$$\left. \tilde{k} \sqrt{\left(\frac{\chi^2_{v\,,\,\alpha/2}+2}{v+1}-1\right)\left(\tilde{k}\right)^2+\frac{\chi^2_{v\,,\,\alpha/2}}{v}} \right) \tag{6}$$

where $\chi^2_{v\,,\,1-\alpha/2}$ and $\chi^2_{v\,,\,\alpha/2}$ $100(1-\alpha/2)$ th and $100(\alpha/2)$ th percentile of the chi-square distribution with $v = (n-1)$ degrees of freedom respectively and $\tilde{k}$ is the maximum likelihood estimator defined as $\tilde{k} = \frac{\sqrt{\sum_{i=1}^{n}(X_i-\bar{X})^2}}{\sqrt{n}\,\bar{X}}$.

3.5 Miller's Confidence Interval

Miller [9] proposed a confidence interval based on the sample coefficient of variation ($\hat{CV}$) approximating an asymptotic normal distribution. Miller's method has the following confidence interval:

$$CI_{Mill} = \left(\left(\frac{S}{\bar{X}}\right) - Z_{1-\alpha/2} \sqrt{\frac{\left(S/\bar{X}\right)^2}{n-1}\left(0.5+\left(\frac{S}{\bar{X}}\right)^2\right)}, \right.$$

$$\left. \left(\frac{S}{\bar{X}}\right) + Z_{1-\alpha/2} \sqrt{\frac{\left(S/\bar{X}\right)^2}{n-1}\left(0.5+\left(\frac{S}{\bar{X}}\right)^2\right)} \right) \tag{7}$$

where $Z_{1-\alpha/2}$ is the $100(1-\alpha/2)$ th percentile of the standard normal distribution.

3.6 Sharma & Krishna's Confidence Interval

Sharma and Krishna [18] proposed following CI for the population CV:

$$CI_{S\&K} = \left(\left[\left(\frac{\bar{X}}{S} \right) - \frac{\phi^{-1}_{(\alpha/2)}}{\sqrt{n}} \right]^{-1} , \left[\left(\frac{\bar{X}}{S} \right) + \frac{\phi^{-1}_{(\alpha/2)}}{\sqrt{n}} \right]^{-1} \right) \tag{8}$$

where ϕ is the cumulative standard normal distribution.

3.7 Curto and Pinto's Confidence Interval

Curto and Pinto [15] proposed a confidence interval for the population CV, given as follows:

$$CI_{C\&P} = \left(\left(\frac{S}{\bar{X}} \right) - Z_{1-\alpha/2} \sqrt{\frac{1}{n} \left(\left(\frac{S}{\bar{X}} \right)^4 + \frac{1}{2} \left(\frac{S}{\bar{X}} \right)^2 \right)}, \right.$$
$$\left. \left(\frac{S}{\bar{X}} \right) + Z_{1-\alpha/2} \sqrt{\frac{1}{n} \left(\left(\frac{S}{\bar{X}} \right)^4 + \frac{1}{2} \left(\frac{S}{\bar{X}} \right)^2 \right)} \right) \tag{9}$$

where $Z_{1-\alpha/2}$ is the $100(1-\alpha/2)$ th percentile of the standard normal distribution.

3.8 Gulhar, Kibria, Albatineh and Ahmed's Confidence Interval

Gulhar et al. [21], engrossing the method for calculating the confidence interval for σ^2 proposed following CI for normal population CV

$$CI_{GKA\&A} = \left(\frac{\sqrt{n-1}\ (S/\bar{X})}{\sqrt{\chi^2_{v\ ,\ 1-\alpha/2}}} \ , \ \frac{\sqrt{n-1}\ (S/\bar{X})}{\sqrt{\chi^2_{v\ ,\ \alpha/2}}} \right) \tag{10}$$

where $\chi^2_{v\ ,\ 1-\alpha/2}$ and $\chi^2_{v\ ,\ \alpha/2}$ are respectively the $100(1-\alpha/2)$ th and $100(\alpha/2)$ th percentile of the chi-square distribution with $v = (n-1)$ degrees of freedom respectively.

3.9 MOVER's Confidence Interval

The MOVER CI for population CV is described as [1]:

$$CI_{MOV} = \left(\left(\bar{X} - \sqrt{\max\left\{ 0\, ,\ \bar{X}^2 + a\, c\, (a-2) \right\}} \right) \left(\frac{S}{c} \right), \right.$$
$$\left. \left(\bar{X} + \sqrt{\max\left\{ 0\, ,\ \bar{X}^2 + b\, c\, (b-2) \right\}} \right) \left(\frac{S}{c} \right) \right) \tag{11}$$

where $a = \sqrt{(n-1) \big/ \chi^2_{(n-1)\,,\,1-\alpha/2}}$, $b = \sqrt{(n-1) \big/ \chi^2_{(n-1)\,,\,\alpha/2}}$ and $c = \bar{X}^2 - \frac{Z^2_{\alpha/2}\, s^2}{n}$.

