Nonexistence results of global solutions for fractional order integral equations and systems

Bouguetof Khaoula
Haouam Kamel*
Rebiai Belgacem
Laboratory of Mathematics
Informatics and Systems
University of Larbi Tebessi
Tebessa 12002
Algeria
haouam@yahoo.fr

Abstract. The main objective of this study is to demonstrate the absence of the nontrivial global solutions of some fractional evolution integral equations and systems. We show that the critical exponent of Fujita type coincides with that obtained by S. Fuqin and S. Peihu in the classical case.
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1. Introduction

The most development of the fractional calculus was into the turn of the 20th century, because it has been used to describe diverse phenomenon in sciences and engineering for its special fundamental properties. Caputo [23] reformulated the most definitions of Riemann derivatives in order to solve some fractional order differential equations and systems from the point of view of various considerations, mainly to ensure that the problem is well posed, then in the majority of cases, the obtained results generalize or at least extend those obtained in the classical case.

In the first part of this study, we deal with the following Cauchy problem:

$$\begin{cases} \mathbf{D}^{\beta}_{0|t}u + (-\triangle)^m u = \frac{1}{\Gamma(1-\gamma)}\int_0^t (t-\tau)^{-\gamma} \mid u \mid^p d\tau, \\ u(0,x) = u_0(x), \end{cases} \tag{1}$$

where $m > 1$, $p > 1$, $0 < \beta < 1$ and $0 < \gamma < 1$. $\mathbf{D}^{\beta}_{0|t}$ denotes the Caputo fractional derivative of order β.

The fractional Laplacien operator is defined as

$$(-\Delta)^{\beta/2}u(t,x) = \mathcal{F}^{-1}(|\xi|^{\beta}\mathcal{F}(u)(\xi))(x), \quad u \in H^{\beta},$$

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*. Corresponding author

where H^{β} is the homogeneous Sobolev defined by

$$H^{\beta}(\mathbb{R}^N) = \{u \in S' \, : \, (-\Delta)^{\beta/2} u \in L^2(\mathbb{R}^N)\}, \quad \beta \in \mathbb{N},$$
$$H^{\beta}(\mathbb{R}^N) = \{u \in L^2(\mathbb{R}^N) \, : \, (-\Delta)^{\beta/2} u \in L^2(\mathbb{R}^N)\}, \quad \beta \notin \mathbb{N},$$

S' is the Schwartz space. $\mathcal{F}$ is the standard Fourier transform and $\mathcal{F}^{-1}$ its inverse.

We observe that the last problem can also be reformulated as follow

$$\begin{cases} \mathbf{D}^{\beta}_{0|t} u + (-\triangle)^m u = D^{-\alpha}_{0|t} \mid u \mid^p ; \alpha = 1 - \gamma \\ u(0,x) = u_0(x), \end{cases} \tag{2}$$

and we was motived mathematically by many interesting works which deal with nonexistence and blow up of solutions of differential integral equations and systems, we cite particularly the problem treated by establishment of a critical exponent, which has been used as a very effective tool for studying the nonexistence of solutions for evolution equations and systems. Our interest is to prove the absence of nontrivial global weak solutions for the problem (1) by looking at critical exponents. The found result is more general than the one established by Sun and Shi [15], when they treated the following Cauchy problem

$$\begin{cases} u_t + (-\triangle)^m u = \frac{1}{\Gamma(1-\gamma)} \int_0^t (t-\tau)^{-\gamma} \mid u \mid^p d\tau, \\ u(0,x) = u_0(x), \end{cases} \tag{3}$$

they proved that if

$$1 < p \leqslant \max \left\{ 1 + \frac{2m(2-\gamma)}{(N - 2m + 2m\gamma)_+}, \frac{1}{\gamma} \right\}$$

then the problem does not accept any nontrivial solution. We cite also the paper of Fino and Kirane [3], the authors consider the following problem

$$\begin{cases} u_t + (-\triangle)^{\beta/2} u = \frac{1}{\Gamma(1-\gamma)} \int_0^t (t-\tau)^{-\gamma} \mid u \mid^{p-1} d\tau, \\ u(0,x) = u_0(x), \end{cases}$$

where $0 < \beta \leq 2$ and $0 < \gamma < 1$. They proved that if

$$1 < p \leq p^* = 1 + \frac{\beta(2+\gamma)}{N - \beta + \beta\gamma} \text{ or } p < \frac{1}{\gamma},$$

where in the case $p = p^*$ and $\beta \in (0,2)$, they take

$$p \geq \frac{N}{N-\beta}, \quad N > \beta,$$

then the solution of the posed problem blows up in a finite time for $u_0 \in C_0(\mathbb{R}^N)$.

