More on some generalized soft mappings in soft topological spaces

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Abstract. In this paper, we continue to study the properties of soft pu-semi-continuous and soft pu-semi-open(closed) functions[7] and initiate the notions of soft pu-irresolute function, soft pre-semi-open(closed) functions and explore it properties. Moreover, we develop the characterization, which gives us the relationship between these mappings.

Keywords: Soft topology, soft semi-open(closed) sets, soft semi-interior(closure), soft pu-semi-continuous function, soft pu-semi-open(closed) functions, soft pu-irresolute function, soft pre-semi-open(closed) functions.

1. Introduction

To deal with complex problems involving imprecisions, vagueness and uncertainty, it is observed that the tools of classical methods are not always successful. One of the reasons of failure of classical methods to solve the problems having uncertainties is having not enough information. Many researches proceeding towards new theories and ideas are developed in recent days and a lot of material is available in the literature.

Molodtsov et. al [30-31] introduced soft sets theory as a new general mathematical approach to deal with uncertain data and not clear objects and applied successfully this approach for modelling the problems having uncertainties. Maji et. al initiated and studied soft sets and explored its applications in decision making problems [28-29]. Xiao et. al [35] and Pei et. al [33] discussed the relationship among soft sets and information systems. Kostek [26] introduced the criteria to measure sound quality using approach of soft sets. Mushrif et. al [32] developed the remarkable method for the classification of natural textures using soft sets. Many researches have been done to add different notions in terms of weak and strong forms and fuzzification in the tool box of soft sets in terms of algebraic structures of soft sets, fuzzy soft sets, intutionistic fuzzy soft sets, weak structures of soft sets, soft topological spaces, fuzzy soft topological spaces, intutionistic fuzzy soft topological spaces as well as its applications [1], [4-24], [27], [34] and [36].

In [25], Kharral and Ahmad and then Zorlutana [36] explored and studied the mappings on soft classes in soft topological spaces. Later, Hussain [6] established fundamental and important characterizations of soft pu-continuous functions, soft pu-open functions and soft pu-closed functions.

Chen [2-3] presented and discussed soft semi-open(closed) sets in soft topological spaces. Hussain [4] continued to study the properties of soft semi-open sets and soft semi-closed sets in soft topological spaces.

In 2016, Hussain [7] generalized the soft functions studied in [6], [36] and introduced and explored new form of continuity called soft pu-semi-continuity via soft semi-open set in soft topological spaces. Moreover the concepts of soft-pu-semi-open and soft pu-semi-closed functions introduced and many of their characterizations and properties were discussed.

In this paper, we continue to study the properties of soft pu-semi-continuous and soft pu-semi-open(closed) functions [7] and initiate the notions of soft pu-irresolute function, soft pre-semi-open (closed) functions and explore it properties. Moreover, we develop the characterization, which gives us the relationship between these mappings.

2. Preliminaries

To make our paper self contained, first we recall some definitions and results defined and discussed in [1], [2-7], [19-20], [28-31], [34] and [36].

Definition 2.1. Let X be an initial universe and E be a set of parameters. Let $P(X)$ denotes the power set of X and A be a non-empty subset of E. A pair (F, A) is called a soft set over X, where F is a mapping given by $F : A \to P(X)$. In other words, a soft set over X is a parameterized family of subsets of the universe X. For $e \in A$, $F(e)$ may be considered as the set of e-approximate elements of the soft set (F, A).

Definition 2.2. For two soft sets (F, A) and (G, B) over a common universe X, we say that (F, A) is a soft subset of (G, B), if

(1) $A \subseteq B$ and

(2) for all $e \in A$, $F(e)$ and $G(e)$ are identical approximations. We write $(F, A)\tilde{\subseteq}(G, B)$.

(F, A) is said to be a soft super set of (G, B), if (G, B) is a soft subset of (F, A). We denote it by $(F, A)\tilde{\supseteq}(G, B)$.

Definition 2.3. Two soft sets (F, A) and (G, B) over a common universe X are said to be soft equal, if (F, A) is a soft subset of (G, B) and (G, B) is a soft subset of (F, A).

Definition 2.4. The union of two soft sets of (F, A) and (G, B) over the common universe X is the soft set (H, C), where $C = A \cup B$ and for all $e \in C$,

$$H(e) = \begin{cases} F(e), & \text{if } e \in A - B \\ G(e), & \text{if } e \in B - A \ . \\ F(e) \cup G(e), & \text{if } e \in A \cap B \end{cases}$$

We write $(F, A)\tilde{\cup}(G, B) = (H, C)$.

Definition 2.5. The intersection (H, C) of two soft sets (F, A) and (G, B) over a common universe X, denoted $(F, A)\tilde{\cap}(G, B)$, is defined as $C = A \cap B$, and $H(e) = F(e)\tilde{\cap}G(e)$, for all $e \in C$.

Definition 2.6. The difference (H, E) of two soft sets (F, E) and (G, E) over X, denoted by $(F, E)\tilde{\backslash}(G, E)$, is defined as $H(e) = F(e)\backslash G(e)$, for all $e \in E$.

Definition 2.7. Let (F, A) be a soft set over X and Y be a non-empty subset of X. Then the sub soft set of (F, A) over Y denoted by (Y_F, A), is defined as follows: $F_Y(\alpha) = Y\tilde{\cap}F(\alpha)$, for all $\alpha \in A$. In other words $(Y_F, A) = \tilde{Y}\tilde{\cap}(F, A)$.

