A hybrid DESA-MOLP method for finding most preferred solution

Marzieh Moradi Daleni
Department of Mathematics
Zarghan Branch
Islamic Azad University
Zarghan
Iran
marziehmoradi99@yahoo.com

Abstract. Data Envelopment Scenario Analysis (DESA) commonly discussed past performance and Multiple Objective Linear Programming (MOLP) well known to plan for future performance goals. The aim of our study is showing an equivalence relation between MOLP and DESA model with using a direction distance function designed to account for desirable and undesirable inputs and outputs together with uncontrollable variables. In addition, we will use the DESA model because in this model we can see decrease all inputs and increases all outputs and estimate one problem instead of n problems. This equivalence model can be effectively used to support interactive processes and performance measures designed to establish future performance goals while taking into account the preferences of decision makers (DMs). In particular, it allows DMs to consider different efficiency improvement strategies when subject to budgetary restrictions. In this context, we tried to solve IDESA models with interactive STOM procedure. The STOM method is the responsible method, because it can estimate any efficient solution, and it indicates Most Preferred Solution (MPS).

Keywords: Data envelopment analysis, Uncertain data, STOM alghorithm, Multi objective linear, programming.

1. Introduction

The first investigations into Data Envelopment Analysis (DEA) was conducted in 1978 by Charnes et al. also in the last few years, much more information on Multi Objective Linear Programming (MOLP) has become available. DEA and MOLP are useful for management to draw a scheme for future. DEA is used to evaluate the past preferences while MOLP is used for future preferences.

A hybrid DEA-MOLP model has been identified as being Tavana et al (2017). Recent developments in this model have led to using interactive MOLP to solve DEA model and finding Most Preferred Solution (MPS).

Data Envelopment Scenario Analysis (DESA) model was chosen because it is one of the most practical ways to this study. Whit this in mind we tried to show a hybrid Imprecise DESA-MOLP and use the interactive MOLP method with the name of STOM method (M) to solve the IDESA.

Imprecise DEA(IDEA) is defined by Cooper et al. (1999) to mean that data were vague, which results in a non-linear DEA model and we can transform it into a linear problem. Also, we obtain a double of interval DEA models that will be utilized for interval data rather than for crisp data. In addition, we would nd the MPS in the IDESA models by the interactive algorithm like STOM method.

Assurance that the manuscript has neither been published nor submitted for publication elsewhere.

2. Data Envelopment Scenario Analysis (DESA)

Thanassoulis and Dyson (1992) developed on a new method for DEA and concluded that discuss the uselessness of radial projection in target setting. In this model, we have $I = \{1, \cdots, m\}$ showing that I is a set of inputs and $O = \{1, \cdots, s\}$ showing that O is a set of outputs. By substituting $I \cong I_g \cup \bar{I}_g$ and $O \cong O_g \cup \bar{O}_g$, where O_g and I_g are used to show outputs and inputs where borders of success are used in the target model. The DESA problem can be expressed as follows:

$$
\begin{array}{lll}
\min\limits_{\lambda_j, Z_k, \theta_i} & \sum\limits_{k \in O} P_k^{+} Z_k - \sum\limits_{i \in I} P_i^{-} \theta_i & \\
\text{s.t} & \sum\limits_{j=1}^{n} \lambda_j x_{ij} = \theta_i x_{io} & i \in I \\
& \sum\limits_{j=1}^{n} \lambda_j y_{kj} = Z_k y_{ko} & k \in O \\
& \theta_i x_{io} \geq G_i & i \in I_g \\
& Z_k y_{ko} \leq G'_k & k \in O_g \\
& A_i \leq \theta_i \leq 1/B_i, & A_i, B_i \in [0,1], \quad \forall i \in I \\
& \Gamma_k \leq 1/Z_k \leq 1/\Delta_k, & \Gamma_k, \Delta_k \in [0,1], \quad \forall k \in O
\end{array}
\tag{1}
$$

P_i^{-} and P_k^{+} are the decision makers' preferences for the recovery of inputs and outputs. θ_i is the contraction rate of input i and Z_k is the development rate of output k. G_i and G'_k are borders for ith input and kth output, respectively. (A_i, B_i) and (Γ_k, Δ_k) are remarked as the width and height borders for and θ_i, Z_k respectively.