3.10 Mahmoudvand & Hassani's Confidence Intervals

Mahmoudvand and Hassani [26] introduced two CIs for the population CV when data are normally distributed. By using the normal approximation, their first CI is given as follows:

$$CI_{M\&H(I)} = \left(\left(\frac{S}{\bar{X}}\right) \left(2 - c_n - Z_{\,1-\alpha/2}\sqrt{1-c_n^2}\right)^{-1}, \right. \tag{12}$$
$$\left. \left(\frac{S}{\bar{X}}\right) \left(2 - c_n + Z_{\,1-\alpha/2}\sqrt{1-c_n^2}\right)^{-1} \right)$$

where $c_n = \sqrt{\frac{2}{n-1}} \left[\frac{\Gamma(n/2)}{\Gamma((n-1)/2)}\right]$ and $Z_{\,1-\alpha/2}$ is the $100(1-\alpha/2)$ th percentile of the standard normal distribution. Further they introduced an asymptotically unbiased point estimator as $\hat{\tau} = \frac{(S/\bar{X})}{2-c_n}$. The resulting CI based on $\hat{\tau}$ is

$$CI_{M\&H(II)} = \left(\hat{\tau} - \frac{\hat{\tau}}{2-c_n} Z_{\,1-\alpha/2} \sqrt{(1-c_n^2) + \frac{\hat{\tau}^{\,2}}{n}}, \right. \tag{13}$$
$$\left. \hat{\tau} + \frac{\hat{\tau}}{2-c_n} Z_{\,1-\alpha/2} \sqrt{(1-c_n^2) + \frac{\hat{\tau}^{\,2}}{n}} \right),$$

where c_n and $Z_{\,1-\alpha/2}$ are defined above.

4. The proposed simple confidence intervals for the population CV

In this section, we propose two approximate and simple confidence intervals for estimating the population CV in non-normal and skewed distributions. The first is based on the Bonett [6] formula for calculating an approximate CI for the variance (σ^2) of the non-normal distributions. The second proposed CI is based on the Niwitpong and Kirdwichai [29] formula for calculating an adjusted Bonett [6] confidence interval for the variance of the non-normal distributions. Let $X_1, X_2, X_3, \ldots, X_n$ be continuous, iid random variables with $\sigma^2 = Var(X_i) > 0$, $E(X_i) = \mu$ and finite fourth moment, then the two proposed confidence interval methods can be derived as follows:

4.1 Abu-Shewiesh, Khurshid & Akyüz's (AK&A(I)) Confidence Interval

The Bonett [6] proposed $(1-\alpha)\,100\,\%$ CI for the population variance (σ^2) is given as follows:

$$CI_{Bonett} = \left(\exp\left\{\ln\left(cS^2\right) - Z_{\,1-\alpha/2}\, Se\right\}, \exp\left\{\ln\left(cS^2\right) + Z_{\,1-\alpha/2}\, Se\right\}\right) \tag{14}$$

where $Z_{1-\alpha/2}$ is the $100(1-\alpha/2)$ th percentile of the standard normal distribution, $c = \frac{n}{(n-Z_{1-\alpha/2})}$, $Se = c\,[\frac{\{\hat{\gamma}_4^* - \frac{(n-3)}{n}\}}{(n-1)}]^{1/2}$ and $\hat{\gamma}_4^* = \frac{n_0\tilde{\gamma}_4 + n\tilde{\gamma}_4}{n_0+n}$.

Here $\tilde{\gamma}_4$ is a prior estimate of γ_4 obtained from a larger sample of size n_0 and $\bar{\gamma}_4 = \frac{n\sum_{i=1}^{n}(X_i-m)^4}{\left[\sum_{i=1}^{n}(X_i-\hat{\mu})^2\right]^2}$, $\hat{\mu} = \bar{X}$, where m is a trimmed mean with trim-proportion of $1/\left\{2\,(n-4)^{1/2}\right\}$. However, if $\tilde{\gamma}_4$ is not available then $\hat{\gamma}_4^* = \tilde{\gamma}_4$. From equation (14), we have:

$$\exp\left\{\ln\left(cS^2\right) - Z_{1-\alpha/2}\,Se\right\} < \sigma^2 < \exp\left\{\ln\left(cS^2\right) + Z_{1-\alpha/2}\,Se\right\}. \tag{15}$$

Assuming that $\mu \neq 0$, dividing CI obtained in equation (15) by μ^2results in

$$\frac{\exp\left\{\ln\left(cS^2\right) - Z_{1-\alpha/2}\,Se\right\}}{\mu^2} < \left(\frac{\sigma}{\mu}\right)^2 < \frac{\exp\left\{\ln\left(cS^2\right) + Z_{1-\alpha/2}\,Se\right\}}{\mu^2}. \tag{16}$$

Since μ is unknown, we can replace it by the unbiased estimator of μ which is $\hat{\mu} = \bar{X}$ resulting in:

$$\frac{\exp\left\{\ln\left(cS^2\right) - Z_{1-\alpha/2}\,Se\right\}}{\bar{X}^{\,2}} < CV^2 < \frac{\exp\left\{\ln\left(cS^2\right) + Z_{1-\alpha/2}\,Se\right\}}{\bar{X}^{\,2}}. \tag{17}$$

Taking the square root, in the CI of equation (17) we obtain

$$\frac{\sqrt{\exp\left\{\ln(cS^2) - Z_{1-\alpha/2}\,Se\right\}}}{\bar{X}} < CV < \frac{\sqrt{\exp\left\{\ln(cS^2) + Z_{1-\alpha/2}\,Se\right\}}}{\bar{X}} \tag{18}$$

which can be written as

$$CI_{AK\&A(I)} = \left(\frac{\sqrt{\exp\left\{\ln(cS^2) - Z_{1-\alpha/2}\,Se\right\}}}{\bar{X}},\; \frac{\sqrt{\exp\left\{\ln(cS^2) + Z_{1-\alpha/2}\,Se\right\}}}{\bar{X}}\right). \tag{19}$$