Definition 2.8. The relative complement of a soft set (F, A) is denoted by $(F, A)^{'}$ and is defined by $(F, A)^{'} = (F^{'}, A)$ where $F^{'} : A \to P(U)$ is a mapping given by $F^{'}(\alpha) = U\backslash F(\alpha)$, for all $\alpha \in A$.

Definition 2.9. Let τ be the collection of soft sets over X, then τ is said to be a soft topology on X if:

(1) Φ , $\tilde{X}$ belong to τ.
(2) the union of any number of soft sets in τ belongs to τ.
(3) the intersection of any two soft sets in τ belongs to τ.
The triplet (X, τ, E) is called a soft topological space over X.

Definition 2.10. Let (X, τ, E) be a soft topological space over X, then soft interior of soft set (F, E) over X is denoted by $(F, E)^{\circ}$ and is defined as the union of all soft open sets contained in (F, E). Thus $(F, E)^{\circ}$ is the largest soft open set contained in (F, E). A soft set (F, E) over X is said to be a soft closed set in X, if its relative complement $(F, E)^{'}$ belongs to τ.

Definition 2.11. Let (X, τ, E) be a soft topological space over X and (F, E) be a soft set over X. Then the soft closure of (F, E), denoted by $\overline{(F, E)}$ is the intersection of all soft closed super sets of (F, E). Clearly $\overline{(F, E)}$ is the smallest soft closed set over X which contains (F, E).

Definition 2.12. Let (X, τ, E) be a soft topological space over X and (F, E) be a soft set over X. Then (F, E) is called soft semi-open set if and only if there exists a soft open set (G, E) such that $(G, E)\tilde{\subseteq}(F, E)\tilde{\subseteq}\overline{(G, E)}$. The set of all soft semi-open sets is denoted by $S.S.O(X)$. Note that every soft open set is soft semi-open set.

A soft set (F, E) is said to be soft semi-closed if its relative complement is soft semi-open . Equivalently there exists a soft closed set (G, E) such that $(G, E)^\circ \tilde{\subseteq} (F, E) \tilde{\subseteq} (G, E)$. Note that every soft closed set is soft semi-closed set.

Definition 2.13. Let (X, τ, E) be a soft topological space over X and (F, E) be a soft set over X. The soft set (F, E) is called a soft point in $\tilde{X}$, denoted by e_F, if for the element $e \in E$, $F(e) \neq \phi$ and $F(e') = \phi$, for all $e' \in E - \{e\}$.

Definition 2.14. The soft point e_F is said to be in the soft set (G, E), denoted by $e_F \tilde{\in} (G, E)$, if for the element $e \in E$, $F(e) \subseteq G(e)$.

Proposition 2.15. Let $e_F \tilde{\in} \tilde{X}$ and (G, E) be a soft set. If $e_F \tilde{\in} (G, E)$, then $e_F \tilde{\notin} (G, E)^c$.

Definition 2.16. A soft set (F, A) over X is said to be a null soft set, denoted by Φ_A , if for all $e \in A$, $F(e) = \phi$.

Definition 2.17. A soft set (F, A) over X is said to be an absolute soft set, denoted by $\tilde{X}_A$, if for all $e \in A$, $F(e) = X$. Clearly, $\tilde{X}_A^c = \Phi_A$ and $\Phi_A^c = \tilde{X}_A$.

Definition 2.18. Let (X, τ, E) be a soft topological space over X and $A \subseteq E$. Then:

(i) soft semi-interior of soft set (F, A) over X is denoted by $sint^s(F, A)$ and is defined as the union of all soft semi-open sets contained in (F, A).

(ii) soft semi-closure of (F, A) over X is denoted by $scl^s(F, A)$ is the intersection of all soft semi-closed super sets of (F, A).

Definition 2.19. Let $SS(X)_A$ and $SS(Y)_B$ be families of soft sets. $u : X \to Y$ and $p : A \to B$ be mappings. Then a soft function $f_{pu} : SS(X)_A \to SS(Y)_B$ defined as:

(1) Let (F, A) be a soft set in $SS(X)_A$. The image of (F, A) under f_{pu} , written as $f_{pu}(F, A) = (f_{pu}(F), p(A))$, is a soft set in $SS(Y)_B$ such that

$$f_{pu}(F)(y) = \begin{cases} \bigcup_{x \in p^{-1}(y) \cap A} u(F(x)), & p^{-1}(y) \cap A \neq \phi \\ \phi, & \text{otherwise} \end{cases},$$

for all $y \in B$.

(2) Let (G, B) be a soft set in $SS(Y)_B$. Then the inverse image of (G, B) under f_{pu} , written as $f_{pu}^{-1}(G, B) = (f_{pu}^{-1}(G), p^{-1}(B))$, is a soft set in $SS(X)_A$ such that

$$f_{pu}^{-1}(G)(x) = \begin{cases} u^{-1}(G(p(x))), & p(x) \in B \\ \phi, & \text{otherwise} \end{cases},$$

for all $x \in A$.

The soft function f_{pu} is called soft surjective, if p and u are surjective. The soft function f_{pu} is called soft injective, if p and u are injective. The soft function f_{pu} is called soft bijective, if p and u are bijective.