3. DESA method with imprecise data

DESA model with imprecise data has many valuable applications, the key advantage is to calculate one model instead of n models, this method can be formulated as follows:

$$
\begin{aligned}
\max_{\lambda_j, Z_k, \theta_i} \quad & \sum_{k\in O} P_k^+ Z_k - \sum_{i\in I} P_i^- \theta_i - M \sum_{i\in I_f} s_i' - M \sum_{k\in O_f} s_k'' \\
\text{s.t.} \quad & \sum_{r=1}^{n}\sum_{j=1}^{n} \lambda_{jr} x_{ij} = \theta_i \sum_{j=1}^{n} x_{ij} && i\in I \\
& \sum_{r=1}^{n}\sum_{j=1}^{n} \lambda_{jr} y_{kj} = Z_k \sum_{j=1}^{n} y_{kr} && k\in O \\
& \sum_{j=1}^{n} \theta_i x_{ij} \geq G_i^L - s_i' && i\in I_f \\
& \sum_{r=1}^{n} Z_k y_{kr} \leq G_k'^U - s_k'' && k\in O_f \\
& \sum_{j=1}^{n} \lambda_{jr} = 1 && \forall r \\
& \lambda_j \geq 0 \quad \forall j, \quad \theta_i \text{ free}, \qquad Z_k \text{ free}, \quad k\in O \\
& s_i' \geq 0 \quad \forall i\in I_f, \qquad s_k'' \geq 0 \quad \forall k\in O_f
\end{aligned} \tag{2}
$$

$\left|G_i^L, G_i^U\right|$ and $\left|G_k'^L, G_k'^U\right|$ are the interval resources for all input consumption and all output generation, respectively. s_i', s_k'' are intended for the legalization of all diminution and all generation, respectively. M shows a penalty factor that has to be intended by the decision maker.

4. Imprecise DESA model based on interval arithmetic

The first systematic report on change DEA model to two models was carried out in 2005 by Wang, so that we could be able to use this method for changing the DESA model to two new models with the names of undesirable model and desirable model. The most remarkable result to emerge from data is that best lower bound and best upper bound for each DMU, where

$$x_{ij} \in \left\lfloor x_{ij}^L, x_{ij}^U \right\rfloor,\ y_{kj} \in \left\lfloor y_{kj}^L, y_{kj}^U \right\rfloor,\ \theta_i \in \left\lfloor \theta_i^L, \theta_i^U \right\rfloor,\ Z_k \in \left\lfloor Z_k^L, Z_k^U \right\rfloor. \tag{3}$$

G_i^U is the limit to all use of input x_{ij}^U, and $G_k'^L$ is the limit to total production of output y_{kr}^L

$$
\begin{aligned}
\max_{\lambda_j, Z_k, \theta_i} \quad & \sum_{k\in O} P_k^+ Z_k^L - \sum_{i\in I} P_i^- \theta_i^L - M \sum_{i\in I_f} s_i' - M \sum_{k\in O_f} s_k'' \\
\text{s.t.} \quad & \sum_{r=1}^{n}\sum_{j=1}^{n} \lambda_{jr} x_{ij}^L = \theta_i^L \sum_{j=1}^{n} x_{ij}^U && i\in I
\end{aligned}
$$

$$
\begin{aligned}
&\sum_{r=1}^{n}\sum_{j=1}^{n}\lambda_{jr}y_{kj}^{U}=Z_k^L\sum_{j=1}^{n}y_{kr}^{L} && k\in O\\
&\sum_{j=1}^{n}\theta_i^L x_{ij}^{U}\geq G_i^U-s_i' && i\in I_f\\
&\sum_{r=1}^{n}Z_k^L y_{kr}^{L}\leq G_k'^L-s_k'' && k\in O_f \qquad (4)\\
&\sum_{j=1}^{n}\lambda_{jr}=1 && \forall r\\
&\lambda_{jr}\geq 0\quad \forall j,\quad \theta_i^L \text{ free},\qquad Z_k^L \text{ free},\quad k\in O\\
&s_i'\geq 0\quad \forall i\in I_f,\qquad\qquad s_k''\geq 0\quad \forall k\in O_f.
\end{aligned}
$$

Where $G_i^L$is the limit to total consumption of input x_{ij}^L and $G_k'^U$ is the limit to total production of output y_{kr}^U. According to the above definition, model (3) is the lower bound of the best possible relative efficiency and model (4) is the upper bound of the best possible relative efficiency.