The absence of the nontrivial global solutions was also considered for a time-space fractional evolution systems, in [29] Xu and Tan shows that any solution (u,v) to the next problem

$$\begin{cases} u_t + (-\triangle)^{\beta_1/2}u = \frac{1}{\Gamma(1-\gamma)}\int_0^t (t-\tau)^{-\gamma} \mid v \mid^{p-1} d\tau, \\ v_t + (-\triangle)^{\beta_2/2}v = \frac{1}{\Gamma(1-\gamma)}\int_0^t (t-\tau)^{-\delta} \mid u \mid^{p-1} d\tau, \\ u(0,x) = u_0(x), v(0,x) = v_0(x), \end{cases}$$

blows up in a finite time for $u_0, v_0 \in C_0(\mathbb{R}^N)$ (with $u_0, v_0 \geq 0, u_0, v_0 \not\equiv 0$) under appropriate conditions as

$$N < \max\left\{\frac{(2-\delta)p + (1-\gamma)pq + 1}{pq-1}, \frac{(2-\gamma)q + (1-\delta)pq + 1}{pq-1}\right\},$$

or

$$p < \frac{1}{\delta} \text{ and } q < \frac{1}{\gamma},$$

where $0 < \gamma, \delta < 1$, $p, q > 1$ and $0 < \beta_1, \beta_2 \leq 2$. In the second part of this study, we extend the above result by replacing u_t and v_t respectively by $\mathbf{D}^{\beta}_{0|t}u$ and $\mathbf{D}^{\beta}_{0|t}v$ where $0 < \beta < 1$.

The paper is organized as follows. In section 1, we present some definitions of fractional calculus that will be used throughout this paper. In section 2, we present the our first main result concerning the absence of nontrivial global weak solutions for the problem (1). In section 3, we extend the first result to studying the blow-up of solutions for a time-space fractional evolution systems.

2. Preliminaries

In this section, we present some basic notions of fractional derivatives and integrations that will be required in later sections and which can been founded in [23].

Definition 1. *For $0 < \alpha < 1$, the Riemann-Liouville derivatives of order α for a continuous function $f : [0,\infty) \to \mathbb{R}$, are defined by*

$$D^{\alpha}_{0|t}f(t) = \frac{1}{\Gamma(1-\alpha)}\frac{d}{dt}\int_0^t (t-\tau)^{-\alpha} f(\tau)d\tau,$$

$$D^{\alpha}_{t|T}f(t) = -\frac{1}{\Gamma(1-\alpha)}\frac{d}{dt}\int_t^T (\tau-t)^{-\alpha} f(\tau)d\tau.$$

Definition 2. *For $0 < \alpha < 1$, the Caputo derivative of order α for a differentiable function $f : [0,\infty) \to \mathbb{R}$ can been written as*

$$\mathbf{D}^{\alpha}_{0|t}f(t) = \frac{1}{\Gamma(1-\alpha)}\frac{d}{dt}\int_0^t (t-\tau)^{-\alpha} f'(\tau)d\tau.$$