Definition 2.20. Let (X, τ, A) and (Y, τ^*, B) be soft topological spaces over X and Y respectively and $u : X \to Y$ and $p : A \to B$ be mappings. Then the soft function $f_{pu} : SS(X)_A \to SS(Y)_B$ is soft pu-semi-continuous if and only if for any soft open set (G, B) in $SS(Y)_B$, $f_{pu}^{-1}(G, B)$ is a soft semi-open set in $SS(X)_A$.

Clearly it follows from the definition that the soft function $f_{pu} : SS(X)_A \to SS(Y)_B$ is soft pu-semi-continuous if and only if for any soft closed set (G, B) in $SS(Y)_B$, $f_{pu}^{-1}(G, B)$ is a soft semi-closed set in $SS(X)_A$.

Definition 2.21. A soft set (F, A) in $SS(X)_A$ is said to be a soft semi-nbd of a soft point $e_F \tilde{\in} \tilde{X}_A$, if there exists a soft semi-open set (H, A) such that $e_F \tilde{\in} (H, A) \tilde{\subseteq} (F, A)$.

Definition 2.22. Let (X, τ, A) and (Y, τ^*, B) be soft topological spaces over X and Y respectively and $u : X \to Y$ and $p : A \to B$ be mappings. Then the soft function $f_{pu} : SS(X)_A \to SS(Y)_B$ is soft pu-semi-open if and only if for any soft open set (F, A) in $SS(X)_A$, $f_{pu}(F, A)$ is soft semi-open in $SS(Y)_B$.

Definition 2.23. Let (X, τ, A) be a soft topological spaces over X, (F, A) be a soft set in $SS(X)_A$ and soft point $e_F \tilde{\in} \tilde{X}_A$. Then e_F is called a soft semi-limit point of a soft set (F, A), if $(H, A) \tilde{\cap} ((F, A) \tilde{\setminus} \{e_F\}) \tilde{\neq} \tilde{\phi}$, for any soft semi-open set (H, A) such that $e_F \tilde{\in} (H, A)$. The set of all soft semi-limit point of (F, A) is called as soft semi-derived set of (F, A) and is denoted by $sd^s(F, A)$. Note that if $(F, A) \tilde{\subseteq} (H, A)$ then $sd^s(F, A) \tilde{\subseteq} sd^s(H, A)$. Clearly e_F is a soft semi-limit point of (F, A) if and only if $e_F \tilde{\in} scl^s((F, A) \tilde{-} \{e_F\})$.

Theorem 2.24. Let (X, τ, A) be a soft topological spaces over X and (F, A) be a soft set in $SS(X)_A$. Then

(1) $scl^s(F, A) \tilde{=} (F, A) \tilde{\cup} sd^s(F, A)$.

(2) $sd^s((F, A) \tilde{\cup} (H, A)) \tilde{=} sd^s(F, A) \tilde{\cup} sd^s(H, A)$.

In general,

(3) $\bigcup_i sd^s(F, A_i) \tilde{=} sd^s(\bigcup_i (F, A_i))$.

(4) $sd^s(sd^s(F, A)) \tilde{\subseteq} sd^s(F, A)$.

(5) $scl^s(sd^s(F, A)) \tilde{=} sd^s(F, A)$.

3. Soft pu-irresolute mappings

Definition 3.1. Let (X, τ, A) and (Y, τ^*, B) are soft topological spaces over X and Y respectively and $u : X \to Y$ and $p : A \to B$ are mappings. Then the soft function $f_{pu} : SS(X)_A \to SS(Y)_B$ is soft pu-irresolute if and only if for any soft semi-open set (G, B) in $SS(Y)_B$, $f_{pu}^{-1}(G, B)$ is a soft semi-open set in $SS(X)_A$.

Theorem 3.2. *Let (X, τ, A) and (Y, τ^*, B) are soft topological spaces over X and Y respectively and e_F be a soft point in $SS(X)_A$. Then the soft function $f_{pu} : SS(X)_A \to SS(Y)_B$ is soft pu-irresolute if and only if for each soft point e_F in $SS(X)_A$, the soft inverse of every soft semi-nbd of $f_{pu}(e_F)$ is a soft semi-nbd of e_F.*

Proof. Let e_F be a soft point in $SS(X)_A$ and (F, B) a soft semi-nbd of $f_{pu}(e_F)$. By definition of soft semi-nbd, there exists soft semi-open set (G, B) in $S.S.O(Y)$ such that $f_{pu}(e_F)\tilde{\in}(G, B)\tilde{\subseteq}(F, B)$. This follows that $e_F\tilde{\in}f_{pu}^{-1}(G, B)\tilde{\subseteq}f_{pu}^{-1}(F, B)$. Since f_{pu} is soft pu-irresolute, then $f_{pu}^{-1}(G, B)$ is a soft semi-open set in $SS(X)_A$. Therefore $f_{pu}^{-1}(F, B)$ is a soft semi-nbd of e_F.

Conversely, suppose that (F, B) be a soft semi-open sets in $S.S.O(Y)$. Take $(G, B)\tilde{=}f_{pu}^{-1}(F, B)$.

Let $e_H\tilde{\in}(G, B)$, then $f_{pu}(e_H)\tilde{\in}(F, B)$. Clearly, (F, B) (being soft semi-open) is a soft semi-nbd of $f_{pu}(e_H)$. So by hypothesis, $(G, B)\tilde{=}f_{pu}^{-1}(F, B)$ is a soft semi-nbd of e_H. Therefore, by definition, there exists soft semi-open set$(G, B)_{e_H}$ in $S.S.O(X)$ such that $e_H\tilde{\in}(G, B)_{e_H}\tilde{\subseteq}(G, B)$. Thus, $(G, B)\tilde{=}\tilde{\bigcup}_{e_H\tilde{\in}(G,B)}(G, B)_{e_H}$. It follows that (G, B) is soft semi-open in $S.S.O(X)$. Therefore the soft function f_{pu} is soft pu-irresolute. This completes the proof.