5. A hybrid imprecise DESA- MOLP model

This paper focuses on relation between multi-objective linear program (MOLP) and DESA model with imprecise data, suppose that:

$$\bar{f}_k(\lambda)=\sum_{r=1}^{n}\sum_{j=1}^{n}\lambda_{jr}y_{kj}^{U} \qquad k=1,\cdots,s, \tag{5}$$

$$\tilde{f}_i(\lambda)=\sum_{r=1}^{n}\sum_{j=1}^{n}\lambda_{jr}y_{kj}^{L} \qquad i=1,\cdots,s. \tag{6}$$

So that

$$\lambda=(\lambda_{11},\lambda_{jr},\cdots,\lambda_{nn})^T.$$

It is assumed that $f_k'=\bar{f}_k(\lambda^*)$ is the maximum possible rate for all the kth outputs and $f''(i)=\tilde{f}_i(\lambda^*)$ is the minimum possible rate for all the ith inputs. where λ^* can be computed by finding the lower part of the models:

$$
\begin{aligned}
\max \quad & \sum_{r=1}^{n}\sum_{j=1}^{n}\lambda_{jr}y_{kj}^{U}\\
\text{s.t.} \quad & \lambda\in\Omega
\end{aligned} \tag{7}
$$

$$
\begin{aligned}
\min \quad & \sum_{r=1}^{n}\sum_{j=1}^{n}\lambda_{jr}y_{ij}^{L}\\
\text{s.t.} \quad & \lambda\in\Omega.
\end{aligned} \tag{8}
$$

Such that $f' = [f'_1, \cdots, f'_s]$ and $f'' = [f''_1, \cdots, f''_m]$ indicate the desirable points of model (5) and model (6). Suppose that:

$$\begin{aligned} &\sum_{j=1}^{n} x_{ij} > 0 \qquad i = 1, \cdots, m, \\ &\sum_{r=1}^{n} y_{kr} > 0 \qquad k = 1, \cdots, s, \\ &w_k = \frac{1}{\sum_{r=1}^{n} y_{kr}^U} \qquad \forall k, \\ &v_i = \frac{1}{\sum_{j=1}^{n} x_{ij}^L} \qquad \forall i. \end{aligned} \tag{9}$$

And

$$\begin{aligned} &\bar{f}_k^* = \frac{F^{\max}}{w_k} = F^{\max} \sum_{r=1}^{n} y_{kr}^U, \quad \tilde{f}_i^* = \frac{H^{\min}}{v_i} = H^{\min} \sum_{j=1}^{n} x_{ij}^L \\ &\tau_i = \theta_i^U - H^{\min}, \quad \gamma_k = F^{\max} - Z_k^U. \end{aligned} \tag{10}$$

Such that

$$F^{\max} = \max_{1 \le k \le s} \{w_k f'_k\} = \max_{1 \le k \le s} \left\{ \frac{f'_k}{\sum_{r=1}^{n} y_{kr}^U} \right\}, \tag{11}$$

$$H^{\min} = \min_{1 \le i \le m} \{v_i f''_i\} = \min_{1 \le i \le m} \left\{ \frac{f''_i}{\sum_{j=1}^{n} x_{ij}^L} \right\}. \tag{12}$$

We can write the equivalence model as the following model:

$$\begin{aligned} \max_{\lambda_j, Z_k, \theta_i} \quad & \sum_{k \in O} P_k^+ Z_k^U - \sum_{i \in I} P_i^- \theta_i^U \\ \text{s.t.} \quad & \tilde{f}_i(\lambda) - \theta_i^U \frac{1}{v_i} = 0 && i = 1, \cdots, m \\ & Z_k^U \frac{1}{w_k} - \bar{f}_k(\lambda) = 0 && k = 1, \cdots, s \\ & g_i v_i \le \theta_i^U && i \in I_f \\ & g'_k w_k \ge Z_k^U && k \in O_f, \ \lambda \in \Omega. \end{aligned} \tag{13}$$