4.2 Abu-Shewiesh, Khurshid & Akyüz's (AK&A(II)) Confidence Interval

The Niwitpong and Kirdwichai [29] adjusted Bonett [6] CI for the population variance σ^2 and proposed the following:

$$CI_{Adjusted\,Bonett} = \left(\exp\left\{\ln\left(cS^2\right) - t_{(\alpha/2,n-1)}\,Se\right\},\; \exp\left\{\ln\left(cS^2\right) + t_{(\alpha/2,n-1)}\,Se\right\}\right), \tag{20}$$

where $t_{(\alpha/2\,,\,n-1)}$ is the $100(1-\alpha/2)$ th percentile of the student t-distribution with $(n-1)$ degrees of freedom, where

$$c = \frac{n}{\left(n - t_{(\alpha/2,\,n-1)}\right)},\; Se = c\left[\frac{\left\{\hat{\gamma}_4^* - \frac{(n-3)}{n}\right\}}{(n-1)}\right]^{1/2} \text{ and } \hat{\gamma}_4^* = \frac{n_0\tilde{\gamma}_4 + n\tilde{\gamma}_4}{n_0+n}.$$

Here, $\tilde{\gamma}_4$ is a prior estimate of γ_4 obtained from a larger sample of size n_0 and $\gamma_4^{/} = \frac{n\sum_{i=1}^{n}(X_i-MD)^4}{\left[\sum_{i=1}^{n}(X_i-\hat{\mu})^2\right]^2}$, $\hat{\mu} = \bar{X}$, where MD is the median, which is a good estimator of μ when the data are from asymmetric and skewed leptokurtic distributions. However, if $\tilde{\gamma}_4$ is not available then $\gamma_4^* = \tilde{\gamma}_4$. From equation (20), we have

$$\exp\left\{\ln\left(cS^2\right) - t_{(\alpha/2\,,n-1)}\,Se\right\} < \sigma^2 < \exp\left\{\ln\left(cS^2\right) + t_{(\alpha/2\,,n-1)}\,Se\right\}. \tag{21}$$

Assuming that $\mu \neq 0$, dividing this confidence interval by μ^2results in

$$\frac{\exp\left\{\ln\left(cS^2\right) - t_{(\alpha/2\,,n-1)}\,Se\right\}}{\mu^2} < \left(\frac{\sigma}{\mu}\right)^2 < \frac{\exp\left\{\ln\left(cS^2\right) + t_{(\alpha/2\,,n-1)}\,Se\right\}}{\mu^2}. \tag{22}$$

Since μ is unknown, we can replace it by the unbiased estimator of μ which is $\hat{\mu} = \bar{X}$resulting in

$$\frac{\exp\left\{\ln\left(cS^2\right) - t_{(\alpha/2\,,n-1)}\,Se\right\}}{\bar{X}^{\,2}} < CV^2 < \frac{\exp\left\{\ln\left(cS^2\right) + t_{(\alpha/2\,,n-1)}\,Se\right\}}{\bar{X}^{\,2}}. \tag{23}$$

Taking the square root in (23) we have

$$\frac{\sqrt{\exp\left\{\ln\left(cS^2\right) - t_{(\alpha/2\,,n-1)}\,Se\right\}}}{\bar{X}} < CV < \frac{\sqrt{\exp\left\{\ln\left(cS^2\right) + t_{(\alpha/2\,,n-1)}\,Se\right\}}}{\bar{X}}. \tag{24}$$

That is

$$CI_{AK\&A(II)} = \left(\frac{\sqrt{\exp\left\{\ln\left(cS^2\right) - t_{(\alpha/2\,,n-1)}\,Se\right\}}}{\bar{X}},\ \frac{\sqrt{\exp\left\{\ln\left(cS^2\right) + t_{(\alpha/2\,,n-1)}\,Se\right\}}}{\bar{X}}\right). \tag{25}$$

5. Simulation study

To judge the performance of the CIs discussed in Sections 3 and 4 a Monte-Carlo simulation under the same simulation conditions, was conducted using the statistical software MATLAB R2016a. The performance of the confidence intervals was considered for various CV values, sample sizes (n) and probability distributions.

5.1 The Simulation Technique

In this simulation, we study the behavior of small and large sample sizes by using $n =$ 15, 25, 50, 100. If the sample size (n) is increased further, this will be examined by using a second simulation for a sample size of $n =$ 500. The data were generated from: (i) a normal distribution with parameters, with a known population mean $\mu = 10$ and known population standard deviation $\sigma = 1$, 3, 5. (ii) Chi-Square distribution with degrees of freedom (df) $\nu =$ 200, 22, 8.
(iii) Gamma distribution with parameters $\alpha = 100$, 11.11, 4 and $\beta = 2$.

The number of simulation replications was $M = 50,000$ for each case. The coefficient of variation and type I error were considered as $CV = 0.10$, 0.30, 0.50 and $\alpha = 0.05$, respectively. The CV was calculated for each one of the three distributions by utilizing the characteristics provided in Table 1.

Table 1: The CV and skewness of data from Normal, Chi-square and Gamma distributions

Distribution	CV	Skewness
$N(\mu, \sigma^2)$	σ/μ	0
Chi-Square (ν)	$\sqrt{2/\nu}$	$2\sqrt{2/\nu}$
Gamma $(\alpha, 2)$	$1/\sqrt{\alpha}$	$2/\sqrt{\alpha}$

The estimated coverage probability (CP) and expected width (EW) for two-sided CIs for CV were obtained as follows [13]:

$$CP = \frac{\sum_{i=1}^{50,000} I\ (L_i \leq CV \leq U_i)}{50,000}$$

$$EW = \frac{\sum_{i=1}^{50,000} (U_i - L_i)}{50,000}$$

where $I = 1$ if $(L \leq CV \leq U)$, and $I = 0$, otherwise.

The simulated coverage probabilities and expected widths for confidence intervals using Normal, Chi-Square and Gamma distributions for grid of values are presented in Tables 2, 3 and 4 respectively, whilst the results for $n = 500$ being provided in Table 5. Each table gives results for the various sample sizes and CV values previously mentioned.

