A fully fuzzy DEA approach for network cost efficiency measurement based on ranking functions

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Abstract. The purpose of this paper is to evaluate the cost-efficiency of the fully fuzzy network data envelopment analysis. Since actual measurement of real-world data is practically impossible, assuming that data is accurate in solving problems is not a valid hypothesis. One of the ways to deal with inaccurate data is fuzzy data. In this paper, linear ranking functions are used to transform the fully fuzzy cost efficiency model into a precise linear programming problem. By assuming triangular fuzzy numbers, the fuzzy cost efficiency of the decision makers is measured. Finally, a numerical example is shown for the proposed method.

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1. Introduction

Data Envelopment Analysis (DEA) is one of the most effective tools for measuring the efficiency and performance of organizations. On the other hand, in many DEA models, intra-organizational relationships have been ignored, and decision makers are thought as a black box, which only considers the inputs and outputs of the system. Unlike the black box model, the Network Data Envelopment Analysis (NDEA) model regards all internal processes in performance evaluation. For example, many companies are composed of several divisions with linked activities as in Diagram 1. In this case, the company is comprised of three divisions, each using its input resources to generate the outputs. However, there are linked activities or intermediate products that are shown by the links $1 \longrightarrow 2$ and $1 \longrightarrow 3$, and the link $2 \longrightarrow 3$. The link $1 \longrightarrow 2$ shows that some outputs of division 1 are used as inputs in division 2. In the current DEA models, each activity must belong to an input or output, not both, so these models cannot be formulated with intermediate products. For the first time, in

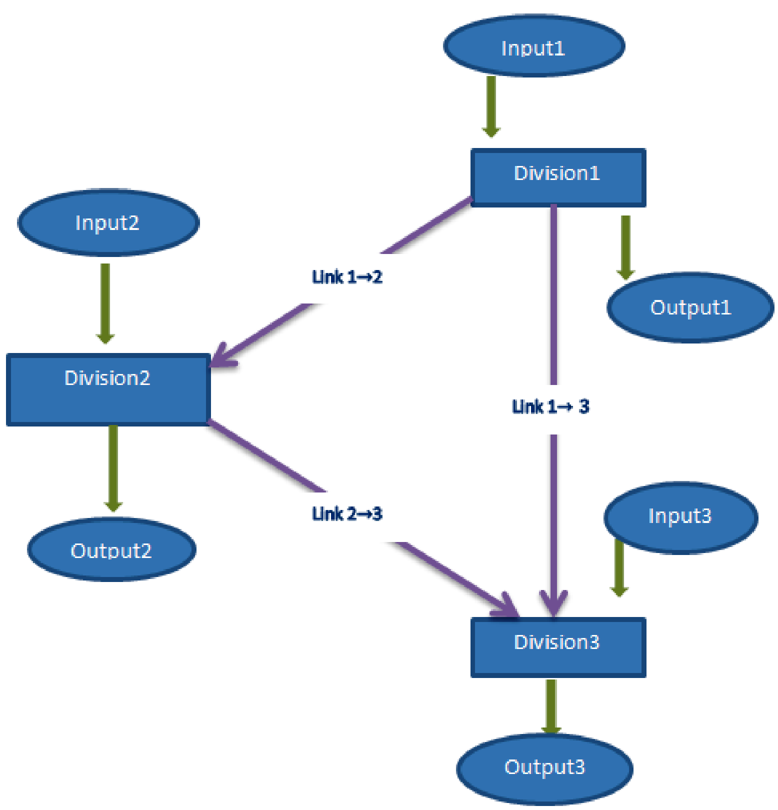

Figure 1: A company with three linked activities

2000, Fare and Grosskopf provided Data Envelopment Analysis models. Later, their models were expanded by several authors. Lewis and Sexton [9] presented a multi-stage NDEA model in 2004 as an extension of the Lewis and Sexton's two-step DEA model [4]. This article solves a DEA model independently for each node. Based on the Slack Based Measure (SBM) model, Tone and Tsutsui [11] further presented a NDEA model in 2009.

As a DEA model, Cost Efficiency (CE), evaluates the ability of a Decision Making Unit (DMU) to generate current outputs at the lowest cost and with assumed input prices. In fact, cost efficiency is a useful choice of inputs in terms of their price used to minimize production costs. Given the fact that in the real world we are dealing with network data envelopment analysis, it is important for managers to evaluate the revenue efficiency in NDEA. In 2013, Bani-Hashemi and Tohidi [1] proposed a model for assessing the cost efficiency of network data envelopment analysis models.

In the classical models, it is assumed that all data is accurate. However, accurate data is not always available, because the nature of data can be vague and unclear. Hence, one of the important ways to deal with inaccurate data is to consider fuzzy data. Only in [7] and [8], the fuzzy CEs (FCEs) are discussed with the fuzzy input- outputs and fuzzy input prices. But in none of these studies, fuzzy cost efficiency measurements in fully fuzzy NDEA (FFNDEA) have not been cited. In this article, the fully fuzzy models of network data envelopment analysis (fuzzy input-outputs and fuzzy input prices) were used in evaluating the fuzzy cost efficiency. Here, the ranking function method is used. Therefore, the ranking functions transform the full fuzzy model of network cost efficiency into a precise linear programming problem for measuring the fuzzy network cost efficiency.

The rest of the article will be as follows. In Section 2, the fuzzy clauses are discussed. In the next section, the problem of fuzzy linear programming and its transformation into a precise problem are studied. In Section 4 will measure the cost-effectiveness in NDEA. In Section 5, the proposed method for measuring fuzzy cost efficiency in FFNDEA is presented and, based on the proposed method, a numerical example is solved in the last section.

2. Fuzzy premises

2.1 Basic definitions of fuzzy

In this section, the basic definitions and the symbols of the fuzzy sets [12, 13], fuzzy Numbers, ranking function [5], and the concept of Fully Fuzzy Linear Programming (FFLP) used in this article are introduced.

Definition 2.1 ([12]). A fuzzy set $\tilde{A}$ is defined in the reference set X with $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in X\}$ where $\mu_{\tilde{A}} : X \longrightarrow [0,1]$ is the membership function and $\mu_{\tilde{A}}(x)$ is the degree of x in A.

Definition 2.2 ([13])**.** Assuming that X is a reference set, a fuzzy set $\tilde{A}$ is said to be convex X if and only if for every $x_1, x_2 \in X$:

$$\mu_{\tilde{A}}\big(\lambda x_1 + (1-\lambda)x_2\big) \geq \min\big(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\big) \qquad \forall\lambda \in [0,1]$$

Definition 2.3. Assuming that X is the reference set, then the fuzzy set A is called normal provided that there exist $x \in X$ so that $\mu_{\tilde{A}}(x) = 1$.