Remark 3.3. Note that soft semi-nbd of e_F may be replaced by soft semi-open-nbd of e_F in above Theorem 3.2.

Theorem 3.4. *Let (X, τ, A) and (Y, τ^*, B) be soft topological spaces over X and Y respectively and e_F be a soft point in $SS(X)_A$. Then the soft function $f_{pu} : SS(X)_A \to SS(Y)_B$ is soft pu-irresolute if and only if for each soft point e_F in $SS(X)_A$ and each soft semi-nbd (G, B) of $f_{pu}(e_F)$, there exists a soft semi-nbd (F, A) of e_F such that $f_{pu}(F, A)\tilde{\subseteq}(G, B)$.*

Proof. Suppose e_F be a soft point in $SS(X)_A$ and (G, B) be a soft semi-nbd of $f_{pu}(e_F)$. Then there exists soft semi-open set $(H, B)_{f_{pu}(e_F)}$ in $SS(Y)_B$ such that $f_{pu}(e_F)\tilde{\in}(H, B)_{f_{pu}(e_F)}\tilde{\subseteq}(G, B)$.

This follows that $e_F\tilde{\in}f_{pu}^{-1}((H, B)_{f_{pu}(e_F)})\tilde{\subseteq}f_{pu}^{-1}(G, B)$.

By hypothesis, $f_{pu}^{-1}((H, B)_{f_{pu}(e_F)})$ is soft semi-open in $SS(X)_A$. Consider we take $(F, A) = f_{pu}^{-1}(G, B)$. Then it follows that (F, A) is soft semi-nbd of e_F and $f_{pu}(F, A) = f_{pu}(f_{pu}^{-1}(G, B))\tilde{\subseteq}(G, B)$.

Conversely, let (U, B) be a soft semi-open set in $SS(Y)_B$. Take $(H, A)\tilde{=} f_{pu}^{-1}(U, B)$. Let $e_F\tilde{\in}(H, A)$, then $f_{pu}(e_F)\tilde{\in}(U, B)$. Thus (U, B) is a soft semi-nbd of $f_{pu}(e_F)$. So by hypothesis, there exists a soft semi-nbd $(V, A)_{e_F}$ of e_F such that $f_{pu}((V, A)_{e_F})\tilde{\subseteq}(U, B)$. Thus it follows that $e_F\tilde{\in}(V, A)_{e_F}\tilde{\subseteq}f_{pu}^{-1}(f_{pu}(V, A)_{e_F})\tilde{\subseteq} f_{pu}^{-1}(U, B)\tilde{=}(H, A)$. Therefore, $(V, A)_{e_F}$ is a soft semi-nbd of e_F, which follows that there exists a soft semi-open set $(H, A)_{e_F}$ in $SS(X)_A$ such that $e_F\tilde{\in}(H, A)_{e_F}\tilde{\subseteq}(H, A)$. Thus $(H, A)\tilde{=}\tilde{\bigcup}_{e_F\tilde{\in}(H,A)}(H, A)_{e_F}$. It follows that (H, A)

is soft semi-open in $SS(X)_A$ [4]. Therefore, f_{pu} is soft pu-irresolute. Hence the proof.

The following theorem gives the characterization of soft pu-irresolute function in terms of soft semi-derived sets.

Theorem 3.5. *Let (X,τ,A) and (Y,τ^*,B) are soft topological spaces over X and Y respectively. Then the soft function $f_{pu}: SS(X)_A \to SS(Y)_B$ is soft pu-irresolute if and only if $f_{pu}(sd^s(G,A))\tilde{\subseteq} scl^s(f_{pu}(G,A))$, for all soft sets (G,A) in $SS(X)_A$.*

Proof. Suppose soft function $f_{pu}: SS(X)_A \to SS(Y)_B$ is soft pu-irresolute. Let (G,A) be a soft set in $SS(X)_A$, and $e_F \tilde{\in} sd^s(G,A)$.

Assume that $f_{pu}(e_F)\tilde{\notin} f_{pu}(G,A)$ and let (V,B) denote a soft semi-nbd of $f_{pu}(e_F)$. Since f_{pu} is soft pu-irresolute, then by Theorem 3.4, there exists a soft semi-nbd (U,A) of e_F such that $f_{pu}(U,A)\tilde{\subseteq}(V,B)$. Now $e_F\tilde{\in} sd^s(G,A)$ follows that $(U,A)\tilde{\bigcap}(G,A)\tilde{\neq}\tilde{\Phi}$. Therefore, there exists at least one soft point $e_H\tilde{\in}(U,A)\tilde{\bigcap}(G,A)$ such that $f_{pu}(e_H)\tilde{\in} f_{pu}(G,A)$ and $f_{pu}(e_H)\tilde{\in} f_{pu}(U,A)$. Since $f_{pu}(e_F)\tilde{\notin} f_{pu}(G,A)$, we have $f_{pu}(e_F)\tilde{\neq} f_{pu}(e_H)$. Thus every soft semi-nbd of $f_{pu}(e_F)$ contains a soft point $f_{pu}(e_H)$ of $f_{pu}(G,A)$ different from $f_{pu}(e_F)$.