Table 2: Estimated coverage probabilities (CP) and expected widths (EW) of the 95% confidence interval using Normal distribution

Measure	Coefficient of Variation Confidence Intervals												
	HR	McK	MMcK	Panich	Mill	S&K	C&P	GKA&A	MOV	M&H(I)	M&H(II)	AK&A(I)	AK&A(II)
n = 15 , CV = 0.10													
CP	0.9225	0.9105	0.9061	0.8845	0.9126	0.2038	0.9061	0.9459	0.9488	0.9444	0.8948	0.9578	0.9742
EW	0.0770	0.0728	0.0711	0.0702	0.0736	0.0102	0.0711	0.0831	0.0840	0.0801	0.0704	0.0966	0.1099
n = 15 , CV = 0.30													
CP	0.9133	0.9091	0.9063	0.8844	0.9136	0.5370	0.9063	0.9287	0.9536	0.9280	0.8943	0.9453	0.9653
EW	0.2319	0.2398	0.2318	0.2283	0.2400	0.0951	0.2318	0.2502	0.2735	0.2412	0.2280	0.2911	0.3315
n = 15 , CV = 0.50													
CP	0.8872	0.9028	0.9032	0.8803	0.9090	0.7193	0.9032	0.8887	0.9536	0.8888	0.8913	0.9164	0.9423
EW	0.3913	0.4934	0.4489	0.4531	0.4647	0.2919	0.4489	0.4222	0.5387	0.4070	0.4379	0.4924	0.5601
n = 25 , CV = 0.10													
CP	0.9333	0.9272	0.9245	0.9113	0.9289	0.2066	0.9245	0.9465	0.9492	0.9455	0.9179	0.9420	0.9623
EW	0.0577	0.0562	0.0554	0.0550	0.0566	0.0078	0.0554	0.0604	0.0610	0.0591	0.0551	0.0629	0.0702
n = 25 , CV = 0.30													
CP	0.91934	0.9262	0.9237	0.9096	0.9282	0.5468	0.9237	0.9281	0.9504	0.9272	0.9167	0.9254	0.9494
EW	0.1738	0.1839	0.1801	0.1787	0.1838	0.0723	0.1801	0.1817	0.1981	0.1780	0.1784	0.1894	0.2113
n = 25 , CV = 0.50													
CP	0.8880	0.9209	0.9219	0.9070	0.9258	0.7284	0.9219	0.8891	0.9518	0.8881	0.914	0.8871	0.9188
EW	0.2920	0.3639	0.3446	0.3485	0.3517	0.2121	0.3446	0.3053	0.3819	0.2990	0.3397	0.3180	0.3545
n = 50 , CV = 0.10													
CP	0.9416	0.9395	0.9382	0.9314	0.9402	0.2116	0.9382	0.9479	0.9507	0.9479	0.9347	0.9432	0.9538
EW	0.0400	0.0396	0.0394	0.0392	0.0398	0.0055	0.0394	0.0408	0.0412	0.0404	0.0393	0.0417	0.0437
n = 50 , CV = 0.30													
CP	0.9242	0.9391	0.9373	0.9299	0.9394	0.5507	0.9373	0.9274	0.9497	0.9270	0.9336	0.9241	0.9351
EW	0.1202	0.1291	0.1276	0.1274	0.1289	0.0505	0.1276	0.1228	0.1337	0.1216	0.1270	0.1254	0.1315
n = 50 , CV = 0.50													
CP	0.8916	0.9416	0.9391	0.9336	0.9412	0.7380	0.9391	0.8907	0.9512	0.8910	0.9351	0.8900	0.9045
EW	0.2009	0.2504	0.2417	0.2454	0.2441	0.1437	0.2417	0.2054	0.2539	0.2033	0.2400	0.2097	0.2198
n = 100 , CV = 0.10													
CP	0.9423	0.9422	0.9415	0.9378	0.9427	0.2151	0.9415	0.9458	0.9483	0.9456	0.9399	0.9426	0.9495
EW	0.0279	0.0280	0.0279	0.0278	0.0280	0.0039	0.0279	0.0282	0.0285	0.0281	0.0278	0.0284	0.0292
n = 100 , CV = 0.30													
CP	0.9266	0.9448	0.9432	0.9409	0.9444	0.5525	0.9432	0.9286	0.9491	0.9286	0.9412	0.9247	0.9318
EW	0.0840	0.0909	0.0903	0.0903	0.0907	0.0355	0.0903	0.0849	0.0924	0.0845	0.0901	0.0853	0.0877
n = 100 , CV = 0.50													
CP	0.8911	0.9486	0.9446	0.9444	0.9456	0.7414	0.9446	0.8911	0.9515	0.8916	0.9429	0.8866	0.8967
EW	0.1403	0.1752	0.1704	0.1735	0.1713	0.0998	0.1704	0.1419	0.1746	0.1412	0.1698	0.1424	0.1465

Table 3: Estimated coverage probabilities (CP) and expected widths (EW) of the 95% confidence interval using Chi-Square distribution