Definition 2.4. A fuzzy number $\tilde{A}$ is a convex normalized fuzzy set $\tilde{A}$ of the real line R such that

1. it exists exactly one $x_0 \in R$ $\mu_{\tilde{A}}(x_0) = 1$ (x_0 is called the mean value of $\tilde{A}$).
2. $\mu_{\tilde{A}}(x)$ is piecewise continuous.

Definition 2.5 ([13])**.** A triangular fuzzy number (TFN), $\tilde{A} = (a^l, a^m, a^u)$ is a fuzzy number with the given membership function $\mu_{\tilde{A}}$

$$\mu_{\tilde{A}}(x) = \begin{cases} (x-a^l)/(a^m-a^l), & a^l < x \leq a^m \\ (x-a^u)/(a^u-a^m), & a^m \leq x < a^u \\ 0, & otherwise. \end{cases}$$

Definition 2.6. A triangular fuzzy number $\tilde{A} = (a^l, a^m, a^u)$ is called a non-negative number if and only if a $a^l \geq 0,\ a^m - a^l \geq 0,\ a^u - a^m \geq 0$ and it is a positive number if and only if $a^l > 0,\ a^m - a^l \geq 0,\ a^u - a^m \geq 0$.

Definition 2.7. The support of a fuzzy set $\tilde{A}$, $S(\tilde{A})$ is the crisp set of all $x \in X$ such that . $\mu_{\tilde{A}}(x) > 0$. The (crisp) set of elements that belong to the fuzzy set $\tilde{A}$ at least to the degree α is called the α-cut set: $A_\alpha = \{x \in X | \mu_{\tilde{A}}(x) \geq \alpha\}$

Definition 2.8 ([5])**.** Suppose $\mathfrak{F}$ a set of all triangular fuzzy numbers. If $\tilde{A} \in \mathfrak{F}$ and $[A^1_\alpha, A^u_\alpha]$, $\alpha \in [0,1]$ the α- cut is $\tilde{A}$. Then, the ranking function of a function $\mathfrak{R} : \mathfrak{F} \longrightarrow \mathbb{R}$ is defined as:

$$\mathfrak{R}(\tilde{A}) = \frac{1}{2}\int_0^1 (A^l_\alpha + A^u_\alpha) d\alpha.$$

If $\tilde{A} = (a^l, a^m, a^u)$ is a triangular fuzzy number, then

$$\mathfrak{R}(\tilde{A}) = \frac{1}{4}(a^l + 2a^m + a^u).$$

Definition 2.9 ([5])**.** If $\tilde{A} = (a^l, a^m, a^u)$ and $\tilde{B} = (b^l, b^m, b^u)$ are two triangular fuzzy numbers, then order of $\tilde{A}$ and $\tilde{B}$ based on the ranking function $\mathfrak{R}$ will be as follows:

(i) $\tilde{A} \preccurlyeq \tilde{B} \Longleftrightarrow \mathfrak{R}(\tilde{A}) \leq \mathfrak{R}(\tilde{B})$

(ii) $\tilde{A} \succcurlyeq \tilde{B} \Longleftrightarrow \mathfrak{R}(\tilde{A}) \geq \mathfrak{R}(\tilde{B})$

(iii) $\tilde{A} \approx \tilde{B} \Longleftrightarrow \mathfrak{R}(\tilde{A}) = \mathfrak{R}(\tilde{B})$

And the features of Linearity will be:

$$\mathfrak{R}(k\tilde{A} + \tilde{B}) = k\mathfrak{R}(\tilde{A}) + \mathfrak{R}(\tilde{B}), \qquad k \in \mathbb{R}.$$

2.2 Math operations on triangular fuzzy numbers

If $\tilde{A} = (a^l, a^m, a^u)$ and $\tilde{B} = (b^l, b^m, b^u)$ are two triangular fuzzy numbers, then the mathematical operations on triangular fuzzy numbers will be as follows:

(i) $\tilde{A} \oplus \tilde{B} \approx (a^l + b^l, a^m + b^m, a^u + b^u)$ Addition

(ii) $\tilde{A} \ominus \tilde{B} \approx (a^l - b^u, a^m - b^m, a^u - b^1)$ Subtraction

(iii) $\tilde{A} \otimes \tilde{B} \approx (a^l b^l, a^m b^m, a^u b^u), \quad \tilde{A}, \tilde{B} \succcurlyeq \tilde{0}$ Multiplication

(iv) $\dfrac{\tilde{A}}{\tilde{B}} \approx \dfrac{(a^l, a^m, a^u)}{(b^l, b^m, b^u)} \approx \left(\dfrac{a^l}{b^u}, \dfrac{a^m}{b^m}, \dfrac{a^u}{b^l}\right), \quad \tilde{A}, \tilde{B} \succ \tilde{0}$ Division

(v) $\forall k \in \mathbb{R},\ k\tilde{A} \approx \begin{cases} (ka^l, ka^m, ka^u), & k > 0 \\ (ka^u, ka^m, ka^l), & k < 0 \end{cases}$ Scalar multiplication

3. Fuzzy linear programming problem

A linear programming problem with fuzzy coefficients and variables is called a full fuzzy linear programming problem. A full fuzzy linear programming problem with m constraints and n fuzzy variables are defined by the following model:

$$\begin{aligned} &\tilde{Z} = \max\,(or\,\min)(\tilde{C}^T \otimes \tilde{X}) \\ &\text{subject to} \quad \tilde{A} \otimes \tilde{X} \preccurlyeq, \approx, \succcurlyeq \tilde{b};\ \tilde{X} \succcurlyeq \tilde{0} \qquad (P1) \end{aligned}$$

where $\tilde{C} = [\tilde{c}_j]_{n\times 1}$, $\tilde{X} = [\tilde{x}_j]_{n\times 1}$, $\tilde{A} = [\tilde{a}_{ij}]_{m\times n}$, $\tilde{b} = [\tilde{b}_j]_{m\times 1}$, and $\tilde{a}_{ij}$, $\tilde{c}_j$, $\tilde{b}_i \in \mathfrak{F}$, $\tilde{x}_j$ are non-negative fuzzy numbers and $\tilde{0} = (0, 0, 0)$.