Consequently, $f_{pu}(e_H)\tilde{\in} sd^s(f_{pu}(G,A))$. This proves the necessity.

Conversely, suppose that f_{pu} is not soft pu-irresolute. Then by Theorem 3.4, there exists a soft point e_F in $SS(X)_A$ and a soft semi-nbd (V,B) of $f_{pu}(e_F)$ such that every soft semi-nbd (U,A) of e_F contains at least one soft element e_H in (U,A) for which $f_{pu}(e_H)\tilde{\notin}(V,B)$. Put $(G,A)\tilde{=}\{e_H\tilde{\in} SS(X)_A : f_{pu}(e_H)\tilde{\notin}(V,B)\}$. Since $f_{pu}(e_F)\tilde{\in}(V,B)$, therefore $e_F\tilde{\notin}(G,A)$ and hence $f_{pu}(e_F)\tilde{\notin} f_{pu}(G,A)$. Since $f_{pu}(G,A)\tilde{\cap}((V,B)\tilde{\setminus} f_{pu}(e_F))\tilde{=}\tilde{\Phi}$, therefore $f_{pu}(e_F)\tilde{\notin} sd^s(f_{pu}(G,A))$. This implies that $f_{pu}(e_F)\tilde{\in} f_{pu}(sd^s(G,A))\tilde{\setminus}(((f_{pu}(G,A))\tilde{\cup} sd^s(f_{pu}(G,A))))\tilde{\neq}\tilde{\Phi}$. This is a contradiction to the given condition. This proves sufficiency. This completes the proof.

Theorem 3.6. *Let (X,τ,A) and (Y,τ^*,B) are soft topological spaces over X and Y respectively and soft function $f_{pu}: SS(X)_A \to SS(Y)_B$ is soft one to one. Then the soft function f_{pu} is soft pu-irresolute if and only if $f_{pu}(sd^s((G,A)))\tilde{\subseteq} sd^s(f_{pu}(G,A))$, for all soft sets (G,A) in $SS(X)_A$.*

Proof. Consider soft sets (G,A) in $SS(X)_A$, $e_F\tilde{\in} sd^s(G,A)$ and (V,B) be a soft semi-nbd of $f_{pu}(e_F)$. Since f_{pu} is soft pu-irresolute, then by Theorem 3.4, there exists a soft semi-nbd (U,A) of e_F such that $f_{pu}(U,A))\tilde{\subseteq}(V,B)$. But $e_F\tilde{\in} sd^s(G,A)$ gives there exists a soft element $e_F\tilde{\in}((U,A)\tilde{\cap}(G,A))$ such that $e_F\tilde{\neq} e_H$. Clearly $f_{pu}(e_H)\tilde{\in} f_{pu}(G,A)$ and since f_{pu} is soft one to one, $f_{pu}(e_F)\tilde{\neq} f_{pu}(e_H)$. Thus every soft semi-nbd (V,B) of $f_{pu}(e_F)$ contains an element $f_{pu}(e_H)$ of $f_{pu}(G,A)$ different from $f_{pu}(e_F)$. Consequently, $f_{pu}(e_F)\tilde{\in} sd^s(f_{pu}(G,A))$.

Therefore, we have $f_{pu}(sd^s(G,A))\tilde{\subseteq} sd^s(f_{pu}(G,A))$. This proves the necessity.

Sufficiency follows from Theorem 3.5. Hence the proof.

4. Soft pre-semi-open mappings

Definition 4.1. Let (X, τ, A) and (Y, τ^*, B) are soft topological spaces over X and Y respectively and $u : X \to Y$ and $p : A \to B$ are mappings. Then the soft function $f_{pu} : SS(X)_A \to SS(Y)_B$ is soft pre-semi-open if and only if for each soft semi-open set (F, A) in $SS(X)_A$, $f_{pu}(F, A)$ is a soft semi-open set in $SS(Y)_B$.

Remark 4.2. The class of soft pre-semi-open functions is a subclass of class of soft semi-open functions defined in [7].

The proof of following theorem is easy and therefore omitted:

Theorem 4.3. *A soft function $f_{pu} : SS(X)_A \to SS(Y)_B$ is soft pre-semi-open if and only if for each soft point e_F in $SS(X)_A$ and for every soft semi-open set (H, A) in $S.S.O(X)$ such that $e_F \tilde{\in} (H, A)$, there exists soft semi-open set (G, B) in $S.S.O(Y)$ such that $f_{pu}(e_F) \tilde{\in} (G, B)$ and $(G, B) \tilde{\subseteq} f_{pu}(H, A)$.*

Theorem 4.4. *A soft function $f_{pu} : SS(X)_A \to SS(Y)_B$ is soft pre-semi-open if and only if for each for each soft point e_F in $SS(X)_A$ and for every soft semi-nbd (U, A) of e_F in $SS(X)_A$, there exists a soft semi-nbd (V, B) of $f_{pu}(e_F)$ in $SS(Y)_B$ such that $(V, B) \tilde{\subseteq} f_{pu}(U, A))$.*

Proof. Suppose (U, A) be a soft semi-nbd of e_F in $SS(X)_A$. Then by definition there exists soft semi-open set (W, A) in $S.S.O(X)$ such that $e_F \tilde{\in} (W, A) \tilde{\subseteq} (U, A)$. Then $f_{pu}(e_F) \tilde{\in} f_{pu}(W, A) \tilde{\subseteq} f_{pu}(U, A)$. Since soft function f_{pu} is soft pre-semi-open, therefore $f_{pu}(W, A)$ is soft semi-open sets in $S.S.O(Y)$. Hence, $(V, B) \tilde{=} f_{pu}(W, A)$ is a soft semi-nbd of $f_{pu}(e_F)$ and $(V, B) \tilde{\subseteq} f_{pu}(U, A)$.