Measure	Coefficient of Variation Confidence Intervals												
	HR	McK	MMcK	Panich	Mill	S&K	C&P	GKA&A	MOV	M&H(I)	M&H(II)	AK&A(I)	AK&A(II)
n = 15 , CV = 0.10													
CP	0.9247	0.9125	0.9080	0.8879	0.9152	0.2063	0.9080	0.9505	0.9533	0.9486	0.8973	0.9669	0.9775
EW	0.0768	0.0726	0.0710	0.0700	0.0735	0.0101	0.0710	0.0829	0.0838	0.0799	0.0702	0.1005	0.1109
n = 15 , CV = 0.30													
CP	0.9243	0.9199	0.9182	0.8950	0.9236	0.5663	0.9182	0.9473	0.9670	0.9451	0.9060	0.9778	0.9835
EW	0.2314	0.2387	0.2310	0.2274	0.2391	0.0941	0.2310	0.2496	0.2725	0.2406	0.2272	0.3339	0.3568
n = 15 , CV = 0.50													
CP	0.9154	0.9223	0.9227	0.8997	0.9293	0.7759	0.9227	0.9430	0.9764	0.9402	0.9113	0.9799	0.9870
EW	0.3808	0.4651	0.4296	0.4302	0.4447	0.2688	0.4296	0.4109	0.5129	0.3961	0.4194	0.6114	0.6555
n = 25 , CV = 0.10													
CP	0.93194	0.9264	0.9235	0.9104	0.9279	0.2136	0.9235	0.9473	0.9500	0.9462	0.9169	0.9615	0.9642
EW	0.0577	0.0561	0.0553	0.0549	0.0565	0.0078	0.0553	0.0603	0.0609	0.0590	0.0550	0.0697	0.0709
n = 25 , CV = 0.30													
CP	0.9296	0.9340	0.9325	0.9185	0.9363	0.5719	0.9325	0.9438	0.9622	0.9421	0.9249	0.9734	0.9737
EW	0.1737	0.1835	0.1798	0.1784	0.1835	0.0720	0.1798	0.1816	0.1978	0.1778	0.1781	0.2344	0.2324
n = 25 , CV = 0.50													
CP	0.9195	0.9389	0.9401	0.9259	0.9433	0.7836	0.9401	0.9368	0.9746	0.9349	0.9327	0.9775	0.9781
EW	0.2868	0.3524	0.3352	0.3380	0.3421	0.2023	0.3352	0.2999	0.3710	0.2937	0.3306	0.4359	0.4376
n = 50 , CV = 0.10													
CP	0.9412	0.9389	0.9375	0.9303	0.9397	0.2122	0.9375	0.9479	0.9504	0.9474	0.9340	0.9560	0.9550
EW	0.0399	0.0396	0.0393	0.0392	0.0397	0.0055	0.0393	0.0408	0.0412	0.0404	0.0392	0.0445	0.0443
n = 50 , CV = 0.30													
CP	0.9340	0.9459	0.9449	0.9374	0.9469	0.5751	0.9449	0.9423	0.9619	0.9419	0.9409	0.9699	0.9690
EW	0.1203	0.1293	0.1277	0.1275	0.1290	0.0506	0.1277	0.1230	0.1338	0.1217	0.1271	0.1503	0.1486
n = 50 , CV = 0.50													
CP	0.9215	0.9545	0.9542	0.9476	0.9559	0.7853	0.9542	0.9286	0.9724	0.9274	0.9500	0.9736	0.9721
EW	0.1992	0.2468	0.2385	0.2419	0.2410	0.1407	0.2385	0.2036	0.2505	0.2015	0.2369	0.2852	0.2894
n = 100 , CV = 0.10													
CP	0.9447	0.9447	0.9442	0.9407	0.9451	0.2182	0.9442	0.9482	0.9502	0.9479	0.9425	0.9546	0.9539
EW	0.0279	0.0280	0.0279	0.0278	0.0280	0.0039	0.0279	0.0282	0.0285	0.0281	0.0278	0.0298	0.0297
n = 100 , CV = 0.30													
CP	0.9383	0.9554	0.9544	0.9506	0.9554	0.5736	0.9544	0.9420	0.9607	0.9419	0.9521	0.9685	0.9691
EW	0.0843	0.0913	0.0906	0.0907	0.0911	0.0357	0.0906	0.0852	0.0927	0.0848	0.0904	0.1009	0.1016
n = 100 , CV = 0.50													
CP	0.9214	0.9621	0.9600	0.9588	0.9608	0.7865	0.9600	0.9246	0.9690	0.9248	0.9584	0.9734	0.9754
EW	0.1396	0.1737	0.1691	0.1721	0.1700	0.0986	0.1691	0.1411	0.1733	0.1404	0.1686	0.1937	0.2016

Table 4: Estimated coverage probabilities (CP) and expected widths (EW) of the 95% confidence interval using Gamma distribution