Definition 3.1 ([6])**.** The fuzzy optimal solution to the full fuzzy linear programming problem $(P1)$ will be $\tilde{X} = [\tilde{x}_j]_{n\times 1}$. if it fits with the following conditions:

1) $\tilde{x}_j$ is a non-negative fuzzy number,

2) $\tilde{A} \otimes \tilde{X} \preccurlyeq, \approx, \succcurlyeq \tilde{b}$,

and 3) If there exist any non-negative fuzzy number such as $\tilde{Y} = [\tilde{y}_j]_{n\times 1}$, in case $\tilde{A} \otimes \tilde{X} \preccurlyeq, \approx, \succcurlyeq \tilde{b}$, then $\mathfrak{R}(\tilde{C}^T \otimes \tilde{X}) \geq \mathfrak{R}(\tilde{C}^T \otimes \tilde{Y})$ for the maximization problem and $\mathfrak{R}(\tilde{C}^T \otimes \tilde{X}) \leq \mathfrak{R}(\tilde{C}^T \otimes \tilde{Y})$ for the minimization problem.

Definition 3.2 ([6])**.** Assume that $\tilde{X} = [\tilde{x}_j]_{n\times 1}$ is the fuzzy optimal solution for full fuzzy linear problem $(P1)$. If there exist any non-negative fuzzy number such as $\tilde{Y} = [\tilde{y}_j]_{n\times 1}$, so that $\tilde{A} \otimes \tilde{Y} \preccurlyeq, \approx, \succcurlyeq \tilde{b}$, and $\mathfrak{R}(\tilde{C}^T \otimes \tilde{X}) = \mathfrak{R}(\tilde{C} \otimes \tilde{Y})$, then $\tilde{Y} = [\tilde{y}_j]_{n\times 1}$ is called a fuzzy optimal solution of $(P1)$. Suppose $\tilde{c}_j = (c_j^l, c_j^m, c_j^u)$, $\tilde{x}_j = (x_j^l, x_j^m, x_j^u)$, $\tilde{a}_{ij} = (a_{ij}^l, a_{ij}^m, a_{ij}^u)$ and $\tilde{b}_i = (b_i^l, b_i^m, b_i^u)$ represents triangular fuzzy numbers. Then, the fuzzy decision parameters and variables in the model $(P1)$ are converted as follows:

$$\tilde{Z} = \max(or\ \min)\Big(\sum_{j=1}^{n}(c_j^l, c_j^m, c_j^u) \otimes (x_j^l, x_j^m, x_j^u)\Big)$$

$$\text{subject to} \quad \sum_{j=1}^{n}(a_{ij}^l, a_{ij}^m, a_{ij}^u) \otimes (x_j^l, x_j^m, x_j^u) \preccurlyeq, \approx, \succcurlyeq\ (b_i^l, b_i^m, b_i^u)\ \forall i;\ (x_j^l, x_j^m, x_j^u) \succcurlyeq \tilde{0}\ \forall j \qquad (p2)$$

After performing the mathematical operations discussed in Section 2-2, the model $(P2)$ is converted to the following form:

$$\tilde{Z} = \max(or\ \min)\ \Big(\sum_{j=1}^{n} c_j^l x_j^l, \sum_{j=1}^{n} c_j^m x_j^m, \sum_{j=1}^{n} c_j^u x_j^u\Big)$$

$$\text{subject to}\ \Big(\sum_{j=1}^{n} a_{ij}^l x_j^l, \sum_{j=1}^{n} a_{ij}^m x_j^m, \sum_{j=1}^{n} a_{ij}^u x_j^u\Big) \preccurlyeq, \approx, \succcurlyeq (b_i^l, b_i^m, b_i^u)\ \forall i;\ (x_j^l, x_j^m, x_j^u) \succcurlyeq \tilde{0} \quad \forall\, j \qquad (P3)$$

Now, using Nasseri et al.'s algorithm [6] and the ranking method, the FFLP $(P2)$ turns into a precise linear programming problem. The steps in the algorithm are briefly summarized below:

Step 1. Transform full fuzzy objective function using its ranking function $\Big(\Re\big(\sum_{j=1}^{n} c_j^l x_j^l, \sum_{j=1}^{n} c_j^m x_j^m, \sum_{j=1}^{n} c_j^u x_j^u\big)\Big)$ into the exact form.

Step 2. Full fuzzy constraints of the model $(P2)$ using the following ranking functions are as follows:

$$\sum_{j=1}^{n} a_{ij}^l x_j^l \leq, =, \geq b_i^l \qquad \forall i$$

$$\sum_{j=1}^{n} a_{ij}^m x_j^m \leq, =, \geq b_i^m \qquad \forall i$$

$$\sum_{j=1}^{n} a_{ij}^u\, x_j^u \leq, =, \geq b_i^u \qquad \forall i$$

Step 3. The non-negative Fuzzy constraints, i.e., $(x_j^l, x_j^m, x_j^u) \succcurlyeq \tilde{0}\ \forall j$ in the model $(P2)$, which guarantees the decision variables assessment as non-triangular fuzzy numbers, will be as follows:

$$x_j^l \geq 0,\ x_j^m - x_j^l \geq 0,\ x_j^u - x_j^m \geq 0,\ \forall j.$$

Thus, using the above steps, the model ($P2$) turns into the exact linear programming problem:

$$\begin{aligned} Z = \max\,(or\ \min)\,&\Re\Big(\sum_{j=1}^{n} c_j^l x_j^l, \sum_{j=1}^{n} c_j^m x_j^m, \sum_{j=1}^{n} c_j^u x_j^u\Big) \\ \text{subject to} \quad & \sum_{j=1}^{n} a_{ij}^l x_j^l \leq,=\geq b_i^l \quad \forall i \\ & \sum_{j=1}^{n} a_{ij}^m x_j^m \leq,=\geq b_i^m \quad \forall i \qquad\qquad (P4) \\ & \sum_{j=1}^{n} a_{ij}^u x_j^u \leq,=\geq b_i^u \quad \forall i \\ & x_j^l \geq 0,\ x_j^m - x_j^l \geq 0,\ x_j^u - x_j^m \geq 0, \quad \forall\, j \end{aligned}$$

Theorem 3.3. *Each feasible solution in the model (P4) is also a feasible solution in the model (P3). The proof is provided in [8].*

Theorem 3.4. *The optimal solution of the model (P4) is the optimal solution for the model (P3) as well. The proof is provided in [8].*

4. Cost efficiency evaluation in network data envelopment analysis

There are drawbacks in the traditional methods of evaluating cost efficiency. These shortcomings are due to the set of defined production possibility P.