Conversely, let (U, A) be a soft semi-open set in $S.S.O(X)$ and e_F be a soft point in (U, A). Then (U, A) is a soft semi-nbd of e_F. So by hypothesis, there exists a soft semi-nbd (V, B) of $f_{pu}(e_F)$ such that $f_{pu}(e_F) \tilde{\in} (V, B) \tilde{\subseteq} f_{pu}(U, A)$. That is, $f_{pu}(U, A)$ is a soft semi-nbd of $f_{pu}(e_F)$. Thus $f_{pu}(U, A)$ is a soft semi-nbd of each of its soft points. Thus $f_{pu}(U, A)$ is soft semi-open. Hence f_{pu} is soft pre-semi-open. This completes the proof.

Theorem 4.5. *A soft function $f_{pu} : SS(X)_A \to SS(Y)_B$ is soft pre-semi-open if and only if $f_{pu}(sint^s(F, A)) \tilde{\subseteq} sint^s(f_{pu}(F, A))$, for all soft sets (F, A) in $SS(X)_A$.*

Proof. Suppose $e_F \tilde{\in} sint^s(F, A)$. Then there exists soft semi-open set (U, A) in $S.S.O.(X)$ such that $e_F \tilde{\in} (U, A) \tilde{\subseteq} (F, A)$. This implies that $f_{pu}(e_F) \tilde{\in} f_{pu}(U, A) \tilde{\subseteq} f_{pu}(F, A)$. Since f_{pu} is soft pre-semi-open, then $f_{pu}(U, A)$ is soft semi-open in $SS(Y)_B$.

Hence $f_{pu}(e_F) \tilde{\in} sint^s(f_{pu}(F, A))$. Thus, $f_{pu}(sint^s(F, A)) \tilde{\subseteq} sint^s(f_{pu}(F, A))$. This proves necessity.

Conversely, let (U, A) be a soft semi-open sets in $S.S.O(X)$. Then by hypothesis, $f_{pu}(U, A) \tilde{=} f_{pu}(sint^s(U, A)) \tilde{\subseteq} sint^s(f_{pu}(U, A)) \tilde{\subseteq} f_{pu}(U, A)$ or $f_{pu}(U, A) \tilde{\subseteq}$

$sint^s(f_{pu}(U,A))\tilde{\subseteq}f_{pu}(U,A)$. This follows that $f_{pu}(U,A)$ is soft semi-open in $SS(Y)_B$. So the soft function f_{pu} is soft pre-semi- open. Hence the proof.

We use Theorem 4.5 and prove:

Theorem 4.6. *A soft function $f_{pu} : SS(X)_A \to SS(Y)_B$ is soft -pre-semi-open if and only if $sint^s(f_{pu}^{-1}(G,B))\tilde{\subseteq}f_{pu}^{-1}(sint^s(G,B))$, for all soft set (G,B) in $SS(Y)_B$.*

Proof. Let (G,B) be any soft set in $SS(Y)_B$. Clearly, $sint^s(f_{pu}^{-1}(G,B)$ is soft semi-open in $SS(Y)_B$. Also $f_{pu}(sint^s(f_{pu}^1(G,B))\tilde{\subseteq}f_{pu}(f_{pu}^1(G,B))\tilde{\subseteq}(G,B)$.

Since f_{pu} is soft pre-semi-open, by Theorem 4.5, we have $f_{pu}(sint^s(f_{pu}^1(G,B)))$ $\tilde{\subseteq}sint^s(G,B)$.

Therefore, $sint^s(f_{pu}^{-1}(G,B))\tilde{\subseteq}f_{pu}(f_{pu}^{-1}(sint^s(f_{pu}^{-1}(G,B))))\tilde{\subseteq}f_{pu}^{-1}(sint^s(G,B))$ or $sint^s(f_{pu}^{-1}(G,B))\tilde{\subseteq}f_{pu}^{-1}(sint^s(G,B))$. This proves necessity.

Conversely, let (F,A) be any soft subset in $SS(X)_A$. By hypothesis, we have $sint^s(F,A)\tilde{\subseteq}sint^s(f_{pu}^1(f_{pu}(F,A)))\tilde{\subseteq}f_{pu}^1(sint^s f_{pu}(F,A))$. This implies that

$$\begin{aligned}&f_{pu}(sint^s(F,A))\tilde{\subseteq}f_{pu}(sint^s(f_{pu}^1(f_{pu}(F,A))))\\&\tilde{\subseteq}f_{pu}(f_{pu}^1(sint^s f_{pu}(F,A)))\tilde{\subseteq}sint^s f_{pu}(F,A).\end{aligned}$$

Consequently, $f_{pu}(sint^s(F,A))\tilde{\subseteq}sint^s f_{pu}(F,A)$, for all soft set (F,A) in $SS(X)_A$. By Theorem 4.5, f_{pu} is soft pre-semi-open. This completes the proof.