Measure	Coefficient of Variation Confidence Intervals												
	HR	McK	MMcK	Panich	Mill	S&K	C&P	GKA&A	MOV	M&H(I)	M&H(II)	AK&A(I)	AK&A(II)
n = 15 , CV = 0.10													
CP	0.9236	0.9124	0.9082	0.8870	0.9146	0.2077	0.9082	0.9501	0.9530	0.9474	0.8969	0.9659	0.9769
EW	0.0769	0.0726	0.0710	0.0701	0.0735	0.0101	0.0710	0.0829	0.0838	0.0799	0.0702	0.1006	0.1110
n = 15 , CV = 0.30													
CP	0.9205	0.9161	0.9142	0.8923	0.9202	0.5635	0.9142	0.9456	0.9646	0.9435	0.9029	0.9762	0.9836
EW	0.2298	0.2369	0.2292	0.2257	0.2372	0.0929	0.2292	0.2479	0.2703	0.2390	0.2254	0.3313	0.3545
n = 15 , CV = 0.50													
CP	0.9137	0.9204	0.9207	0.8972	0.9272	0.7739	0.9207	0.9395	0.9750	0.9366	0.9088	0.9788	0.9857
EW	0.3805	0.4647	0.4293	0.4299	0.4443	0.2686	0.4293	0.4105	0.5125	0.3957	0.4191	0.6090	0.6528
n = 25 , CV = 0.10													
CP	0.9335	0.9280	0.9255	0.9120	0.9295	0.2089	0.9255	0.9494	0.9518	0.9482	0.9186	0.9628	0.9659
EW	0.0576	0.0561	0.0553	0.0549	0.0564	0.0078	0.0553	0.0603	0.0609	0.0590	0.0550	0.0697	0.0709
n = 25 , CV = 0.30													
CP	0.9309	0.9350	0.9335	0.9192	0.9371	0.5691	0.9335	0.9453	0.9639	0.9448	0.9263	0.9746	0.9748
EW	0.1729	0.1826	0.1789	0.1776	0.1826	0.0714	0.1789	0.1808	0.1968	0.1771	0.1772	0.2328	0.2308
n = 25 , CV = 0.50													
CP	0.9177	0.9380	0.9390	0.9243	0.9427	0.7805	0.9390	0.9333	0.9744	0.9311	0.9316	0.9773	0.9779
EW	0.2869	0.3526	0.3354	0.3381	0.3423	0.2025	0.3354	0.3000	0.3712	0.2937	0.3307	0.4370	0.4388
n = 50 , CV = 0.10													
CP	0.9404	0.9381	0.9365	0.9294	0.9392	0.2150	0.9365	0.9479	0.9503	0.9473	0.9326	0.9549	0.9556
EW	0.0399	0.0396	0.0394	0.0392	0.0398	0.0055	0.0394	0.0408	0.0412	0.0404	0.0392	0.0444	0.0443
n = 50 , CV = 0.30													
CP	0.9363	0.9483	0.9468	0.9399	0.9489	0.5757	0.9468	0.9438	0.9614	0.9425	0.9430	0.9709	0.9696
EW	0.1198	0.1286	0.1271	0.1268	0.1284	0.0501	0.1271	0.1224	0.1331	0.1212	0.1265	0.1493	0.1477
n = 50 , CV = 0.50													
CP	0.9194	0.9537	0.9534	0.9464	0.9547	0.7870	0.9534	0.9277	0.9705	0.9267	0.9492	0.9732	0.9720
EW	0.1992	0.2468	0.2385	0.2419	0.2409	0.1407	0.2385	0.2036	0.2505	0.2015	0.2369	0.2851	0.2893
n = 100 , CV = 0.10													
CP	0.9459	0.9460	0.9453	0.9418	0.9465	0.2143	0.9453	0.9478	0.9503	0.9478	0.9438	0.9544	0.9543
EW	0.0279	0.0280	0.0279	0.0278	0.0280	0.0039	0.0279	0.0283	0.0285	0.0281	0.0278	0.0298	0.0297
n = 100 , CV = 0.30													
CP	0.9380	0.9539	0.9528	0.9492	0.9537	0.5744	0.9528	0.9409	0.9606	0.9405	0.9509	0.9680	0.9683
EW	0.0839	0.0908	0.0901	0.0902	0.0905	0.0353	0.0901	0.0848	0.0922	0.0844	0.0899	0.1003	0.1009
n = 100 , CV = 0.50													
CP	0.9211	0.9632	0.9614	0.9586	0.9624	0.7845	0.9614	0.9255	0.9707	0.9238	0.9587	0.9741	0.9758
EW	0.1396	0.1738	0.1692	0.1721	0.1700	0.0987	0.1692	0.1412	0.1733	0.1405	0.1686	0.1936	0.2015

5.2 The simulation results

In this section, the simulation results will be discussed by the distribution type.

5.3 Normal distribution results

In Table 2, it is seen that coverage probabilities and average widths of proposed confidence intervals for Normal Distribution when $CV = 0.10, 0.30, 0.50$ and $\alpha = 0.05$ are quite close to the nominal confidence level even in small sample sizes. The coverage probabilities of all confidence intervals except S&K confidence interval are close to the nominal confidence level for each value of the coefficient of variation. Proposed two methods have performed very well in terms of average widths. As the value of the coefficient of variation decreases, the average width of confidence intervals decreases. For large samples, i.e. $n = 500$, from Table 5 is seen that the coverage probabilities of proposed confidence intervals for Normal distribution are similar to the coverage probabilities of confidence intervals based on skewed distributions. The coverage probabilities of proposed confidence intervals are quite close to the nominal confidence level for all the sample sizes and coefficient of variations when $\alpha = 0.05$.

5.4 Non-normal (skewed) distributions results

The coverage probabilities of proposed confidence intervals based on Chi-square and Gamma distributions are quite close to the nominal confidence level for $\alpha = 0.05$. Average widths of the confidence intervals shrunk as the sample size increases. The S&K confidence interval has the narrowest average widths in all cases. The proposed confidence intervals performed as well as the other confidence intervals (Tables 3-4).

The proposed methods are robust, but slightly wider than the existing confidence intervals even in very small samples. This can be easily seen from the Figures (1-4).

6. Empirical applications

In this section, we consider two real-life data to illustrate the performance of the proposed simple CIs.

6.1 Example 1: Infants Weights Data

The first data set was obtained from Ziegler et al. [8] (cited in Ledolter and Hogg [14]).

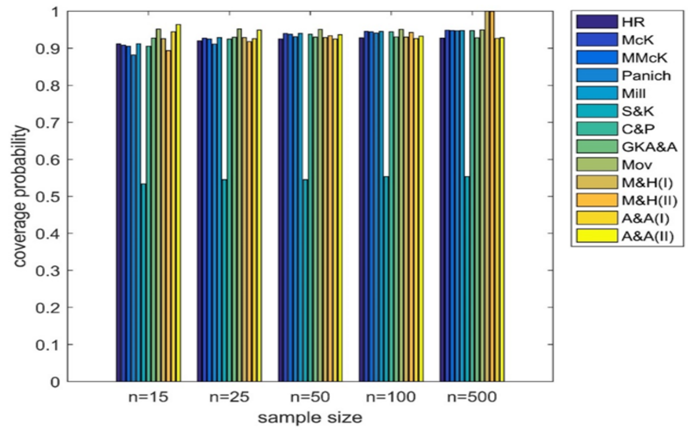

Figure 1: Sample size vs. coverage probability for normal distribution when CV = 0.30