Also, we can write

$$\tilde{f}_i(\lambda) - \theta_i^U \frac{1}{v_i} = 0 \Leftrightarrow v_i \tilde{f}_i(\lambda) = \theta_i^U \tag{14}$$
$$\Leftrightarrow v_i \tilde{f}_i(\lambda) - H^{\min} = \theta_i^U - H^{\min} \Leftrightarrow v_i\big(\tilde{f}_i(\lambda) - \tilde{f}_i^*\big) = \theta_i^U - H^{\min}.$$

Therefore,

$$Z_k^U \frac{1}{w_k} \bar{f}_k(\lambda) = 0 \Leftrightarrow -w_k \bar{f}_k(\lambda) = -Z_k^U \tag{15}$$
$$\Leftrightarrow F^{\max} - w_k \bar{f}_k(\lambda) = F^{\max} - Z_k^U \Leftrightarrow w_k\big(\bar{f}_k^* - \bar{f}_k(\lambda)\big) = F^{\max} - Z_k^U.$$

Such that $P_k^+ = P_i^- = 1$, $k \in O$, $i \in I$. Accordingly, above changes lead to the following model:

$$\max_{\lambda_j Z_k, \theta_i} \Big(\sum_{k\in O} Z_k^U - \sum_{i \in I} \theta_i^U\Big) = \min_{\lambda_j, Z_k, \theta_i} \Big(\sum_{i\in I} \theta_i^U + \sum_{k \in O}(-Z_k^U)\Big)$$
$$\Longrightarrow \min\Big(\sum_{i\in I}(\theta_i^U - H^{\min}) + \sum_{k\in O}(F^{\max} - Z_k^U)\Big) = \min\Big(\sum_{i \in I} \tau_i + \sum_{k\in O} \gamma_k\Big).$$

Also, for any $\lambda \in \Omega$, we have:

$$\begin{array}{ll}
F^{\max} - Z_k^U \geq w_k \bar{f}_k^* - Z_k^U \geq w_k \bar{f}_k(\lambda) - Z_k^U = 0 & k = 1, \cdots, s \\
\theta_i^U - H^{\min} \geq \theta_i^U - v_i \tilde{f}_i^* \geq \theta_i^U - v_i \tilde{f}_i(\lambda) = 0 & i = 1, \cdots, m \\
\bar{f}_k^* = \dfrac{F^{\max}}{w_k} \geq \dfrac{w_k f_k'}{w_k} = f_k' = \max\limits_{\lambda\in\Omega} \bar{f}_k(\lambda) & k = 1, \cdots, s \\
f_i^* = \dfrac{H^{\min}}{v_i} \leq \dfrac{v_i f_i''}{v_i} = f_i'' = \min\limits_{\lambda \in \Omega} \tilde{f}_i(\lambda) & i = 1, \cdots, m.
\end{array}$$

Now imprecise DESA model can be write as a minimax formulation as follows:

$$\begin{array}{lll}
\min & \sum\limits_{i\in I} \tau_i \sum\limits_{k\in O} \gamma_k & \\
\text{s.t.} & v_i\big(\tilde{f}_i(\lambda) - \tilde{f}_i^*\big) = \tau_i & i \in I \\
& w_k\big(\bar{f}_k^* - \bar{f}_k(\lambda)\big) = \gamma_k & k \in k = O \\
& g_i v_i - H^{\min} \leq \tau_i & i \in I_f \\
& g_k' w_k - M \leq \gamma_k & k \in O_f \\
& \sum\limits_{j=1}^{n} \lambda_{jr} = 1,\ \forall r\ \lambda_{jr} \geq 0,\ M \geq 0 & j, r = 1, \cdots, n.
\end{array} \tag{16}$$

Now imprecise DESA model and minimax MOLP formulation are equivalent, at this point an interactive method can be used for solve the imprecise DESA models and find the most preferred solution.

6. STOM algorithm for solving DESA model with imprecise data

STOM method is an interactive method, this method helps to DM for search the MPS on the efficient frontier by systematically changing the weighting parameters. STOM algorithm is defined as follows:

Step 1. During the first phase, solve the combined-oriented DESA model (7), (8) for all DMUs, so that we would be able to obtain the inefficient and efficient DMUs

Step 2. In the second step, we could be to check MPS with collect the target unit based on formulation (16), if the DM admit the target unit obtained as MPS, then stop. Elsewhere, go to step 3.