We use Theorem 4.6 and prove:

Theorem 4.7. *A soft function $f_{pu} : SS(X)_A \to SS(Y)_B$ is soft -pre-semi-open if and only if $f_{pu}^{-1}(scl^s(G,B))\tilde{\subseteq}scl^s(f_{pu}^{-1}(G,B))$, for all soft set (G,B) in $SS(Y)_B$.*

Proof. Let (G,B) be a soft sets in $SS(Y)_B$.

By Theorem 4.6, $sint^s(f_{pu}^1((G,B)^c))\tilde{\subseteq}f_{pu}^{-1}(sint^s((G,B)^c))$. This implies that $sint^s((f_{pu}^{-1}(G,B))^c)\tilde{\subseteq}f_{pu}^{-1}(sint^s((G,B)^c))$. As $sint^s(G,B)\tilde{=}(scl^s((G,B)^c))^c$ [8], therefore $(scl^s(f_{pu}^{-1}(G,B)))^c\tilde{\subseteq}f_{pu}^{-1}((scl^s(G,B))^c)$, or

$$(scl^s(f_{pu}^{-1}(G,B)))^c\tilde{\subseteq}(f_{pu}^{-1}((scl^s(G,B))))^c.$$

Hence

$$f_{pu}^{-1}(scl^s(G,B))\tilde{\subseteq}scl^s(f_{pu}^{-1}(G,B)).$$

This proves necessity.

Conversely, (G,B) be a soft sets in $SS(Y)_B$. By hypothesis,

$$f_{pu}^{-1}(scl^s((G,B)^c))\tilde{\subseteq}scl^s(f_{pu}^{-1}((G,B)^c)).$$

This implies $(scl^s(f_{pu}^{-1}((G,B)^c))^c\tilde{\subseteq}(f_{pu}^{-1}(scl^s((G,B)^c)))^c$. Hence,

$$(scl^s((f_{pu}^{-1}((G,B)^c))))^c\tilde{\subseteq}f_{pu}^{-1}((scl^s((G,B)^c))^c).$$

This gives $sint^s(f_{pu}^{-1}(G,B))\tilde{\subseteq}f_{pu}^{-1}(sint^s(G,B))$. Now from Theorem 4.6, it follows that f_{pu} is soft pre-semi-open. This completes the proof.

5. Soft pre-semi-closed mappings

Definition 5.1. Let (X, τ, A) and (Y, τ^*, B) are soft topological spaces over X and Y respectively and $u : X \to Y$ and $p : A \to B$ are mappings. Then the soft function $f_{pu} : SS(X)_A \to SS(Y)_B$ is soft pre-semi-closed if and only if for each soft semi-closed set (F, A) in $SS(X)_A$, $f_{pu}(F, A)$ is a soft semi-closed set in $SS(Y)_B$.

Theorem 5.2. *A soft function $f_{pu} : SS(X)_A \to SS(Y)_B$ is soft pre-semi-closed if and only if $scl^s(f_{pu}(F, A)) \tilde{\subseteq} f_{pu}(scl^s(F, A))$, for every soft subset (F, A) of $SS(X)_A$.*

Proof. Suppose f_{pu} is soft pre-semi-closed and let (F, A) be a soft subset in $SS(X)_A$. Since f_{pu} is soft pre-semi-closed, then $f_{pu}(scl^s(F, A))$ is soft semi-closed in $SS(Y)_B$. Since $f_{pu}(F, A) \tilde{\subseteq} f_{pu}(scl^s(F, A))$, then we have $scl^s(f_{pu}(F, A)) \tilde{\subseteq} f_{pu}(scl^s(F, A))$. This proves necessity.

Conversely, suppose (F, A) is a soft semi-closed set in $SS(X)_A$. Using hypothesis, we have $f_{pu}(F, A) \tilde{\subseteq} scl^s(f_{pu}(F, A)) \tilde{\subseteq} f_{pu}(scl^s(F, A)) \tilde{=} f_{pu}(F, A)$. Hence, $f_{pu}(F, A) \tilde{=} scl^s(f_{pu}(F, A))$. Thus $f_{pu}(F, A)$ is soft semi-closed set in $SS(Y)_B$. This proves that f_{pu} is soft pre-semi-closed. This completes the proof.

Theorem 5.3. *A soft function $f_{pu} : SS(X)_A \to SS(Y)_B$ is soft pre-semi-closed if and only if $\{\overline{f_{pu}(F, A)}\}^0 \tilde{\subseteq} f_{pu}(scl^s(F, A))$, for every soft subset (F, A) of $SS(X)_A$.*

Proof. Suppose f_{pu} is soft pre-semi-closed and (F, A) is any soft subset of $SS(X)_A$. Then $f_{pu}(scl^s(F, A))$ is soft semi-closed in $SS(Y)_B$. This implies that there exists a soft closed subset (G, B) of $SS(Y)_B$ such that

$$(G, B)^0 \tilde{\subseteq} f_{pu}(scl^s(F, A)) \tilde{\subseteq} (G, B).$$

This gives that $\{\overline{(f_{pu}(scl^s(F, A)))}\}^0 \tilde{\subseteq} (G, B)^0 \tilde{\subseteq} f_{pu}(scl^s(F, A))$.

Then $\{\overline{(f_{pu}(F, A))}\}^0 \tilde{\subseteq} (\overline{(f_{pu}(scl^s(F, A)))})^0$ gives $\{\overline{f_{pu}(F, A)}\}^0 \tilde{\subseteq} f_{pu}(scl^s(F, A))$. This proves necessity.