$$P = \big\{(x, y) | x \geq X\lambda, y \leq Y\lambda, \lambda \geq 0\big\} \tag{1}$$

The production possibility set P is defined only on the basis of the technical factors $X = (x_1, \cdots, x_n) \in \mathbb{R}^{m\times n}$ and $Y = (y_1, \cdots, y_n) \in \mathbb{R}^{s\times n}$ and has no relationship with the input price, $C = (c_1, \cdots, c_n)$. Tone [11] defines a set of production possibility set based on cost as follows:

$$P_c = \big\{(\bar{x}, y) | \bar{x} \geq \bar{X}\lambda, y \leq Y\lambda, \lambda \geq 0\big\} \tag{2}$$

where $\bar{X} = (\bar{x}_1, \cdots, \bar{x}_n)$ and $\bar{x}_j = (c_{1j}x_{1j}, \cdots, c_{mj}x_{mj})^T$. Here, it is assumed that the matrices C and X are non-negative, and all inputs are cost-related. It is also assumed that the elements $\bar{x}_{ij} = (c_{ij}x_{ij})\,\big(\forall(i, j)\big)$ are inhomogeneous units such as dollar, in the sense that the multiplication of these elements is significant.

Based on the definition of the set of new production possibility set Pc, the new technical efficiency $\bar{\theta}^*$ is given as the optimal solution to the linear programming problem:

$$\begin{aligned} \bar{\theta}^* = \min\ & \bar{\theta} \\ \text{s.t}\quad & \bar{\theta}\bar{x}_0 \geq \bar{X}\lambda \\ & y_0 \leq Y\lambda \\ & \lambda \geq 0. \end{aligned}$$

The new cost efficiency $\bar{\gamma}^*$ is as follows:

$$\bar{\gamma}^* = e\bar{x}_0^*/e\bar{x}_0, \tag{3}$$

where $e \in \mathbb{R}^m$, is a row vector with the elements 1 and $\bar{x}_0^*$ is the optimal solution to the linear programming problem below:

$$\begin{aligned} [Ncost]\quad \min\ & e\bar{x} \\ \text{s.t}\quad & \bar{x} \geq \bar{X}\lambda \\ & y_0 \leq Y\lambda \\ & \lambda \geq 0. \end{aligned}$$

5. Network data envelopment analysis based on SBM model

The commonly used DEA models measure the relative efficiency of multiple input/output decision maker units. One of the drawbacks of such models is the neglect of intermediate products or linked activities. In this section, the network data envelopment and the characteristics of the production possibility set is analyzed.

Suppose n is the decision maker available in Section K. m_k and r_k are the numbers of inputs and outputs in the k^{th} section. The link from division k to division h is represented by (h,k) and the set of all links is shown by L. The observed data is $\{x_j^k \in R_+^{m_k}\}(j = 1,\cdots,n, k = 1,\cdots,K)$, $\{y_j^k \in R_+^{r_k}\}(j = 1,\cdots,n, k = 1,\cdots,K)$ and $\{z_j^{(k,h)} \in R_+^{t_{(k,h)}}\}(j = 1,\cdots,n,(k,h) \in L)$.

The set of production possibility set in network data envelopment analysis is defined as follows:

$$P = \Big\{(x^k, y^k, z^{(h,k)})\big| x^k \geq X^k\lambda^k, y^k \leq Y^k\lambda^k, z^{(k,h)} = z^{(k,h)}\lambda^k\,(as\,outputs\,k), z^{(k,h)} = z^{(k,h)}\lambda^h(as\,inputs\,h), \lambda \geq 0\Big\}.$$

Assume that the following model (with input-oriented) has a return to the variable scale and DMU_o, $(o = 1,\cdots,n)$ unit under evaluation. Since the SBM model needs positive data, this paper assumes that all data are positive.

$$
\begin{aligned}
[NSBM] \quad \theta_0 = \min \sum_{k=1}^{K} w^k & \left[1 - \frac{1}{m_k} \Big(\sum_{i=1}^{m_k} \frac{s_i^{k-}}{x_{io}^k} \Big) \right] \\
\text{s.t} \quad & x_o^k = X^k \lambda^k + s^{k-} \\
& y_o^k = Y^k \lambda^k - s^{k+} \\
& \lambda^k, \lambda^h, s^{k-}, s^{k+} \geq 0 \\
& z_o^{(k,h)} = z^{(k,h)} \lambda^k \ \big(\forall (k,h)\big), \qquad (a) \\
& z_o^{(k,h)} = z^{(k,h)} \lambda^h \ \big(\forall (k,h)\big), \\
& \text{or} \\
& z_o^{(k,h)} \lambda^k = z^{(k,h)} \lambda^k \ \big(\forall (k,h)\big), \qquad (b),
\end{aligned}
$$

where $y^k = (y_1^k, \cdots, y_n^k) \in R^{r_k \times n}$, $X^k = (x_1^k, \cdots, x_n^k) \in R^{m_k \times n}$, $z^{(k,h)} = \big(z_1^{(k,h)}, \cdots, z_n^{(k,h)}\big) \in R^{t_{(k,h)} \times n}$, s^{k-} (s^{k+}) are the auxiliary vectors of the input (output). Given the linked constraints, there are several choices that can be made in two possible ways:

(a) In the first case, the values of fixed intermediate current are taken into account.

$$
\begin{aligned}
& z_o^{(k,h)} = z^{(k,h)} \lambda^k \ \big(\forall (k,h)\big), \qquad (a) \\
& z_o^{(k,h)} = z^{(k,h)} \lambda^h \ \big(\forall (k,h)\big).
\end{aligned}
$$

(b) In the latter case, the average flow rates in the link can be freely reduced or increased.

$$z^{(k,h)} = z^{(k,h)} \lambda^h \ \big(\forall (k,h)\big).$$

6. Cost efficiency in network data envelopment analysis

The production possibility set based on cost for NDEA will be as follows [1]:

$$
\begin{aligned}
P_c = \Big\{ & (\bar{x}^k, y^k, \bar{z}^{(k,h)}) \big| \bar{x}^k \geq \bar{X}^k \lambda^k, y^k \leq Y^k \lambda^k, z^{(k,h)} = z^{(k,h)} \lambda^k (as\, outputs\, k), \bar{z}^{(k,h)} \\
& = \bar{z}^{(k,h)} \lambda^h (as\, inputs\, h), \ \lambda \geq 0 \Big\},
\end{aligned}
$$

where

$$
\begin{aligned}
& \bar{X}^k = (\bar{x}_1^k, \cdots, \bar{x}_n^k), && \bar{x}_j^k = (c_{1j}^k x_{1j}^k, \cdots, c_{mj}^k x_{mj}^k) \\
& \bar{z}^{(k,h)} = \big(\bar{z}_1^{(k,h)}, \cdots, \bar{z}_n^{(k,h)}\big), && \bar{z}_j^{(k,h)} = \big(c_{1j}^k z_{1j}^{(k,h)}, \cdots, c_{mj}^k z_{mj}^{(k,h)}\big).
\end{aligned}
$$