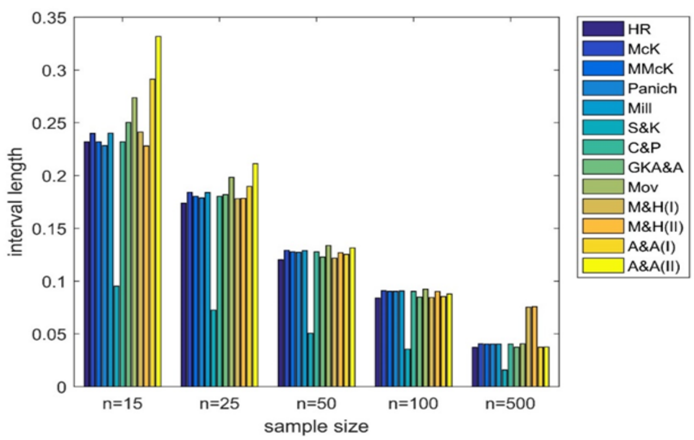

Figure 2: Sample Size vs. Interval Length for Normal Distribution when CV = 0.30

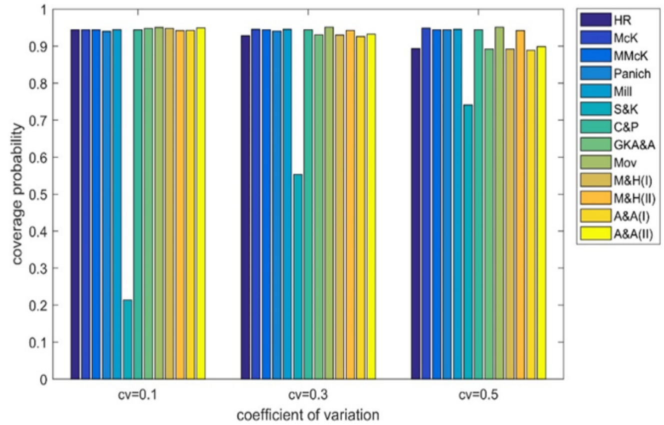

Figure 3: Coefficient of Variation vs. Coverage Probability for Normal Distribution when n = 100

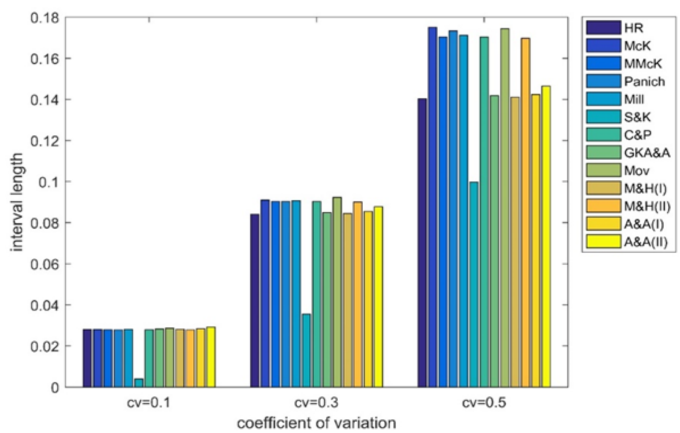

Figure 4: Coefficient of Variation vs. Interval Length for Normal Distribution when n = 100

Table 5: Estimated coverage probabilities (CP) and expected widths (EW) of the 95% confidence interval using Normal, Chi-Square and Gamma distributions (n=500)

	Coefficient of Variation Confidence Intervals												
	HR	McK	MMcK	Panich	Mill	S&K	C&P	GKA&A	MOV	M&H(I)	M&H(II)	AK&A(I)	AK&A(II)
Normal Distribution													
CV = 0.10													
CP	0.9445	0.9460	0.9459	0.9455	0.9460	0.2166	0.9459	0.9457	0.9481	0.9998	0.9997	0.9450	0.9462
EW	0.0124	0.0125	0.0125	0.0125	0.0125	0.0017	0.0125	0.0124	0.0125	0.0250	0.02476	0.0124	0.0125
CV = 0.30													
CP	0.9281	0.9502	0.9493	0.9491	0.9496	0.5536	0.9493	0.9288	0.9503	0.9997	0.9995	0.9274	0.9292
EW	0.0372	0.0405	0.0403	0.0405	0.0404	0.0158	0.0403	0.0373	0.0405	0.0752	0.0757	0.0373	0.0375
CV = 0.50													
CP	0.8888	0.9533	0.9485	0.9520	0.9488	0.7422	0.9485	0.8895	0.9502	0.9984	0.9987	0.8883	0.8906
EW	0.0621	0.0776	0.0759	0.0774	0.0760	0.0439	0.0759	0.0622	0.0763	0.1254	0.1310	0.0622	0.0626
Chi-Square Distribution													
CV = 0.10													
CP	0.9467	0.9485	0.9484	0.9474	0.9486	0.2148	0.9484	0.9483	0.95072	0.9999	0.99974	0.9537	0.9531
EW	0.0124	0.0125	0.0125	0.0125	0.0125	0.0017	0.0125	0.0124	0.0125	0.0250	0.0247	0.0128	0.0127
CV = 0.30													
CP	0.9387	0.9585	0.9578	0.9576	0.9579	0.5751	0.9578	0.9398	0.9595	0.9997	0.9998	0.9686	0.9725
EW	0.0374	0.0407	0.0406	0.0407	0.0406	0.0159	0.0406	0.0375	0.0408	0.0756	0.0761	0.0434	0.0446
CV = 0.50													
CP	0.9210	0.9698	0.9665	0.9690	0.9667	0.7875	0.9665	0.9214	0.96814	0.9994	0.9994	0.9791	0.9853
EW	0.0620	0.0775	0.0758	0.0773	0.0759	0.0438	0.0758	0.0622	0.0762	0.1254	0.1309	0.0846	0.0905
Gamma Distribution													
CV = 0.10													
CP	0.9474	0.9495	0.9493	0.9485	0.9496	0.2187	0.9493	0.9493	0.9514	0.9999	0.9997	0.9539	0.9535
EW	0.0124	0.0125	0.0125	0.0125	0.0125	0.0017	0.0125	0.0124	0.0125	0.0250	0.0247	0.0128	0.0128
CV = 0.30													
CP	0.9405	0.9588	0.9578	0.9577	0.9580	0.5793	0.9578	0.9412	0.9603	0.9998	0.9997	0.9689	0.9725
EW	0.0372	0.0405	0.0403	0.0404	0.0404	0.0157	0.0403	0.0373	0.0405	0.0752	0.0757	0.0431	0.0443
CV = 0.50													
CP	0.9208	0.9699	0.9666	0.9689	0.9668	0.7863	0.9666	0.9213	0.9687	0.9994	0.9995	0.9798	0.9860
EW	0.0620	0.0775	0.0758	0.0773	0.0759	0.0438	0.0758	0.0622	0.0762	0.1254	0.1309	0.0846	0.0905