Conversely, suppose (F, A) is a soft semi-closed set in $SS(X)_A$. Then by hypothesis, $\{\overline{(f_{pu}(F, A))}\}^0 \tilde{\subseteq} f_{pu}(scl^s(F, A)) \tilde{=} f_{pu}(F, A)$. Put $(G, B) \tilde{=} \overline{(f_{pu}(F, A))}$. Clearly, (G, B) is soft closed in $SS(Y)_B$. This implies that $(G, B)^0 \tilde{\subseteq} f_{pu}(F, A) \tilde{\subseteq} (G, B)$. Hence $f_{pu}(F, A)$ is soft semi-closed in $SS(Y)_B$. This implies f_{pu} is soft pre-semi-closed.

Theorem 5.4. *A soft bijective soft function $f_{pu} : SS(X)_A \to SS(Y)_B$ is soft pre-semi-closed if and only if for each subset (G, B) of $SS(Y)_B$ and each soft semi-open set (F, A) in $SS(X)_A$ containing $f_{pu}^{-1}(G, B)$, there exists a soft semi-open set (H, B) in $SS(Y)_B$ soft containing (G, B) such that $f_{pu}^{-1}(H, B) \tilde{\subseteq} (F, A)$.*

Proof. Let $(H,B)\tilde{=}(f_{pu}((F,A)^c))^c$. Then $(H,B)^c\tilde{=}f_{pu}((F,A)^c)$. Since f_{pu} is soft pre-semi-closed, so (H,B) is soft semi-open. Since $f_{pu}^{-1}(G,B)\tilde{\subseteq}(F,A)$, we have $(H,B)^c\tilde{=}f_{pu}((F,A)^c)\tilde{\subseteq}f_{pu}(f_{pu}^{-1}((G,B)^c))\tilde{\subseteq}(G,B)^c$. Hence, $(G,B)\tilde{\subseteq}(H,B)$, and thus (H,B) is a soft semi-open nbd of (G,B).

Further, $(F,A)^c\tilde{\subseteq}f_{pu}^{-1}(f_{pu}((F,A)^c))\tilde{=}f_{pu}^{-1}((H,B)^c)\tilde{=}(f_{pu}^{-1}((H,B)))^c$. This proves that $f_{pu}^{-1}((H,B))\tilde{\subseteq}(F,A)$.

Conversely, suppose (F,A) is a soft semi-closed set in $SS(Y)_B$. Let $e_G\tilde{\in}(f_{pu}(F,A))^c$. Then $f_{pu}^{-1}(e_G)\tilde{\subseteq}(f_{pu}^{-1}(f_{pu}(F,A)))^c\tilde{\subseteq}(F,A)^c$ and $(F,A)^c$ is soft semi-open in $SS(X)_A$. Hence by hypothesis, there exists a soft semi-open set (H,B) soft containing e_G such that $f_{pu}^{-1}((H,B))\tilde{\subseteq}(F,A)^c$. Since f_{pu} is soft one-one, we have $e_G\tilde{\in}(H,B)\tilde{\subseteq}(f_{pu}(F,A))^c$. Thus $(f_{pu}(F,A))^c\tilde{=}\tilde{\bigcup}_{e_G\tilde{\in}(f_{pu}(F,A))^c}(H,B)$. Hence, $(f_{pu}(F,A))^c$ is soft semi-open set [4]. This proves that f_{pu} is soft pre-semi-closed. Hence the proof.

We use Theorem 4.7 and give the following characterizations:

Theorem 5.5. *Let $f_{pu}:SS(X)_A\to SS(Y)_B$ be a soft bijective soft function. Then the following are equivalent:*

(1) *f_{pu} is soft pre-semi-closed.*

(2) *f_{pu} is soft pre-semi-open.*

(3) *f_{pu}^{-1} is soft pu-irresolute.*

Proof. (1) $\Rightarrow$ (2). It is straightforward.

(2) $\Rightarrow$ (3) Let (F,A) be soft set in $SS(X)_A$. Since f_{pu} is soft pre-semi-open, by Theorem 4.7, $f_{pu}^{-1}(scl^s(f_{pu}(F,A)))\tilde{\subseteq}scl^s(f_{pu}^{-1}(f_{pu}(F,A)))$ implies

$$scl^s(f_{pu}(F,A))\tilde{\subseteq}f_{pu}(scl^s(F,A)).$$

Thus $scl^s((f_{pu}^{-1})^{-1})(F,A)$ is soft contained in $(f_{pu}^{-1})^{-1}(scl^s(F,A))$, for every soft subset (F,A) of $SS(X)_A$. Then again by Theorem 4.7, it follows that f_{pu}^{-1} is soft pu-irresolute.

(3) $\Rightarrow$ (1). Let (F,A) be a soft semi-closed set in $SS(X)_A$. Then $(F,A)^c$ is soft semi-open in $SS(X)_A$. Since f_{pu}^{-1} is pu-irresolute, $(f_{pu}^{-1})^{-1}((F,A)^c)$ is soft semi-open in $SS(Y)_B$. But $(f_{pu}^{-1})^1((F,A)^c)\tilde{=}f_{pu}((F,A)^c)\tilde{=}(f_{pu}(F,A))^c$. Thus $f_{pu}(F,A)$ is soft semi-closed in $SS(Y)_B$. This proves that f_{pu} is soft pre-semi-closed. This completes the proof.

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