Based on this set, a new production possibility, $\bar{\gamma}^{*k}$, is obtained from the following linear programming problem:

$$
\begin{aligned}
[NNCost] \quad & \min \sum_{k=1}^{K} \bar{x}^k + \sum_{h} \bar{z}^{(k,h)} \\
& \text{s.t} \quad \bar{x}^k \geq \bar{X}^k \lambda^k, \qquad k = 1, \cdots, K \\
& \quad y_o \leq Y^k \lambda^k, \qquad k = 1, \cdots, K \\
& \quad z_o^{(k,h)} = z^{(k,h)} \lambda^k \left(\forall (k,h)\right), \qquad (a) \\
& \quad \bar{z}_o^{(k,h)} = \bar{z}^{(k,h)} \lambda^h \left(\forall (k,h)\right) \\
& \quad \text{or} \\
& \quad z_o^{(k,h)} = \bar{z}^{(k,h)} \lambda^h \left(\forall (k,h)\right) \qquad (b) \qquad (P5) \\
& \quad \lambda^k,\ \lambda^h \geq 0.
\end{aligned}
$$

The new network cost efficiency for each division k is obtained from the following equation:

$$
\bar{\gamma}^{*k} = \sum_{k=1}^{K} \bar{x}_o^{*k} + \sum_{h} \bar{z}_o^{*(k,h)} \Bigg/ \sum_{k=1}^{K} \bar{x}_o^{k} + \sum_{h} \bar{z}_o^{(k,h)},
$$

where $e \in R^m$ is a row vector with elements is 1.

7. Proposed fuzzy cost efficiency method in fully fuzzy network data envelopment analysis

In the real world, input-output data and their corresponding costs are not accurately observed and may be available in inappropriate forms such as fuzzy numbers, in particular triangular fuzzy numbers. Many researchers have investigated the cost efficient with fuzzy and intermediate data. Yet, in these studies, only the decision parameters are considered as fuzzy and the decision variables are precise quantifiers. However, in this paper, the fully-fuzzy models of NDEA are used to measure the cost-effectiveness of a fully fuzzy environment in which all the decision making parameters and variables are represented by triangular fuzzy numbers.

To measure Fuzzy cost efficiency in NDEA, the model $(P4)$ was expanded to a Fully-Fuzzy environment. Suppose n decision maker unit is available in division K. m_k and r_k are the number of Fuzzy inputs and outputs in the k^{th} division. The link from division k to division h is represented by (k,h) and the set of all links with L. The observed Fuzzy data $\tilde{x}_j^k$, $\tilde{y}_j^k$, $\tilde{z}_j^{(k,h)}$ and $\tilde{c}_j^k$ (where $j = 1, \cdots, n,\ k = 1, \cdots, K$) show fuzzy inputs, fuzzy outputs in each division, fuzzy linked activities from division k to division h, and fuzzy input units in each division, respectively. If these data are triangular fuzzy numbers, we will have:

$$\begin{aligned}
&\tilde{x}_j^k = \big(x_j^{l,k}, x_j^{m,k}, x_j^{u,k}\big), && j=1,\cdots,n, \quad k=1,\cdots,K\\
&\tilde{y}_j^k = \big(y_j^{l,k}, y_j^{m,k}, y_j^{u,k}\big), && j=1,\cdots,n, \quad k=1,\cdots,K\\
&\tilde{c}_j^k = \big(c_j^{l,k}, c_j^{m,k}, c_j^{u,k}\big), && j=1,\cdots,n, \quad k=1,\cdots,K\\
&\tilde{z}_j^{(k,h)} = \big(z_j^{l,(k,h)}, z_j^{m,(k,h)}, z_j^{u,(k,h)}\big), && j=1,\cdots,n, \quad (k,h)\in L.
\end{aligned}$$

As mentioned earlier, the model $(P5)$ will become a fully fuzzy model as follows:

$$\begin{aligned}
[FFNNCost] \quad \min\ & \sum_{k=1}^{K} \tilde{\bar{x}}^k \oplus \sum_h \tilde{\bar{z}}^{(k,h)}\\
& \tilde{\bar{x}}^k \succcurlyeq \sum_{j=1}^{n} \tilde{\bar{X}}_j^k \otimes \tilde{\lambda}_j^k, \quad k=1,\cdots,K\\
& \tilde{y}_o^k \preccurlyeq \sum_{j=1}^{n} \tilde{y}_j^k \otimes \tilde{\lambda}_j^k, \quad k=1,\cdots,K \qquad (P6)\\
& \tilde{z}_o^{(k,h)} \approx \sum_{j=1}^{n} \tilde{z}_j^{(k,h)} \otimes \tilde{\lambda}_j^k, \quad \forall (k,h)\\
& \tilde{\bar{z}}_o^{(k,h)} \approx \sum_{j=1}^{n} \tilde{\bar{z}}_j^{(k,h)} \otimes \tilde{\lambda}_j^h, \quad \forall (k,h) \qquad (a)\\
& \text{or}\\
& \sum_{j=1}^{n} \tilde{z}_j^{(k,h)} \otimes \tilde{\lambda}_j^k \approx \sum_{j=1}^{n} \tilde{\bar{z}}_j^{(k,h)} \otimes \tilde{\lambda}_j^h, \quad \forall (k,h) \qquad (b)\\
& \tilde{\lambda}_j^k,\ \tilde{\lambda}_j^h \succcurlyeq \tilde{0} \quad j,k,
\end{aligned}$$

where $\tilde{\lambda}_j^k = (\lambda_j^{l,k}, \lambda_j^{m,k}, \lambda_j^{u,k})$ are non-negative triangular Fuzzy variables and

$$\begin{aligned}
&\tilde{\bar{X}}^k = (\tilde{\bar{x}}_1^k, \cdots, \tilde{\bar{x}}_n^k), && \tilde{\bar{x}}_j^k = (\tilde{\bar{c}}_{1j}^k \tilde{\bar{x}}_{1j}^k, \cdots, \tilde{\bar{c}}_{mj}^k \tilde{\bar{x}}_{mj}^k)\\
&\tilde{\bar{z}}^{(k,h)} = \big(\tilde{\bar{z}}_1^{(k,h)}, \cdots, \tilde{\bar{z}}_n^{(k,h)}\big), && \tilde{\bar{z}}_j^{(k,h)} = \big(\tilde{\bar{c}}_{1j}^k \tilde{\bar{z}}_{1j}^{(k,h)}, \cdots, \tilde{\bar{c}}_{mj}^k \tilde{\bar{z}}_{mj}^{(k,h)}\big).
\end{aligned}$$