It is clear that

$$\mathbf{D}^{\alpha}_{0|t}f(t) = D^{\alpha}_{0|t}[f(t) - f(0)].$$

On the other hand, for every differentiable functions $f, g : [0, \infty) \to \mathbb{R}$, we have the following formula of integration by parts

$$\int_0^T D^{\alpha}_{0|t}f.g(t) = \int_0^T D^{\alpha}_{t|T}g.f(t),$$

and for $f \in C^2[0, \infty)$, we have

$$-DD^{\alpha}_{t|T}f(t) = D^{\alpha+1}_{t|T}f(t).$$

For $\eta \gg 1$ and $0 < \alpha < 1$. Let

$$f(t) = \begin{cases} (1 - \frac{t}{T})^{\eta}, & 0 < t \leqslant T, \\ 0, & t \geqslant T, \end{cases} \tag{4}$$

then

$$D^{\alpha}_{t|T}f(t) = \frac{(1 - \alpha + \eta)B(1 - \alpha; \eta - 1)}{\Gamma(1 - \alpha)}T^{-\alpha}(1 - \frac{t}{T})^{\eta-\alpha},$$

and

$$\int_0^T f(t)^{-p'/p}|D^{\alpha}_{t|T}f(t)|^{p'} = CT^{1-p'\alpha}.$$

3. Nonexistence results

Definition 3. *Let $\beta = 2, u_0 \in C_0(\mathbb{R}^N)$. We calle u is a mild solution of (2), if*

$$u = P_{\alpha}(t)u_0 + \int_0^t (t - s)^{\alpha-1}S_{\alpha}(t - s)D^{-(1-\gamma)}_{0|t}|u|^p. \tag{5}$$

for more detail see [28]

Definition 4 (Weak solution)**.** *Let $u \in C[(0, T), L^p(\mathbb{R}) \cap L^1_{loc}((0, T) \times \mathbb{R}^n) \cap L^p_{loc}((0, T) \times \mathbb{R}^n))]$ is a local weak solution to problem (2), if*

$$\begin{aligned} &\int_{Q_T} u_0\, D^{\beta}_{t|T}\varphi(t, x)dxdt + \int_{Q_T} \varphi(t, x)D^{-\alpha}_{0|t}|u|^p \\ &= \int_{Q_T} u\, D^{\beta}_{t|T}\varphi(t, x)dxdt + \int_{Q_T} (-\Delta)^m u\varphi(t, x) \end{aligned}$$

where $Q_T = [0, T] \times \Omega$; $\Omega = supp\varphi$,the test function $\varphi \in C^{1,2}_{t,x}(Q_T), \varphi(t, x) = 0$, if $T = \infty$ the solution is global.

Theorem 1. *Let $p > 1, 0 \leq \alpha + \beta \leq 1$ and $u_0 > 0$, assume that $p > \frac{2m}{N\beta - 2m} + 1$ with $N > \frac{2m}{\beta}$ is satisfied if*

$$p \leq max\left\{\frac{2m(\alpha+\beta)}{2m + N\beta - 2m(\alpha+\beta)} + 1, \frac{1}{\gamma}\right\}.$$

The problem (2) does not admit nontrivial global weak solution.

Proof. On the contrary, suppose that u is is nontrivial nonnegative solution for all time $t > 0$. Choose the test function

$$\varphi(t,x) = D^{\alpha}_{t|T}\tilde{\varphi} = \varphi_1^l(x) D^{\alpha}_{t|T}\varphi_2(t), \quad l > \frac{p}{p-1},$$

where

$$\varphi_1(x) = \Phi\left(\frac{|x|}{T^{\frac{\beta}{2m}}}\right) = \Phi(\xi) = \begin{cases} 1, & 0 < \xi \leqslant 1, \\ 0, & \xi \geqslant 2, \end{cases}$$

and

$$\varphi_2(t) = \left(1 - \frac{t}{T}\right)^{\eta}, \eta > \max\left\{\frac{p(\alpha+\beta)}{p-1} - 1, \alpha+\beta+1\right\}.$$

Using the Definition 4, we obtain

$$\begin{aligned} &\int_{Q_T} u_0 \, D^{\beta}_{t|T} D^{\alpha}_{t|T}\tilde{\varphi} dxdt + \int_{Q_T} D^{\alpha}_{t|T}\tilde{\varphi} \, D^{-\alpha}_{0|t}|u|^p \\ &= \int_{Q_T} u \, D^{\beta}_{t|T} D^{\alpha}_{t|T}\tilde{\varphi} dxdt + \int_{Q_T} (-\Delta)^m u \, D^{\alpha}_{t|T}\tilde{\varphi}. \end{aligned}$$