Step 3. Set the initial weighting parameters for all DMUs, and reach the initial solution of the decision variables.
Suppose for given positive weight vectors $w^t = \{w_1^t, \cdots, w_k^t, \cdots, w_s^t\}$ and $v^t = \{v_1^t, \cdots, v_i^t, \cdots, v_m^t\}$.

The optimal solution of the minimax model is given by $\lambda^t = \{\lambda_{11}^t, \cdots, \lambda_{jr}^t, \cdots, \lambda_{nn}^t\}$, whichmust be anefficientsolution. The optimal values of the dual variable of the kth and ith objective constraint $w_k\big(\bar{f}_k^* - \bar{f}_k(\lambda)\big) = \gamma_k$, $v_i\big(\tilde{f}_i(\lambda) - \tilde{f}_i^*\big) = \tau_i$ are given by β_k^t and α_i^t Iteration $t = 0$. Select an initial point $\lambda^0 = [\lambda_{11}^0, \cdots, \lambda_{nn}^0]$. An initial solution can be obtained by solving minimax model by setting

$$v_i^t = \Big(\frac{1}{X_i^* - f_i''}\Big) \quad \forall i = 1, \cdots, m$$

and

$$w_k^t = \Big(\frac{1}{f_k' - Y_k'^*}\Big) \quad \forall\, k = 1, \cdots, s.$$

Also, the initial dual variable values $\beta^t = [\beta_1^t, \cdots, \beta_s^t]^T$, $\alpha^t = [\alpha_1^t, \cdots, \alpha_m^t]^T$ for the first s and i constraint on the upper bounds of outputs and lower bounds of inputs.

Step 4. we are now ready to find the new reference point by $\bar{q}^t = (\bar{q}_1^t, \cdots, \bar{q}_s^t)$ and $\tilde{q}^t = (\tilde{q}_1^t, \cdots, \tilde{q}_m^t)$ Where $\tilde{q}_i^t = f_i^t + \tilde{\tau}^t \cdot \Delta f_i''^t \quad \forall i = 1, \cdots, m$,
$\bar{q}_k^t = f_k^t + \bar{\gamma}^t \cdot \Delta f_k'^t \quad \forall k = 1, \cdots, s$

$$\bar{\mu}_k^t = \frac{1}{|\bar{q}_k^t - \bar{f}_k^t|} \qquad \forall\, k = 1, \cdots, s,$$
$$\tilde{\mu}_i^t = \frac{1}{|\tilde{q}_i^t - \tilde{f}_i^t|} \qquad \forall\, i = 1, \cdots, m.$$

Let (λ^*, f^*) be a solution.

Step 5. given the new point of reference obtained in step 4, the new weights are computed as follows:

$$w_k^t = \frac{1}{f_k' - Y_k'^*} \qquad \forall\, k = 1, \cdots, s_1,$$
$$w_k^t = \frac{1}{f_k' - Y_k'^*} \qquad \forall\, k = 1, \cdots, s_2,$$
$$V_i^t = \frac{1}{f_i' - x_i'^*} \qquad \forall\, i = 1, \cdots, m_1,$$
$$V_i^t = \frac{1}{f_i' - Y_i'^*} \qquad \forall\, i = 1, \cdots, m_2.$$

Step 6. after determined the new weights in step 5, we could solve the super ideal point method (16) and obtain the optimal value.

Step 7. select the most preferred solutions by the decision maker.

with the completion of these steps and solve formulation (16), we are now ready to obtain the new target unit of DMU_p. Decision maker select the best solutions, otherwise we go to step 4.

7. Numerical example

The following table summarizes the data pertaining to seven banks. These data were prepared as described by Yang (2004), the sample was selected on the basis of four inputs and two outputs. This sample was solved by STOM algorithm, the method has several benefits, for instance, it searches for the MPS on the efficient frontier. We noted from Table 1 that four inputs are branches, number of ATMs, staff and asset size. The two outputs are customer satisfaction and total revenue. On the other hand, for this study, it is assumed that the data have been changed from certain to imprecise, as shown in Table 2.