Model $(P6)$ is a fully fuzzy cost efficiency model in fuzzy NDEA. After replacing the triangular fuzzy variables and parameters in model $(P6)$ and using mathematical operations on triangular fuzzy numbers and the steps of the Nasseri algorithm, the fully fuzzy linear programming model $(P6)$ becomes the exact linear programming:

$$\min\ \frac{1}{4}\left[\sum_{k=1}^{K} \bar{x}^{l,k} + \sum_h \bar{z}^{l,(k,h)} + 2\Big(\sum_{k=1}^{K} \bar{x}^{m,k} + \sum_h \bar{z}^{m,(k,h)}\Big) + \sum_{k=1}^{K} \bar{x}^{u,k} + \sum_h \bar{z}^{u,(k,h)}\right]$$

$$
\begin{aligned}
\text{s.t.}\quad & \bar{x}^{l,k} \geq \sum_{j=1}^{n} \bar{X}_j^{l,k}\lambda_j^{l,k}, && k=1,\cdots,K \\
& \bar{x}^{m,k} \geq \sum_{j=1}^{n} \bar{X}_j^{m,k}\lambda_j^{l,k}, && k=1,\cdots,K \\
& \bar{x}^{u,k} \geq \sum_{j=1}^{n} \bar{X}_j^{u,k}\lambda_j^{l,k}, && k=1,\cdots,K \\
& y_o^{l,k} \leq \sum_{j=1}^{n} y_j^{l,k}\lambda_j^{l,k}, && k=1,\cdots,K \qquad (P7) \\
& y_o^{m,k} \leq \sum_{j=1}^{n} y_j^{m,k}\lambda_j^{m,k}, && k=1,\cdots,K \\
& y_o^{u,k} \leq \sum_{j=1}^{n} y_j^{u,k}\lambda_j^{u,k}, && k=1,\cdots,K \\
& z_o^{l,(k,h)} = \sum_{j=1}^{n} z_j^{l,(k,h)}\lambda_j^{l,k}, && \forall(k,h) \\
& z_o^{m,(k,h)} = \sum_{j=1}^{n} z_j^{m,(k,h)}\lambda_j^{m,k}, && \forall(k,h) \\
& z_o^{u,(k,h)} = \sum_{j=1}^{n} z_j^{u,(k,h)}\lambda_j^{u,k}, && \forall(k,h) \\
& \bar{z}_o^{l,(k,h)} = \sum_{j=1}^{n} \bar{z}_j^{l,(k,h)}\lambda_j^{l,h}, && \forall(k,h) \qquad (a) \\
& \bar{z}_o^{m,(k,h)} = \sum_{j=1}^{n} \bar{z}_j^{m,(k,h)}\lambda_j^{m,h}, && \forall(k,h) \\
& \bar{z}_o^{u,(k,h)} = \sum_{j=1}^{n} \bar{z}_j^{u,(k,h)}\lambda_j^{u,h}, && \forall(k,h) \\
& \text{or} \\
& \sum_{j=1}^{n} z_j^{l,(k,h)}\lambda_j^{l,k} = \sum_{j=1}^{n} \bar{z}_j^{l,(k,h)}\lambda_j^{l,h}, && \forall(k,h) \\
& \sum_{j=1}^{n} z_j^{m,(k,h)}\lambda_j^{m,k} = \sum_{j=1}^{n} \bar{z}_j^{m,(k,h)}\lambda_j^{m,h}, && \forall(k,h) \qquad (b) \\
& \sum_{j=1}^{n} z_j^{u,(k,h)}\lambda_j^{u,k} = \sum_{j=1}^{n} \bar{z}_j^{u,(k,h)}\lambda_j^{u,h}, && \forall(k,h)
\end{aligned}
$$

$$\lambda_j^{l,k} \geq 0,\ \lambda_j^{m,k} - \lambda_j^{l,k} \geq 0, \lambda_j^{u,k} - \lambda_j^{m,k} \geq 0 \quad \forall j, k$$
$$\lambda_j^{l,k} \geq 0,\ x_j^{m,k} - x_j^{l,k} \geq 0, x_j^{u,k} - x_j^{m,k} \geq 0 \quad \forall j, k.$$

Theorem 7.1. *The optimal solution for the model* ($P7$) *will be a model optimization solution* ($P6$). *The proof of this is similar to the proof of Theorem* 3.3.

Definition 7.2. The fuzzy cost efficiency of the i^{th} DMU in the FFDEA is defined as the ratio of the minimum fuzzy cost to the observed fuzzy cost of DMU_i:

$$\begin{aligned}
\tilde{\bar{\gamma}}_i^{*k} &= \frac{\sum_{k=1}^K \tilde{\bar{x}}_i^{*k} \oplus \sum_h \tilde{\bar{z}}_i^{*(k,h)}}{\sum_{k=1}^K \tilde{\bar{x}}_i^{k} \oplus \sum_h \tilde{\bar{z}}_i^{*(k,h)}} \\
&= \frac{\left(\sum_{k=1}^K \bar{x}_i^{l,k*} + \sum_h \bar{z}_i^{l,(k,h)*}, \sum_{k=1}^K \bar{x}_i^{m,k*} + \sum_h \bar{z}_i^{m,(k,h)*}, \sum_{k=1}^K \bar{x}_i^{u,k*} + \sum_h \bar{z}_i^{u,(k,h)*}\right)}{\left(\sum_{k=1}^K \bar{x}_i^{l,k} + \sum_h \bar{z}_i^{l,(k,h)}, \sum_{k=1}^K \bar{x}_i^{m,k} + \sum_h \bar{z}_i^{m,(k,h)}, \sum_{k=1}^K \bar{x}_i^{u,k} + \sum_h \bar{z}_i^{u,(k,h)}\right)} \\
&= \left(\frac{\sum_{k=1}^K \bar{x}_i^{l,k*} + \sum_h \bar{z}_i^{l,(k,h)*}}{\sum_{k=1}^K \bar{x}_i^{u,k} + \sum_h \bar{z}_i^{u,(k,h)}}, \frac{\sum_{k=1}^K \bar{x}_i^{m,k*} + \sum_h \bar{z}_i^{m,(k,h)*}}{\sum_{k=1}^K \bar{x}_i^{m,k} + \sum_h \bar{z}_i^{m,(k,h)}}, \frac{\sum_{k=1}^K \bar{x}_i^{u,k*} + \sum_h \bar{z}_i^{u,(k,h)*}}{\sum_{k=1}^K \bar{x}_i^{l,k} + \sum_h \bar{z}_i^{l,(k,h)}}\right).
\end{aligned}$$