A simple computation yields $D^{\beta}_{t|T}(D^{\alpha}_{t|T}\tilde{\varphi}) = D^{\alpha+\beta}_{t|T}\tilde{\varphi}$, we obtain

$$\begin{aligned} &cT^{1-(\alpha+\beta)} \int_{\mathbb{R}^N} u_0 \varphi_1^l(x) dx + \int_{Q_T} \tilde{\varphi}|u|^p \\ &= \int_{Q_T} u\varphi_1^l \, D^{\alpha+\beta}_{t|T}\varphi_2(t) dxdt + \int_{Q_T} u(-\Delta)^m \varphi_1^l(x) D^{\alpha}_{t|T}\varphi_2. \end{aligned}$$

The application of Ju inequality

$$(-\Delta)^m \varphi_1^l \leq l\varphi_1^{l-1}(-\Delta)^m \varphi_1,$$

implies that

$$\int_{Q_T} \tilde{\varphi}|u|^p \leq c\int_{Q_T} u\varphi_1^{l-1}(-\Delta)^m \varphi_1(x) D^{\alpha}_{t|T}\varphi_2 + \int_{Q_T} u\varphi_1^l \, D^{\alpha+\beta}_{t|T}\varphi_2(t) dxdt.$$

For estimating the second member of the above inequality, we write

$$\int_{Q_T} u\varphi_1^{l-1}(-\Delta)^m \varphi_1 D^{\alpha}_{t|T}\varphi_2 = \int_{Q_T} u\tilde{\varphi}^{\frac{1}{p}} \varphi_1^{l-1}(-\Delta)^m \varphi_1 D^{\alpha}_{t|T}\varphi_2 \, \tilde{\varphi}^{\frac{-1}{p}}.$$

According to $\varepsilon-$Young inequality

$$XY \leq \varepsilon X^p + c(\varepsilon) Y^{p'}, p + p' = p.p',$$

we have

$$\int_{Q_T} u(-\Delta)^m \varphi_1 D^{\alpha}_{t|T} \varphi_2 \leq \varepsilon \int_{Q_T} |u|^p \tilde{\varphi} + c_1(\varepsilon) \int_{Q_T} \varphi_1^{l-1} |(-\Delta)^m \varphi_1 D^{\alpha}_{t|T} \varphi_2|^{p'} \tilde{\varphi}^{\frac{-p'}{p}}.$$

In the same way, we get

$$\int_{Q_T} u\varphi_1^l \ D^{\alpha+\beta}_{t|T} \varphi_2 \leq \varepsilon \int_{Q_T} |u|^p \tilde{\varphi} + c_2(\varepsilon) \int_{Q_T} |\varphi_1^l D^{\alpha+\beta}_{t|T} \varphi_2|^{p'} \tilde{\varphi}^{\frac{-p'}{p}}.$$

When ε is small, we obtain

$$\int_{Q_T} |u|^p \tilde{\varphi} \leq c \left(\int_{Q_T} \varphi_1^{l-p'} \varphi_2^{-\frac{1}{p-1}} |(-\Delta)^m \varphi_1 D^{\alpha}_{t|T} \varphi_2|^{p'} + \int_{Q_T} |D^{\alpha+\beta}_{t|T} \varphi_2|^{p'} \varphi_1^l \varphi_2^{\frac{-1}{p-1}} \right).$$

The change of variables

$$\tau = \frac{t}{T}, \xi = \frac{x}{T^{\frac{\beta}{2m}}}.$$

Implies that

$$\int_{Q_T} |u|^p \tilde{\varphi} \leqslant cT^{-(-1+(\alpha+\beta)p' - \frac{N\beta}{2m})} \left[\int_{|\xi|<2} \Phi |(-\Delta)^m \Phi(\xi)|^{p'} d\xi + \int_{|\xi|<2} \Phi^l d\xi \right].$$

Then, we have the estimate

$$\int_{Q_T} |u|^p \tilde{\varphi} < CT^{-((\alpha+\beta)p' - 1 - \frac{N\beta}{2m})}.$$

We put $\delta = -((\alpha+\beta)p' - 1 - \frac{N\beta}{2m})$, then

In the case $\delta < 0$, let $T \longrightarrow +\infty$, we obtain $\int_{Q_T} |u|^p \tilde{\varphi} < 0$. This implies that $u \equiv 0$.