Table 6: The 95% confidence intervals for the population coefficient of variation of the weights of one-month old infants

Method	Confidence Interval Limits		
	Lower Limit	Upper Limit	Width
HR	0.1132	0.1633	0.0501
McK	0.1130	0.1633	0.0503
MMcK	0.1133	0.1633	0.0500
Panich	0.1121	0.1620	0.0499
Mill	0.1130	0.1635	0.0505
S&K	0.1336	0.1432	0.0096
C&P	0.1133	0.1633	0.0500
GKA&A	0.1173	0.1683	0.0510
MOV	0.1169	0.1689	0.0520
M&H (I)	0.1169	0.1675	0.0506
M&H (II)	0.1127	0.1626	0.0499
AK&A (I)	0.1160	0.1703	0.0543
AK&A (II)	0.1156	0.1710	0.0554

The summary statistics for infants weight data are as follows: $n = 61$, $mean = 4431$, $median = 4368$ grams,$sd = 612.9$ grams,$cv = 0.1383$ and $skewness = -0.27$. The Kolmogorov-Smirnov (K-S) goodness-of-fit test with p-value greater than 0.05 suggested that the data follow a normal distribution. The resulting 95% CIs and their corresponding widths are calculated and reported in Table 6. From this table, we notice that the width of confidence intervals AK&A(I) and AK&A(II) are wider than all other methods but they are performs well and still very close to other widths.

6.2 Example 2: Postmortem Interval (PMI) data

The second data set was obtained from Banik and Kibria [28]. The data represents the postmortem interval (PMI) which is defined as the elapsed time between death and an autopsy. The summary statistics for PMI data are as follows: $n = 22$, $mean = 7.3$, $median = 6.15$, $sd = 3.185$, $cv = 0.4363$ and $skewness = 1.14$.According to Banik and Kibria [28], using the Kolmogorov-Smirnov (K-S) goodness-of-fit test, the PMI data follow a gamma distribution with shape parameter $\alpha = 5.25$, and scale parameter, $\beta = 1.39$. The resulting 95% CIs and corresponding widths are given in Table 7.

Table 7: The 95% confidence intervals for the population coefficient of variation of the postmortem interval (PMI)

Method	Confidence Interval Limits		
	Lower Limit	Upper Limit	Width
HR	0.2995	0.5730	0.2735
McK	0.2718	0.5863	0.3144
MMcK	0.2848	0.5877	0.3029
Panich	0.2727	0.5746	0.3018
Mill	0.2812	0.5913	0.3100
S&K	0.3690	0.5335	0.1645
C&P	0.2848	0.5877	0.3029
GKA&A	0.3356	0.6234	0.2878
MOV	0.3199	0.6586	0.3387
M&H (I)	0.3324	0.6134	0.2809
M&H (II)	0.2818	0.5805	0.2986
AK&A (I)	0.3006	0.6950	0.3944
AK&A (II)	0.2669	0.7875	0.5206

From this table, we notice that the width of confidence intervals AK&A(I) and AK&A(II) are still wider than all other methods but the method AK&A(I) performed better than the method AK&A(II). The results of the two numerical examples given in this section shows that the two CIs proposed in this paper are good competitors compared with the other eleven confidence intervals studied.

7. Summary and concluding remarks

In this study, two simple approximate confidence interval methods namely, AK&A(I) and AK&A(II), were developed for the population coefficient of variation (CV). The study is related to the performance comparison of the available and proposed methods for population coefficient of variation. We express that the proposed two methods are good competitors compared with the other eleven confidence intervals studied in this paper. We clearly obtained that the coverage probabilities of proposed confidence intervals for all distributions when CV = 0.10, 0.30, 0.50 and $\alpha = 0.05$ are quite close to the nominal confidence level even in small sample sizes. Also, these confidence intervals performed as well as the other confidence intervals both coverage probability and average width. Furthermore, the proposed two methods are consistent to existing methods for normal distribution. However, for non-normal and skewed distributions, the average widths of the AK&A(I) confidence interval are tighter than the average widths of AK&A(II) confidence interval. Therefore, it is recommended to use the AK&A(I) confidence interval for skewed distributions. In addition to the simulation study, two real life data numerical examples are analyzed for illustrating the findings of the paper which reinforced the findings of the simulation study to some extent.

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