In other words, using the definition 2.8

$$\Re\Big(\sum_{k=1}^{k} \tilde{\bar{x}}_i^{*k} \oplus \sum_h \tilde{\bar{z}}_i^{*(k,h)}\Big) \approx \Re\Big(\sum_{k=1}^{k} \tilde{\bar{x}}_i^{k} \oplus \sum_h \tilde{\bar{z}}_i^{(k,h)}\Big).$$

Definition 7.3. i^{th} DMU in the network data envelopment analysis is called Fuzzy Cost Efficiency if the observed Fuzzy Cost and the minimum Fuzzy Cost equal DMU_i, i.e.,

$$\sum_{k=1}^{k} \tilde{\bar{x}}_i^{*k} \oplus \sum_h \tilde{\bar{z}}_i^{*(k,h)} \approx \sum_{k=1}^{k} \tilde{\bar{x}}_i^{k} \oplus \sum_h \tilde{\bar{z}}_i^{(k,h)}.$$

In other words, using definition 2.8

$$\Re\Big(\sum_{k=1}^{k} \tilde{\bar{x}}_i^{*k} \oplus \sum_h \tilde{\bar{z}}_i^{*(k,h)}\Big) \approx \Re\Big(\sum_{k=1}^{k} \tilde{\bar{x}}_i^{k} \oplus \sum_h \tilde{\bar{z}}_i^{(k,h)}\Big).$$

8. Numerical example

In this section, an illustrative example of electric power companies are presented for describing network DEA. As we know, the vertically integrated electric power companies consist of several divisions such as generation, transmission and distribution. For illustrative purpose, ten vertically integrated electric power companies in the U.S in 1994 [16]. The inputs, outputs and links are as follows:

Generation (Div1):
Input1 = Labor input (number of employees)
Transmission (Div2):
Input2 = Labor input (number of employees)
Output2 = Electric power sold to large customers
Distribution (Div3):
Input3 = Labor input (number of employees)
Output3 = Electric power sold to small customers
Link (1-2) = Electric power generated (output from Generation Devision and input to Transmission Devision)
Link (2-3) = Electric power sent (output from Transmission Devision and input to Distribution Devision)

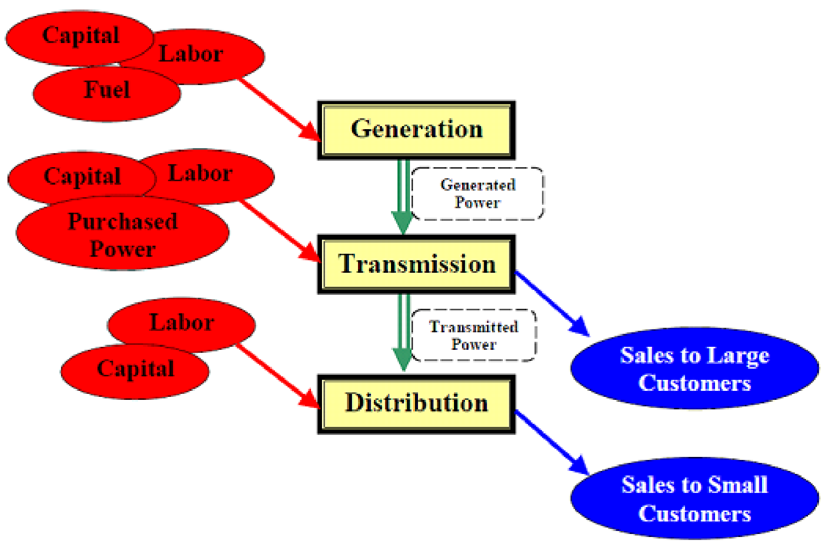

Figure 2: Vertically integrated electric power companies

Table 1: Fuzzy inputs, fuzzy outputs, fuzzy input cost in three divisions

	Div1		Div2			Div3		
DMU	Input1	C1	Input2	C2	Output2	Input3	C3	Output3
A	(0.836,0.838,0.840)	(248,250,253)	(0.275,0.277,0.279)	(118,120,123)	(0.876,0.879,0.881)	(0.960,0.962,0.965)	(218,220,223)	(0.335,0.337,0.340)
B	(1.231,1.233,1.235)	(497,500,502)	(0.130,0.132,0.133)	(178,180,183)	(0.535,0.538,0.540)	(0.440,0.443,0.445)	(208,210,213)	(0.15,0.18,0.20)
C	(0.318,0.321,0.323)	(123,125,128)	(0.042,0.045,0.048)	(289,290,293)	(0.909,0.911,0.914)	(0.482,0.485,0.487)	(137,140,142)	(0.195,0.198,0.200)
D	(1.480.1.483,1.485)	(110,113,115)	(0.110,0.111,0.113)	(57,60,62)	(0.55,0.57,0.59)	(0.465,0,467,0.470)	(147,150,153)	(0.488,0.491,0.495)
E	(1.590,1.592,1.595)	(48,50,53)	(0.205,0.208,0.211)	(83,85,88)	(1.085,1.086,1.089)	(1.070,1.073,1.075)	(197,200,202)	(0.370,0.372,0.375)
F	(0.76,0.79,0.81)	(42,45,48)	(0.136,0.139,0.141)	(93,95,98)	(0.720,0.722,0.724)	(0.543,0.545,0.548)	(80,85,89)	(0.250,0.253,0.255)
G	(0.449,0.451,0.454)	(92,95,96)	(0.073,0.075,0.077)	(98,100,102)	(0.507,0.509,0.511)	(0.365,0.366,0.368)	(147,150,153)	(0.238,0.241,0.244)
H	(0.405,0.408,0.410)	(447,450,452)	(0.072,0.074,0.076)	(138,140,143)	(0.617,0.619,0.621)	(0.226,0.229,0.231)	(228,230,235)	(0.095,0.097,0.099)
I	(1.860,1.864,1.865)	(198,200,203)	(0.059,0.061,0.063)	(129,130,135)	(1.021,1.023,1.025)	(0.689,0.691,0.693)	(158,160,165)	(0.35,0.38,0.40)
J	(1.220,1.222,1.225)	(8,10,14)	(0.147,0.149,0.151)	(43,45,47)	(0.765,0.769,0.771)	(0.336,0.337,0.339)	(448,450,453)	(0.175,0.178,0.180)