In the case $\delta = 0$, let $\varphi_1(x) = \Phi\left(\frac{|x|}{(\frac{T}{\mu})^{\frac{\beta}{2m}}} \right)$ and using $p > \frac{2m}{N\beta - 2m} + 1$, we get

$$\int_{Q_T} |u|^p \tilde{\varphi} < c\mu^{p' - \frac{N\beta}{2m}} + c\mu^{-\frac{N\beta}{2m}}.$$

This implies that $u \equiv 0$ when $\mu \longrightarrow \infty$. $\square$

Remark 2. If $\beta = 1$ we go back to classical problem that was treated by Sun and Shi [15].

4. Nonexistence result for fractional evolution systems

This section is devoted to the study of the nonexistence of a solution in a fractional evolution system that is the core of our study.

$$(6)\qquad \begin{cases} D^{\beta}_{0|t}(u-u_0)+(-\Delta)^m u = D^{-\alpha}_{0|t}|v|^p, \alpha = 1-\gamma \\ D^{\delta'}_{0|t}(v-v_0)+(-\Delta)^{m'} v = D^{-\delta}_{0|t}|u|^q, \delta = 1-\beta'. \end{cases}$$

Theorem 3. *Let $p, q > 1$. The system (6) does not admit nontrivial global weak solution for all*

$$N < \left\{ \frac{(\frac{\delta'+\delta}{q}) + (\beta+\alpha) - (1-\frac{1}{pq})}{(\frac{\delta'}{2p'm'q}) + (\frac{(\beta)}{2mq'})}, \frac{(\frac{\beta+\alpha}{p}) + (\delta'+1-\delta) - (1-\frac{1}{pq})}{(\frac{\beta}{2q'mp}) + (\frac{(\delta')}{2m'p'})} \right\}.$$

Proof. The proof proceeds by contradiction. Let

$$\xi_1(t,x) = D^{\alpha}_{t|T}\varphi' = D^{\alpha}_{t|T}(\varphi_1^l(x)\varphi_2(t)),$$

and

$$\xi_2(t,x) = D^{\delta}_{t|T}\varphi' = D^{\delta}_{t|T}(\varphi_1^l(x)\varphi_2(t))$$

such that

$$\varphi_1(x) = \Phi(y) \quad \text{and} \quad \varphi_2(t) = \left(1-\frac{t}{T}\right)^{\eta},$$

where $l > max\{p', q'\}$ and $\eta > max\left\{\frac{q'(\alpha+\beta)-1}{q'-\frac{-1}{q-1}}, \frac{p'(\delta+\delta')-1}{p'-\frac{-1}{p-1}}\right\}$.

The weak solution to system (6) such that

$$cT^{1-(\alpha+\beta)}\int_{\mathbb{R}^N} u_0\varphi_1^l(x)dx + \int_{Q_T}\varphi'|v|^p$$
$$= \int_{Q_T} u\varphi_1^l\, D^{\alpha+\beta}_{t|T}\varphi_2(t)dxdt + \int_{Q_T} u(-\Delta)^m\varphi_1^l(x)D^{\alpha}_{t|T}\varphi_2,$$

and

$$cT^{1-(\delta+\delta')}\int_{\mathbb{R}^N} v_0\varphi_1^l(x)dx + \int_{Q_T}\varphi'|u|^q$$
$$= \int_{Q_T} v\varphi_1^l\, D^{\delta+\delta'}_{t|T}\varphi_2(t)dxdt + \int_{Q_T} v(-\Delta)^{m'}\varphi_1^l(x)D^{\alpha}_{t|T}\varphi_2.$$