Table 1: Original Data Set

DMU	Bank	Branches	ATMs	Staff	Asset size	Customer	Total
1	Abby	0.77	2.18	2.35	2.96	6.79	10.57
2	Barclay	1.95	3.19	8.43	3.53	2.55	13.35
3	Halifax	0.80	2.10	3.21	2.41	9.17	8.14
4	HSBC	1.75	4.00	13.30	4.85	5.82	23.67
5	Lloyds TSB	2.50	4.30	9.27	2.40	6.57	14.01
6	Nat West	1.73	3.30	7.70	3.09	4.86	12.04
7	RBS	0.65	1.73	2.65	1.34	7.28	7.36

8. Conclusions

The evidence from this study intimates that a hybrid DESA-MOLP model developed to consider strategic environments in which the value of inputs and outputs are uncertain and the basis on which to apply the STOM interactive

Table 2: Interval Data Set

DMU	Bank	Branch	ATMs	Staff	Asset size	Customer	Total
1	Abby	0.77	[2.18,2.2]	[2.35,2.50]	[2.96,3]	6.79	10.57
2	Barclay	1.95	[3.19,3.20]	[8.43,8.50]	[3.53,4]	2.55	13.35
3	Halifax	0.80	[2.10,2.2]	[3.21,3.50]	[2.41,2.5]	9.17	8.14
4	HSBC	1.75	[4.00,4.1]	[13.3,13.5]	[4.85,5]	5.82	23.67
5	Lloyds TSB	2.50	[4.30,4.7]	[9.27,9.3]	[2.40,2.43]	6.57	14.01
6	Nat West	1.73	[3.30,3.40]	[7.7,7.8]	[3.09,3.2]	4.86	12.04
7	RBS	0.65	[1.73,1.80]	[2.65,2.7]	[1.34,1.4]	7.28	7.36

method so as to achieve the MPS along the efficient frontier for each DMU. We are confident that our procedure might be useful for finding any efficient solution. Also, this approach results in decreasing total input and increasing total output at the same time. Also, it handles one problem instead of solving n independent linear programming models.

References

[1] A. Charnes, W.W. Cooper, E. Rhodes, *Measuring the efficiency of decision making units*, European Journal of Operational Research, 1978, 429-444.

[2] W.W. Cooper, K.S. Park, G. Yu, *IDEA and ARIDEA: models for dealing with imprecise data in DEA*, Management Science, 45 (1999), 597-607.

[3] A. Ebrahimnejad, M. Tavana, *An interactive MOLP method for identifying target units in output-oriented DEA models: the NATO enlargement problem*, Measurement, 52 (2014), 124-134.

[4] M. Luque, J. Yang, B.Y. Wong, *Project method for multiobjective optimization based on gradient projection and reference points*, IEEE Transactions on Systems, Man, and Cybernetics-part A: System and Humans, 39 (2009), 864879.

[5] M. Luque, L.A. Lopez-Agudo, O.D. Marcenaro-Gutierrez, *Equivalent reference points in multiobjective programming*, Expert Systems with Applications, 42 (2015), 22052212.

[6] A.D. Thanassoulis, R.G. Dyson, *Estimation preferred target input-output levels using data envelopment analysis*, European Journal of Operational Research, 56 (1992), 80-97.

[7] M. Tavana, A. Ebrahimnejad, F.J. Santos-Arteaga, S.M. Mansourzadeh, R.K. Matin, *A hybrid DEA-MOLP model for public school assessment and closure decision in the city of Philadelphia*, Socio-Economic Planning Sciences (in press), 2017.

[8] J.B. Yang, D.L. Xu, *Interactive minimax optimization for integrated performance analysis and resource planning*, Computer & Operations Research, 46 (2014), 78-90.

[9] Y.M. Wang, R. Greatbanksa, J.B. Yang, *Interval efficiency assessment using data envelopment analysis fuzzy sets*, Fuzzy Set and Systems, 153 (2005), 347-370.

[10] B.Y. Wong, J.B. Yang, R. Greatbank, *Using DEA and ER approach for performance measurement of UK retail banks*, MCDM 2004, Whistler, Canada.

Accepted: 6.06.2019