Table 2: Fuzzy unit input link cost

Link			
Link12	Lc1	Link23	Lc2
(0.891,0.894,0.897)	(340,345,348)	(0.360,0.362,0.365)	(160,162,165)
(0.675,0.678,0.780)	(210,212,215)	(0.185,0.188,0.190)	(120,123,125)
(0.835,0.836,0.838)	(170,173,175)	(0.205,0.207,0.210)	(340,345,347)
(0.865,0.869,0.872)	(398,400,405)	(0.514,0.516,0.520)	(452,456,460)
(0.690,0.693,0.695)	(25,27,30)	(0.405,0.407,0.410)	(65,67,70)
(0.961,0.966,0.970)	(94,96,100)	(0.265,0.269,0.273)	(75,78,80)
(0.645,0.647,0.650)	(80,83,85)	(0.255,0.257,0.259)	(185,189,192)
(0.752,0.756,0.760)	(285,289,290)	(0.101,0.103,0.105)	(85,90,92)
(1.189,1.191,1.194)	(100,104,107)	(0.400,0.402,0.405)	(17,19,23)
(0.790,0.792,0.795)	(45,47,50)	(0.185,0.187,0.190)	(325,327,330)

Here, it is assumed that the intermediate flow values in the link can be freely reduced or increased; thus, the proposed model for evaluating fuzzy cost efficiency is as follows:

$$\min \frac{1}{4}\left[\sum_{k=1}^{K}\bar{x}^{l,k}+\sum_{h}\bar{z}^{l,(k,h)}+2\Big(\sum_{k=1}^{K}\bar{x}^{m,k}+\sum_{h}\bar{z}^{m,(k,h)}\Big)+\sum_{k=1}^{K}\bar{x}^{u,k}+\sum_{h}\bar{z}^{u,(k,h)}\right]$$

$$\begin{aligned}
\text{s.t.}\quad & \bar{x}^{l,k} \geq \sum_{j=1}^{n}\bar{X}_j^{l,k}\lambda_j^{l,k}, && k=1,\cdots,K\\
& \bar{x}^{m,k} \geq \sum_{j=1}^{n}\bar{X}_j^{m,k}\lambda_j^{m,k}, && k=1,\cdots,K\\
& \bar{x}^{u,k} \geq \sum_{j=1}^{n}\bar{X}_j^{u,k}\lambda_j^{u,k}, && k=1,\cdots,K\\
& y_o^{l,k} \leq \sum_{j=1}^{n}y_j^{l,k}\lambda_j^{l,k}, && k=1,\cdots,K\\
& y_o^{m,k} \leq \sum_{j=1}^{n}y_j^{m,k}\lambda_j^{m,k}, && k=1,\cdots,K\\
& y_o^{u,k} \leq \sum_{j=1}^{n}y_j^{u,k}\lambda_j^{u,k}, && k=1,\cdots,K\\
& \sum_{j=1}^{n}z_j^{l,(k,h)}\lambda_j^{l,k} = \sum_{j=1}^{n}z_j^{l,(k,h)}\lambda_j^{l,h}, && \forall(k,h)\\
& \sum_{j=1}^{n}z_j^{m,(k,h)}\lambda_j^{m,k} = \sum_{j=1}^{n}z_j^{m,(k,h)}\lambda_j^{m,h}, && \forall(k,h)
\end{aligned}$$

$$\sum_{j=1}^{n} z_j^{u,(k,h)} \lambda_j^{u,k} = \sum_{j=1}^{n} z_j^{u,(k,h)} \lambda_j^{u,h}, \qquad \forall (k,h)$$
$$\lambda_j^{l,k} \geq 0, \quad \lambda_j^{m,k} - \lambda_j^{l,k} \geq 0, \quad \lambda_j^{u,k} - \lambda_j^{m,k} \geq 0 \qquad \forall j,k$$
$$\bar{x}_j^{l,k} \geq 0, \quad \bar{x}_j^{m,k} - \bar{x}_j^{l,k} \geq 0, \quad \bar{x}_j^{u,k} - \bar{x}_j^{m,k} \geq 0, \qquad \forall j,k$$
$$\bar{z}_j^{l,k} \geq 0, \quad \bar{z}_j^{m,k} - \bar{z}_j^{l,k} \geq 0, \quad \bar{z}_j^{u,k} - \bar{z}_j^{m,k} \geq 0, \qquad \forall j,k$$

The above model is solved using GAMS software and the results are shown in Table 3 As shown in the table and according to Definition 7.3, DMU_D is cost

Table 3: Evaluating and ranking revenue efficiency

$DMUs$	$\tilde{\bar{\gamma}}^{*k}$	$R(\tilde{\bar{\gamma}}^{*k})$	Rank
A	(0.326,0.375,0.391)	0.375	8
B	(0.150,0.178,0.195)	0.178	10
C	(0.565,0.607,0.781)	0.607	3
D	(0.972,1,1.0002)	1	1
E	(0.370,0.457,0.463)	0.457	6
F	(0.542,0.666,0.730)	0.666	2
G	(0.479,0.556,0.630)	0.556	4
H	(0.345,0.418,0.504)	0.418	7
I	(0.206,0.239,0.274)	0.239	9
J	(0.300,0.447,0.486)	0.447	5

efficient. In the last column of Table 3, the cost efficiency of the decision making units is arranged based on the ranking function of Definition 2.8.

Conclusion

In this paper, a new idea was developed for expanding the classic cost efficiency model into a fully fuzzy environment in a network data envelopment analysis. In the fuzzy cost efficiency model, it is assumed that the inputs and outputs and their corresponding costs are fuzzy triangular numbers. Then, using the ranking function method, the fully fuzzy cost efficiency model is transformed into a precise model, and ultimately, the final fuzzy cost function is obtained as a triangular fuzzy number.

Since cost efficiency sensitivity analysis helps the manager or decision maker to modify the amount of inputs under evaluation to minimize cost. Therefore, future work can include sensitivity analysis of performance, as well as finding the appropriate stability area to maintain cost efficiency in precise and imprecise network data envelopment analysis.

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