By the Hölder inequality, we may write

$$\int_{Q_T} u\varphi_1^l\, D^{\alpha+\beta}_{t|T}\varphi_2(t)dxdt \le \left[\int_{Q_T}|u|^q\varphi'\right]^{\frac{1}{q}}\left[\int_{Q_T}\varphi_1^l(x)\varphi_2^{\frac{-1}{q-1}}|D^{\alpha+\beta}_{t|T}\varphi_2|^{q'}\right]^{\frac{1}{q'}},$$

and

$$\int_{Q_T} u(-\Delta)^m \varphi_1^l(x) D^\alpha_{t|T}\varphi_2 \leq \left[\int_{Q_T} |u|^q \varphi'\right]^{\frac{1}{q}} \left[\int_{Q_T} \varphi_1^{l-q'} \varphi_2^{\frac{-1}{q-1}} |(-\Delta)^m \varphi_1 D^\alpha_{t|T}\varphi_2|^{q'}\right]^{\frac{1}{q'}}.$$

Then

$$\int_{Q_T} |v|^p \varphi' \leq \left[\int_{Q_T} |u|^q \varphi'\right]^{\frac{1}{q}} A,$$

where

$$A = \left[\int_{Q_T} \varphi_1^l(x) \varphi_2^{\frac{-1}{q-1}} |D^{\alpha+\beta}_{t|T}\varphi_2|^{q'}]^{\frac{1}{q'}} + [C\int_{Q_T} \varphi_1^{l-q'} \varphi_2^{\frac{-1}{q-1}} |(-\Delta)^m \varphi_1 D^\alpha_{t|T}\varphi_2|^{q'}]^{\frac{1}{q'}}\right].$$

Similarly ,we obtain the estimate

$$\int_{Q_T} |u|^q \varphi' \leq \left[\int_{Q_T} |v|^p \varphi'\right]^{\frac{1}{p}} .B,$$

where

$$B = \left[\int_{Q_T} \varphi_1^l(x) \varphi_2^{\frac{-1}{p-1}} |D^{\delta+\delta'}_{t|T}\varphi_2|^{p'}]^{\frac{1}{p'}} + [C\int_{Q_T} \varphi_1^{l-p'} \varphi_2^{\frac{-1}{p-1}} |(-\Delta)^{m'} \varphi_1 D^\delta_{t|T}\varphi_2|^{p'}]^{\frac{1}{p'}}\right].$$

Consequently we can write

$$\left[\int_{Q_T} |v|^p \xi_1\right]^{1-\frac{1}{pq}} \leqslant B^{\frac{1}{q}}.A,$$

$$\left[\int_{Q_T} |u|^q \xi_2\right]^{1-\frac{1}{pq}} \leqslant A^{\frac{1}{p}}.B.$$

If we use the change of variables as

$$t = R\tau \quad \text{and} \quad x = R^{\frac{\beta}{2m}} y \quad \text{in} \quad A,$$

and

$$t = R\tau \quad \text{and} \quad x = R^{\frac{\delta'}{2m'}} y \quad \text{in} \quad B.$$

We find the following estimation

$$\left[\int_{Q_T} |v|^p \xi_1\right]^{1-\frac{1}{pq}} \leq C(R^{-l_1})^{\frac{1}{q}}.R^{-l_2}$$

where

$$l_1 = (\delta' + \delta) - \frac{1}{p'}\left(\frac{N\delta'}{2m'} + 1\right),$$

$$l_2 = (\beta + \alpha) - \frac{1}{q'}\left(\frac{N\beta}{2m} + 1\right).$$

The case $\frac{l_1}{q} + l_2 > 0$ is equivalent to

$$N < \frac{(\frac{\delta'+\delta}{q}) + (\beta + \alpha) - (1 - \frac{1}{pq})}{\frac{\delta'}{2p'm'q} + \frac{\beta}{2mq'}}.$$

Similarly

$$N < \frac{(\frac{\beta+\alpha}{p}) + (\delta' + 1 - \delta) - (1 - \frac{1}{pq})}{\frac{\beta}{2q'mp} + \frac{\delta'}{2m'p'}}).$$

That's why to obtain contradiction it suffices to assume

$$N < \max\left\{\frac{(\frac{\delta'+\delta}{q}) + (\beta + \alpha) - (1 - \frac{1}{pq})}{\frac{\delta'}{2p'm'q} + \frac{\beta)}{2mq'}}; \frac{(\frac{\beta+\alpha}{p}) + (\delta' + 1 - \delta) - (1 - \frac{1}{pq})}{\frac{\beta}{2q'mp} + \frac{\delta'}{2m'p'}}\right\}$$

□

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